RAPID COMMUNICATIONS

PHYSICAL REVIEW B VOLUME 48, NUMBER 6 1 AUGUST 1993-II

Effect of strong scattering on the low-temperature penetration depth of a d-wave superconductor

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(Received 7 May 1993)

For a pure superconductor in a d-wave-like state at temperatures T well below the critical temperature T_c, the deviation $\Delta\lambda$ of the penetration depth from its zero-temperature value $\lambda(0)$ is proportional to T. When the concentration n_i of strongly scattering impurities is nonzero, $\Delta\lambda \propto T^n$, where $n=2$ for $T<T^* \ll T_c$ and $n=1$ for $T^*<T\ll T_c$. The crossover temperature T^* and the increase in $\lambda(0)$ scale as $\sqrt{n_i}$ up to logarithmic corrections when resonant scattering is dominant. We argue that this case is relevant to recent measurements on $YBa_2Cu_3O_{7-\delta}$ and present specific results for a model pairing state with $d_{x^2-y^2}$ symmetry.

Measurements of the electromagnetic penetration depth λ at low temperature T are beginning to yield a consistent picture of the pairing state of the high-temperature superconductors.[1,2] Until recently it had been thought that the most credible data exhibited an exponential temperature dependence at low temperatures, but a reanalysis[3] of the data of Fiory *et al.*[4] clearly showed that the deviation $\Delta\lambda$ from the zero-temperature value $\lambda(0)$ was quadratic in temperature. Subsequently, this finding has been confirmed in a number of studies[5–7] and reanalyses[8,9] of existing data.[10] Remarkably, the available data are well described over the entire temperature range below the critical temperature T_c by the empirical formula $\lambda(T)=\lambda(0)/\sqrt{1-t^2}$, where $t\equiv T/T_c$.

In contrast to this body of data are equally credible studies of the penetration depth in $Tl_2CaBa_2Cu_2O_{8-\delta}$ single crystals[11,12] and in Y-Ba-Cu-O single crystals.[13] In the case of the $Tl_2CaBa_2Cu_2O_{8-\delta}$ single crystals, all independent components of the penetration depth tensor were measured. These studies report that at the lowest temperatures measured, the penetration depth varied linearly with temperature, not quadratically. If confirmed, these latter observations would be consistent with the prediction[3] that for an unconventional singlet superconductor with the point group symmetry of Y-Ba-Cu-O, the only nodes allowed are line nodes, distributed on the Fermi surface in such a way that *all* components of the penetration depth should exhibit a linear temperature dependence at low temperatures.

Annett, Goldenfeld, and Renn also argued[3] that $\Delta\lambda \propto T^2$ could still be indicative of an unconventional singlet state, given that it has been known for some time that scattering and Fermi-liquid effects may have this effect on superconductors with line nodes.[14,15] Furthermore, in discussing the discrepancy between early data on Y-Ba-Cu-O and their own data, Hardy *et al.* postulate[13] that the intrinsic behavior, found in crystals as pure as those in their study, is indeed linear, but that impurities and other defects present in thin films and poorer quality crystals change the low-temperature power law from linear to quadratic.

The purpose of the present paper is to consider this explanation in some detail. We propose that the dominant scattering mechanism in thin films and single crystals of Y-Ba-Cu-O is resonant scattering,[16] which is consistent with the phenomenology in two distinct ways. First, if the scattering responsible for the change in the power law of the penetration depth were Born scattering, there would be a large suppression of T_c, which is simply not observed.[17] Secondly, as noted previously,[1] the low-temperature thermal conductivity κ of single crystals of Y-Ba-Cu-O changes from $\kappa \sim T^2$ to $\kappa \sim T$ for temperatures below about 0.3 K.[18] This is apparently accompanied by a linear term in the specific heat.[19] These temperature dependences are consistent with those predicted by resonant scattering processes.[16] If resonant scattering is indeed responsible for the quadratic temperature dependence of λ in impure samples, then the precise nature of the crossover can be predicted, and compared with experiments in which the impurity concentration is systematically varied. Such a prediction is the principal result of the present paper.

Our calculation given below is valid for a two-dimensional $d_{x^2-y^2}$ superconductor, but ignores possible effects due to the presence of localized states. Such states have been argued to cause the asymptotic temperature dependence to be exponential rather than power law,[20] but it is not clear whether or not an intermediate quadratic temperature regime will still exist. Furthermore, the effect of localized states on thermodynamic properties should be confined to temperatures below a mobility gap which is found to scale with $\exp[-(E_F/\Delta_0)]$. Although the ratio E_F/Δ_0 may be considerably smaller than in ordinary superconductors, it still appears possible that the mobility gap is so small so as to make localization effects irrelevant for experiments.

To determine the penetration depth or superfluid density in a model d-wave superconductor, we calculate the electromagnetic response tensor K, relating the current density $\mathbf{j}$ to an applied vector potential $\mathbf{A}$: $j=-K\,\mathbf{A}$. If the electromagnetic response tensor K_{ij} is diagonal, it is simply related to the eigenvalues of the penetration depth tensor $(4\pi/c)K_{ii}=\lambda_i^{-2}$, where λ_i is the penetration depth for current flow in the direction i. In the simplest BCS-like model for an anisotropic superconducting state in the presence of elastic scattering, the response tensor is given by

$$K_{ij}=\frac{e^2}{c}\left\langle v_i(\mathbf{k})v_j(\mathbf{k})\int_0^\infty d\omega \tanh\frac{\omega}{2T}\mathrm{Re}\frac{\Delta_k^2}{(\tilde{\omega}^2-\Delta_k^2)^{3/2}}\right\rangle , \tag{1}$$

where $\langle\cdots\rangle\equiv 2(2\pi)^{-d}\int dS_F/|\mathbf{v}(\mathbf{k})|\cdots$ represents an angular average over an arbitrary Fermi surface in a d-dimensional metal, and $\mathbf{v}(\mathbf{k})$ is the Fermi velocity. The renormalized frequency $\tilde{\omega}$ is defined by $\tilde{\omega}=\omega-\Sigma_0$, where the impurity self-energy Σ_0 in the s-wave (t-matrix) scattering approximation is given by $\Sigma_0=\Gamma G_0/(c^2-G_0^2)$, and $\Gamma=n_i n/(\pi N_0)$ is a scattering rate parameter dependent only on the impurity concentration n_i, the electron density n, and the density of states N_0. The parameter c characterizes the strength of the interaction in an individual scattering event, varying between weak ($c\gg 1$, Born approximation) and strong ($c\ll 1$, unitarity limit) scattering. Finally, G_0 is the integrated diagonal Green's function averaged over disorder, $G_0=-i\langle\tilde{\omega}/(\tilde{\omega}^2-\Delta_k^2)^{1/2}\rangle$. We note that in Eq. (1) and what follows, we have specialized to those unconventional superconducting states for which the gap renormalization vanishes ($\tilde{\Delta}_k=\Delta_k$) for symmetry reasons; this includes the $d_{x^2-y^2}$ state of current interest.

As shown by Gor'kov[21] and by Ueda and Rice,[22] an infinitesimal amount of disorder in unconventional superconducting states with line nodes in three dimensions leads to a nonzero density of states at zero energy $N(0)$ [the same conclusion holds for point nodes in two dimensions (2D)]. This implies the existence of a "gapless" temperature regime, where all superconducting properties reflect the temperature dependence of the corresponding normal-state Fermi-liquid properties, albeit with reduced coefficients varying with $N(0)$. For the case of the penetration depth, which has no normal-state analog, it was pointed out in Ref. 14 that a low-temperature behavior of the form $\lambda\simeq\tilde{\lambda}_0+c_2T^2$ was to be expected for 3D line node states in the presence of disorder. The shift $\tilde{\lambda}_0-\lambda_0$ from the pure London penetration depth $\lambda_0=[4\pi mc^2/(ne^2)]^{1/2}$ (spherical Fermi surface), as well as the range over which the characteristic gapless temperature dependence $\lambda-\tilde{\lambda}_0\sim T^2$ is observed, are predicted to scale with $N(0)$. Here we explicitly estimate both quantities as functions of impurity concentration in order to facilitate comparison of the theory with experiment.

The crossover temperature is estimated crudely by assuming that an impurity-dominated regime exists at low temperatures, where the penetration depth obeys the T^2 relation discussed above, and that at somewhat higher temperatures (still considerably less than T_c) the penetration depth displays the temperature dependence of the pure state, $\lambda\simeq\lambda_0+c_1T$. In fact, such a clean separation of energy scales is possible only in the strong scattering limit,[16] but we will nevertheless define a rough measure of the crossover temperature for the general case by interpolating between the low- and intermediate-temperature regimes as $\lambda=\tilde{\lambda}_0+bT^2/(T^*+T)$. Fitting this form in the two cases $T\ll T^*$ and $T\gg T^*$ leads to the result $T^*\simeq c_1/c_2$. The coefficient c_1 is of order λ_0/T_c. To calculate c_2, we first extract the temperature-dependent part of Eq. (1), defining $K(T)=K(0)+\delta K(T)$, with

$$\delta K_{ij}(T)=\frac{-2e^2}{c}\left\langle v_i(\mathbf{k})v_j(\mathbf{k})\int_0^\infty d\omega\, f(\omega)\mathrm{Re}\frac{\Delta_k^2}{(\tilde{\omega}^2-\Delta_k^2)^{3/2}}\right\rangle , \tag{2}$$

where $f(\omega)$ is the Fermi function. The branch of the square root in Eq. (2) is defined such that for $\omega>0$, $(\tilde{\omega}^2-\Delta_k^2)^{1/2}=\eta|\tilde{\omega}^2-\Delta_k^2|^{1/2}e^{i\theta/2}$, where $\eta=1$ if $\mathrm{Re}\tilde{\omega}>|\Delta_k|$ and $\eta=i$ if $\mathrm{Re}\tilde{\omega}<|\Delta_k|$. The angle θ is given by

$$\theta=\tan^{-1}[\mathrm{Im}\tilde{\omega}/(\mathrm{Re}\tilde{\omega}-\Delta_k)] +\tan^{-1}[\mathrm{Im}\tilde{\omega}/(\mathrm{Re}\tilde{\omega}+\Delta_k)] .$$

To make further analytic progress, we note that in the impurity-dominated "gapless" regime, the renormalized frequency $\tilde{\omega}$ takes the limiting form $\tilde{\omega}\to i\gamma+a\omega$, where γ is a constant dependent on impurity concentration and scattering strength, and the constant $a\simeq O(1)$.

The Fermi function in the integral in Eq. (2) restricts the important integration range to small frequencies at low temperatures. We may thus replace the renormalized frequency $\tilde{\omega}$ everywhere by its low-frequency limiting form. Careful treatment of the branch cut then leads to the final result

$$\delta K_{ij}(T)\simeq\frac{-e^2}{c}\frac{\pi^2}{2}\gamma a T^2\left\langle v_i(\mathbf{k})v_j(\mathbf{k})\frac{\Delta_k^2}{(\gamma^2+\Delta_k^2)^{5/2}}\right\rangle . \tag{3}$$

The angular average may now be performed easily for any model superconducting state and Fermi surface. It is clear, however, that for any order parameter Δ_k which vanishes linearly in the neighborhood of line nodes (point nodes in 2D) the angular average in Eq. (3) varies as γ^{-2}, and the coefficient c_2 therefore varies typically as γ^{-1}. As discussed below, γ is a monotonically increasing function of impurity concentration; the result for c_2 may therefore seem somewhat surprising at first glance. It is important to note, however, that in the limit of vanishingly small impurity concentrations $\gamma\to 0$, the range of gapless behavior $\delta\lambda\sim T^2$ is restricted to a vanishingly small range below $T^*\sim c_1/c_2\sim\gamma$.

We now examine the gapless solution $i\gamma$ to the zero-frequency transcendental equation for $\tilde{\omega}$. For general scattering parameters Γ and c, γ satisfies

$\gamma=\Gamma n_0/(c^2+n_0^2)$, where $n_0=N(0)/N_0=\langle\gamma/(\gamma^2+\Delta_k^2)^{1/2}\rangle$ is the normalized residual density of states at the Fermi level. In the Born limit, $c\gg1$, this equation may be solved to yield $\gamma\simeq\Delta_0 e^{-\Delta_0/\Gamma_N}$, where Δ_0 is the gap maximum over the Fermi surface, and $\Gamma_N=\Gamma/(1+c^2)$. It is thus clear that in order to obtain an experimentally observable T^2 contribution to the penetration depth in this limit, scattering rates $\Gamma_N\sim\Delta_0$ are necessary. As the critical temperature for all scattering strengths in the *s*-wave *t*-matrix approximation for unconventional superconducting states obeys an Abrikosov-Gorkov[23] relation, $\ln(T_c/T_{c0})=\psi(\frac{1}{2})-\psi(\frac{1}{2}+\Gamma_N/(2\pi T_c))$, where ψ is the digamma function and T_{c0} is the pure T_c, such large scattering rates must perforce lead to a large $[O(T_{c0})]$ suppression of the critical temperature, which is apparently not observed in experiments.

The situation is quite different in the presence of strong scattering, $c\ll1$. In this case a closed form solution for $\gamma=\Gamma/n_0$ is not generally possible, but it is easy to estimate that $\gamma\sim(\Gamma\Delta_0)^{1/2}$ up to a logarithmic correction. The crucial point is that a relatively small concentration of defects can lead to a substantial residual density of states of γ, within a range where the relative T_c suppression $(T_{c0}-T_c)/T_{c0}$ is of order $\Gamma/\Delta_0\ll1$. We estimate that resonant defect concentrations of order 1% would lead to gapless behavior $(\delta\lambda\sim T^2)$ over a temperature range T^* of order 10% of T_c, but negligible (<1%) T_c suppression. Concentrations of order 0.1% would also lead to negligible T_c suppression, and the gapless range would be restricted to temperatures below 2–3 % of T_c, below the lowest temperatures where penetration depth experiments on high-T_c superconducting materials have been performed at this writing.

For concreteness, we now present explicit results for a $d_{x^2-y^2}$ state in a system with cylindrical Fermi surface. The model order parameter then takes the form $\Delta_k=\Delta_0(\hat{k}_x^2-\hat{k}_y^2)=\Delta_0\cos2\phi$. A low-temperature analytic estimate for $\mathbf{A},\mathbf{j}$ in the basal plane then yields a pure penetration depth $\lambda_1(T)\simeq\lambda_0+c_1T$, with $c_1=(\ln2)\lambda_0/\Delta_0$. The constant γ satisfies the exact self-consistency relation $\gamma=\Gamma/n_0$, where $n_0=2\gamma\mathbf{K}(\Delta_0/\sqrt{\gamma^2+\Delta_0^2})/\sqrt{\gamma^2+\Delta_0^2}$, and $\mathbf{K}$ is the complete elliptic integral of the first kind. For low impurity concentrations $\gamma\ll\Delta_0$, we find $n_0\simeq2\gamma\ln(4\Delta_0/\gamma)/(\pi\Delta_0)$. The numerically determined dependence of γ on the scattering rate Γ in the $d_{x^2-y^2}$ state is shown in Fig. 1 for resonant scattering, $c=0$, along with a fit of the form $\gamma\simeq0.63(\Gamma\Delta_0)^{1/2}$. Evaluation of Eq. (3) yields $c_2\simeq\pi\lambda_0/(6\gamma\Delta_0)$, and a crossover temperature estimate of $T^*\simeq6\ln(2)\gamma/\pi\simeq0.83(\Gamma\Delta_0)^{1/2}$, using the result of Fig. 1.

An independent estimate of the effect of impurity scattering may be obtained by accepting the interpolation formula $\lambda=\tilde{\lambda}_0+aT^2/(T^*+T)$ given above, and extrapolating a fit to the linear-T penetration depth data in the intermediate-temperature range down to $T=0$. The difference $\tilde{\lambda}_0-\lambda_0>0$ between this intercept and the actual limiting low-temperature value of the penetration depth may thus be estimated even in some experiments

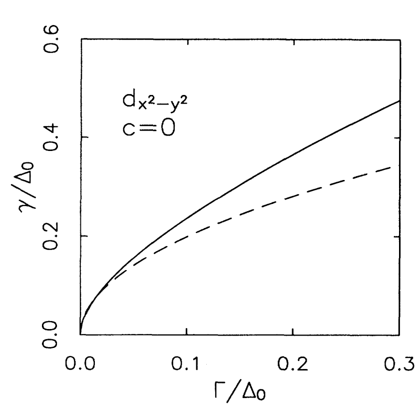

FIG. 1. Dependence of γ on Γ for a $d_{x^2-y^2}$ pairing state. The full curve is the numerical evaluation of the elliptic function, while the dashed curve is an empirical fit $\gamma\approx0.63\Gamma^{1/2}$.

which do not determine the absolute value of $\lambda(T=0)$. Here we give results for $\tilde{\lambda}_0-\lambda_0$ only in the resonant scattering limit $c\ll1$. By setting $T=0$ in Eq. (1) and deforming the contour in the complex ω plane onto the imaginary axis, then changing variables $\omega\to\tilde{\omega}$, we find

$$K_{ij}'\equiv K_{ij}(T=0,\Gamma)-K_{ij}(T=0,\Gamma=0)$$
$$=\frac{e^2}{c}\left\langle v_i(\mathbf{k})v_j(\mathbf{k})\frac{\Delta_k^2}{(\gamma^2+\Delta_k^2)^{1/2}}\right\rangle . \quad (4)$$

For the $d_{x^2-y^2}$ state, we arrive at

$$(\tilde{\lambda}_0-\lambda_0)/\lambda_0\simeq[\gamma/(\pi\Delta_0)]\ln(4\Delta_0/\gamma)\simeq\Gamma/(2\gamma) ,$$

where the last result follows from the self-consistency equation for γ. Using the empirical fit $\gamma\simeq0.63(\Gamma\Delta_0)^{1/2}$, we find finally $(\tilde{\lambda}_0-\lambda_0)/\lambda_0\simeq0.79(\Gamma/\Delta_0)^{1/2}$. Thus the limiting low-temperature value of the penetration depth also scales roughly with the size of the gapless region, or $N(0)$.

The range of applicability of our analytic estimates is illustrated by examining exact numerical evaluations of Eq. (1), as given in Fig. 2 where we plot the normalized superfluid density $n^s/n\equiv(\lambda_0/\lambda)^2$ for various values of the scattering parameters c and Γ. For resonant scatter-

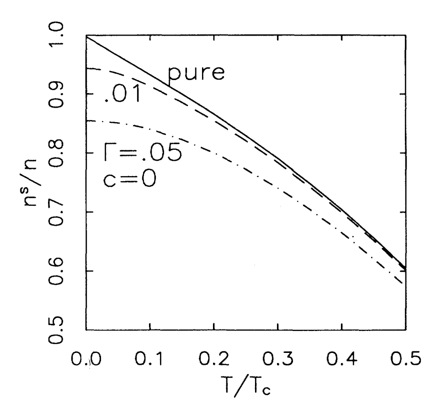

FIG. 2. Normalized superfluid density n^s/n vs reduced temperature T/T_c. Solid line: $\Gamma/T_c=0$; dashed $c=0$, $\Gamma/T_c=0.01$; dashed-dotted; $c=0$, $\Gamma/T_c=0.05$.

ing $c=0$ and $\Gamma/T_c=0.01$, our estimate of the crossover temperature above yields $T^*\simeq 0.12T_c$, using $\Delta_0/T_c\simeq 2.14$ for a $d_{x^2-y^2}$ state. This in fact corresponds roughly to the range of temperatures where the superfluid density deviates substantially from the pure case. For the larger scattering rate, $\Gamma/T_c=0.05$, the deviations from the pure result above the estimated crossover $T^*\simeq 0.27T_c$ are substantially larger, and there is no temperature range over which a linear-T ("pure") behavior is realized. Thus the crude calculation we have described can be expected to give a useful approximation to exact results only for concentrations $n_i \lesssim 10^{-2}T_c/E_F$, where E_F is the effective Fermi energy for the system. As T_c/E_F is not too much smaller than one in the CuO materials, the estimates made here are probably appropriate for nominally clean samples. In addition, we expect our *qualitative* predictions for the rough scaling of various quantities with concentration to hold over a much larger range. The theory could thus be used to analyze systematic doping studies of penetration with concentrations of nonmagnetic impurities in the CuO planes at the few-percent level.

Confirmation of this picture of penetration depth measurements awaits a consistent description of other low-temperature properties within the same framework. In addition, we are lacking a persuasive explanation of the origin of resonant impurity scattering in the high-T_c materials. We hope to address these questions in a future publication.

One of us (N.G.) gratefully acknowledges the support of the National Science Foundation through Grant No. NSF-DMR-90-15791. P.J.H. is grateful to W. O. Putikka for assistance with some numerical calculations, and for helpful discussions.

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