Hubbard model: Field theory and critical phenomena

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(Received 19 November 1992)

In this work we use field-theoretic and renormalization-group methods to study the finite-temperature ferromagnetic phase transition in the three-dimensional Hubbard model. We show that the nature of the ferromagnetic transition may strongly depend on the constraint imposed on the system. For example, if the system is allowed to exchange particles with a reservoir, the stability criterion for the occurrence of a continuous phase transition can be calculated using the mean-field approximation. On the other hand, if the total charge is conserved and charge- and spin-density fluctuations are considered in the theory, a different criterion, quite distinct from the mean-field one, is found, resulting in a fluctuation-induced renormalized Heisenberg tricritical point. The tricritical behavior is studied by calculating the one-loop renormalized free energy and using scaling analysis to determine the tricritical exponents. Finally, the exact solution of the model in the spherical limit and near the ferromagnetic transition is presented, in which case no fluctuation-induced tricritical point is found.

I. INTRODUCTION

The Hubbard model[1] has been the prototype model used to describe a variety of Fermi systems such as normal-liquid ^{3}He, low-dimensional conductors, magnetism of itinerant electrons, and high-T_c superconductivity.[2]

In this work we focus our attention on a long-standing problem related to the one-band Hubbard model, namely the existence and nature of a finite-temperature ferromagnetic phase transition in three dimensions. In one[3] and two[4] dimensions the continuous spin symmetry of the Hubbard model cannot be broken and thus no ferromagnetism is found. Numerical simulations[5] in three-dimensional cubic lattices with first-neighbor hopping have been restricted to the half-filled band case where ferromagnetism is also not expected.[6] Ferromagnetism has been found only away from half filling,[7,8] in which case the so-called Nagaoka theorem[9] asserts that, for appropriate values of the model parameters, the ground state on square and cubic lattices with one hole in a half-filled band is a fully aligned ferromagnetic state in the infinite U (intra-atomic Coulomb repulsion) limit. Recently, extensions of Nagaoka's theorem to finite values of hole density,[10] Coulomb repulsion,[11] and temperature[12] have been reported. Moreover, despite the great effort[13] to overcome the drawbacks of the Hartree-Fock-Stoner theory of itinerant electron ferromagnetism, a satisfactory description of the pertinent critical phenomena in three dimensions is still not available. In particular, early renormalization-group studies[14] concluded, in agreement with mean-field arguments, that the spin-charge coupling in any case restricts the possibility of occurrence of first-order transition. Contrary to this assertion, in this work we show that the coupling of charge- and spin-density fluctuations in the three-dimensional one-band Hubbard model, with *total charge fixed,* severely restricts the occurrence of a continuous ferromagnetic transition.

The question of the nature of a ferromagnetic transition in the Hubbard model is in fact related to a more general class of problem, namely, the occurrence of a phase transition under a constraint imposed on some "hidden" (noncritical) variable coupled to the critical (magnetic) degrees of freedom. In this context one refers to Fisher's celebrated paper[15] on the renormalization of critical exponents by hidden variables in which an interesting model of a two-dimensional "mobile-electron Ising ferromagnet" is exactly solved. He proved that, if the chemical potential is chosen to ensure overall electroneutrality (half-filled band case), the Onsager-Ising (I) critical exponents get renormalized (R), i.e., $\alpha_R = -\alpha_I(1-\alpha_I)^{-1}, \nu_R = \nu_I(1-\alpha_I)^{-1}, \ldots$. Fisher's theory of critical phenomena in constrained systems was later generalized[16,17] to include the possibility of first-order phase transitions. In particular, it was found[14] that if $\alpha < 0$, the renormalized behavior occurs only at a single point, namely, the tricritical point, in contrast to the case of a divergent specific heat $(\alpha > 0)$[16] in which the λ-line exponents are renormalized and the tricritical exponents are those of the unconstrained system.

For the constrained three-dimensional Hubbard model, that is, total charge held fixed, we find a fluctuation-induced tricritical point, whose nature is precisely that proposed by Dohm,[17] and identify the region of fluctuation-induced first-order transition in the space of parameters of the Hubbard Hamiltonian. The tricritical behavior is studied by calculating the one-loop renormalized free energy using field-theoretic and renormalization-group techniques. Moreover, a tricritical scaling analysis permits to obtain the tricritical exponents which are directly related to the fixed-point critical exponents (renormalized Heisenberg in the present

0163-1829/93/48(6)/3755(15)/$06.00

case), with surprisingly interesting results.[18] It is also shown that if microscopic *and* macroscopic charge fluctuations are allowed on the same footing, i.e., if the system is in contact with a reservoir of particles defined by the chemical potential μ (unconstrained case), the transition remains continuous in the stability region suggested by the mean-field theory. Finally, the exact solution of the model in the spherical limit ($n \to \infty$, where n is the number of spin components) is presented, for which the mean-field region of stability is preserved both in the unconstrained and constrained cases.

II. FIELD-THEORETICAL REPRESENTATION OF THE MODEL AND MEAN-FIELD SOLUTION

The use of functional integral methods to treat both the Hubbard and Anderson models has been hindered by many technical and conceptual difficulties, but recent results[19] have elucidated the main points of controversy. The common feature in the systems of interest, which hopefully is mimicked by the single-band Hubbard model, is a competition between a hopping term (kinetic energy) and a local Coulomb repulsion interaction, i.e.,

$$H = \sum_{i,j,\sigma} t_{ij} c^{\dagger}_{i\sigma} c_{j\sigma} + U \sum_{i} n_{i\uparrow} n_{i\downarrow} , \tag{1}$$

where t_{ij} is the hopping integral between sites i and j, U is the Coulomb coupling strength, $c_{i\sigma}$ ($c^{\dagger}_{i\sigma}$) is the annihilation (creation) operator for a fermion of spin σ at site i, and $n_{i\sigma}$ is the fermion number operator. The following general identity[19] explicitly represents the Hubbard-Coulomb interaction in terms of squares of spin and charge operators

$$n_{i\uparrow} n_{i\downarrow} = \tfrac{1}{2}(1-b_c) n_i - \sum_{\alpha=c}^{x,y,z} b_\alpha (S_i^\alpha)^2 , \tag{2a}$$

$$n_i = n_{i\uparrow} + n_{i\downarrow}, \quad S_i^\alpha = \sum_{\sigma,\sigma'} c^{\dagger}_{i\sigma} S^{\alpha}_{\sigma,\sigma'} c_{i\sigma'} , \tag{2b}$$

where $S^{c}_{\sigma,\sigma'} = (i/2)\delta_{\sigma,\sigma'}$, and $S^{\alpha}_{\sigma,\sigma'}$, with $\alpha = x,y,z$, are the spin-$\frac{1}{2}$ matrix elements; the parameters b_α satisfy the constraint $\sum_\alpha b_\alpha = 2$. Using the above identities and the Stratanovich-Hubbard transformation,[20] the partition function,

$$Z = \mathrm{Tr}\exp[-\beta(H - \mu \hat{N}_e)] , \tag{3}$$

where $\beta = 1/kT$ and $\hat{N}_e$ is the total electron number operator, is written as a functional integral over the Fourier transforms of the fluctuations of the auxiliary fields conjugate to the charge and spin operators

$$Z = Z_0 \int (D\phi_q)(D\mathbf{S}_q)\exp[-\beta F(\phi_q, \mathbf{S}_q)] . \tag{4}$$

In (4), an expansion around the paramagnetic uniform static saddle-point value is made, where

$$\beta F_0 = -\ln Z_0 = -\sum_{\mathbf{k},\sigma} \ln[1+\exp(-\beta\varepsilon_{\mathbf{k}})] - \beta N b_c \bar{n}^2 , \tag{5a}$$

$$\varepsilon_{\mathbf{k}} = \varepsilon^0_{\mathbf{k}} - \mu - \frac{U}{2}(b_c - 1 + \bar{n}) , \tag{5b}$$

$$\beta F\{\phi_q, \mathbf{S}_q\} = \frac{1}{2}\left[\sum_q \left[\frac{1}{2b_c}\phi_q\phi_{-q} + \sum_{\alpha=x}^{y,z}\frac{1}{2b_\alpha} S_q^\alpha S_{-q}^\alpha\right]\right] + \mathrm{Tr}_{(k,\sigma)} \ln(1 - \delta\underline{V}\,\underline{G}^0) , \tag{5c}$$

$\varepsilon^0_{\mathbf{k}}$ are the band energies and $\bar{n} = N_e/N$ is the average on-site charge per particle (N_e may or may not be conserved according to the possibility of exchanging particles with a reservoir); $(2b_c)^{-1/2}\phi_q$ and $(2b_\alpha)^{-1/2}S^\alpha_q$ are the Fourier transforms of the fluctuations [around the saddle-point values $\bar{\phi}_{q=0} = i(\beta b_c N U/2)^{1/2}\bar{n}$ and $\mathbf{S}_{q=0} = 0$] of the auxiliary fields (commuting variables) conjugate to the charge S_i^c and spin $\mathbf{S}_i$ operators, $q \equiv (\mathbf{q}, \omega_\nu)[k \equiv (\mathbf{k}, \omega_n)]$ are wave vectors and boson (fermion) Matsubara frequencies; $\delta\underline{V}$ and $\underline{G}^0$ are defined by the matrix elements

$$\delta V^{\sigma,\sigma'}_{k,k'} = -(\beta U/N)^{1/2}(\phi_{k-k'=q} S^c_{\sigma,\sigma'} + \mathbf{S}_{k-k'=q} \cdot \mathbf{S}_{\sigma,\sigma'}) , \tag{6a}$$

$$G^{0\sigma,\sigma'}_{k,k'} = G^0_k \delta_{k,k'} \delta_{\sigma,\sigma'}, \; G^0_k = (i\omega_n - \beta\varepsilon_{\mathbf{k}})^{-1} . \tag{6b}$$

The unique choice $2b_c = 2b_\alpha = 1$ results in a diagrammatic many-body series derived from (4) that has no spurious diagrams which either violate Pauli's principle (if $b_c \neq b_z$) or spin conservation (if $b_x \neq b_y$).[19] In addition, by performing the trace in Eq. (5c) we verify that $F(\phi_q, S_q)$ is spin-rotational invariant to all orders of perturbation theory and represents a field theory involving a critical vector spin field coupled to a noncritical scalar charge field. The interaction vertices of the theory are of type $r = r_c + r_{\mathrm{nc}}$, where r_c (r_{nc}) is the number of critical fields (noncritical fields) present in the interaction.

We are now concerned with the infrared divergencies of the theory as one approaches the transition. Therefore, to generalize the power counting[21] of a divergent graph constructed using both the critical field $\mathbf{S}$ and the noncritical field ϕ, one must keep in mind that the noncritical propagators of internal lines do not contribute to the power counting. In this way, a diagram of order $n = \sum_r n_r$ of a vertex function [see Eq. (20)] with E external lines, L loops, and I internal lines constructed using n_r momentum-independent interactions of type r, behaves asymptotically as Λ^δ, where Λ is the infrared momentum cutoff and δ is given by

$$\delta = Ld - 2I = -\sum_r n_r \delta_r + [d + E_c - (d/2)E] . \tag{7}$$

In (7), E_c (E_{nc}) is the number of critical (noncritical) external lines ($E = E_c + E_{\mathrm{nc}}$), d is the system dimensionality, and

$$\delta_r = d + r_c - (d/2) r \tag{8}$$

is the dimension of the coupling constant of the interaction r. The critical dimension, obtained from $\delta_r = 0$ and

$r=r_c=4$, is $d_c=4$, as in a ϕ^4 theory.[21] At $d=d_c$, the relevant couplings have positive dimensionality $\delta_r>0$. Conversely, the irrelevant ones have $\delta_r<0$ and need not be considered in the determination of the asymptotic critical behavior of the system. Since the momentum dependence of the interaction softens the infrared behavior, the coupling of the relevant interaction of type r, proportional to the generalized paramagnetic susceptibility[22,19]

$$\chi_0(q_1,q_2,\ldots,q_{r-1})=-(\beta/N)\sum_k G^0(k+q_1)G^0(k+q_2)\cdots G^0\left[k+\sum_{i=1}^{r-1}q_i\right], \tag{9}$$

is evaluated at zero momenta, $q_1=q_2=\cdots=q_{r-1}=0$. Finally, assuming that the ferromagnetic critical temperature satisfies $0\ll kT_c\ll E_f$, so that quantum effects[23] are negligible (the frequency mode of interest is the zero-frequency mode), the functional $F[(\phi_{\mathbf{q}},S_{\mathbf{q}})]$ is cast in the form[24]

$$\begin{aligned}\beta F(\phi_{\mathbf{q}},\mathbf{S}_{\mathbf{q}})=&\tfrac{1}{2}\sum_{\mathbf{q}}(r_s+q^2)\mathbf{S}_{\mathbf{q}}\cdot\mathbf{S}_{-\mathbf{q}}+\lambda_s\\&\times\sum_{\mathbf{q}_1,\mathbf{q}_2,\mathbf{q}_3}\mathbf{S}_{q_1}\cdot\mathbf{S}_{q_2}\mathbf{S}_{q_3}\cdot\mathbf{S}_{-(\mathbf{q}_1+\mathbf{q}_2+\mathbf{q}_3)}\\&+\frac{r_c}{2}\sum_{\mathbf{q}}\phi_{\mathbf{q}}\phi_{-\mathbf{q}}+\lambda_{sc}\sum_{\mathbf{q},\mathbf{k}}\mathbf{S}_{\mathbf{k}}\cdot\mathbf{S}_{\mathbf{q}-\mathbf{k}}\phi_{-\mathbf{q}},\end{aligned} \tag{10a}$$

$$r_s=[1-(U/2)N_T(E_f)],\quad r_c=[1+(U/2)N_T(E_f)], \tag{10b}$$

$$\begin{aligned}\lambda_s&=-\left[\frac{U^2}{192\beta N}\right]N_T''(E_f),\\ \lambda_{sc}&=-\frac{i}{8}\left[\frac{U^3}{\beta N}\right]^{1/2}N_T'(E_f),\end{aligned} \tag{10c}$$

where $N_T(E_f)$ is the electronic density of states near the Fermi level averaged over an energy range of order $kT(\ll E_f)$ through the factor $(\partial f/\partial E)$, $f(E)$ is the Fermi distribution function, and prime means differentiation. Note that the couplings λ_s and λ_{sc} depend on both the Hubbard-Coulomb interaction and the derivatives (shape) of the density of states at the Fermi level. In particular, λ_{sc} is pure imaginary and brings new features in comparison with formally similar problems such as the metamagnet[25] and the axial next-nearest-neighbor Ising (ANNI)[26] model in a field and the interaction of spin with elastic degrees of freedom.[27] Note also that expansion (10) is strictly valid only if $N_T(E)$ is a *smooth* function of E near E_f, i.e., E_f cannot be placed at a value of E for which $N_T(E_f)$ develops a Van Hove singularity.

The partition function (3) is fully defined only when the appropriated boundary condition or constraint is specified.[28,29] For example, if the total charge of the system is held fixed, the total charge fluctuation vanishes, i.e.,

$$\phi=\phi_{\mathbf{q}=0}=\int dx\,\phi(x)=0\ . \tag{11}$$

In this case the spin fluctuations are *not* coupled to the macroscopic (uniform) mode of charge fluctuations and the integration in Eq. (4) must be evaluated under the delta constraint $\delta(\phi_{\mathbf{q}=0})$. On the other hand, if the system exchanges particles with a reservoir defined by the chemical potential μ, both macroscopic *and* microscopic $(\mathbf{q}\neq0)$ charge fluctuations are allowed with no restriction, interacting with the spin fluctuations according to Eq. (10). To comprise the constraints of interest, we write the interaction term of Eq. (10) in the form[28]

$$\lambda_{sc}\sum_{\mathbf{q}}\mathbf{S}_{\mathbf{q}}^2\phi_{-\mathbf{q}}\rightarrow\lambda_{sc}^{(0)}\mathbf{S}_0^2\phi_0+\lambda_{sc}^{(1)}\sum_{\mathbf{q}\neq0}\mathbf{S}_{\mathbf{q}}^2\phi_{-\mathbf{q}}, \tag{12}$$

where the $\mathbf{q}=0$ mode is singled out and

$$\mathbf{S}_{\mathbf{q}}^2=\sum_{\mathbf{k}}\mathbf{S}_{\mathbf{k}}\cdot\mathbf{S}_{\mathbf{q}-\mathbf{k}}\ . \tag{13}$$

This prescription allows the two boundary conditions to manifest through solutions satisfying either $\lambda_{sc}^{(0)}=\lambda_{sc}^{(1)}$, if the total charge fluctuates, or $\lambda_{sc}^{(0)}=0$, if the total charge is held fixed. Now after integrating over the charge field in Eq. (4), using Eqs. (10)–(13), we obtain a generalized $0(n)$ symmetric ϕ^4 theory as follows:

$$\begin{aligned}\beta F\{\mathbf{S}_{\mathbf{q}}\}=&\tfrac{1}{2}\sum_{\mathbf{q}}(r_s+q^2)\mathbf{S}_{\mathbf{q}}.\mathbf{S}_{-\mathbf{q}}\\&+\lambda_s\sum_{\mathbf{q}_1,\mathbf{q}_2,\mathbf{q}_3}\mathbf{S}_{\mathbf{q}_1}\cdot\mathbf{S}_{\mathbf{q}_2}\mathbf{S}_{\mathbf{q}_3}\cdot\mathbf{S}_{-(\mathbf{q}_1+\mathbf{q}_2+\mathbf{q}_3)}\\&-\lambda_c^{(0)}\mathbf{S}_0^4-\lambda_c^{(1)}\sum_{\mathbf{q}\neq0}\mathbf{S}_{\mathbf{q}}^2\mathbf{S}_{-\mathbf{q}}^2,\end{aligned} \tag{14a}$$

$$\lambda_c^{(1)}=(\lambda_{sc}^{(1)})^2/2r_c,\quad\lambda_c^{(0)}=(\lambda_{sc}^{(0)})^2/2r_c\ . \tag{14b}$$

The free-energy functional (14a) can be formally made equivalent to a simpler one, containing only two coupling constants, against the substitution $-\lambda_c'^{(0)}=-\lambda_c^{(0)}+\lambda_s$, $\lambda_c'^{(1)}=-\lambda_c^{(1)}+\lambda_s$, and $\lambda_s'=0$. However, we keep this extended version because it allows us to obtain the two different solutions, corresponding to the boundary conditions mentioned above, within the same formalism. Moreover, this procedure also enables us to explicitly calculate charge susceptibilities, providing essential physical insights concerning the fixed-point solutions and topology of the flux diagrams (see next section).

A mean-field solution of this problem gives the lines of instability, $\lambda_s-\lambda_c^{(0)}=0$, and $\lambda_c^{(0)}=0$ (for the Hubbard model $\lambda_c^{(0)},\lambda^{(1)}\leq0$), dividing the first- and second-order phase transition sectors in the (λ_s,λ_c) plane (see Fig. 1). Recall that only the $\mathbf{q}=0$ mode contributes in the mean-field approximation. At Stoner-type criticality $r_s=0$, and using Eqs. (10b), (10c), and (14b), the above lines of instability give the following criterion for the system to undergo a continuous ferromagnetic transition, if the total charge fluctuates $(\lambda_c^{(0)}\neq0)$:

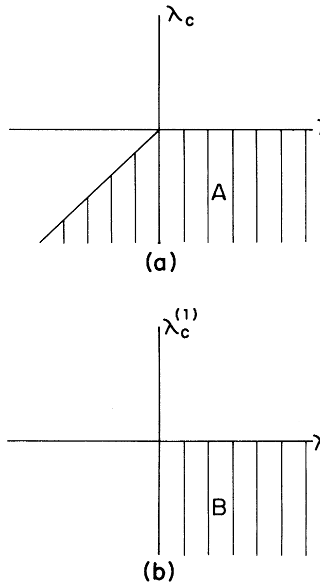

FIG. 1. Mean-field regions of stability: (a) region A, for constant chemical potential, defined by lines $\lambda_s-\lambda_c=0$ and $\lambda_c=\lambda_c^{(0)}=0$; (b) region B, for total charge fixed ($\lambda_c^{(0)}=0$), defined by lines $\lambda_s=0$ and $\lambda_c^{(1)}=0$. Though redundant in the mean-field approximation, we plot $\lambda_c^{(1)}$ in the last case in order to contrast with results when fluctuations are considered in the theory (Fig. 3).

$$\frac{3N_T'(E_f)^2}{2N_T(E_f)}>N_T''(E_f)\ , \tag{15a}$$

whereas a first-order transition occurs for the reverse inequality. On the other hand, in the case of total charge fixed ($\lambda_c^{(0)}=0$), the above condition reduces to

$$N_T''(E_f)<0\ . \tag{15b}$$

We see from Eqs. (15a) and (15b) that, even at the mean-field level, the two boundary conditions lead to distinct criteria for the occurrence of a continuous ferromagnetic transition. In the next section we show that in the constrained case (total charge held fixed), if fluctuations are allowed in the theory, the above mean-field picture is drastically modified, resulting in a different criterion for the occurrence of a continuous ferromagnetic transition.

III. FIXED POINTS AND CRITICAL EXPONENTS

Our main goal in this section is to determine the fixed points and critical exponents associated with a ferromagnetic transition described by Eq. (14), as well as the conditions under which this transition is reached. In order to formulate the problem in the field-theoretic framework, we start this section by defining the relevant quantities necessary to establish the renormalization-group program.

The functional generator for the Green's functions is given by

$$Z(T,\mathbf{h};t)=Z_0\int D(S_{\mathbf{q}})\exp\left[-\beta F(\mathbf{S}_{\mathbf{q}})+\sum_{\mathbf{q},\alpha}h_{\mathbf{q}}^{\alpha}S_{-\mathbf{q}}^{\alpha}+\tfrac{1}{2}\sum_{\mathbf{q},\alpha}t_{\mathbf{q}}^{\alpha}(S_{-\mathbf{q}}^{\alpha})^2\right] \tag{16}$$

and

$$-\beta F(T,\mathbf{h};t)=\ln Z(T,\mathbf{h};t) \tag{17}$$

is the generator for the connected Green's functions; the index α runs over the spin field components. By performing a Legendre transformation with respect to the expectation value;

$$\langle \mathbf{S}_{\mathbf{q}}\rangle\equiv\mathbf{M}_{\mathbf{q}}=-\beta\frac{\delta F}{\delta\mathbf{h}_{-\mathbf{q}}}(T,\mathbf{h};t)\ , \tag{18}$$

we obtain the free-energy functional,

$$\beta F(T,\mathbf{M};t)=\beta F(T,\mathbf{h};t)+\sum_{\mathbf{q}}\mathbf{h}_{\mathbf{q}}\cdot\mathbf{M}_{-\mathbf{q}}\ , \tag{19}$$

which generates the one-particle irreducible part of the connected Green's functions, i.e., the vertex functions

$$\Gamma^{(N,L)}_{\alpha_1\cdots\alpha_N,\beta_1\cdots\beta_L}[T;(\mathbf{k}_i),(\mathbf{p}_i);(\lambda_i),\Lambda]=\beta\left.\frac{\delta F^{N+L}(T,\mathbf{M};t)}{\delta M^{\alpha_1}_{-\mathbf{k}_1}\cdots\delta M^{\alpha_N}_{-\mathbf{k}_N}\delta t^{\beta_1}_{-\mathbf{p}_1}\cdots\delta t^{\beta_L}_{-\mathbf{p}_L}}\right|_{\substack{t=0\\ \mathbf{M}=0}}\ , \tag{20}$$

where (λ_i) denotes the bare coupling constants of the theory [for convenience we replace $\lambda_i\to\lambda_i/4!$ in Eq. (14)], the Brillouin zone is made spherical with cutoff Λ and the magnetization $\mathbf{M}$ of equilibrium is zero because Z in the preceding section is obtained by expanding around the paramagnetic solution.

In order to check the implications of the constraints on the spin-charge coupling, one can introduce in the free-energy functional, a field conjugate to the charge-density fluctuations, through the interacting term $\sum_{\mathbf{q}}\phi_{\mathbf{q}}\varepsilon_{-\mathbf{q}}$. In this way, one obtains

$$\langle\phi_{\mathbf{q}}\rangle=-\beta\left.\frac{\delta F}{\delta\varepsilon_{-\mathbf{q}}(T,\mathbf{M};\varepsilon)}\right|_{\substack{\mathbf{M}=0\\ \varepsilon=0}}=-\frac{\lambda_{sc}}{r_c}\langle S_{\mathbf{q}}^2\rangle\ , \tag{21}$$

where $\lambda_{sc}=\lambda_{sc}^{(0)}\delta_{\mathbf{q},0}+\lambda_s^{(1)}(1-\delta_{\mathbf{q},0})$. It is then clear that if the total charge is held constant, i.e., $\langle\phi_0\rangle=0$, and, as magnetic energy fluctuations $\langle S_0^2\rangle\neq 0$, one must require, as a necessary condition, that $\lambda_{sc}^{(0)}=0$.

The renormalization-group program used here is a generalized massless $0(n)$ symmetric ϕ^4 theory[21] in which the two new quartic couplings with restricted momentum conservation, appearing in Eq. (14), reflect the possibility of imposing a constraint on the total charge of the system. At the critical point $T=T_c$, the renormalized vertex functions are defined (using a shorthand notation) by

$$\Gamma_R^{(N,L)}[(g_i),\kappa]=\lim_{\Lambda\to\infty}Z_{\mathbf{S}}^{N/2}Z_{\mathbf{S}^2}^{L}\Gamma^{(N,L)}[(\lambda_i),\Lambda]\ , \tag{22}$$

where $Z_{\mathbf{S}}$ and $Z_{\mathbf{S}^2}$ are the renormalization functions, (g_i) denotes the renormalized coupling constants, and κ defines the scale of the momenta at which the following normalization conditions are fixed.

$$\Gamma_R^{(2)}(k=0)=0\ , \tag{23a}$$

$$\frac{\partial}{\partial k^2}\Gamma_{R_{\alpha,\beta}}(k)\Bigg|_{k^2=\kappa^2}=\delta_{\alpha,\beta}\ , \tag{23b}$$

$$\Gamma_R^{(2,1)}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{p})\Bigg|_{\mathrm{SP}}=1\ , \tag{23c}$$

and

$$\Gamma^{(4)}_{R_{\alpha_1}\cdots\alpha_4}(\mathbf{k}_1,\ldots,\mathbf{k}_4)\Bigg|_{\mathrm{SP}}=\kappa^{\varepsilon}(u_s\underline{F}+u_c^{(0)}\underline{P}+u_c^{(1)}\underline{T})\ . \tag{23d}$$

In (23) the momenta are chosen at a symmetry point (SP), which for $\Gamma^{(2,1)}$ implies $p^2=(\mathbf{k}_1+\mathbf{k}_2)^2=\kappa^2$ and for $\Gamma^{(4)}$ $\mathbf{k}_i\cdot\mathbf{k}_j=(\kappa^2/4)(4\delta_{i,j}-1)$; the tensors $\underline{F}$, $\underline{P}$, and $\underline{T}$ can be written as

$$\underline{F}=\tfrac{1}{3}(\delta_{\alpha_1,\alpha_2}\delta_{\alpha_3,\alpha_4}+\delta_{\alpha_1,\alpha_3}\delta_{\alpha_2,\alpha_4}+\delta_{\alpha_1,\alpha_4}\delta_{\alpha_2,\alpha_3})\delta_{\sum_i \mathbf{q}_i,0}\ , \tag{24a}$$

$$\underline{P}=\tfrac{1}{3}(\delta_{\alpha_1,\alpha_2}\delta_{\alpha_3,\alpha_4}\delta_{\mathbf{q}_1+\mathbf{q}_2,0}\delta_{\mathbf{q}_3+\mathbf{q}_4,0}+\text{permutations})\ , \tag{24b}$$

$$\underline{T}=\tfrac{1}{3}\left[\delta_{\alpha_1,\alpha_2}\delta_{\alpha_3,\alpha_4}\left[\delta_{\sum_i \mathbf{q}_i,0}-\delta_{\mathbf{q}_1+\mathbf{q}_2,0}\delta_{\mathbf{q}_3+\mathbf{q}_4,0}\right]+\text{permutations}\right]\ . \tag{24c}$$

The above prescriptions insure that the theory is suitable to treat the asymptotic critical behavior of the system since the cutoff Λ (microscopic detail) is made unimportant and the vertex functions developing infrared singularities acquire masses through the normalization conditions; the renormalization functions Z_{S} and Z_{S^2} and the bare coupling constants $\lambda_s,\lambda_c^{(0)}$, and $\lambda_c^{(1)}$ in Eqs. (22) are expanded in powers of the renormalized coupling constants $g_s,g_c^{(0)}$, and $g_c^{(1)}$ (made dimensionless through $g_i\to\kappa^{\varepsilon}g_i$) with $\varepsilon(=4-d)$-dependent coefficients in such a way that the divergences (both infrared and ultraviolet) are removed order by order in perturbation theory.

In the one-loop approximation we take the following working conditions:

$$\lambda_i=g_i+\sum_{j,j'}a_{jj'}g_jg_{j'},\quad i,j=s,c^{(0)},c^{(1)}\ , \tag{25}$$

$$Z_{\mathrm{S}}=1\ , \tag{26}$$

$$\bar{Z}_{\mathrm{S}^2}=Z_{\mathrm{S}}Z_{\mathrm{S}_2}=1+\sum_i b_ig_i\ . \tag{27}$$

Using now the bare vertex functions listed in Appendix A (see Fig. 2) and the normalization conditions, Eqs. (22) and (23), we obtain

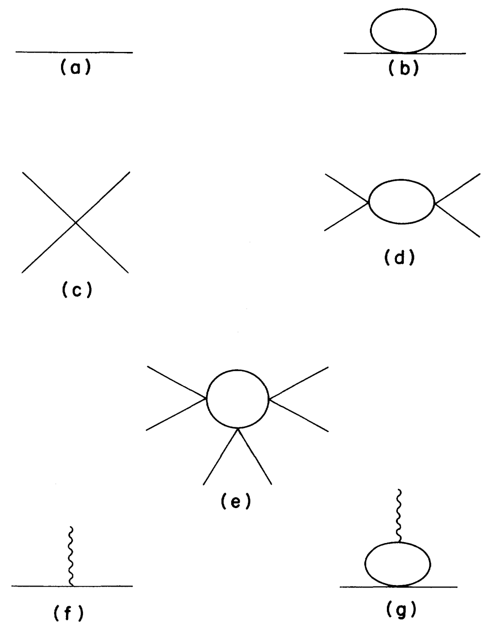

FIG. 2. Diagrams, up to one loop, contributing to the vertex functions $\Gamma^{(2,0)}$ [(a) and (b)], $\Gamma^{(4,0)}$ [(c) and (d)], $\Gamma^{(6,0)}$ (e), and $\Gamma^{(2,1)}$ [(f) and (g)].

$$\lambda_s=g_s+I_{\mathrm{SP}}\left[\frac{(n+8)}{6}g_s^2-2g_sg_c^{(1)}+\frac{2}{3}g_c^{(1)^2}\right]\ , \tag{28a}$$

$$\lambda_c^{(0)}=g_c^{(0)}-I_{\mathrm{SP}}\left[\frac{n}{6}g_c^{(0)^2}-\frac{(n+2)}{3}g_sg_c^{(0)}+\frac{2}{3}g_c^{(1)}g_c^{(0)}\right]\ , \tag{28b}$$

$$\lambda_c^{(1)}=g_c^{(1)}-I_{\mathrm{SP}}\left[\frac{(n+4)}{6}g_c^{(1)^2}-\frac{(n+2)}{3}g_sg_c^{(1)}\right]\ , \tag{28c}$$

and

$$\bar{Z}_{\mathrm{S}^2}=1+I_{\mathrm{SP}}\left[\frac{(n+2)}{6}g_s-\frac{n}{6}g_c^{(0)}-\frac{1}{3}g_c^{(1)}\right]\ , \tag{29}$$

with

$$I_{\mathrm{SP}}=\frac{\kappa^{\varepsilon}}{S_d}J_{\mathrm{SP}}[r_s(T_c)=0,\mathbf{k}_i+\mathbf{k}_j,\Lambda\to\infty]$$
$$=\frac{1}{\varepsilon}\left[1+\frac{\varepsilon}{2}\right]+O(\varepsilon)\ , \tag{30}$$

where the geometrical factor S_D is absorbed in the coupling constants and dimensional regularization is used to evaluate the integral.

The fact that the underlying physical theory is invariant under different renormalization conditions [choice of κ in Eq. (23) or length scale as the cutoff $\Lambda\to\infty$] implies the renormalization-group equation,

$$\left[\kappa\frac{\partial}{\partial\kappa}+\sum_i\beta_i\frac{\partial}{\partial g_i}-\frac{N}{2}\gamma_{\mathrm{S}}+L\gamma_{\mathrm{S}^2}\right]\Gamma_R^{(N,L)}=0\ , \tag{31}$$

where

$$\beta_i(g_s,g_c^{(0)},g_c^{(1)})=\kappa\frac{\partial g_i}{\partial\kappa}\bigg|_{\lambda_s,\lambda_c^{(0)},\lambda_c^{(1)}}, \tag{32}$$

$$\gamma_S[(g_i)]=\kappa\left[\frac{\partial}{\partial\kappa}\ln Z_S\right]\bigg|_{\lambda_s,\lambda_c^{(0)},\lambda_c^{(1)}}, \tag{33}$$

and

$$\gamma_{S^2}[(g_i)]=-\kappa\left[\frac{\partial}{\partial\kappa}\ln Z_S^2\right]\bigg|_{\lambda_s,\lambda_s^{(0)},\lambda_c^{(1)}}. \tag{34}$$

The fixed points of the theory are solutions of the linear set of equations, $\beta_i(g_s^*,g_c^{(0)*},g_c^{(1)*})=0$, and the stability of these solutions is determined by the eigenvalues of the matrix $B_{ij}=\partial\beta_i/\partial g_j^*$. By using $\kappa\partial\lambda_i/\partial\kappa=-\varepsilon\lambda_i$ and $\kappa\partial\lambda_i^2/\partial\kappa=-2\varepsilon\lambda_i^2$ (recall that λ_i was made dimensionless by replacing $\lambda_i\to\kappa^{-\varepsilon}\lambda_i$) and inverting the linear set of equations (28) to obtain $g_i=g_i(\lambda_s,\lambda_s^{(1)},\lambda_s^{(0)})$, we find

$$\beta_s=-\varepsilon g_s+\frac{(n+8)}{6}g_s^2-2g_sg_c^{(1)}+\frac{2}{3}g_c^{(1)^2}, \tag{35a}$$

$$\beta_c^{(0)}=-\varepsilon g_c^{(0)}-\frac{n}{6}g_c^{(0)^2}+\frac{(n+2)}{3}g_sg_c^{(0)}-\frac{2}{3}g_c^{(1)}g_c^{(0)}, \tag{35b}$$

$$\beta_c^{(1)}=-\varepsilon g_c^{(1)}-\frac{(n+4)}{6}g_c^{(1)^2}+\frac{(n+2)}{3}g_sg_c^{(1)}, \tag{35c}$$

from which the matrix B_{ij} is easily calculated. Note that $g_c^{(0)}$ does not contribute to either the renormalization of g_s or $g_c^{(1)}$ (see Appendix A).

Exploiting the solution of Eq. (31) for $\Gamma_R^{(2,0)}$ one concludes[21] that the exponents η and ν are determined by the values of the renormalization functions at their respective fixed points

$$\eta=\gamma_s[(g_i^*)] \tag{36}$$

and

$$2-\nu^{-1}+\eta=\bar\gamma_{S^2}[(g_i^*)], \tag{37}$$

where

$$\bar\gamma_{S^2}[(g_i)]=-\kappa\left[\frac{\partial}{\partial\kappa}\ln\bar Z_{S^2}\right]\bigg|_{\{\lambda_i\}}. \tag{38}$$

Relation (37), in fact, stems from the renormalization-group equation for $\Gamma_R^{(2,0)}$ above T_c in which the last term in Eq. (30) is replaced by $L\gamma_{S^2}\to(L+t\partial/\partial t)\gamma_{S^2}$, where $t=(T-T_c)/T_c$ is the renormalized reduced temperature [it should not be mistaken with t used in Eqs. (16)–(20)].

In spite of the fact the calculation is performed in the one-loop approximation [first order in $\varepsilon=(4-d)$ dimensions], careful inspection of the results, including a second-order similar calculation for a particular case,[30] yields relations between the (FP) values (and eigenvalues) and their critical exponents. Using Eqs. (35)–(38) we display in Fig. 3(a) the flow diagrams for the two boundary conditions of interest. Note that the physical space parameters of the Hubbard model is restricted to $g_c(g_c^{(1)})<0$. At the Heisenberg FP (the most stable one) with eigenvalues $b_1=\varepsilon$ and $b_2=-(\alpha/\nu)$, $N_T''(E_f)<0$ and $N_T'(E_f)=0$, i.e., the Fermi level sits at a maximum of the "renormalized" density of states: magnetic and charge degrees of freedom are decoupled. The interesting feature of this phase diagram though, is the existence of runaway lines attracted to the invariant line, $g_s-g_c^{(1)}=g_H^*$, joining FPs (3) and (6) (g_H^* is the Heisenberg FP value of ϕ^4 coupling constant). This line crosses

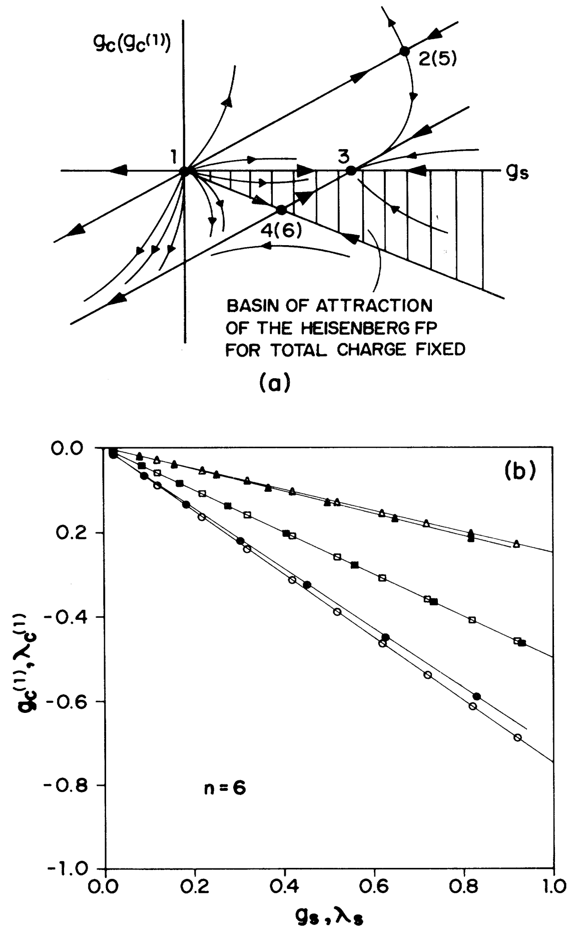

FIG. 3. (a) Flow diagram of the Hubbard model, near a ferromagnetic transition, for constant chemical potential ($g_c^{(0)}=g_c^{(1)}\equiv g_c$) and for total charge fixed ($g_c^{(0)}=0$). The flow lines are identical in both cases. FP1 (FP3) is the trivial Gaussian (Heisenberg) fixed point. For constant chemical potential (fixed charge) FP2 (FP5) with $g_s^*=g_c^*(g_c^{(1)*})$ is a Gaussian (spherical) FP and FP4 (FP6) with $g_s^*=g_H^*+(6\alpha/n\nu)$ and $g_c^*(g_c^{(1)*})=(6\alpha/n\nu)$ is a Heisenberg (renormalized Heisenberg) FP. (b) Circles and black dots indicate associated values of bare and renormalized couplings, in one loop and first order in ε, for which a first-order transition is predicted if the total charge is held fixed. For values indicated by white (bare coupling) and black (renormalized) triangles the system is attracted to the Heisenberg FP3 (continuous transition). Coupling values indicated by white and black squares lie on the same straight line and are attracted to the renormalized Heisenberg FP6 (tricritical behavior). We take $n=6$, so $\alpha<0$ to first order in ε.

the mean-field line of instability $g_s=0$, valid if the total charge is held fixed, and a first-order transition is anticipated. Notice that at the mean-field level (zero-loop order), $g_i \equiv \lambda_i$ [see Eqs. (28)]. Thus, FP(6) with eigenvalues $b_1=\varepsilon$ and $b_2=\alpha/\nu=-\alpha_R/\nu_R$ is a renormalized Heisenberg tricritical point. In this case the basin of attraction of the Heisenberg FP is not defined by the mean-field instability criterion Eq. (15b) but rather by the conditions[31]

$$\frac{g_c^{(1)*}}{g_s^*}\left[\text{or}\left.\frac{\lambda_c^{(1)}}{\lambda_s}\right|_{T_c}\right] > \frac{6\alpha/n\nu}{g_H^*+(6\alpha/n\nu)}+0(\varepsilon)\ , \tag{39a}$$

and

$$g_c^{*(1)}(\lambda_c^{(1)}) \le 0 \text{ (Hubbard model)}\ , \tag{39b}$$

where n is the number of components of the spin field and α and ν are the usual Heisenberg critical exponents. Using Eqs. (10b), (10c), and (14b), conditions (39) can be cast in the form

$$\frac{3N_T'^2(E_f)}{2N_T(E_f)} < \frac{6\alpha/n\nu}{g_H^*+(6\alpha/n\nu)}N_T''(E_f)+0(\varepsilon)\ . \tag{40}$$

Note that the Coulomb coupling U is absent in (40). The condition is utterly defined by the *shape* of the density of states near the Fermi level (local geometrical properties) and by the properties describing the criticality of a Heisenberg system. This severe restriction is a consequence of the coupling between charge- and spin-density fluctuations and is not manifest in the mean-field theory. If the spin-charge coupling is too strong, violating condition (40), ferromagnetic criticality fails to occur and the system might be compelled to undergo a first-order transition.

To further check the instability criterion Eq. (39), we use Eqs. (28)–(28c) to show numerically in Fig. 3(b) that, to first order in ε, the line of instability remains the same both in the bare and in the renormalized coupling spaces; that is, by taking initial values of the bare couplings along a straight line, for which conditions (39) predict a second- (first-) order transition, Eqs. (28a)–(28c) map these points into initial renormalized coupling values, lying in *another* line, for which the renormalized theory predicts also a second- (first-) order transition. The instability line,

$$\left.\frac{\lambda_c^{(1)}}{\lambda_s}\right|_{T_c} = \frac{g_c^{(1)*}}{g_s^*} = \frac{6\alpha/n\nu}{g_H^*+(6\alpha/n\nu)}+0(\varepsilon)\ , \tag{41}$$

is the special line which maps onto itself. The fluctuation-induced tricritical point lies on this line, with coordinates $g_c^*=(6\alpha/n\nu)+0(\varepsilon^2)$ and $g_s^*=g_H^*+(6\alpha/n\nu)+0(\varepsilon^2)$.

IV. TRICRITICAL BEHAVIOR

In the last section we have seen that the presence of the critical runaway lines in the flow diagram of Fig. 3 suggests that FP(6) is a renormalized Heisenberg tricritical point. This aspect deserves further investigation together with the question of the nature of the Heisenberg FP(4). We shall address these problems by calculating the one-loop renormalized free energy, i.e., the effective potential in the field-theoretic language.

The uniform solution of the bare Helmholtz free energy is obtained from Eqs. (19) and (20)

$$\beta F(T,\mathbf{M}) = \sum_{N(\text{even})}\frac{1}{N!}\sum_{\alpha_1\cdots\alpha_N}\Gamma^{(N,0)}_{\alpha_1\cdots\alpha_N}(T;\mathbf{k}_1=0,\ldots,\mathbf{k}_N=0)M_{\alpha_1\cdots\alpha_N}\ , \tag{42}$$

where the vertex functions are calculated at zero external momenta and at a temperature in the vicinity of T_c. Now using the results of Appendix B (see also Fig. 2) the one-loop free energy reads

$$\begin{aligned}\beta F(T,\mathbf{M}) = {}&\frac{1}{2}r_s(T)M^2+\frac{(\lambda_2-\lambda_c^{(0)})}{4!}M^4\\&+\frac{(n-1)}{2}\int dq \ln\left[1+\frac{(\lambda_s-\lambda_c^{(0)})M^2/6}{r_s(T)+q^2}\right]\\&+\frac{1}{2}\int dq\ln\left[1+\frac{(3\lambda_s-\lambda_c^{(0)}-2\lambda_c^{(1)})M^2/6}{r_s(T)+q^2}\right]\ ,\end{aligned} \tag{43}$$

where $M=(\sum_\alpha M_\alpha^2)^{1/2}$. As expected, for $\lambda_c^{(0)}=\lambda_c^{(1)}\equiv\lambda_c$, Eq. (43) reduces to the standard expression for a ϕ^4 theory, with an effective coupling $\lambda_s-\lambda_c$. In the general case, however, the coupling $\lambda_c^{(0)}$ and $\lambda_c^{(1)}$ contribute in a nontrivial way.

To derive the renormalized free energy we replace $r_s(T)$ in Eq. (42) by $r_s(T)=r_s(T_c)+Z_{S^2}t$, where $r_s(T_c)$ is obtained from Eqs. (A1) and (23a) and Z_{S^2} is given by (29); we also eliminate the bare coupling constants using Eq. (27), and recall that since $Z_S=1, M_{\alpha_R}=M_\alpha$. It is easier, however, to first calculate the renormalized equation of state

$$H=\beta\left.\frac{\partial F}{\partial M}\right|_t\ , \tag{44}$$

which is cast in the form

$$H=tM+\frac{(g_s-g_c^{(0)})}{6}M^3+(n-1)K(g_1)+K(g_2)\ , \tag{45a}$$

with

$$K(g)=\tfrac{1}{4}gM(t+gM^2/2)\ln(t+gM^2/2)\ , \tag{45b}$$

$$g_1=(g_s-g_c^{(0)})/3\ , \tag{45c}$$

$$g_2=(3g_s-g_c^{(0)}-2g_c^{(1)})/3\ . \tag{45d}$$

Finally, the renormalized free energy is obtained from

Eq. (45) after integration as follows:

$$\beta F(t,\mathbf{M})-\beta F(t,0)=\frac{1}{2}tM^2+\frac{(g_s-g_c^{(0)})}{4!}M^4 +(n-1)G(g_1)+G(g_2)\ , \quad (46a)$$

where

$$G(g)=\frac{1}{8}\left[t+\frac{g}{2}M^2\right]^2\left[\ln\left[t+\frac{g}{2}M^2\right]-\frac{1}{2}\right]. \quad (46b)$$

Let us first study the case $g_c^{(0)}=g_c^{(1)}\equiv g_c$. It is clear that the invariant line $g_s-g_c=g_H^*$ is parallel to the mean-field line of instability $g_s-g_c=0$, valid when the system is in contact with a reservoir of particles. Notice that at the mean-field (MF) level all renormalized couplings are equal to the bare ones [see Eq. (28)]. Since F in Eq. (46) reduces to the Helmholtz free energy of a ϕ^4 theory with coupling constant g_s-g_c, and in the runaway region $g_s-g_c>0$, no first-order transition is anticipated. In fact, the invariant line $g_s-g_c=g_H^*$ is a critical line with ideal Heisenberg critical exponents and the mean-field criterion [Eq. (15a)] is the correct one to define the region of continuous transitions in the (g_s,g_c) plane. Hence the Heisenberg FP(4) is not a tricritical point as suggested by Achiam.[29] To clarify its meaning one can study the charge-density fluctuation response function[28,14] to the field ε [see Eq. (21)]. Near the ferromagnetic transition, the wave-vector-dependent charge susceptibility is given by

$$\chi_c(\mathbf{q})=r_c^{-1}\left[1+g_c\left[\langle(\mathbf{S})_{\mathbf{q}}^2(\mathbf{S})_{-\mathbf{q}}^2\rangle-\langle(\mathbf{S})_{\mathbf{q}}^2\rangle\langle(\mathbf{S})_{-\mathbf{q}}^2\rangle\right]\right], \quad (47)$$

where $g_c\equiv g_c^{(0)}\delta_{\mathbf{q},0}+g_c^{(1)}(1-\delta_{\mathbf{q},0})$. We see that the line joining the origin to the FP(4) divides the second-order sector into two regions, one in which the lines flow to FP(3) with $g_c^*=0$ and the uniform charge susceptibility is regular, i.e., $\chi_c(0)=r_c^{-1}$. In the second region the lines flow to the invariant line $g_s-g_c=g_H^*$. As the contribution of spin fluctuations to $\chi_c(0)$ is proportional to the specific heat, a weak cusp singularity (≈ -0.1) is expected. Thus FP(4), which does not stand for any new ferromagnetic critical behavior, is in fact the signature of the existence of two disjoint regions in the space of parameters, characterized by distinct charge background responses. This feature could only be made explicit by allowing, even when $g_c^{(0)}=g_c^{(1)}$, fixed-point solutions in which $g_c^{(0)*}=g_c^{(1)*}=0$ or $g_c^{(0)*}=g_c^{(1)*}\neq 0$, both within the same Heisenberg universality class.

We now turn to the case $g_c^{(0)}\neq g_c^{(1)}$. The occurrence of a first-order transition can be examined using Eq. (46) and a prescription devised by Iacobson and Amit.[32] We take a point in the vicinity of the mean-field boundary

$$g_s-g_c^{(0)}=A\varepsilon^2,\quad A>0\ , \quad (48)$$

with

$$g_c^{(0)}-g_c^{(1)}=B\varepsilon,\quad B\geq 0\ . \quad (49)$$

If a first-order transition takes place, a second minimum $(M_1\neq 0)$ is also a solution of the equations

$$(\partial F/\partial M)_{M_1}=0,\quad F(M_1)=F(0)\ . \quad (50)$$

Then, the transition temperature increases, $t_1=b\varepsilon$, $b>0$, and the magnetization displays a finite discontinuity which, in the ε expansion context, is of the form

$$M_1^2=c/\varepsilon,\quad c>0\ . \quad (51)$$

Solving Eq. (46), with the above conditions, yields

$$b=\frac{B^2c}{36},\quad c=\frac{3}{B}\exp\left[-\frac{2A}{B^2}-\frac{1}{3}\right]. \quad (52)$$

It is clear that a first-order transition occurs, i.e., $b,c>0$, except for $B=0$ $(g_c^{(0)}=g_c^{(1)})$ as studied before; the two boundary conditions manifest through different values of the coupling constants. Moreover, one should emphasize that if fluctuations (one-loop contributions) are neglected in Eq. (42), the transition is always continuous, thus characterizing the nature of the first-order transition.

The tricritical behavior governed by the renormalized Heisenberg FP(6) can now be investigated using the scaling[33,16,17] form of the Gibbs free energy,

$$G(t,h,g)\approx|g|^{d\nu_R/\phi}f\left[\frac{t}{|g|^{1/\phi}},\frac{h}{|h|^{\Delta_R/\phi}}\right], \quad (53)$$

where g is the relevant scaling field (eigenvector) along the invariant line $g_s-g_c^{(1)}=g_H^*$, with eigenvalue $b_2=-\alpha_R/\nu_R$, $\Delta_R=\Delta(1-\alpha)^{-1}$, $\phi=-b_2\nu_R=\alpha_R$ is the crossover exponent, and h is the reduced magnetic field. $\phi=\alpha_R$[16,17] distinguishes this type of tricritical point from the so-called[29] ordinary tricritical points, where classical exponents with logarithmic corrections hold in three dimensions. Similar analysis can be made for Ising and renormalized XY systems. For the three cases,[34] one finds the very interesting result of

$$\bar{\alpha}_t=1,\quad c_t=\left.\frac{\partial^2G}{\partial t^2}\right|_{g,h}\sim|g|^{-\bar{\alpha}_t}, \quad (54)$$

which, if experimentally accessible, might easily identify this type of tricritical point. We also find

$$\alpha_t=3-(2/\alpha),\quad c_g=\left.\frac{\partial^2G}{\partial g^2}\right|_{t,h}\sim|g|^{-\alpha t}, \quad (55)$$

and

$$\beta_t=\beta/\alpha,\quad M=\left.\frac{\partial G}{\partial h}\right|_{t,g}\sim g^{\beta_t}, \quad (56)$$

where $M\sim g^{\beta_t}$ is the magnetization discontinuity along the direction of the relevant scaling field.

Note that the tricritical exponents listed above are determined by the standard Heisenberg critical exponents. Note also that since $|\alpha|$ is usually very small these exponents are rather large. This fact can be further explored by direct calculation using Eq. (45) and the scaling form of the equation of state[35]

$$H=M^{\delta_R}h\left[\frac{t^{\beta_R}}{M},\frac{g^{\beta_t}}{M}\right]. \tag{57}$$

Making use of scaling relations, the exponents β_t and α_t are found to be of order ε^{-1} to leading order in ε as follows:

$$\alpha_t=-\frac{4(n+8)}{(4-n)\varepsilon}+0(\varepsilon^0), \tag{58a}$$

$$\beta_t=-\frac{n+8}{(4-n)\varepsilon}+0(\varepsilon^0). \tag{58b}$$

Note that as $\varepsilon\to 0$ (or $\alpha\to 0$), $g^{\beta_t}\to 0$, $g\ll 1$. Thus, for $d\geq 4$ where fluctuations play no role (apart from logarithmic corrections at $d=4$), mean field is exact and the tricritical point disappears.

V. SPHERICAL LIMIT

The spherical model[36] is one of the very few nontrivial examples of an interacting many-body system that can be solved exactly in three dimensions. It is well known that the n vector with infinite dimensional spins is identical to the spherical model.[37] Hence, to lowest order in n^{-1}, all vertices of the ϕ^4 field theory can be calculated in closed form in a self-consistent manner and the emerging theory is quite different from either a Gaussian theory or a loop expansion. In turn, the spherical limit serves as a lowest-order term of a systematic expansion in powers of (n^{-1}).[38] Recently,[39] the spherical model has also been used in reference to itinerant electron systems.

In order to obtain the spherical limit of the Hubbard model near a ferromagnetic transition, one must replace the quartic spin couplings in Eqs. (14a) and (14b) by $\lambda_s\to\lambda_s/n$, $\lambda_c^{(0)}\to\lambda_c^{(0)}/n$, and $\lambda_c^{(1)}\to\lambda_c^{(1)}/n$, where n is the number of components of the spin field. The required renormalization scheme is the same as presented in Sec. III, where as now the contractions of the tensors appearing in Eq. (23d) are given in Appendix C.

The two-point function has only contributions from diagrams with the greatest number of tadpoles at a given order in the perturbation expansion as displayed in Fig. 4. The vertices can either be of types $\underline{F}$ and $\underline{P}$ as shown in Appendix C. The inclusion of fluctuations in the propagator only produces a shift in the critical temperature but does not change the coefficient of the k^2 term (there are no k^2 contributions to leading order in n^{-1}) so the field does not get renormalized, i.e., $Z_s=1$.

In considering higher vertex functions one should recall that, as in the usual spherical model, tadpoles do not change the order in n^{-1} of a given diagram but their insertion in vertices is taken care of by replacing the bare mass by the full one[21] (summing the geometric series of Fig. 4). Then, it is easy to convince ourselves that the n^{-1} leading contributions to the four-point vertex functions are obtained by summing diagrams of the type shown in Fig. 5. Using results of Appendix C, one concludes that the coupling constant λ_s is not coupled to the other two. Conversely, $\lambda_c^{(0)}(\lambda_c^{(1)})$ couples not only to itself but also to λ_s. This situation is best summarized in Figs. 5(a) and 5(b). In either case all diagrams can be added to give a geometric series for λ_s and the sum of two series for $\lambda_c^{(0)}(\lambda_c^{(1)})$, as shown in Appendix C. Then, adding the series and using the renormalization conditions, Eq. (23), one has

$$g_s=\lambda_s(1+\lambda_s I_{\rm SP}/6)^{-1}, \tag{59a}$$

$$g_c^{(0)}=(\lambda_c^{(0)}-\lambda_s)[1+(\lambda_s-\lambda_c^{(0)})I_{\rm SP}/6]^{-1}+\lambda_s(1+\lambda_s I_{\rm SP}/6)^{-1}, \tag{59b}$$

while a third equation is obtained from (59b) by replacing $g_c^{(0)}(\lambda_c^{(0)})$ by $g_c^{(1)}(\lambda_c^{(1)})$. The dimensionless integral $I_{\rm SP}$ is given in Appendix D. Finally, the vertex function $\Gamma^{(2,1)}$ in the spherical limit can also be written as the geometric series presented in Appendix D. (Typical diagrams contributing to this function are depicted in Fig. 6). Summing the series, using the renormalization conditions, Eqs. (23) and Eqs. (59), one obtains

$$Z_{S^2}=[1-(g_s-g_c^{(0)})I_{SP}/6]^{-1}. \tag{60}$$

The Wilson functions (β) [Eq. (32)], are given by

$$\beta_s[(g)]=-\varepsilon g_s(1-g_s I_{\rm SP}/6), \tag{61a}$$

$$\beta_c^{(0)}[(g)]=-\varepsilon g_c^{(0)}[1+(g_c^{(0)}-2g_s)I_{\rm SP}/6]. \tag{61b}$$

A third equation for $\beta_c^{(1)}$ is obtained from (61b) by replacing $g_c^{(0)}$ by $g_c^{(1)}$. Using either boundary condition, total charge fixed or constant chemical potential, one obtains four FPs (g^*), satisfying $[\beta(g^*)]=0$, easily calculated from Eqs. (61). Two of them are Gaussian and the other

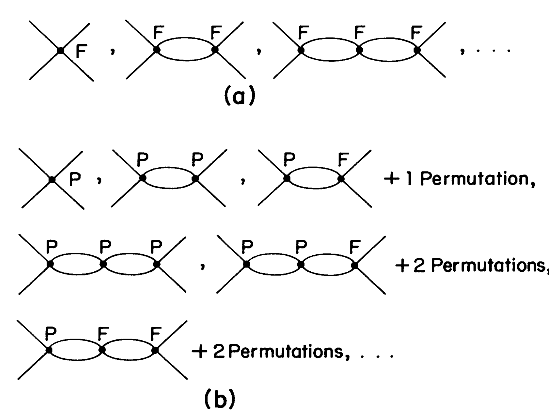

FIG. 5. Diagrams contributing to $\Gamma^{(4)}$: (a) In the spherical limit λ_s (vertex F) only couples to itself and the contributions add to a geometric series; (b) instead, $\lambda_c(\lambda_c^{(0)}$ or $\lambda_c^{(1)})$ couples also to λ_s and now the contributions can be written as the sum of two geometric series. For simplicity we illustrate only the coupling of $\lambda_c^{(0)}$ (vertex P) with λ_s.

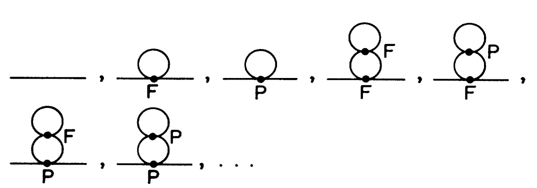

FIG. 4. Diagrams contributing to $\Gamma^{(2)}$ to leading order in n^{-1}. The vertices can either be of type $\underline{F}$ or $\underline{P}$.

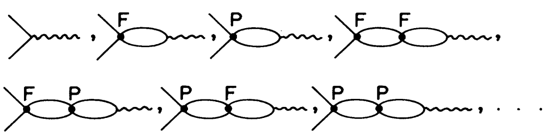

FIG. 6. In the spherical limit the vertex $\Gamma^{(2,1)}$ has contributions of the type shown with $\underline{F}$- and $\underline{P}$-type vertices adding to a geometric series.

two are spherical. This can be verified by calculating the critical indices at each fixed point. Using the fact that $Z_s=1$ and Eq. (60), one has for the Wilson functions (γ), Eqs. (33) and (34),

$$\gamma_s=0\ , \tag{62a}$$

$$\gamma_{s^2}=\varepsilon(g_s-g_c^{(0)})I_{SP}/6\ . \tag{62b}$$

Hence, the exponent $\eta=\gamma_s^*$ vanishes for all fixed points and the value of the correlation length exponent $\nu^{-1}=2-\gamma_{s^2}^*$ is either $\nu=1/2$ at the Gaussian FPs or $\nu^{-1}=2-\varepsilon$ at the spherical FPs.

The spherical limit of the model considered here has a negative value of the specific-heat exponent, $\alpha=-\varepsilon(2-\varepsilon)^{-1},0<\varepsilon<2$, as in the Heisenberg case. Hence, the ferromagnetic spherical FP(3) is the most stable one, with eigenvalues $b_1=b_2=\varepsilon$. On the other hand, the spherical FP(4), appearing in the case of constant chemical potential, and the Gaussian FP(6), for fixed charge, both with eigenvalues $b_1=-b_2=\varepsilon$, move down, lying now on the negative $g_c^{(1)}$ and g_c axis, respectively. Flow diagrams for both boundary conditions are shown in Fig. 7. Note that in the minimal subtraction renormalization scheme $g_{SPH}^*+6(\alpha/\nu)=0$, where $g_{SPH}^*=6\varepsilon$ is the fixed-point value of the spherical coupling constant and α and ν are the usual spherical critical exponents. For constant chemical potential, the previous discussion for the Heisenberg case fully applies. In contrast, the total charge fixed case has no runaway lines in the classical second-order region. Thus the Gaussian FP(6) is no longer a fluctuation-induced tricritical point and the basin of attraction of the ferromagnetic FP(3) is now the full fourth quadrant, in agreement with the mean-field stability condition. Hence, the regions of stability of continuous ferromagnetic transitions for either boundary condition are in accordance with the classical predictions of Fig. 1 and Eqs. (15a) and (15b).

Finally, for completeness, we show in Fig. 8(a) the flux diagram corresponding to the Ising ($n=1$) solution of the generalized $O(n)\phi^4$ theory presented in Sec. III. Since in this instance $\alpha>0$, for the total charge fixed case, the most stable FP is the renormalized Ising FP, with $g_c^{*(1)}>0$, lying therefore in the nonphysical region of the Hubbard model. On the other hand, the Ising FP is a fluctuation-induced tricritical point, in which case any

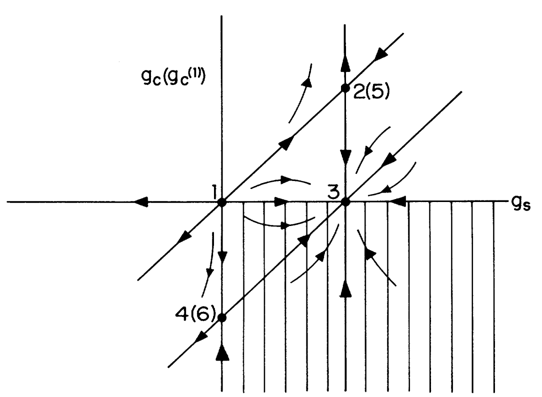

FIG. 7. Flow diagram of the Hubbard model in the spherical limit and near a ferromagnetic transition. FPs 1 and 2 (1 and 6) are of Gaussian type and FP 3 and 4 (3 and 5) are spherical ones for constant chemical potential (total charge fixed). For either boundary condition, the region of stability of a continuous ferromagnetic transition is in accordance with the classical theory, Fig. 1. For total charge fixed, the fourth quadrant, indicated with lines, is the basin of attraction of the ferromagnetic spherical FP3.

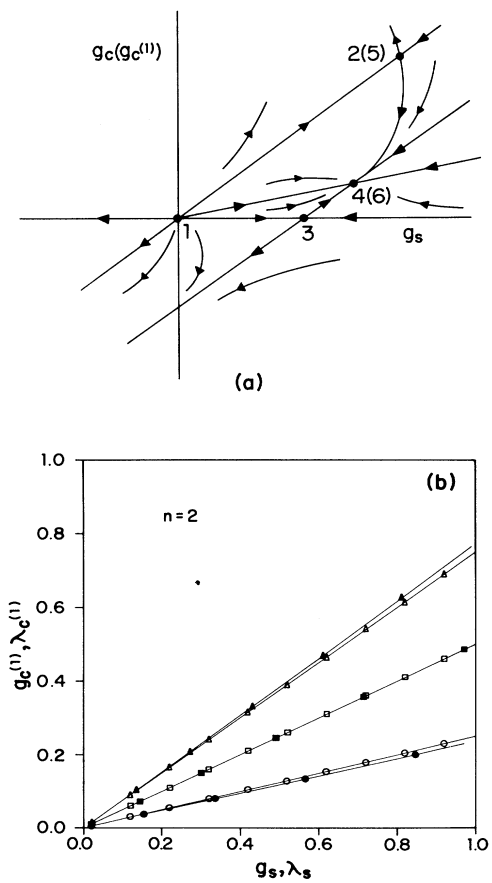

FIG. 8. (a) Flow diagram of the Hubbard model, near a ferromagnetic transition, in the case where one assumes an Ising-ferromagnetic transition, for which $\alpha>0$. The FPs are as in Fig. 3(a) if one reads Ising instead of Heisenberg. The basis of attraction of the ferromagnetic Ising FP3 now reduces to the line joining it to FP1. (b) Similar to Fig. 3(b), but now all initial coupling values are attracted to FP4(6) lying in the nonphysical region of the Hubbard model. We take $n=2$, so $\alpha>0$ to first order in ε.

small charge-density fluctuation drives the system away from a continuous ferromagnetic phase transition. To first order in $\varepsilon,\alpha>0$ for $n<4$; in Fig. 8(b) we show initial bare and renormalized couplings for $n=2$.

VI. DISCUSSION AND CONCLUSIONS

We have shown that the coupling of charge- and spin-density fluctuations in the three-dimensional one-band Hubbard model, with total charge fixed, severely restricts the occurrence of a continuous ferromagnetic transition. We find that the renormalized Heisenberg fixed point with nonzero spin-charge coupling is a fluctuation-induced tricritical point, resulting in a novel instability criterion for the occurrence of this transition. The tricritical exponents have been determined by scaling analysis and by direct computation to leading order in $\varepsilon=(4-d)$ dimensions, revealing very interesting results. We have also shown that if the system is allowed to exchange particles with a reservoir, the mean-field criterion for the occurrence of a continuous transition persists, even when fluctuations are considered in the theory. Moreover, the exact solution of the model in the spherical limit, and near the ferromagnetic transition, shows that no fluctuation-induced tricritical point is found and that the regions of stability of continuous ferromagnetic transition for either boundary condition are in agreement with the mean-field criteria.

The different criterion, Eq. (40), is of particular interest to numerical studies of the Hubbard model in which the total charge of the system is kept constant. Under these circumstances, the occurrence of a continuous ferromagnetic transition is governed by condition (40), which should prove essential for a proper interpretation of the results. In fact, the nature of the ferromagnetic transition, *first or second order,* might differ according to the constraint used in the calculation, though this distinction might be felt only when the correlation length is of the order of the sample size.[40] Nonetheless, the effect predicted in this paper is a statistical-thermodynamic result in the sense that it is derived in the thermodynamic limit of the system. Any finite-size effect must be added to the discussion regardless of the nature of the transition.[41]

Some remarks about the above conclusions are instructive. It is believed that the thermodynamic properties of a physical system may be equally computed from any of the various ensembles used in statical mechanics,[42] though average values of fluctuations are, in general, ensemble-dependent quantities.[43] Near a phase transition, care must be exercised, particularly when the system is subjected to constraints. In fact, as emphasized by Griffiths,[44] besides nonanalytic dependence and discontinuity of thermodynamic functions, an alternative definition for what one means by a phase transition is precisely the fact that "the properties in the interior of a large system can be influenced by what is happening at the boundaries, even when the boundaries become infinitely far away." It is with this in mind that we state in Sec. II that the partition function or free energy of our system is fully defined only when the appropriated constraint is specified. Given a specific constraint, the renormalization-group program of Secs. III and IV manifests this choice by generating specific fixed-point values for the coupling constants. By changing the constraint, new coupling-constant values are generated and thus, the nature of the transition might change under this new situation. One should note, however, that under the *same* constraint, the Helmholtz and Gibbs free energies, $F(T,\mathbf{M})$ and $G(T,\mathbf{H})$, are connected by a Legendre transformation, since T, $\mathbf{M}$ or $T,\mathbf{H}$ are the only relevant thermodynamic variables near the ferromagnetic transition.

Finally, though the main motivation of our work is toward the understanding of the fundamental aspects of the Hubbard model, we would like to add some closing remarks on the experimental side of the problem studied here.

First, it is interesting to stress that the most stable fixed point of the Hubbard model is a Heisenberg one,[14] with critical exponents identical to those found in localized spin systems. Thus the ferromagnetic critical properties of iron[45] and, say, EuO,[46] should be the same, as indeed found experimentally. We believe that the universal properties characterizing the ferromagnetism of the model will not change if, instead, a multiband version is used.[47] In fact our description can be generalized to account for this feature by just properly redefining an effective bare electronic density of states.

The second point of discussion is the possibility of the observation of the fluctuation-induced tricritical point reported here. As emphasized in Sec. IV our description of the tricritical behavior include Ising, XY, Heisenberg, and the renormalized versions of these fluctuation-induced tricritical points. The tricritical nature of these critical points appears when the relevant scaling field driving the first-order transition is brought into the discussion [see Eqs. (49)–(53)]. Although several previous discussions[16,17,35] have pointed out some aspects of this problem, to the best of our knowledge a complete determination of these tricritical exponents had not yet been reported. We thus believe that these very interesting results will stimulate the experimentalists and enhance the search for such types of tricritical behavior among the various classes of systems in which they might occur. As far as itinerant electron systems are concerned, fluctuation-induced first-order transition and tricritical behavior could be searched for in alloys or compounds of transition metals.[48] In these cases one must observe, however, that the hybridization of the d electrons with s and p bands might inhibit the first-order transition because the latter bands may play the role of a reservoir of particles.

ACKNOWLEDGMENTS

This work was partially supported by CNPq, CAPES, and FINEP (Brazilian Agencies).

APPENDIX A

In this appendix we list some formulas which are useful in the calculation of the fixed points and critical exponents (see Sec. III).

As the critical point and in the one-loop approximation, we have the bare vertex functions (see Fig. 2):

$$\Gamma^{(2,0)}_{\alpha_1\alpha_2}[\mathbf{k}_1,\mathbf{k}_2;(\lambda_i),\Lambda]=\left\{[r_s(T_c)+k_1^2]+\left[\frac{(n+2)}{6}\lambda_s-\frac{n}{6}\lambda_c^{(0)}-\frac{1}{3}\lambda_c^{(1)}\right]D_1[r_s(T_c),\Lambda]\right\}\delta_{\mathbf{k}_1,\mathbf{k}_2}\delta_{\alpha_1,\alpha_2}\ , \tag{A1}$$

$$\Gamma^{(2,1)}(\mathbf{k}_1,\mathbf{k}_2;\mathbf{p};\{\lambda_i\},\Lambda)|_{SP}=1-\left.\left[\frac{(n+2)}{6}\lambda_s-\frac{n}{6}\lambda_c^{(0)}-\frac{1}{3}\lambda_c^{(1)}\right]J_{SP}[r_s(T_c);\mathbf{p},\Lambda)]\right|_{SP}\ , \tag{A2}$$

$$\begin{aligned}\Gamma^{(4,0)}_{\alpha_1\cdots\alpha_4}(\mathbf{k}_1,\ldots\mathbf{k}_4);\{\lambda_i\}\Lambda|_{\mathrm{SP}}=&\left.\left\{\lambda_s-\left[\frac{(n+8)}{6}\lambda_s^2-2\lambda_s\lambda_c^{(1)}+\frac{2}{3}\lambda_c^{(1)2}\right]J_{SP}(r_s(T_c),\mathbf{k}_i+\mathbf{k}_j,\Lambda)\right\}\underline{F}\right|_{SP}\\&-\left\{\lambda_c^{(0)}+\left[\frac{n}{6}\lambda_c^{(0)2}-\frac{(n+2)}{3}\lambda_s\lambda_c^{(0)}+\frac{2}{3}\lambda_c^{(1)}\lambda_c^{(0)}\right]J_{SP}[r_s(T_c),\mathbf{k}_i+\mathbf{k}_j,\Lambda]\right\}\underline{P}|_{SP}\\&-\left\{\lambda_c^{(1)}+\left[\frac{(n+4)}{6}\lambda_c^{(1)2}-\frac{(n+2)}{3}\lambda_s\lambda_c^{(1)}\right]J_{SP}[r_s(T_c),\mathbf{k}_i+\mathbf{k}_j,\Lambda]\right\}\underline{T}|_{SP}\ ,\end{aligned} \tag{A3}$$

where

$$D_1[r_s(T_c),\Lambda]=\int^\Lambda dq[r_s(T_c)+q^2]^{-1}\ , \tag{A4}$$

$$J_{SP}[r_s(T_c),\mathbf{k}_i+\mathbf{k}_j,\Lambda]=\int^\Lambda dq[r_s(T_c)+q^2]^{-1}[r_s(T_c)+(\mathbf{k}_i+\mathbf{k}_j-q)^2]^{-1}|_{SP}\ . \tag{A5}$$

The choices $i,j(=1,\ldots 4)$ in Eq. (A3) follows the permutation of the wave vectors as in Eqs. (24), $r_s(T_c)$ is the bare mass at the critical point and the following tensor contractions have been used:

$$\sum_\beta \underline{X}^{\alpha_1\alpha_2\beta\beta}_{\mathbf{k}_1,\mathbf{k}_2,\mathbf{q},-q}=[\underline{X}]\frac{(n+2)}{3}\delta_{\alpha_1,\alpha_2}\delta_{\mathbf{k}_1,\mathbf{k}_2}\ , \tag{A6}$$

with

$$[\underline{F}]=\frac{n+2}{3},\quad [\underline{P}]=\frac{n}{3},\quad [\underline{T}]=\frac{2}{3} \tag{A7}$$

and

$$\left.\left[\sum_{\beta\beta'}\underline{X}^{\alpha_1\alpha_2\beta\beta'}_{\mathbf{k}_1,\mathbf{k}_2,-\mathbf{q},-\mathbf{q}'}\underline{Y}^{\beta\beta'\alpha_3\alpha_4}_{\mathbf{q},\mathbf{q}',\mathbf{k}_3,\mathbf{k}_4}+2\text{ permutations}\right]\right|_{SP}$$

$$=[(\underline{X}\,\underline{Y})_F\underline{F}+(\underline{X}\,\underline{Y})_P\underline{P}+(\underline{X}\,\underline{Y})_T\underline{T}]\delta_{\mathbf{q}+\mathbf{q}',\mathbf{k}_1}+\mathbf{k}_j|_{SP}\ , \tag{A8}$$

with

$$(\underline{P}\,\underline{P})_P=\frac{n}{3},\quad (\underline{P}\,\underline{P})_F=(\underline{P}\,\underline{P})_T=0\ , \tag{A9}$$

$$(\underline{P}\,\underline{F})_P=\frac{(n+2)}{3},\quad (\underline{P}\,\underline{F})_F=(\underline{P}\,\underline{F})_T=0\ , \tag{A10}$$

$$(\underline{P}\,\underline{T})_P=\frac{2}{3},\quad (\underline{P}\,\underline{T})_F=(\underline{P}\,\underline{T})_T=0\ , \tag{A11}$$

$$(\underline{F}\,\underline{F})_F=\frac{(n+8)}{3},\quad (\underline{F}\,\underline{F})_T=(\underline{F}\,\underline{F})_P=0\ , \tag{A12}$$

$$(\underline{F}\,\underline{T})_F=2,\quad (\underline{F}\,\underline{T})_T=\frac{(n+2)}{3},\quad (\underline{F}\,\underline{T})_P=0\ , \tag{A13}$$

$$(\underline{T}\,\underline{T})_F=\frac{3}{4},\quad (\underline{T}\,\underline{T})_T=\frac{(n+4)}{3},\quad (\underline{T}\,\underline{T})_P=0\ . \tag{A14}$$

Note that $\underline{P}$ acts as a projector with respect to $\underline{F}$ and $\underline{T}$; thus the interaction of $\lambda_c^{(0)}$ with either λ_s or $\lambda_c^{(1)}$ does not contribute to the renormalization of the later couplings.

APPENDIX B

In order to calculate the bare Helmholtz free energy in Eq. (42) we need to evaluate the one-loop bare vertex functions at zero external momenta and at a temperature in the vicinity of T_c. We thus have (see Fig. 2) the following:

$$\Gamma^{(2,0)}_{\alpha_1,\alpha_2}(T;0)=\left\{r_s(T)+\frac{1}{2}[(n-1)\lambda_1+\lambda_2]\int\frac{dq}{r_s(T)+q^2}\right\}\delta_{\alpha_1,\alpha_2}\ , \tag{B1}$$

$$\Gamma^{(4,0)}_{\alpha_1\cdots\alpha_4}(T;0)=\left\{(\lambda_s-\lambda_c^{(0)})+\frac{3}{2}[-(n-1)\lambda_1^2-\lambda_2^2]\int\frac{dq}{(r_s(T)+q^2)^2}\right\}\delta_{\alpha_1,\alpha_2}\delta_{\alpha_3,\alpha_4}\ , \tag{B2}$$

$$\Gamma^{(6,0)}_{\alpha_1\cdots\alpha_6}(T;0)=\left\{15[(n-1)\lambda_1^3+\lambda_2^3]\int\frac{dq}{[r_s(T)+q^2]^3}\right\}\delta_{\alpha_1,\alpha_2}\delta_{\alpha_3,\alpha_4}\delta_{\alpha_5,\alpha_6}\ , \tag{B3}$$

where

$$\lambda_1=\tfrac{1}{3}(\lambda_s-\lambda_c^{(0)}),\quad \lambda_2=\tfrac{1}{3}(3\lambda_s-\lambda_c^{(0)}-2\lambda_c^{(1)}) \ . \tag{B4}$$

To evaluate $\Gamma^{(2,0)}$ and $\Gamma^{(4,0)}$, the results of Appendix A have been used and for $\Gamma^{(6,0)}$ the following tensor contractions are needed.

$$\sum_{\beta\beta'\beta''} \underline{X}^{\alpha_1\alpha_2\beta\beta'}_{0,0,\mathbf{q},-\mathbf{q}'}\underline{Y}^{\alpha_3\alpha_4\beta\beta''}_{0,0,\mathbf{q}',-\mathbf{q}'}\underline{Z}^{\alpha_5\alpha_6\beta'\beta''}_{0,0,\mathbf{q}'',-\mathbf{q}'} =(\underline{X}\ \underline{Y}\ \underline{Z})\delta_{\alpha_1,\alpha_2}\delta_{\alpha_3,\alpha_4}\delta_{\alpha_5,\alpha_6}\delta_{q,q'}\delta_{q,q''} \ , \tag{B5}$$

with

$$(FFF)=\tfrac{5}{9}(n+2),(FFP)=\tfrac{5}{3}(n+8) \ , \tag{B6}$$

$$(FFT)=30,\quad (FPP)=\tfrac{5}{3}(n+2) \ , \tag{B7}$$

$$(FPT)=20,\quad (FTT)=20 \ , \tag{B8}$$

$$(PPP)=\frac{5}{9}n,\quad (PPT)=\frac{10}{3} \ , \tag{B9}$$

$$(PTT)=\frac{20}{3},\quad (TTT)=\frac{40}{9} \ . \tag{B10}$$

From (B1)–(B3) we infer the series

$$\beta F(T,M)=\frac{1}{2}r_s(T)M^2+\frac{\lambda_1}{4!}M^4-\frac{1}{2}\sum_{m=1}^{\infty}\frac{1}{m}\left[(n-1)\left[-\frac{\lambda_1}{2}M^2\right]^m+\left[-\frac{\lambda_2}{2}M^2\right]^m\right]\int\frac{dq}{[r_s(T)+q^2]^m} \ , \tag{B11}$$

which results in Eq. (43).

APPENDIX C

In this appendix we outline the calculation of the full vertex functions in the spherical limit (see Sec. V).

We use the following contractions (valid in the $n\to\infty$ limit) of the tensors $\underline{F}$, $\underline{P}$ and $\underline{T}$.

$$\sum_{\beta}\underline{X}^{\alpha_1\alpha_2\beta\beta}_{\mathbf{k}_1,\mathbf{k}_2,\mathbf{q},-\mathbf{q}}=\tfrac{1}{3}[\underline{X}]\delta_{\alpha_1,\alpha_2}\delta_{\mathbf{k}_1,\mathbf{k}_2} \ , \tag{C1a}$$

where $\underline{X}$ stands for any of the above tensors with

$$(\underline{F})=(\underline{P})=\tfrac{1}{3},(\underline{T})=0 \ . \tag{C1b}$$

We also need

$$\left.\left[\sum_{\beta\beta'}\underline{X}^{\alpha_1\alpha_2\beta\beta'}_{\mathbf{k}_1,\mathbf{k}_2,-\mathbf{q}\mathbf{q}'}\underline{Y}^{\beta\beta'\alpha_3\alpha_4}_{\mathbf{q}',\mathbf{q},\mathbf{k}_3,\mathbf{k}_4}+2\text{ permutations}\right]\right|_{\rm SP} =\left.[(\underline{X}\ \underline{Y})_F\underline{F}+(\underline{X}\ \underline{Y})_P\underline{P}+(\underline{X}\ \underline{Y})_T\underline{T}]\delta_{\mathbf{q}+\mathbf{q}',\mathbf{k}_i+\mathbf{k}_j}\right|_{\rm SP} \ , \tag{C2a}$$

where the only nonvanishing coefficients are

$$(\underline{F}\ \underline{F})_F=(\underline{P}\ \underline{P})_P=(\underline{T}\ \underline{T})_T=(\underline{F}\ \underline{P})_P=(\underline{F}\ \underline{T})_T=\tfrac{1}{3} \ . \tag{C2b}$$

With the help of Fig. 4 and Eqs. (C1) and (C2) it is easy to show that only the vertices $\underline{F}$ and $\underline{P}$ contribute to the self-energy ($\Gamma^{(2)}$).

The diagrams of Fig. 5 give the following geometric series for the $\Gamma^{(4)}$ vertex function:

$$\text{(i) [diagrams Fig. 5(a)]}\Longrightarrow\lambda_s[1-(\lambda_s I_{\rm SP}/6)+(\lambda_s I_{\rm SP}/6)^2+\cdots] \ , \tag{C3}$$

$$\text{(ii) [diagrams Fig. 3]}\Longrightarrow\lambda_c^{(0)}-\lambda_c^{(0)^2}I_{\rm SP}/6+\lambda_c^{(0)}\lambda_s I_{\rm SP}/3-(\lambda_c^{(0)^3}-3\lambda_c^{(0)^2}\lambda_s+3\lambda_c^{(0)}\lambda_s^2)(I_{\rm SP}/6)^2+\cdots) \ , \tag{C4}$$

where the dimensionless integral $I_{\rm SP}$ is given by

$$I_{\rm SP}=\left.\frac{\kappa^\varepsilon}{S^d}\int dq\frac{1}{q^2(q+k)^2}\right|_{\rm SP} =\frac{\Gamma(d/2)\Gamma(2-d/2)\Gamma^2(d/2)-1)}{2\Gamma(d-2)} \ , \tag{C5}$$

with Γ denoting the usual gamma function. Adding and subtracting (C1), (C5) can be rewritten as the sum of two geometric series. A third equation for the full $\lambda_c^{(1)}$ vertex is obtained from (C5) by replacing $\lambda_c^{(0)}$ by $\lambda_c^{(1)}$.

The diagrams of Fig. 7 give the following contributions to $\Gamma^{(2,1)}$:

$$\text{(diagrams Fig. 4)}\Longrightarrow 1+(\lambda_s\kappa^{-\varepsilon}I_{\rm SP}/6)-(\lambda_c^{(0)}\kappa^{-\varepsilon}I_{\rm SP}/6)+(\lambda_s^2+\lambda_c^{(0)^2}-2\lambda_s\lambda_c^{(0)})\kappa^{-2\varepsilon}(I_{\rm SP}/6)^2+\cdots \ , \tag{C6}$$

which can also be recast as a geometric series.

[1]J. Hubbard, Proc. R. Soc. London Ser. A **276**, 238 (1963).

[2]See, e.g., *Strongly Correlated Electron Systems II,* edited by G. Baskaran, A. E. Ruckestein, E. Tosatti, and Y. Lu (World Scientific, Singapore, 1991).

[3]E. H. Lieb and F. Y. Wu, Phys. Rev. Lett. **25**, 1445 (1968); Phys. Rev. **125**, 164 (1962).

[4]J. E. Hirsch, Phys. Rev. Lett. **51**, 1900 (1983); Phys. Rev. B **31**, 4403 (1985); S. Rudin and D. C. Mattis, Phys. Lett. **110A**, 273 (1985); J. E. Hirsch and S. Tang, Phys. Rev. Lett. **62**, 591 (1989). For recent discussions on the magnetic properties of

the two-dimensional Hubbard model, see, e.g., L. Chen, C. Bourbonnais, T. Li, and A.-M. S. Tremblay, Phys. Rev. Lett. **65**, 1470 (1991); D. Poiblanc, Phys. Rev. B **45**, 10 775 (1992); W. O. Putikka, M. U. Luchini, and M. Ogata, Phys. Rev. Lett. **69**, 2288 (1992).

[5]J. E. Hirsch, Phys. Rev. B **35**, 1851 (1987); R. T. Scalettar, D. J. Scalapino, R. T. Sugar, and D. Toussaint, *ibid.* **36**, 8632 (1987); **39**, 4711 (1989); H. Yokoyama and H. Shiba, J. Phys. Soc. Jpn. **56**, 1490 (1987); **56**, 3582 (1987); see also Y. Kakehashi and H. Hasegawa, Phys. Rev. B **37**, 7777 (1988); W. Nolting and W. Borgiel, Phys. Rev. B **39**, 6962 (1989).

[6]Recently E. H. Lieb [Phys. Rev. Lett. **62**, 1201 (1989)] studied the half-filled Hubbard model in bipartite lattices (that is, the lattice sites are divided into two disjoint sets A and B such that the hopping term only connects sites of A and B) and proved that the total spin is given by $2S_T=|B|-|A|$; thus the ground state is ferromagnetic only if $|A|\neq|B|$. See also A. Mielke, J. Phys. A **2**, 3311 (1991); H. Tasaki, Phys. Rev. Lett. **69**, 1608 (1992).

[7]See, e.g., B.-H. Zhao, H-Q. Nie, K-Y. Zhang, K. A. Chao, and R. Micnas, Phys. Rev. B **36**, 2321 (1987). Recent studies based on high-temperature series expansions, diagrammatic technique, and exact diagonalization have also reported ferromagnetic solutions on cubic lattices for high values of U and electron density close to half-filling: D. F. B. ten Haaf and J. M. J. van Lewwuwen, Phys. Rev. B **46**, 6313 (1992); J. A. Henderson, J. Oitmaa, and M. C. B. Ashley, *ibid.* **46**, 6328 (1992); Yu. A. Izyumov, B. M. Letfulov, E. V. Shipitsyn, M. Bartkowiak, and K. A. Chao, *ibid.* **46**, 15 697 (1992); A. M. S. Macêdo, C. A. Macêdo, and M. D. Coutinho-Filho, J. Magn. Magn. Mater. **104-107**, 679 (1992).

[8]Recent results based on the Gutzwiller-type variational ansatz have been reported: B. S. Shastry, H. R. Krishnamurthy, and P. W. Anderson, Phys. Rev. B **41**, 2375 (1990); A. G. Basile and E. Elser, *ibid.* **41**, 4842 (1990); W. Von der Linden and D. M. Edwards, J. Phys. Condens. Matter. **3**, 4917 (1991). See, however, interesting remarks by T. Kopp, A. E. Ruckenstein, and S. Schmitt-Rink, Phys. Rev. B **42**, 6850 (1990); M. W. Long and R. Fehrenbacher, J. Phys. Condens. Matter **2**, 10 343 (1990).

[9]Y. Nakaoka, Phys. Rev. **147**, 392 (1966).

[10]G.-S. Tian, Phys. Rev. B **44**, 4444 (1991); J. Phys. **24**, 513 (1991); S. A. Trugman, **42**, 6612 (1990); A. Barbieri, J. A. Riera, and A. P. Young, Phys. Rev. B **41**, 11 697 (1990). See also, X. Y. Zhang, E. Abrahams, and G. Kotliar, Phys. Rev. Lett. **66**, 1236 (1991); Y. Fang, A. E. Ruckenstein, E. Dagotto, and S. Schmidt-Rink, Phys. Rev. B **40**, 7406 (1989); B. Doucot and X. G. Wen, *ibid.* **40**, 2719 (1989); J. A. Riera and A. P. Young, *ibid.* **40**, 5285 (1989). The main findings are that the Nagaoka state is thermodynamically stable only in the limit of vanishing hole concentration and that the holes do not form bound states.

[11]H. Tasaki, Phys. Rev. B **40**, 9192 (1989); see also, E. Dagotto, A. Moreo, and T. Barnes, *ibid.* **40**, 6721 (1989).

[12]M. Aizenman and E. H. Lieb, Phys. Rev. Lett. **65**, 1470 (1990).

[13]Studies of the Hubbard or simplified versions of the model in infinite dimensions have revealed several interesting features, in particular some characteristics of an exact quantum mean-field theory of correlated electron systems: W. Metzner and D. Vollhardt, Phys. Rev. Lett. **62**, 324 (1989); P. G. J. van Dongen and D. Vollhardt, *ibid.* **65**, 1663 (1990); P. G. J. van Dongen, Phys. Rev. B **45**, 2267 (1992); E. Müller-Hartmann, Z. Phys. B **74**, 507 (1989); **76**, 211 (1990); P. Fazekas, B. Menge, and E. Müller-Hartmann, *ibid.* **78**, 69 (1990); B. Menge and E. Müller-Hartmann, *ibid.* **82**, 237 (1991); U. Brandt and C. Mielsch, *ibid.* **75**, 365 (1989); **79**, 295 (1990); **82**, 37 (1991); C. J. Thompson, Y. S. Yang, A. J. Guttmann, and M. F. Sykes, J. Phys. **24**, 1261 (1991); Y. S. Yang and C. J. Thompson, *ibid.* **24**, L279 (1991); Q. Si, G. Kotliar, and A. Georges, Phys. Rev. B **45**, 6479 (1992); A. Georges and G. Kotliar, *ibid.* **45**, 6479 (1992).

[14]A. A. Gomes and P. Lederer, J. Phys. (Paris) **38**, 231 (1977); C. M. Chaves, P. Lederer, and A. A. Gomes, J. Phys. **10**, 3367 (1977).

[15]M. W. Fisher, Phys. Rev. **176**, 257 (1968).

[16]L. Gunther, D. J. Bergman, and Y. Imry, Phys. Rev. Lett. **27**, 558 (1971); Y. Imry, O. Entin-Wohlman, and D. J. Bergman, J. Phys. C **6**, 2846 (1973); **7**, 1621 (1974).

[17]V. Dohm, J. Phys. C **7**, L174 (1974). See also V. Dohm and P. J. Kortman, Phys. Rev. B **9**, 4775 (1974); F. J. Wegner, J. Phys. C **7**, 2109 (1974).

[18]Preliminary accounts of these results can be found in the following: M. D. Coutinho-Filho, M. L. Lyra, and A. M. Nemirovsky, Solid State Commun. **74**, 1175 (1990); M. D. Coutinho-Filho, J. Phys. (Paris) Colloq. **49**, C8-49 (1988). See also M. L. Lyra and M. D. Coutinho-Filho, Solid State Commun. **81**, 883 (1992); Phys. Lett. A **165**, 96 (1992).

[19]C. A. Macêdo and M. D. Coutinho-Filho, Europhys. Lett. **3**, 387 (1987), Phys. Rev. B **43**, 13 515 (1991); C. A. Macêdo, M. D. Coutinho-Filho, and M. A. de Moura, Phys. Rev. B **25**, 5965 (1982), and references therein. Alternative formulations are found in the following: G. Kotliar and A. E. Ruckenstein, Phys. Rev. Lett. **57**, 1362 (1986); J. E. Hirsch, Phys. Rev. B **28**, 4059 (1983); **34**, 3216 (1986). See also U. Wolf, Nucl. Phys. B **225** [FS9], 391 (1983).

[20]R. L. Stratonovich, Dokl. Akad. Nauk. SSSR **115**, 1094 (1957) [Sov. Phys. Dokl. **2**, 416 (1958)]; J. Hubbard, Phys. Rev. Lett. **3**, 77 (1959).

[21]See, e.g., D. J. Amit, *Field Theory, The Renormalization Group and Critical Phenemena* (World Scientific, Singapore, 1984).

[22]J. A. Hertz and M. A. Klenin, Phys. Rev. B **10**, 1084 (1974).

[23]J. A. Hertz, Phys. Rev. B **14**, 525 (1976).

[24]Similar equations have been found also in Ref. 14. Notice, however, that these authors did not derive an expansion around the saddle point and that in their expressions for the spin and charge mass terms, the 1/2 factors are missing; thus, random-phase approximation (RPA) cannot be obtained in the quadratic approximation, using $2b_c=2b_\alpha=1$ in Eq. (5c), as suggested by them. A full discussion of this point is given by C. A. Macêdo and M. D. Coutinho-Filho in Ref. 19.

[25]D. R. Nelson and M. E. Fisher, Phys. Rev. B **11**, 1030 (1975).

[26]C. S. Yokoi, M. D. Coutinho-Filho, and S. R. Salinas, Phys Rev. B **31**, 4502 (1985).

[27]J. Bruno and J. Sak, Phys. Rev. B **22**, 3302 (1980); J. F. Nicoll, *ibid.* **24**, 388 (1981). See also D. J. Bergman and B. I. Halperin, *ibid.* **13**, 2145 (1976).

[28]Y. Achiam and Y. Imry, Phys. Rev. B **12**, 2768 (1976).

[29]Y. Achiam, J. Phys. C **10**, 1491 (1977).

[30]M. A. de Moura, T. C. Lukensky, Y. Imry, and A. Aharony, Phys. Rev. B **13**, 2176 (1976).

[31]To simplify the notation, the coupling constants in Eq. (39) are those calculated using renormalization by minimal subtraction. See, e.g., J. C. Le Guillon and J. Zinn-Justin, Phys. Rev. B **21**, 3976 (1980); A. A. Vladimirov, D. I. Kazakov, and O. V. Tarasov, Zh. Eksp. Teor. Fiz. **77**, 1035 (1979) [Sov. Phys. JETP **50**, 521 (1979)].

[32]H. H. Iacobson and D. J. Amit, Ann. Phys. (N.Y.) **133**, 57

(1981); see also *Field Theory, The Renormalization Group and Critical Phenomena* (Ref. 21), p. 330.

[33]See, e.g., I. D. Lawrie and S. Sarbach, in *Phase Transitions and Critical Phenomena,* edited by C. Domb and J. L. Lebowitz (Academic, London, 1983), Vol. 9.

[34]For the Heisenberg tricritical point $\phi=-\alpha$ and $\bar{\alpha}_t=-1$.

[35]See also the tricritical behavior induced by cubic anisotropy, in which case a further symmetry-breaking field might restore the continuous transition. J. Rudnik, Phys. Rev. B **18**, 1406 (1978); E. Domany, D. Mukamel, and M. E. Fisher, *ibid.* **15**, 5432 (1977); O. G. Mouritsen, S. J. Knak Jensen, and P. Back, Phys. Rev. Lett. **39**, 629 (1977); S. J. Knak Jensen, O. G. Mouritsen, E. Kjaersgaarrd Hansen, and P. Bak, Phys. Rev. B **19**, 5886 (1979), and references therein. Experiments have shown that impurities can also turn the fluctuation-driven first-order transition second order. V. J. Cox, A. Gibaud, and R. A. Cowley, Phys. Rev. Lett. **61**, 982 (1988).

[36]T. H. Berlin and M. Kac, Phys. Rev. **86**, 821 (1952).

[37]M. Kac and C. J. Thompson, Proc. Norwegian Acad. Sci. **5**, 163 (1971); E. H. Stanley, *Introduction to Phase Transitions and Critical Phenomena* (Clarendon, Oxford, 1971), p. 128.

[38]S. K. Ma, Phys. Rev. Lett. **29**, 1311 (1972); Phys. Rev. A **7**, 2172 (1973).

[39]M. Grilli, R. Raimondi, C. Castellani, C. Di Castro, and G. Kotliar, Phys. Rev. Lett. **67**, 259 (1991).

[40]See, e.g., D. J. Bergman and B. I. Halperin, in Ref. 17. Very fine experiments could, in principle, distinguish the cause of the specific-heat rounding. See, e.g., W. Huhn and V. Dohm, Phys. Rev. Lett. **61**, 1368 (1988).

[41]See, e.g., V. Privman and M. E. Fisher, J. Stat. Phys. **33**, 385 (1983).

[42]M. E. Fisher, Arch. Ratl. Mech. Anal. **17**, 377 (1964); R. B. Griffiths, J. Math. Phys. **6**, 1447 (1965). See also, D. Rulle, *Statistical Mechanics* (Benjamin, New York, 1969), and references therein.

[43]J. L. Lebowitz, J. K. Percus, and L. Verlet, Phys. Rev. **153**, 250 (1967); C. Moukarzel and N. Parga, J. Phys. A **22**, 943 (1989).

[44]R. B. Griffiths, in *Statistical Mechanics: New Concepts, New Problems, New Applications,* edited by S. A. Rice, K. F. Freed, and J. C. Light (The University of Chicago Press, Chicago, 1972), p. 31. See also old arguments by Pippard, stating that the "... thermodynamic functions must be regarded as a property of the system and of its constraint, . . .", A. B. Pippard, *Classical Thermodynamics* (Cambridge University, Cambridge, England, 1966), p. 97.

[45]N. Stüsser, M. Th. Rekveldt, and T. Spruijt, Phys. Rev. B **31**, 5905 (1985).

[46]J. Als-Nielsen, O. W. Dietrich, W. Kunnmann, and L. Passel, Phys. Rev. Lett. **27**, 741 (1971); J. Als-Nielsen and O. W. Dietrich, Phys. Rev. B **14**, 4908 (1976).

[47]It may be, however, that a detailed microscopic description of the ferromagnetism of the transition metals needs a multiband model: see, e.g., R. Mota and M. D. Coutinho-Filho, Phys. Rev. B **33**, 7724 (1986), and references therein.

[48]See, e.g., D. Block, D. M. Edwards, M. Shimizu, and J. Voiron, J. Phys. F **5**, 1217 (1975); A. Z. Men'shikov, G. A. Takzei, Yu. A. Dorofeev, V. A. Kazantsev, A. K. Kostyshin, and I. I. Sych, Zh. Eksp. Teor. Fiz. **89**, 1269 (1989) [Sov. Phys. JETP **62**, 734 (1985)]; E. Fawcett, R. B. Roberts, R. Doy, and G. K. White, Europhys. Lett. **1**, 473 (1986).