Resistance resonances for resonant-tunneling structures of quantum dots

M. Sumetskiĭ
Institut für Energieverfahrenstechnik, Forschungszentrum Jülich G.m.b.H. (KFA), D-5170 Jülich, Germany
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By use of the generalized Breit-Wigner formula we find a broad class of non-one-dimensional resonant-tunneling structures of quantum dots or atoms having exponentially high and narrow resistance resonances and conductance zero points. It is shown that in definite cases they correspond to the exponentially narrow backscattering conductance resonances near the levels of quantum dots. As examples, the structures of two, three, and four quantum dots are considered. The possibility of observing the resistance resonances by using the scanning tunneling microscope is discussed.

I. INTRODUCTION

Resonant tunneling is of particular interest both from the physical standpoint and with regard to its application in the future nanometer electronics. It has been theoretically studied and experimentally observed in planar heterostructures,[1] for field emission through adsorbates,[2] and also for systems of few atoms,[3,4] and quantum dots.[5–7] The principal feature of resonant tunneling is the existence of narrow conductance peaks corresponding to the localized levels of the structure. It is well known that such resonances appear from the *poles* of the transmission amplitude that are exponentially close to the real axis in the complex energy plane. Below, we consider the resonant-tunneling structures consisting of a few atoms, defects, or quantum dots, and study another interesting property of their conductance behavior: Besides the *conductance resonances,* the non-one-dimensional resonant-tunneling structure can have similar exponentially narrow and exponentially high *resistance resonances.* Resistance resonances come from the *zeros* of the transmission amplitude that are exponentially close to the real axis in the complex energy plane. In specific cases the zeros may correspond to the real energies and, hence, to the absolute reflection. The latter effect was found for a number of other non-one-dimensional structures (oscillating potential barrier,[8] *T*-shaped microjunction,[9] waveguide with defects[10]) and discussed in Refs. 11 and 12.

We apply the generalized Breit-Wigner formula[6] to find a broad class of non-one-dimensional resonant-tunneling structures having exponentially high and narrow resistance resonances and transmission zeros. In general, the narrow resistance resonances do not correspond to the narrow backscattering conductance resonances. However, it is shown below that in definite cases the *exponentially narrow backscattering resonances* appear near the levels of quantum dots. At first we suggest general patterns of resonant-tunneling structures having transmission (conductance) zeros and narrow backscattering resonances. Then the example of two, three, and four quantum dot structures are considered. The possibility of observing the resistance resonances with use of the scanning tunneling microscope is discussed.

II. CONDUCTANCE ZEROS AND RESISTANCE RESONANCES

Consider the resonant-tunneling structure with N quantum dots and two electrodes shown in Fig. 1(a). Each dot j is suggested to have a resonant level E_j close to the incident electron energy E. Assume that the values $|E-E_j|$ ($j=1,\ldots,N$), are small compared to the characteristic electron energy in quantum dots. Then, using the multidimensional Landauer approach,[13] one can find the conductance of this structure by the generalized Breit-Wigner formula:[6]

$$G(E)=\frac{e^2}{\pi\hbar}\mathrm{Tr}(\boldsymbol{\Gamma}_1\mathbf{Q}(E)\boldsymbol{\Gamma}_2\mathbf{Q}^\dagger(E))=\frac{e^2}{\pi\hbar}\sum_{j,k=1}^{N}|q_{jk}|^2\Gamma_j^{(1)}\Gamma_k^{(2)} \quad (1)$$

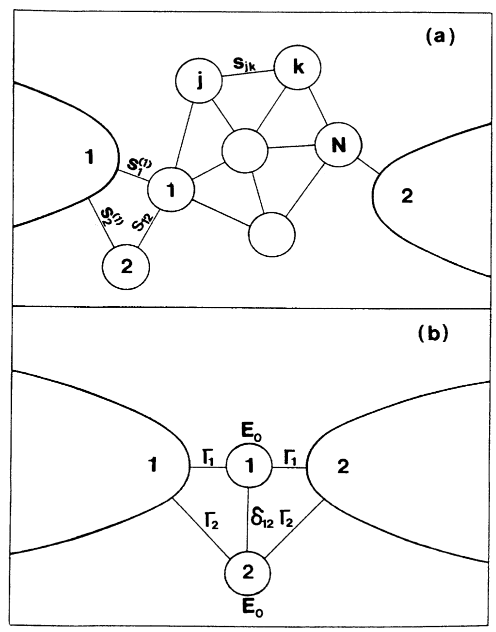

FIG. 1. (a) A resonant-tunneling structure with two electrodes and N quantum dots considered. (b) A similar structure with two quantum dots.

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with matrices

$$\mathbf{Q}(E)=\|q_{jk}\|=(E\mathbf{I}-\mathbf{H}-\tfrac{1}{2}\mathbf{\Gamma})^{-1} ,$$
$$\mathbf{\Gamma}=\mathbf{\Gamma}_1+\mathbf{\Gamma}_2 ,$$
$$\mathbf{H}=\begin{vmatrix} E_1 & \delta_{12} & \cdots & \delta_{1N} \\ \delta_{21} & E_2 & \cdots & \delta_{2N} \\ \vdots & \vdots & & \vdots \\ \delta_{N1} & \delta_{N2} & \cdots & E_N \end{vmatrix} ,$$
$$\mathbf{\Gamma}_m=\begin{vmatrix} \Gamma_1^{(m)} & 0 & \cdots & 0 \\ 0 & \Gamma_2^{(m)} & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & \Gamma_N^{(m)} \end{vmatrix} , \quad (2)$$

where $\mathbf{I}$ is the unit matrix, $\mathbf{\Gamma}$ is the matrix of total level widths, and $\mathbf{H}$ is the symmetric energy matrix. The nondiagonal elements of the energy matrix, δ_{jk}, are the flux overlap integrals between eigenstates of dots j and k, and $\Gamma_j^{(m)}$ is the partial width of decay from the state in dot j into electrode m.[6] For the potential, which is constant in the barrier region,

$$\delta_j \propto \exp(-\gamma s_{jk}) , \quad \Gamma_l^{(m)} \propto \exp(-2\gamma s_l^{(m)}) , \quad (3)$$

where γ is the absolute value of the wave number in the barrier region, s_{jk} is the distance between dot j and dot k, and $s_l^{(m)}$ is the distance between dot l and electrode m [see Fig. 1(a)]. According to Eq. (3), values δ_{jk} and $\Gamma_l^{(m)}$ are of the same order if the distance s_{jk} is twice as large as the distance $s_l^{(m)}$. The equation $\det[E\mathbf{I}-\mathbf{H}-(i/2)\mathbf{\Gamma}]=0$ defines eigenvalues of the system for $\mathbf{\Gamma}=\mathbf{0}$ and conductance resonances for small $\mathbf{\Gamma}$. For the single-particle resonant-tunneling problems, Eqs. (1) and (2) have some advantages compared to the most commonly used tight-binding Hamiltonian approach,[14] because they give the answer for the conductance in a closed analytical form, simply taking into account the coupling between quantum dots and electrodes.

The effect in question is readily found from Eqs. (1) and (2). Assume that coupling $\Gamma_j^{(1)}$ between electrode 1 and the quantum dots is negligible for all dots except for the quantum dot with $j=1$ nearest to electrode 1. Similarly, let us neglect all the decay widths $\Gamma_j^{(2)}$ in Eqs. (1) and (2) except $\Gamma_N^{(2)}$ that defines coupling between electrode 2 and the nearest quantum dot N [see Fig. 1(a)]. Equations (1) and (2) then give

$$G(E)=\frac{e^2}{\pi\hbar}\frac{|M_{1N}(E)|^2\Gamma_1^{(1)}\Gamma_N^{(2)}}{|\det(E\mathbf{I}-\mathbf{H}-\frac{i}{2}\mathbf{\Gamma})|^2} \quad (4)$$

where $M_{1N}(E)$ is the minor of matrix $E\mathbf{I}-\mathbf{H}-(i/2)\mathbf{\Gamma}$ that does not contain its line 1 and column N. Hence it does not contain decay widths $\Gamma_1^{(1)}$ and $\Gamma_N^{(2)}$ which are the only not negligible in matrix $\mathbf{\Gamma}$. Thus, under the propositions made the function $M_{1N}(E)$ is a real polynomial of the $N-2$ order. Evidentially, the zeros of $M_{1N}(E)$ define the conductance zeros or the poles of resistance $R(E)=1/G(E)$. Let $E=E^{(0)}$ be such a singularity. If we take into account small but finite values of $\Gamma_j^{(m)}$ for $j\neq 1,N$, then in a small vicinity of $E^{(0)}$ we find that $M_{1N}(E)=\mu(E-E^{(0)})+\varepsilon+i\kappa)$ with small real values ε and κ. Substituting the latter expression into Eq. (3), we obtain the Breit-Wigner *resistance resonance*

$$R(E)=1/G(E)=\frac{(\pi\hbar/e^2)C}{(E-E_R)^2+\Gamma_R^2} , \quad (5)$$

with $E_R=E^{(0)}-\varepsilon$ and $\Gamma_R=|\kappa|$. Here C is a slow function of energy that is approximately constant in the vicinity of E_R.

Notice that the existence of resistance resonances is a peculiar feature of systems with *non-one-dimensional* relative positions of quantum dots. Actually, it is easy to see from Eqs. (1) and (2) that function $M_{1N}(E)$ calculated in the nearest-neighbor approximation for the simple chain of quantum dots is equal to constant $\delta_{12}\delta_{23}\cdots\delta_{N-1,N}$. Distinguish between the case of a *few* quantum dots considered and the infinite crystal structure where the arbitrarily narrow forbidden energy gaps also could exist for the one-dimensional case.

III. EXPONENTIALLY NARROW BACKSCATTERING CONDUCTANCE RESONANCES

Let us bring one more dot, say dot number 2, close to electrode 1 [see Fig. 1(a)]. Suppose that $\Gamma_2^{(1)}$ and δ_{12} are the only connections of dot 2 to the system that are not negligible. For simplicity let all parameters of the system $|E-E_j|$, δ_{jk}, and $\Gamma_j^{(m)}$, except maybe $\Gamma_2^{(1)}$ and δ_{12}, have the same order δ_0. Then calculations by Eqs. (1) and (2) give the following expression for the conductance:

$$G(E)=\frac{e^2}{\pi\hbar}D\frac{(E-E_2)^2+\frac{1}{4}(\Gamma_2^{(1)})^2+|f_1(E)|\Gamma_2^{(1)}\delta_{12}^2}{(E-E_2+f_2(E)\delta_{12}^2)^2+(\frac{1}{4}\Gamma_2^{(1)}+f_3(E)\delta_{12}^2)^2} , \quad (6)$$

where $D\sim 1$ and real functions $f_j(E)$ have the order of δ_0^{-1} and are independent of E_2, δ_{12}, and $\Gamma_2^{(1)}$. Now let us move quantum dot 2 in a direction away from the structure. Then the values δ_{12} and $\Gamma_2^{(1)}$ become exponentially small compared to δ_0. According to estimates (3), we have $\Gamma_2^{(1)}\propto\delta_{12}^2$ for the increasing and approximately equal distances s_{12} and $s_2^{(1)}$ [see Fig. 1(a)]. It can be seen from Eq. (6) that if $\Gamma_2^{(1)}\gtrsim\delta_{12}^2/\delta_0$, then $G(E)\sim e^2/\pi\hbar$ in the entire energy interval $|E-E_j|\sim\delta_0$ considered. However, if $\Gamma_2^{(1)}\ll\delta_{12}^2\delta_0$, then estimate $G(E)\sim e^2/\pi\hbar$ will be valid only outside the small interval $|E-E_2|\lesssim\delta_{12}^2/\delta_0\ll\delta_0$. Inside this interval, conductance $G(E)$ has an *exponentially narrow gap*. In the much smaller interval $|E-E_2|\lesssim(\Gamma_2^{(1)}/\delta_0)^{1/2}\delta_{12}\ll\delta_{12}^2/\delta_0$, this resonance has the Breit-Wigner form (5) with $E_R=E_2$, $\Gamma_R=2(f_1(E_2)\Gamma_2^{(1)})^{1/2}\delta_{12}$, and $C\sim\delta_0^2$.

We would like to emphasize the difference between the general conclusion about the existence of resistance resonances made from Eq. (5) and the conclusion found from Eq. (6). According to Eq. (5), the function $G(E)$ near the resistance resonance is a parabola having a minimum

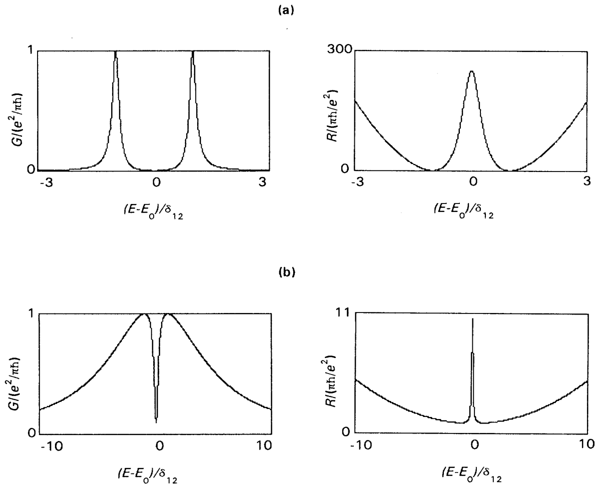

FIG. 2. Conductance $G(E)$ and resistance $R(E)$ dependencies for the two quantum dot structure shown in Fig. 1(b).

(a) $\Gamma_1/\delta_{12}=0.2$, $\Gamma_2/\delta_{12}=0.01$;

(b) $\Gamma_1/\delta_{12}=5$, $\Gamma_2/\delta_{12}=0.01$.

very close to zero. However, this parabola is not narrow in general, its minimum curvature radius being the same as the characteristic dimensions of the $G(E)$ graph [see, for example, Fig. 2(a)]. With the help of Eq. (6), we find additionally that the resistance resonance may be an exponentially narrow dip (backscattering resonance) in the conductance near the level of quantum dot 2 as, e.g., shown in Fig. 2(b). Note that the resistance graph contains narrow peaks in both cases shown in Fig. 2.

IV. TWO-QUANTUM-DOT STRUCTURE

In general, dots 1 and N in Fig. 1(a) can coincide. It is just the case for the simplest structure of two quantum dots shown in Fig. 1(b) that we present as an example. For simplicity, consider equal quantum dots each having level E_0 and set $\Gamma_1^{(1)}=\Gamma_1^{(2)}=\Gamma_1$ and $\Gamma_2^{(1)}=\Gamma_2^{(2)}=\Gamma_2$. In line with the general scheme described, let the decay width Γ_2 be relatively small so that $\Gamma_2 \ll \Gamma_1$ and $\Gamma_2 \ll \delta_{12}^2/\Gamma_1$. Then, according to Eqs. (1) and (2), the conductance of this structure is

$$G(E)=\frac{e^2}{\pi\hbar}\frac{(E-E_0)^2\Gamma_1^2+2\delta_{12}^2\Gamma_1\Gamma_2}{[(E-E_0)^2-\delta_{12}^2]^2+(E-E_0)^2\Gamma_1^2} . \tag{7}$$

One can readily see that the numerator in Eq. (7) will be close to zero for $E\approx E_0$. Under the propositions made, the resistance of the structure in a small vicinity of E_0, where $|E-E_0|\ll\delta_{12}$, has the Breit-Wigner representation (5) with

$$E_R=E_0 , \quad \Gamma_R=2(2\Gamma_2/\Gamma_1)^{1/2}\delta_{12} , \quad C=\delta_{12}^4/\Gamma_1^2 . \tag{8}$$

Evidently, the width of the resistance resonance Γ_R can be exponentially small if the distance between dot 2 and the electrodes is large enough. In general, it can be larger, of the same order, or less than the characteristic width Γ_1 of the conductance resistance. If there are two conductance resonances [it follows from Eq. (7) that for $\Gamma_1\ll\delta_{12}$ they correspond to $E_0\pm\delta_{12}$] then the resistance resonance lies between them and is represented by a sharp peak on the $R(E)$ dependence and by a well with exponentially small minimum on the $G(E)$ dependence, as shown in Fig. 2(a). If, following the preceding, we additionally propose that dot 2 is relatively far from dot 1 so that $\delta_{12}\ll\Gamma_1$, then a narrow dip appears on the conductance graph. This gap splits the conductance resonance, as shown in Fig. 2(b).

Note that the two-quantum-dot structure considered is similar to the model of the **T**-shaped structure treated analytically in Ref. 11. The difference is that in Ref. 10 the decay channels of quantum dot 2 into the electrodes $\Gamma_2^{(1)}$ and $\Gamma_2^{(2)}$ were disregarded and, as a result, the absolute reflection was found. It is simple to find from Eqs. (1) and (2) that, for $\Gamma_2^{(1)}=\Gamma_2^{(2)}=0$, the absolute reflection energy coincides with the level of the second quantum dot E_2 independently of the other parameters of the structure.

V. TRANSMISSION ZEROS FOR THE THREE- AND FOUR-QUANTUM-DOT STRUCTURES

Let us consider a structure of three quantum dots. Again, according to the general scheme described, let electrode 1 be connected with dot 1 only and, similarly, let electrode 2 be connected with dot 3 only [see Fig. 3(a)]. It is clear from Sec. II that the zero conductance for such a structure corresponds to the real energy and, hence, to the absolute reflection. This energy is simply found from Eqs. (1) and (2):

$$E^{(0)}=E_2-(\delta_{12}\delta_{23}/\delta_{13}) \tag{9}$$

and is independent of levels E_1 and E_3. The conductance zero $E^{(0)}$ coincides with level E_2 if quantum dot 2 is weakly connected to one of the other dots, i.e., if δ_{12} or δ_{23} is small.

Consider now that the electrodes are connected with quantum dot 1 only, as shown in Fig. 3(b). In this case

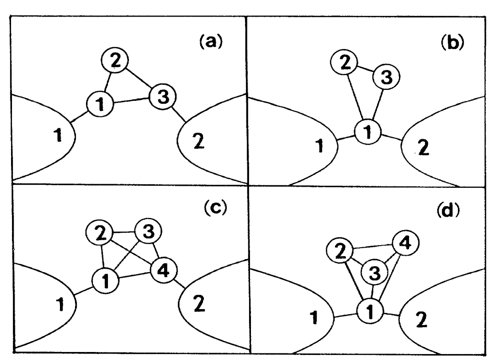

FIG. 3. Three- and four-quantum-dot structures. The nonnegligible connections between quantum dots and electrodes are denoted by straight lines.

the system may have two conductance zeros $E^{(0)}_{1,2}$ defined by the equation

$$(E^{(0)}-E_2)(E^{(0)}-E_3)-\delta_{23}^2=0 \; . \tag{10}$$

Notice the independence of zeros $E^{(0)}_{1,2}$ on level E_1 and couplings δ_{12} and δ_{13}. Evidentially, for small δ_{23} we obtain two conductance zeros coinciding with levels E_2 and E_3, respectively.

Let us turn to the four-quantum-dot structure. Consider the situation shown in Fig. 3(c) when electrodes 1 and 2 have connections with quantum dots 1 and 4, respectively, and quantum dots 2 and 3 have no direct connection with the electrodes. Then the conductance zeros are the roots of the following quadratic equation:

$$\begin{aligned}(E^{(0)}-E_2)(E^{(0)}-E_3)\delta_{14}+(E^{(0)}-E_2)\delta_{13}\delta_{34}\\ +(E^{(0)}-E_3)\delta_{12}\delta_{24}+\delta_{12}\delta_{23}\delta_{34}+\delta_{13}\delta_{23}\delta_{24}\\ -\delta_{14}\delta_{23}^2=0\end{aligned} \tag{11}$$

that is independent of energies E_1 and E_4. It is easy to verify from Eq. (11) that, moving dot 2 (or 3) aside, we obtain one zero similar to the zero of Eq. (9) and another one that coincides with the level of quantum dot 2 (or 3).

The configuration of the structure shown in Fig. 3(d) is similar to the one in Fig. 3(b). From Eqs. (1) and (2), it is not difficult to find the cubic equation defining conductance zeros for this case. We will not dwell on this consideration. In general, the conductance zeros of the N quantum-dot structure are defined by the algebraic equation of order $N-2$ or $N-1$ for the structures similar to the ones shown in Figs. 3(a) and 3(c) or 3(b) and 3(d).

VI. INDEPENDENCE OF RESISTANCE RESONANCES ON THE ELECTRODES STRUCTURE

The model that leads to the generalized Breit-Wigner formula (1) and (2) considers electrodes in the free-electron approximation (see Refs. 6 and 15). Then the interior structure of electrodes has no influence on the positions of either the zeros or the poles of transmission amplitude in the complex plane of energy. It can be seen from the concrete examples of Sec. III and also from the general expressions of Sec. II that the transmission zeros are also independent of the value of levels E_1 and E_N of quantum dots that have direct contact with electrodes. However, the electrodes can have their own fine energy structure, ignored by the free-electron model, that could affect these zeros. Here we prove that the latter does not happen.

Assume that the tips of electrodes A and B are prepared from a number of quantum dots, and each of these electrodes makes contact with quantum-dot structure C via a single quantum dot, as shown in Fig. 4. Then the conductance zeros that come from the structure C are *independent* of the interior parameters E_i, δ_{jk}, and $\Gamma_j^{(m)}$ of electrodes A and B. In other words, they are independent of the structure of the electrodes. This property of resistance resonances is quite interesting because it shows that these resonances define the intrinsic features of the quantum-dot structure under investigation but not of the whole system in general. Note that the poles of the transmission amplitude do not possess such a property.

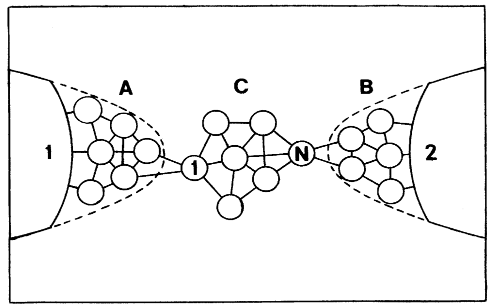

FIG. 4. In this figure, we show electrodes that consist partly of quantum dots, i.e., have their own fine energy structure. The free-electron approximation is assumed now only for the parts 1 and 2 of the electrodes.

To prove the above treatment, we use an expression for conductance of a structure consisting of two quantum-dot structures that are connected in series via one quantum dot.[6] Introduce matrix $\mathbf{Q}^{(D)}$ of a structure D similar to the matrix $\mathbf{Q}$ defined in Sec. II for the structure shown in Fig. 1(a). At first, we define the substructure C_1 which coincides with substructure C without dot 1 (see Fig. 4). The substructure A is connected in series with substructure $B\cup C_1$ via single quantum dot 1. The conductance of the structure shown in Fig. 4 can be expressed via matrices $\mathbf{Q}^{(A)}$ and $\mathbf{Q}^{(B\cup C_1)}$ by Eq. (1) with

$$q_{jk}=\frac{\left[\sum_{m\in A} q_{jm}\delta_{1m}\right]\left[\sum_{n\in C_1} q_{kn}^{(B\cup C_1)}\right]}{E-E_1-\sum_{m,n\in A} q_{nm}^{(A)}\delta_{1m}\delta_{1n}-\sum_{m,n\in C_1} q_{mn}^{(B\cup C_1)}\delta_{1m}\delta_{1n}} \tag{12}$$

[see Eq. (8) in Refs. 6 and 16], where the letters j and k relate to the dots which are connected with electrodes 1 and 2, respectively. In turn, the matrix elements $q_{kn}^{(B\cup C_1)}$ in the numerator of Eq. (12) are defined by the expression

$$q_{kn}^{(B\cup C_1)}=\frac{\lambda_n\sum_{n\in B} q_{kp}^{(B)}\delta_{Np}}{E-E_2-\sum_{l,p\in C_0} q_{lp}^{(C_0)}\delta_{Nl}\delta_{Np}-\sum_{l,p\in B} q_{lp}^{(B)}\delta_{Nl}\delta_{Np}} \, ,$$

$$\lambda_n=\sum_{l\in C_0} q_{nl}^{(C_0)}\delta_{Nl} \quad \text{for } n\neq N \, , \tag{13}$$

$$\lambda_n=1 \quad \text{for } n=N \, ,$$

where C_0 is the structure C without dots 1 and N. As a result of Eqs. (12) and (13), we find that, for all j and k considered,

$$\begin{aligned} q_{jk} &\propto \sum_{n,l\in C_0} q_{nl}^{(C_0)}\delta_{\ln}\delta_{Nl}+\delta_{lN}\\ &=q_{lN}^{(C)}\det(\mathbf{Q}^{(C_0)}/\det(\mathbf{Q}^{(C)})\\ &=M_{lN}^{(C)}(E)\det(\mathbf{Q}^{(C_0)}) \, . \end{aligned} \tag{14}$$

Thus transmission zeros of structure C, being defined from equation $M_{1N}(E)=0$, coincide with zeros of all q_{jk}, i.e., with transmission zeros of the system as a whole.

VII. DISCUSSION

The effect of resistance resonance can manifest itself in the current-voltage curve most clearly when the incoming electrons are monoenergetic to a good accuracy, so that their energy is concentrated near some energy $E_{\rm inc}$ in a small interval $\Delta E \ll \Gamma_R$. The current through the structure in this case is proportional to conductance $G(E)$. Being tuned to the starting position with $E_{\rm inc}=E_R$, the structure under consideration could be treated as a sensitive nanometer detector and a rectifier of relatively small incoming signals, changing the applied voltage or shifting the structure levels. Actually, due to the steep slopes of the current-voltage curve at both sides of E_R, the relatively weak variation of system parameters changing the difference $E_{\rm inc}-E_R$ will switch the device, i.e., will cause the abrupt positive increasing of the current independently of the $E_{\rm inc}-E_R$ sign. The situation described relates mostly to the future ballistic nanoelectronic device working on the approximately monoenergetic electron fluxes.

As of now, the observation of resistance resonances could best be done by studying the conductance-voltage curve of the structure. That could be an artificial structure of quantum dots[7] and, probably, of atoms.[17] Another possibility is to search for such resonances in the conductance curves of tunneling media with accidentally interspersed resonant states. One could observe resistance resonances investigating, e.g., a thin insulating film on a conducting substrate with a scanning tunneling microscope (STM). If there is only one defect with a resonant level disposed inside the film under the STM tip, then the usual resonant-tunneling effect could be observed. However, in the neighborhood of this defect, there could be other defects with resonant levels, as shown in Fig. 5. Comparing this figure with Figs. 1(b) and 3, one finds a clear similarity among the structure configurations shown in Figs. 1(b), 3(a), and 3(c) and the ones shown in Figs. 5(a), 5(b), and 5(c), respectively. Thus the resistance resonances might be observed for such kinds of structures.

In summary, we have studied a property of resonant-tunneling structures consisting of quantum dots or atoms.

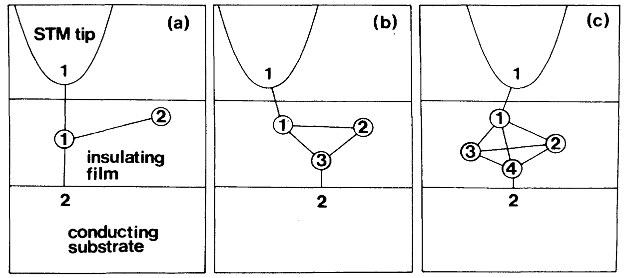

FIG. 5. The STM tip over the thin insulating film with defects lying on the conducting substrate. The configurations of structures (a), (b), and (c) are similar to the ones shown in Figs. 1(b), 3(a), and 3(c), respectively.

This property is restricted to systems with non-one-dimensional relative positions of quantum dots and electrodes. It becomes evident starting from the two-quantum-dot structure. It implies that the resonant-tunneling structure can have exponentially narrow and high resistance resonances and exponentially narrow backscattering conductance resonances. We find a simple algebraic equation for such resonances. We show that they are not sensitive to the structure of electrodes and hence they define the intrinsic property of the structure under investigation. Under certain conditions, the structure tuned to the resistance resonance may be a sensitive nanometer detector and rectifier of signals. Recent advances in nanotechnology and, in particular, in experimental observation of conductance resonances for systems of few atoms[4] and quantum dots[7] provide the possibility of observing resistance resonances as well as conductance ones.

Note added in proof. Recently [M. Sumetskii (unpublished)] we have found that the exponentially narrow dips in the *current*-voltage curve can exist no matter how large the width of energy spectrum of incident electrons, ΔE, and the applied voltage are. As examples, the two quantum-dot structure with three electrodes and the three quantum-dot structure with two electrodes are considered.

ACKNOWLEDGMENT

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[1] For a review, see *Physics of Quantum Electron Devices,* edited by F. Capasso, Springer Series in Electronics and Photonics Vol. 28 (Springer, New York, 1990).

[2] For a review, see A. Modinos, *Field, Thermionic, and Secondary Electron Emission Spectroscopy* (Plenum, New York, 1984).

[3] A. V. Chaplik and M. V. Entin, Zh. Eksp. Teor. Fiz. **67**, 208 (1974) [Sov. Phys. JETP **40**, 106 (1974)]; I. M. Lifshits and V. Ya. Kirkpichenkov, *ibid.* **77**, 989 (1979) [*ibid.* **50**, 499 (1979)]; M. Yu. Sumetskii, Pis'ma Zh. Eksp. Teor. Fiz. **44**, 287 (1986) [JETP Lett. **44**, 369 (1986)]; T. K. Ng and P. A. Lee, Phys. Rev. Lett. **61**, 1768 (1988).

[4] R. H. Koch and A. Hartstein, Phys. Rev. Lett. **54**, 1848 (1985); S. J. Bending and M. R. Beasley, *ibid.* **55**, 324 (1985); A. B. Fowler, G. L. Timp, J. J. Wainer, and R. A. Webb, *ibid.* **57**, 138 (1986); T. E. Kopley, P. L. McEuen, and R. G. Wheeler, *ibid.* **61**, 1654 (1988).

[5] V. Kalmeyer and R. B. Laughlin, Phys. Rev. B **35**, 9805 (1987); M. Büttiker, IBM J. Res. Dev. **32**, 63 (1988); W. Xue and P. A. Lee, Phys. Rev. B **38**, 3913 (1988); G. W. Bryant, *ibid.* **39**, 3145 (1989); H. C. Liu and G. C. Aers, J. Appl. Phys. **65**, 4908 (1989); H. Shi and G. Kirczenow, J. Phys. Condens. Matter **3**,

955 (1991); Y. Meier, N. S. Wingreen, and P. A. Lee, Phys. Rev. Lett. **66**, 3048 (1991).
[6]M. Sumetskii, J. Phys. Condens. Matter **3**, 2651 (1991).
[7]M. A. Reed, J. N. Randall, and R. J. Aggarwal *et al.*, Phys. Rev. Lett. **60**, 535 (1988); C. G. Smith, M. Pepper, and H. Ahmed *et al.*, J. Phys. C **21**, L893 (1989); L. P. Kouwenhoven, F. W. J. Hekking, and B. J. van Wees *et al.*, Phys. Rev. Lett. **65**, 361 (1990); Bo Su, V. G. Goldman, and J. E. Cunninham, Phys. Rev. B **46**, 7644 (1992); A. T. Jonson, L. P. Kouwenhoven, and W. de Jong *et al.*, Phys. Rev. Lett. **69**, 1592 (1992).
[8]A. K. Kazanskii, V. N. Ostrovskii, and E. A. Solov'ev, Zh. Eksp. Teor. Fiz. **70**, 493 (1976) [Sov. Phys. JETP **43**, 254 (1976)].
[9]F. Sols, M. Macucci, U. Ravaioli, and K. Hess, J. Appl. Phys. **66**, 3892 (1989).
[10]C. S. Chu and R. S. Sorbello, Phys. Rev. B **40**, 5941 (1989); P. F. Bagwell, *ibid.* **41**, 10 354 (1990); A. Kumar and P. F. Bagwell, *ibid.* **43**, 9012 (1991); Y. B. Levinson, M. I. Lubin, and E. V. Sukhorukov, *ibid.* **45**, 11 936 (1992).
[11]W. Porod, Z. Shao, and C. S. Lent, Appl. Phys. Lett. **61**, 1350 (1992).
[12]P. R. Price, Appl. Phys. Lett. **62**, 289 (1993).
[13]M. Büttiker, Y. Imry, R. Landauer, and S. Pinhas, Phys. Rev. B **31**, 6207 (1985); A. D. Stone and A. Szafer, IBM J. Res. Dev. **32**, 384 (1988).
[14]See, e.g., M. K. Schwalm and W. A. Schwalm, Phys. Rev. B **45**, 6868 (1992), and references therein.
[15]M. Yu. Sumetskii, Zh. Eksp. Teor. Fiz. **94**, 7 (1988) [Sov. Phys. JETP **67**, 438 (1988)].
[16]There is a misprint in the denominator of Eq. (8) in Ref. [6]: letter *S* should be changed for letter *D*.
[17]See P. Zeppenfeld, C. P. Lutz, and D. M. Eigler, Ultramicroscopy **42-44**, 128 (1992), and references therein.