Magnetic field of vortices crossing a superconductor surface

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The field distribution of a straight London vortex crossing the flat boundary of a uniaxial superconductor is evaluated. A superconducting half-space and a thin film are considered for a general crystal and vortex orientation relative to the surface. Vortex lattices in a film with a tilted crystal axis are distorted more strongly than in the bulk.

I. INTRODUCTION

Techniques have recently been emerging for studying vortices in superconductors. Improved decoration experiments by Gammel *et al.*[1] yielded a wealth of information on vortices in anisotropic high-T_c materials; part of these data[2] are still to be digested by the theoretical community. The scanning tunneling experiments of Hess, Robinson, and Waszczak[3] have not only revealed the details of the core structure (on the scale of the coherence length ξ), but have also demonstrated a strong correlation between the anisotropy of the material in question ($NbSe_2$) and the flux-line lattices (on the scale of the penetration depth $\lambda \gg \xi$).[4] Recent developments in the Hall-probe technique made it possible to probe the vortex field *distribution* outside the sample;[5] some of the measurements are done with an epitaxial film of LaSrCuO grown with the crystal axis c at an angle (49°) with the normal to the film surface. Recent advancements in magnetic force microscopy[6] promise to yield more data on the vortex field in the near future. All these experiments provide information about the vortex structure and the fundamental intervortex interactions in anisotropic superconductors.

The interpretation of these measurements is, however, complicated by the fact that the structure of vortices and their interaction are strongly affected by the interface between the superconductor and the outer space (vacuum) where the data are taken. The pioneering work on this effect is due to Pearl,[7] who evaluated the field distribution of a vortex out of a thin *isotropic* superconducting film; he also solved the problem of a vortex perpendicular to the flat surface of a superconducting half-space.[8] To describe the experiments mentioned, the Pearl problem is to be solved for an *anisotropic* superconducting slab (or half-space) with arbitrary orientations of the crystal axes and of the vortex itself relative to the slab boundaries. However, being designed for vortices perpendicular to the boundaries of isotropic samples, Pearl's method utilizes the cylindrical symmetry of the vortex field and thus cannot be applied to a general situation of a tilted vortex in anisotropic materials. The approach described in this paper is free of the requirement of cylindrical symmetry.

Basically, we employ the two-dimensional (2D) Fourier transform (FT) in the plane (x, y) of the interface; since both London (for the superconductor) and Laplace (for the outside space) equations are of second order, the remaining dependence on the third coordinate z contains simple exponential functions. A version of this method has been used in Ref. 9 for the problem of a vortex parallel to the boundary; a similar approach has been employed by Marchetti.[10] We demonstrate the method for a tilted straight vortex in anisotropic (uniaxial) half-space and for a thin film with an arbitrarily oriented c axis. In fact, our method can be applied to the more general situation of a flat boundary between either anisotropic superconductors or any combination of anisotropic and isotropic materials. The method is based on London equations and as such is relevant for materials with $\lambda \gg \xi$. One should bear in mind that for layered Josephson coupled superconductors (such as many high-T_c materials) the London approach works well provided $\xi_c(T)$ exceeds the interlayer spacing.

Considering only *straight* vortices, we leave aside the physical question of how in fact the vortices behave in the boundary layer under certain external conditions. Being aware of this weakness of our approach, we note that the main thrust of this work is in providing a method for the evaluation of the field distribution *outside* the sample. Although this distribution is sensitive to the curvature of vortex lines immediately under the surface, the influence of vortex elements decreases exponentially with their depth. We thus expect that the method proposed allows one to distinguish between vortices crossing the boundary at an angle and those which exit perpendicular to the surface. (A brief inspection of our Fig. 2 for the outside field of a tilted vortex strongly supports this claim.)

In the next section we formulate the problem and describe general features of the method. Then we solve the problem of a straight vortex crossing the boundary of an anisotropic half-space; the problem of an anisotropic film follows.

II. MAIN EQUATIONS

As was pointed out, in materials with $\lambda \gg \xi$ the magnetic field $\mathbf{h}$ is described by the London equations (excluding a small region of the vortex core)

$$h_i - \lambda^2 m_{kl} e_{ils} e_{ktj} h_{j,st} = \phi_0 \hat{v}_i \delta(\mathbf{r}_0) \ . \tag{1}$$

The mass tensor m_{ik} is defined in the usual way: For uniaxial materials, $m_a m_b m_c = m_a^2 m_c = 1$, which implies $\lambda^3 = \lambda_a^2 \lambda_c$, $\lambda_\beta = \sqrt{m_\beta}\lambda$ ($\beta = a$ or c). In the following, unless it is stated otherwise, we use the geometric average λ as a unit length. Further, e_{ikl} is the unit antisymmetric tensor, ϕ_0 is the flux quantum, $\hat{v}$ is the direction of the vortex axis, $\mathbf{r}_0$ is the position vector in the plane perpendicular to $\hat{v}$, and $h_{j,st} \equiv \partial^2 h_j / \partial x_s \partial x_t$. For the general case of the c axis at an angle θ to the direction z normal to the boundary plane (or planes), one can choose the x axis in the plane $(\hat{c}, \hat{z})$ and $\hat{y}$ in the (a, b) plane; see Fig. 1(a). Then the nonzero components of the mass tensor are

$$\begin{aligned}
m_{xx} &= m_a \cos^2\theta + m_c \sin^2\theta \,, \\
m_{zz} &= m_a \sin^2\theta + m_c \cos^2\theta \,, \\
m_{yy} &= m_a \,, \ \ m_{xz} = (m_a - m_c)\cos\theta \sin\theta.
\end{aligned} \tag{2}$$

The general solution of Eq. (1) is

$$\mathbf{h} = \mathbf{h}^{(0)} + \mathbf{h}^{(v)} \ , \tag{3}$$

where $\mathbf{h}^{(0)}$ solves the homogeneous [with zero right-hand side (RHS)] equation, whereas $\mathbf{h}^{(v)}$ is a particular solution of the full Eq. (1). In the presence of a vortex at the straight line parallel to $\hat{v}$, the part $\mathbf{h}^{(v)}$ can be taken as the field of an *infinitely* long unperturbed straight vortex; this assures the correct singular behavior of the solution $\mathbf{h}$ at the vortex axis. For a straight vortex, $\mathbf{h}^{(v)}$ was studied in Refs. 11–13.

Because of the presence of flat boundaries parallel to (x, y), it is convenient to perform the FT of Eq. (1) with respect to x, y; then we are left with the *linear second-order ordinary* differential equations for $\mathbf{h}(\mathbf{k}, z) = \int dxdy \mathbf{h}(\mathbf{r}, z) \exp(-i\mathbf{k}\cdot\mathbf{r})$; $\mathbf{r} = (x, y)$, with z-independent coefficients. Thus, components of $\mathbf{h}^{(0)}(\mathbf{k}, z)$ are linear combinations of simple exponentials:

$$\mathbf{h}^{(0)}(\mathbf{k}, z) = \sum_n \mathbf{H}^{(n)} e^{\alpha_n z} \ . \tag{4}$$

Here, the z-independent constants $\alpha_n(\mathbf{k})$ and $\mathbf{H}^{(n)}(\mathbf{k})$ are still to be determined; α should have the real part which assures the decay of $\mathbf{h}^{(0)}$ deep in the material.

Writing explicitly Eq. (1) with zero RHS, one obtains the linear and homogeneous system of equations for the three components of $\mathbf{h}^{(0)}$; each term in the sum (4) should satisfy this system separately. Omitting for brevity the argument $\mathbf{k}$ and the label n we have

$$\Delta_{ij} H_i = 0, \tag{5}$$

with $i, j = x, y, z$ and with a symmetric matrix Δ_{ij}:

$$\begin{aligned}
\Delta_{xx} &= 1 + m_{zz}k_y^2 - m_a\alpha^2, \\
\Delta_{xy} &= -k_y(im_{xz}\alpha + m_{zz}k_x), \\
\Delta_{xz} &= im_a k_x\alpha - m_{xz}k_y^2, \\
\Delta_{yy} &= 1 + m_{zz}k_x^2 - m_{xx}\alpha^2 + 2im_{xz}k_x\alpha, \\
\Delta_{yz} &= k_y(im_{xx}\alpha + m_{xz}k_x), \\
\Delta_{zz} &= 1 + m_a k_x^2 + m_{xx}k_y^2.
\end{aligned} \tag{6}$$

The vanishing determinant Δ of the system (5) provides all possible α_n. In finding the coefficients $\mathbf{H}^{(n)}(\mathbf{k})$ it is useful to observe that having chosen $\mathbf{h}^{(v)}$ as the field of an infinite vortex, we in fact impose the condition $\text{div}\mathbf{h}^{(0)} = 0$ or

$$k_x H_x^{(n)} + k_y H_y^{(n)} - i\alpha_n H_z^{(n)} = 0 \,. \tag{7}$$

Therefore, any one of Eqs. (5) can be replaced with a simpler Eq. (7). After simple but tedious algebra we obtain

$$\begin{aligned}
\alpha_{1,2} &= \pm\left(\frac{1 + m_a k^2}{m_a}\right)^{1/2}, \\
\alpha_{3,4} &= ik_x \frac{m_{xz}}{m_{xx}} \pm \left(\frac{1 + k_x^2/m_a m_{xx} + k_y^2 m_c}{m_{xx}}\right)^{1/2}.
\end{aligned} \tag{8}$$

Now, for each α_n one can express some of $\mathbf{H}^{(n)}(\mathbf{k})$ in terms of others with the help of the homogeneous system (5), thus reducing the number of quantities to be determined by boundary conditions; this procedure is problem specific.

The particular solution of the London equation (1), $\mathbf{h}^{(v)}$, should also be Fourier transformed in the xy plane parallel to the interface. This has been done in Ref. 14 for the interface in the ab plane. In the general case, the vortex orientation is given by two spherical angles: θ_v between $\hat{z}$ and $\hat{v}$ and ϕ_v between the projection of $\hat{v}$ onto xy and $\hat{x}$. Transforming the RHS of Eq. (1), $R_i = \phi_0 \hat{v}_i \delta(x_0)\delta(y_0)$ (x_0 and y_0 are perpendicular to the vortex direction $\hat{v}$), one uses

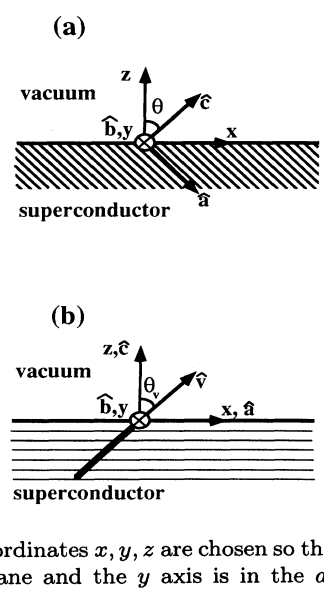

FIG. 1. Coordinates x, y, z are chosen so that the interface is in the xy plane and the y axis is in the ab plane of the crystal. (a) The c axis is oriented arbitrarily with respect to the surface. (b) (ab) planes are parallel to the surface; the vortex orientation $\hat{v}$ is arbitrary.

$$x_0 = (x\cos\phi_v + y\sin\phi_v)\cos\theta_v - z\sin\theta_v,$$
$$y_0 = -x\sin\phi_v + y\cos\phi_v, \quad (9)$$
$$z_0 = (x\cos\phi_v + y\sin\phi_v)\sin\theta_v + z\cos\theta_v,$$

to obtain the vector $\mathbf{R}$ of the RHS:

$$R_i = \phi_0 \begin{Bmatrix} \cos\phi_v\tan\theta_v \\ \sin\phi_v\tan\theta_v \\ 1 \end{Bmatrix} e^{-iz\tan\theta_v(k_x\cos\phi_v + k_y\sin\phi_v)}. \quad (10)$$

To evaluate the 2D Fourier transform $\mathbf{h}^{(v)}(\mathbf{k})$ one can use the fact that in an infinite straight vortex, nothing depends on the longitudinal coordinate z_0 ($\hat{z_0} = \hat{v}$):

$$\partial_{z_0} = \sin\theta_v(\cos\phi_v\partial_x + \sin\phi_v\partial_y) + \cos\theta_v\partial_z = 0. \quad (11)$$

Therefore, in order to obtain the 2D FT one can use the replacement $k_z \to -\tan\theta_v(k_x\cos\phi_v + k_y\sin\phi_v)$ in the 3D FT. Equation (1) then reads

$$\tilde{\Delta}_{ij}h_i^{(v)}(\mathbf{k}) = R_j\,, \quad (12)$$

where $\tilde{\Delta}_{ij}$ is obtained from Δ_{ik} of Eq. (6) by the replacement

$$\alpha \to -i\tan\theta_v(k_x\cos\phi_v + k_y\sin\phi_v)\,. \quad (13)$$

Solving Eq. (12) for $h_i^{(v)}$ one can utilize the known determinant

$$\Delta = m_a m_{xx}\prod_{n=1}^{4}(\alpha - \alpha_n) \quad (14)$$

and use the replacement (13) to obtain $\tilde{\Delta}$. In the general case, expressions for $h_i^{(v)}$ are cumbersome and we do not write them down explicitly. Instead, we will demonstrate the method in a few cases of interest (for which ϕ_v and at least one of the angles θ or θ_v vanish).

The field outside the sample satisfies div$\mathbf{h} = 0$ and curl$\mathbf{h} = \mathbf{0}$, so that one looks for $\mathbf{h} = \nabla\varphi$ with $\nabla^2\varphi = 0$. The general solution of the Laplace equation that vanishes at $|z| \to \infty$ is

$$\varphi(\mathbf{r}, z) = \int \frac{d^2\mathbf{k}}{(2\pi)^2}\varphi(\mathbf{k})e^{i\mathbf{k}\cdot\mathbf{r} - k|z|}\,, \quad (15)$$

where $\mathbf{r} = (x, y)$, $\mathbf{k} = (k_x, k_y)$. The 2D Fourier transform is defined by

$$\varphi(\mathbf{k}) = e^{k|z|}\int d^2\mathbf{r}\varphi(\mathbf{r}, z)e^{-i\mathbf{k}\cdot\mathbf{r}}. \quad (16)$$

The z-independent $\mathbf{H}^{(n)}(\mathbf{k})$ and $\varphi(\mathbf{k})$ are determined by boundary conditions, a particular form of which is problem specific. At boundaries with vacuum, the conditions consist of continuity requirements for $\mathbf{h}$ at the sample surface and of the vanishing field at infinity.

Another type of boundary condition should be satisfied at the interfaces between two superconductors. For example, at grain boundaries with a "perfect" electrical contact and with no suppression of the order parameter near the interface (no Josephson properties), the conditions follow from the fact that London equations should hold on both sides of the interface (see Refs. 15 and 16): Tangential components of the vector

$$Q_i = m_{ik}j_k \quad (17)$$

should be continuous.

III. SUPERCONDUCTING HALF-SPACE

With the z axis directed into vacuum, we have a requirement Re$\alpha > 0$; i.e., the only terms possible in Eq. (4) contain α_1 and α_3. The continuity of $\mathbf{h}$ at the surface $z = 0$ gives

$$ik_x\varphi = h_x^{(v)} + H_x^{(1)} + H_x^{(3)}\,,$$
$$ik_y\varphi = h_y^{(v)} + H_y^{(1)} + H_y^{(3)}\,, \quad (18)$$
$$-k\varphi = h_z^{(v)} + H_z^{(1)} + H_z^{(3)}\,.$$

One can exclude $\mathbf{H}^{(1)}$ (or $\mathbf{H}^{(3)}$) from these equations multiplying them by ik_x, ik_y, α_1 (or α_3) and using Eq. (7):

$$k(k+\alpha_1)\varphi + (\alpha_1 - \alpha_3)H_z^{(3)} = f(\mathbf{h}^{(v)}, \alpha_1)\,,$$
$$k(k+\alpha_3)\varphi + (\alpha_3 - \alpha_1)H_z^{(1)} = f(\mathbf{h}^{(v)}, \alpha_3)\,, \quad (19)$$

where

$$f(\mathbf{h}^{(v)}, \alpha) = -i\mathbf{k}\cdot\mathbf{h}^{(v)} - \alpha h_z^{(v)}\,. \quad (20)$$

Moreover, excluding φ from Eqs. (19) we have a useful relation

$$(k+\alpha_1)H_z^{(1)} + (k+\alpha_3)H_z^{(3)} = -kh_z^{(v)} + i\mathbf{k}\cdot\mathbf{h}^{(v)}. \quad (21)$$

We will now proceed by specifying the orientations of the vortex ($\hat{v}$) and of the crystal ($\hat{c}$) with respect to the boundary.

A. Crystal surface coincides with the *ab* plane

In this case $\theta = 0$ [see Fig. 1(b)] and the mass tensor is diagonal: $m_{xx} = m_{yy} = m_a$, $m_{zz} = m_c$. Let us start with the part $\mathbf{h}^{(0)}$ of the field distribution, Eq. (4). One finds $\Delta = \det\Delta_{ik} = m_a^2(\alpha^2 - \alpha_1^2)(\alpha^2 - \alpha_3^2)$, where $\alpha_1 = \sqrt{(1 + m_a k^2)/m_a}$ and $\alpha_3 = \sqrt{(1 + m_c k^2)/m_a}$. Solving Eq. (5) for $n = 1$ and $n = 3$ we obtain

$$H_x^{(1)} = H_y^{(1)}k_x/k_y = H_z^{(1)}ik_x\alpha_1/k^2\,,$$
$$H_x^{(3)} = -H_y^{(3)}k_y/k_x, \quad H_z^{(3)} = 0\,. \quad (22)$$

Thus, all $\mathbf{H}^{(n)}$ are expressed in terms of $H_z^{(1)}$ and $H_y^{(3)}$.

The field $\mathbf{h}^{(v)}$ of an unperturbed vortex satisfies Eq. (12) with the RHS $\mathbf{R} = \phi_0\exp(-izk_x\tan\theta_v)(\hat{x}\tan\theta_v + \hat{z})$. According to Eqs. (13) and (14), $\tilde{\Delta} = \det\tilde{\Delta}_{ik} = m_a^2(k_x^2\tan^2\theta_v + \alpha_1^2)(k_x^2\tan^2\theta_v + \alpha_3^2)$ or $\tilde{\Delta} = \tilde{\Delta}_1\tilde{\Delta}_2$ with

$$\tilde{\Delta}_1 = 1 + m_a k^2 + m_a k_x^2\tan^2\theta_v\,,$$
$$\tilde{\Delta}_2 = 1 + m_c k^2 + m_a k_x^2\tan^2\theta_v\,. \quad (23)$$

Solving Eq. (12) for $\mathbf{h}^{(v)}$ we obtain

$$h_x^{(v)} = \phi_0\tan\theta_v[1 + m_a(k_x^2\tan^2\theta_v + k_y^2) + m_c k_x^2]/\tilde{\Delta}\,,$$
$$h_y^{(v)} = \phi_0\tan\theta_v(m_c - m_a)k_x k_y/\tilde{\Delta}\,, \quad (24)$$
$$h_z^{(v)} = \phi_0/\tilde{\Delta}_1\,.$$

This representation of the vortex field has been used in Ref. 14.

We now apply the boundary conditions; the first of Eqs. (19) gives the outside potential

$$\varphi(\mathbf{k}) = -\frac{f(\mathbf{h}^{(v)},\alpha_1)}{k(k+\alpha_1)} = -\frac{\phi_0(\alpha_1 + ik_x\tan\theta_v)}{k(k+\alpha_1)\tilde{\Delta}_1}. \tag{25}$$

In particular, for a vortex parallel to the c axis, $\theta_v = 0$, we have

$$\varphi(\mathbf{k}) = -\frac{\phi_0}{m_a k\alpha_1(k+\alpha_1)}. \tag{26}$$

This, along with Eq. (16), provides the outside field distribution in the Hall-probe experiment[17] where the Bi 2:2:1:2 crystal was subject to a small field perpendicular to the ab crystal surface. Setting $m_a = 1$ in Eq. (26) we recover Pearl's result[8] for a vortex perpendicular to the isotropic half-space. Note that setting all masses to unity in Eq. (25), we in fact have a solution for the vortex tilted relative to the boundary of the isotropic half-space, which has not been considered by Pearl.

For the internal problem we obtain

$$H_z^{(1)} = \frac{\phi_0(ik_x\tan\theta_v - k)}{(k+\alpha_1)\tilde{\Delta}_1}, \qquad H_y^{(3)} = \frac{\phi_0 k_x k_y \tan\theta_v}{k^2\tilde{\Delta}_2}. \tag{27}$$

The rest of the H's are given in Eqs. (22). We now have all what is needed for the field evaluation both in and outside the superconductor.

B. Vortex perpendicular to the boundary, c is arbitrary

This is the case of the Hall-probe experiment of Ref. 5; $\theta_v = \varphi_v = 0$ whereas $\theta \neq 0$; see Fig. 1(a). The relevant α_1 and α_3 are given in Eq. (8). Solving the system (5) for $n = 1$, one obtains

$$H_x^{(1)} = H_z^{(1)}\frac{i\alpha_1\Delta_{xy}^{(1)} + k_y\Delta_{xz}^{(1)}}{k_x\Delta_{xy}^{(1)} - k_y\Delta_{xx}^{(1)}}, \qquad H_y^{(1)} = H_z^{(1)}\frac{i\alpha_1\Delta_{xx}^{(1)} + k_x\Delta_{xz}^{(1)}}{k_y\Delta_{xx}^{(1)} - k_x\Delta_{xy}^{(1)}} \tag{28}$$

[we have replaced one of the Eqs. (5) with Eq. (7); $\Delta_{ij}^{(1)}$ stands for Δ_{ij} at $\alpha = \alpha_1$]. The result for $n = 3$ is obtained by the replacement $1 \to 3$ in Eqs. (28). Thus, all $\mathbf{H}^{(n)}$ are expressed in terms of $H_z^{(1)}$ and $H_z^{(3)}$.

The RHS of Eq. (12) is $\mathbf{R} = \phi_0\hat{z}$. According to rule (14), the determinant $\tilde{\Delta} = \Delta(\alpha = 0) = -m_a m_{xx}\alpha_1^2|\alpha_3|^2$; with the help of identity $m_{xx}m_{zz} - m_{xz}^2 = m_a m_c$ we obtain $\tilde{\Delta} = (1 + m_a k^2)(1 + m_{zz}k_x^2 + m_c k_y^2)$. Solving $\tilde{\Delta}_{ij}h_i^{(v)} = \phi_0\hat{z}_j$, we obtain

$$h_x^{(v)} = -h_y^{(v)}k_y/k_x = \phi_0 m_{xz}k_y^2/\tilde{\Delta}, \qquad h_z^{(v)} = \phi_0(1 + m_{zz}k^2)/\tilde{\Delta} \tag{29}$$

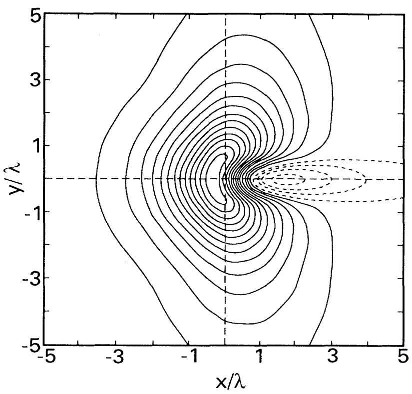

FIG. 2. Contours of the constant field component $h_z(x,y)$ at the distance $z = \lambda_{ab} = \lambda\sqrt{m_a}$ from the surface for the superconducting half-space $z < 0$. The angle θ between c and z is 49°; the anisotropy parameter $\gamma = \sqrt{m_c/m_a} = 4$. Intervals between adjacent contours are $2 \times 10^{-2}\phi_0/4\pi\lambda^2$. Dashed contours correspond to $h_z \leq 0$.

(this could have been taken from Ref. 11).

Turning to boundary conditions (18), we see that after expressing H_x, and H_y in terms of H_z in Eqs. (28), we have three equations for three unknowns: φ, $H_z^{(1)}$, and $H_z^{(3)}$. The explicit formulas, however, are too cumbersome and from this point on we resort to the numerical evaluation. Distribution of the z component of the vortex field, as detected at a distance z from the surface, is shown in Fig. 2 for the parameters indicated in the figure caption. We see that the surface perturbs substantially the vortex field. Indeed, in the bulk the field $h_z(x,y)$ is an even function of x,[12,13,18] whereas near the surface this symmetry is lost.

IV. THIN FILM

A. Single vortex

A thin film can be treated as a slab of a thickness $d \ll \lambda$ ($d \ll 1$ in dimensionless units). Though solvable, the problem of a slab is made cumbersome by the need to determine too many $\mathbf{H}^{(n)}$'s in Eq. (4) for the perturbation of the vortex field by the boundaries: All four α's are relevant. On the other hand, in the thin-film limit, the only role of the London equation (1) is to provide proper boundary conditions for the outside magnetostatic problem. We demonstrate below that this can be done without a complete solution of the slab problem.

Let us consider a vortex perpendicular to the slab boundaries; within our notation $\hat{v} = \hat{z}$ or $\phi_v = \theta_v = 0$. Equation (1) then reads in conventional units

$$h_i - 4\pi\lambda^2 m_{kl}e_{ils}j_{k,s}/c = \phi_0\hat{z}_i\delta(x,y). \tag{30}$$

We now place the origin at the slab middle and integrate Eq. (30) over the slab thickness. In the following we use the notation

$$\frac{1}{d}\int_{-d/2}^{d/2} f(z)dz = \langle f\rangle, \quad f\left(\frac{d}{2}\right) - f\left(-\frac{d}{2}\right) = [f]. \qquad (31)$$

We now make an assumption (confirmed by the result) that all quantities change in the film plane on distances larger than λ; e.g., evaluating $\langle 4\pi j_x/c\rangle$ we write

$$\langle \partial_y h_z - \partial_z h_y\rangle = \partial_y\langle h_z\rangle - [h_y]/d \simeq -[h_y]/d. \qquad (32)$$

Keeping only leading terms, we obtain from Eq. (30) for $i = z$

$$\langle h_z\rangle + \lambda_{\rm eff}(m_{xx}\partial_y[h_y] + m_a\partial_x[h_x]) = \phi_0\delta(x,y), \qquad (33)$$

where the effective average film penetration depth

$$\lambda_{\rm eff} = \lambda^2/d. \qquad (34)$$

The equations for $i = x, y$ involve currents j_x, j_y and, therefore, the normal derivatives of the tangential field *inside* the film; these are needed for the internal problem.

All quantities at the LHS of Eq. (33) vary continuously when one crosses the film along z from one side to another: The continuity of the derivatives in the parentheses follows from the continuity of the tangential field. Further, in the limit $d \to 0$, $\langle h_z\rangle = h_z(d/2) = h_z(-d/2)$. Thus we obtain for the boundary at $z = d/2$

$$h_z + 2\lambda_{\rm eff}(m_{xx}\partial_y h_y + m_a\partial_x h_x) = \phi_0\delta(x,y). \qquad (35)$$

The field outside is the gradient of the potential (15). Performing the FT of Eq. (35) and utilizing the field continuity, we obtain

$$\varphi(\mathbf{k}) = -\frac{\phi_0}{k + 2\lambda_{\rm eff}(m_{xx}k_y^2 + m_a k_x^2)}. \qquad (36)$$

This solves the problem of the field distribution outside the film; in the isotropic case all masses are unity, and Eq. (36) recovers Pearl's result.[7] Thus, in anisotropic film we have two effective lengths to characterize the vortex field, $\lambda^2 m_{xx}/d$ and $\lambda^2 m_a/d$; the first length depends on the c orientation, the second is just λ_{ab}^2/d.

The quantity measured by the Hall probe technique is

$$h_z(\mathbf{r},z) = -\int k\varphi(\mathbf{k})e^{i\mathbf{k}\cdot\mathbf{r}-kz}d^2\mathbf{k}/(2\pi)^2, \qquad (37)$$

where z is the distance of the probe from the sample surface. The contours of $h_z =$ const evaluated numerically for a film with $m_c/m_a = 16$, $\theta = 49°$, and $z = \lambda^2/d$ are shown in Fig. 3. Note that h_z changes sign in a broad domain adjacent to the x axis; still the total flux in z direction is $h_z(\mathbf{k}=0; z) = \phi_0$, the situation similar to the bulk case.[12] For the parameters chosen, h_z reaches a minimum of $\approx -5\times 10^{-4}\phi_0/\lambda_{\rm eff}^2$ at $x \approx 10\lambda_{\rm eff}$, $y = 0$. At the first sight, this feature suggests a peculiar vortex-vortex interaction since in the *bulk* the interaction potential is $\propto h_z$. It can be shown, however, that the interaction in a film is a monotonic function of the intervortex distance, the peculiarity of $h_z(x,y)$ notwithstanding (see below).

We evaluate now the energy of a single vortex. The outside part of it is $\int h^2 dV/8\pi = \int d^2\mathbf{k}k|\varphi(\mathbf{k})|^2/16\pi^3$, where to apply the 2D FT we first transformed $\int h^2 dV =$

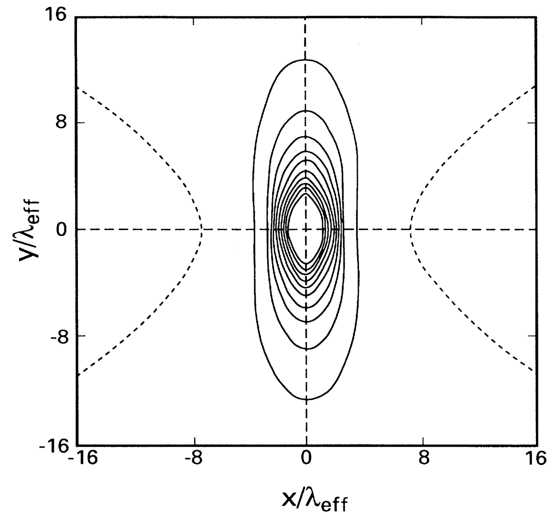

FIG. 3. Contours of the constant field component $h_z(x,y)$ at the distance $z = \lambda_{\rm eff}$ from the film surface. The angle θ between the crystal axis c and the normal to the film is 49°, $\gamma = 4$. Intervals between adjacent contours are $10^{-3}\phi_0/4\pi\lambda_{\rm eff}^2$; the dashed line is $h_z = 0$.

$\int \mathbf{h}\cdot\nabla\varphi dV$ into an integral over the film surface. Then, it easy to see that the magnetic part $\int h^2 dV/8\pi$ of the internal energy can be neglected relative to the kinetic one $\epsilon_{\rm kin} = (2\pi\lambda^2/c^2)\int dV m_{ik}j_i j_k$. Since the integral here is over the *film* volume, i.e., for $d \to 0$ over the xy plane, the current density in the integrand is expressed in terms of the tangential field: $4\pi\mathbf{j}/c = 2\mathbf{z}\times\mathbf{h}(d/2)/d$. We then obtain $\epsilon_{\rm kin} = \lambda_{\rm eff}\int dxdy(m_a h_x^2 + m_{xx}h_y^2)/2\pi = \lambda_{\rm eff}\int d^2\mathbf{k}(m_a k_x^2 + m_{xx}k_y^2)|\varphi(\mathbf{k})|^2/8\pi^3$. Thus, the total energy of a vortex is

$$\epsilon_0 = \frac{\phi_0^2}{16\pi^3}\int\frac{d^2\mathbf{k}}{k + 2\lambda_{\rm eff}(m_a k_x^2 + m_{xx}k_y^2)}. \qquad (38)$$

B. Vortex lattice

For two vortices separated by $\mathbf{a}$, one obtains $\epsilon = 2\epsilon_0 + \epsilon_{\rm int}$, where the interaction part reads

$$\epsilon_{\rm int} = \frac{\phi_0^2}{8\pi^3}\int\frac{d^2\mathbf{k}\cos\mathbf{k}\cdot\mathbf{a}}{k + 2\lambda_{\rm eff}(m_a k_x^2 + m_{xx}k_y^2)}. \qquad (39)$$

Compare this with $h_z(\mathbf{a}, z=0)$ to see that the bulk relation between the interaction and h_z [$\epsilon_{\rm int} = \phi_0 h_z(\mathbf{a})/4\pi$ per unit length] does not hold for a film. Numerical evaluation shows that $\epsilon_{\rm int}$ decreases monotonically with the distance $|\mathbf{a}|$ irrespective of the $\mathbf{a}$ direction.

According to Eq. (36), the field at distances $r \gg \lambda_{\rm eff}$ is nearly isotropic. This is true for the interaction (39) as well; thus in the limit $B \ll \phi_0/\lambda_{\rm eff}^2$ the vortex lattice is expected to be made of equilateral triangles. However, unlike the true isotropic case, the degeneracy with respect to rotations of the lattice as a whole is removed: The plane $(\mathbf{B},\mathbf{c})$ should be a symmetry element.

In the opposite limit, $B \gg \phi_0/\lambda_{\rm eff}^2$ (which covers practically all fields since $\phi_0/\lambda_{\rm eff}^2 \to 0$ for $d \to 0$), the lattice interaction energy per unit area is given by

$$\mathcal{F}_{\rm int} = \frac{B^2}{8\pi\lambda_{\rm eff}} \sum{}' \frac{1}{m_a G_x^2 + m_{xx} G_y^2}\,, \tag{40}$$

where $\sum'$ is extended over all nonzero reciprocal-lattice vectors $\mathbf{G}$. To find the equilibrium lattice one can use the procedure employed for the bulk case (see, e.g., Ref. 19); we obtain that in a film the lattice is made of isosceles triangles with a side-to-base ratio

$$\rho_f = (1 + 3m_{xx}/m_a)^{1/2}/2\,. \tag{41}$$

In the bulk this ratio is given by[19]

$$\rho_b = (1 + 3m_c/m_{zz})^{1/2}/2\,. \tag{42}$$

Although $\rho_f = \rho_b$ at $\theta = 0$ or $\pi/2$, the difference between ρ_f and ρ_b can be substantial; e.g., for $\theta = 45°$ and for the anisotropy parameter $\gamma^2 = m_c/m_a \gg 1$, $(\rho_f/\rho_b)^2 \approx 3\gamma^2/14$; i.e., the lattice in a film is "squeezed" more than in the bulk. In other words, the lattice should undergo transformation if the slab thickness d increases. This feature could be seen if d exceeds somewhat the zero-temperature bulk penetration depth $\lambda(0)$; then, reducing the temperature T one can go from the film situation with $d < \lambda(T)$ to that of the bulk. Thus, the equilibrium vortex lattice structure should depend on T.

In conclusion, we have shown that the superconductor surface affects the vortex field in a nontrivial manner. Examples given in Figs. 2 and 3 show that in general there is no simple way to extract characteristics of the vortex field in the bulk and the material anisotropy parameters looking at the outside field distribution. On the other hand, direct examination of this distribution provides information on the orientation of vortices immediately under the surface, because the distribution is qualitatively different for vortices exiting the sample being tilted or perpendicular to the surface. Also, we found that the structure of flux-line lattices in anisotropic films is different from that in the bulk; it depends on the film thickness via parameter $\lambda(T)/d$.

Noted added in proof. Our formal approach differs from that of Brandt:[20] we do not isolate the contribution of the "vortex image" which helps little even in the isotropic situation. Effects of the superconductor surface upon the vortex core have recently been considered by Fritz *et al.* within the Ginzburg-Landau approach.[21]

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