# Comparison of experimental magnetization and specific-heat data with Landau-Ginzburg theory results for high-temperature superconductors near $H_{c2}$

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(Received 8 March 1993)

We have recently studied the magnetization and specific heat for two- and three-dimensional superconductors, starting from Landau-Ginzburg theory. In this paper we compare the results of this study with available experimental data. All our results have been calculated for a strong magnetic field along the $c$ axis in the region near the $H_{c2}$ line where the lowest-Landau-level approximation is valid. We show that the theoretical two-dimensional magnetization compares very well with that measured for the high-$T_c$ superconductor Bi 2:2:2:3, which is a quasi-two-dimensional system. Our comparison leads to an estimate of $\kappa$, the Ginzburg-Landau parameter as $\kappa \simeq 140$. In three dimensions a reasonable fit to the specific heat for a conventional superconductor (niobium) is obtained, but the anisotropic Landau-Ginzburg theory cannot be made to fit $YBa_2Cu_3O_{7-\delta}$ over any sizable temperature range. It is only when the Lawrence-Doniach model is considered (which takes into account the layered structure of the system) that a fit becomes possible over any temperature range. At high temperatures the form of this function is the same as the two-dimensional Landau-Ginzburg theory and the fact that this fits the experimental data indicates that the layers are acting independently.

## I. INTRODUCTION

The phenomenological Landau-Ginzburg model has long been used in the theoretical investigation of superconductivity. Using the techniques developed in a previous paper[1] we compare the specific-heat and magnetization functions with experimental data for a number of materials, whose superconductivity is either two or three dimensional, in the region near the $H_{c2}$ line where the lowest-Landau-level approximation (LLL) is valid.

We will begin with a brief review of Landau-Ginzburg theory, in order to define our notation. This will be followed by a summary of the techniques which have been used to evaluate the magnetization and specific heat over the complete temperature range. The difficulty in using Landau-Ginzburg theory is that when one goes beyond mean-field theory the only systematic analytical technique available is to expand the free energy in terms of a perturbation series in the coupling parameter.[2,3] This series, which gives us the free energy as a function of the reduced temperature, is asymptotic in nature, so in order to apply it over any temperature range it must be extrapolated. This is done using a resummation technique, usually a Padé or Padé-Borel approximant. However, in Ref. 1 we pointed out that the result of this resummation was highly dependent on the precise technique used. We found that writing the series as an expansion in the entropy and then resumming improved the self-consistency of the series, especially at low temperatures. We will therefore use this technique to calculate the specific heat and magnetization and then compare them with experimental data.

We will consider data for materials whose superconductivity is either quasi-two or three dimensional. An example of the former is a layered superconductor where the coupling between the layers has become small. The data will then be compared with the appropriate two- or three-dimensional Landau-Ginzburg theory results. The data has been plotted against a "reduced" temperature variable, which makes data at different fields and temperatures collapse on a universal curve.

We show that the theoretical two-dimensional magnetization compares very well with the experimental data of Li *et al.*[4] for the high-temperature superconductor Bi 2:2:2:3, which is a quasi-two-dimensional system. Moreover, after fitting the theoretical curve we are left with two relationships for $L_z$, $\kappa$ and, $B'$, the interlayer spacing, Landau-Ginzburg ratio, and the gradient of the $B_{c2}$ line. If we fix $L_z$ at the value determined by neutron scattering of 18.6 Å we obtain $\kappa \simeq 140$ and $B' \simeq 5$ T/K. A previous calculation by Li *et al.*[4] following the method of Hao and Clem[5] gave $\kappa = 170$ and the value of $B'$ used by Li *et al.* is 3.4 T/K, although they say that the plot is fairly insensitive to the value used. The comparison of the two-dimensional theoretical results with the specific-heat data of Urbach *et al.*[6] for the multilayer system of superconducting $Mo_{77}Ge_{23}$ separated by insulating amorphous germanium is less successful. We are unable to obtain agreement between theory and experiment. This may be because the experimental system cannot be adequately described by a disorder free two-dimensional model.

In three dimensions we have a reasonable fit to the specific-heat data for a conventional isotropic superconductor, niobium, as measured by Farrant and Gough.[7] However, we were completely unable to manipulate the anisotropic Ginzburg-Landau theory to fit the $YBa_2Cu_3O_{7-\delta}$ data of Welp *et al.*[8] over any sizable temperature range. We then considered using the Lawrence-Doniach model,[9] which takes the layered structure into account. We found that at high temperatures the system is, in fact, two dimensional, which means physically that the layers have become independent.

Finally, the resistivity data of Worthington *et al.*[10] predicts a melting criterion for the vortex lattice as a function of field and temperature. We have evaluated this criterion in terms of our reduced temperature variable.

## II. REVIEW OF LANDAU-GINZBURG THEORY RESULTS

To fix notations, we briefly describe anisotropic Landau-Ginzburg theory for a superconductor, where $\psi$, the wave function is our spatially dependent order parameter. For a fuller explanation and justification, see Refs. 1–3.

We start with the free-energy functional,

$$\frac{F[\psi(r)]}{k_B T_c}=\int d^3r\left[\alpha(T)|\psi|^2+\beta\frac{|\psi|^4}{2}+\sum_{\mu=1}^{3}\frac{|(-i\hbar\partial_\mu-2eA_\mu)\psi|^2}{2m_\mu}\right]+\frac{B^2}{2\mu_0}\ . \tag{2.1}$$

Here $\alpha(T)$ is a temperature variable, $\beta$ is the coupling constant, and $m_\mu$ is the effective mass. In the cases we consider the masses in the $ab$ plane are taken as equal and are denoted by $m_{ab}$, and the mass in the $c$ direction is written as $m_c$. The temperature dependence of $\alpha(T)$ is taken to be linear, $\alpha(T)=(T-T_c)\alpha'$. We also assume the lowest-Landau-level approximation, which is valid near the $H_{c2}$ line. If fluctuations in the vector potential $A_\mu$ are ignored, which again is a valid approximation near the $H_{c2}$ line, we can write the free energy in terms of the reduced temperature: $\alpha_h=\alpha+e\mu_0H\hbar/m_{ab}$. This is zero along the $H_{c2}$ line. With the use of Feynman diagrams it is now possible to evaluate the free energy as a perturbation series in terms of the coupling constant, $\beta$.[2,3] The free energy per unit volume in two dimensions is now

$$G_{2D}=\frac{k_BTe\mu_0H}{L_z\pi\hbar}g_{2D}(x)\ ,$$

where

$$g_{2D}(x)=\ln\left[\frac{\tilde{\alpha}}{\pi k_BT}\right]+f_{2D}(x)$$

and

$$\tilde{\alpha}=\left[\frac{\beta e\mu_0Hk_BT}{2\pi L_z\hbar x}\right]^{1/2}, \tag{2.2}$$

where $f_{2D}(x)$ is the perturbation series. Similarly in three dimensions,

$$G_{3D}=\left[\frac{e\mu_0Hk_BT\sqrt{2m_c\bar{\alpha}}}{\pi\hbar^2}\right][1+f_{3D}(x)]$$

$$=\left[\frac{e\mu_0Hk_BT\sqrt{m_c}}{\pi\hbar^2}\right]^{4/3}\beta^{1/3}g_{3D}(x)\ ,$$

where

$$g_{3D}(x)=\frac{[1+f_{3D}(x)]}{(2x)^{1/3}}$$

and

$$\bar{\alpha}=\left[\frac{\beta e\mu_0Hk_BT\sqrt{2m_c}}{8\pi\hbar^2x}\right]^{2/3}. \tag{2.3}$$

The field $H$ is in units $A/m$. The reduced temperature variable ($\alpha_h$ or $\alpha_T$, its dimensionless analog) is connected in two dimensions with the perturbation series variable $x$ by

$$\alpha_h=\left[\frac{\beta ek_BT\mu_0H}{L_z\pi\hbar}\right]^{1/2}\alpha_T,\quad \alpha_T=\frac{(1-4x)}{(2x)^{1/2}}\ , \tag{2.4}$$

and in three dimensions by

$$\alpha_h=\left[\frac{\beta e\mu_0Hk_BT\sqrt{2m_c}}{4\pi\hbar^2}\right]^{2/3}\alpha_T,\quad \alpha_T=\frac{(1-8x)}{(2x)^{2/3}}\ . \tag{2.5}$$

We will use the temperature variable $\alpha_T$ in our calculations. (This is similar to the variable $y$ in Refs. 11 and 12.) The temperature variable used on the experimental scaling plots is either this variable or proportional to it, so that the experimental data taken at different fields collapses onto one universal curve. The temperature dependence of the $\alpha_T$ variable is such that high temperature is represented by $\alpha_T\to\infty$, low temperature by $\alpha_T\to-\infty$, and $\alpha_T=0$ corresponds to being on the $H_{c2}$ line. The above treatment is valid where the lowest-Landau-level approximation can be trusted, which probably requires at least that $H>H_{c2}/f$, where $f=3$ according to Tešanovíc *et al.*[13] In general, scaling plots in which data at different fields and temperature collapse onto a single curve are associated with a phase transition but it is not being suggested that one is necessarily present here. The existence of scaling has been recently discussed by Ullah and Dorsey,[14] but was noted many years ago by Bray.[15]

We can now by differentiation of the free energy with respect to $\alpha_T$ obtain the dimensionless entropy, $s(\alpha_T)$ and from there invert the series to obtain $\alpha_T(s)$. The new series when resummed by Padé or Padé-Borel approximants was previously found to have better self-consistency than the original series, especially in the low-temperature regime and also compared most favorably with the available Monte Carlo data.[16] Moreover, in the zero-dimensional case which we could analyze exactly, this manipulation of the series after Padé or Padé-Borel resummation, was found to give the most accurate results, so we follow this procedure here.

The thermodynamic functions that we actually need for comparison with experimental data are the magnetization and specific heat. These are now easily calculable.

The specific heat is just the differential of the entropy with respect to $\alpha_T$. In $n$ dimensions (where $n=2$ or 3) we define the dimensionless entropy by

$$s_{n\mathrm{D}} = -\frac{dg_{n\mathrm{D}}(x)}{dx}\frac{dx}{d\alpha_T} \ . \tag{2.6}$$

The normalized specific heat is then given by

$$\frac{C}{\Delta C} = \beta_a \frac{ds_{n\mathrm{D}}}{dx}\frac{dx}{d\alpha_T} \ , \tag{2.7}$$

where $C$ is the specific heat, $\Delta C$ is the mean-field discontinuity in the specific heat, and $\beta_a$ is the Abrikosov factor. The mean-field theory should be a valid approximation at low temperatures so the normalized specific heat should approach one in this limit.

The magnetization within the LLL approximation is found to be

$$M = -\frac{e\hbar}{m_{ab}}\mu_0 \langle |\psi|^2 \rangle \tag{2.8}$$

as

$$\langle |\psi|^2 \rangle = \frac{\partial G_{n\mathrm{D}}}{\partial \alpha_H} \ . \tag{2.9}$$

In two dimensions this leads to the equation

$$\frac{M}{(TH)^{1/2}} = \left[ \frac{k e^3 \mu_0^3 \hbar}{\beta L_z \pi m_{ab}^2} \right]^{1/2} s_{2\mathrm{D}} \ , \tag{2.10}$$

where $s_{2\mathrm{D}}$ is just the dimensionless entropy. Hence we can use our method of resumming the perturbation series for the entropy to calculate the magnetization (within the LLL approximation). The right-hand side of Eq. (2.10) is a function of $\alpha_T$ only, enabling all experimental magnetization data in a range of fields and temperatures (taken within the LLL regime) to be collapsed onto one curve.

In three dimensions the LLL magnetization is

$$\frac{M}{(TH)^{2/3}} = \left[ \frac{8 e^5 \mu_0^5 k_B^2 m_c}{\hbar \pi^2 m_{ab}^3 \beta} \right]^{1/3} s_{3\mathrm{D}} \tag{2.11}$$

where again the right-hand side is a function of $\alpha_T$ only.

It should be noted that we have calculated all our quantities with appropriate Système International (SI) dimensions and converted the experimental data from cgs to SI.

It has been pointed out by Ikeda and Tsuento,[17] that starting from the Landau-Ginzburg theory of Eq. (2.1) corrections from higher Landau levels are quite large. The corrections they calculate are field dependent such that the right-hand sides of Eqs. (2.10) and (2.11) are no longer functions of $\alpha_T$ only, and so if large, these terms should prevent the experimental data from collapsing onto a single curve. Moreover, there are also corrections because of the limits of validity of Landau-Ginzburg theory, see, for example, Refs. 18–20. However, despite all these possible corrections the experimental data can be seen to scale in the manner suggested by the LLL approximation—indicating that the sum of all corrections must be small.

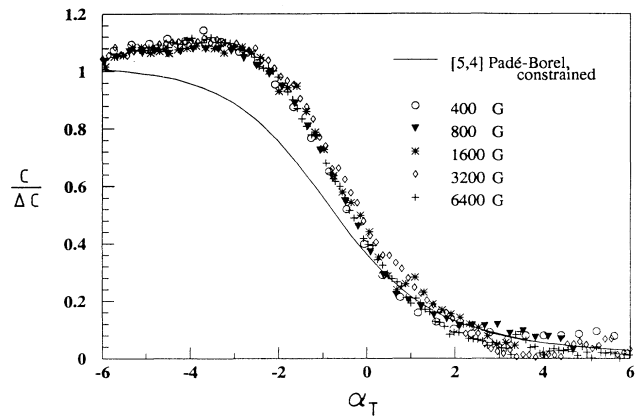


FIG. 1. Specific-heat data for $Mo_{77}Ge_{23}$ multilayer system and two-dimensional [5,4] Padé-Borel Landau-Ginzburg specific-heat curve, showing that the agreement is very poor.

An alternative method of scaling the thermodynamic properties has been used by Salamon *et al.*[21] which uses $XY$-like critical scaling to predict the scaling form for the data. However, in the region that the LLL approximation is valid our scaling collapses the data better and we additionally correctly predict the actual functional form.

## III. TWO-DIMENSIONAL COMPARISON

We have previously compared the two-dimensional Landau-Ginzburg theory with the specific-heat data for the multilayer material of superconducting $Mo_{77}Ge_{23}$ separated by insulating amorphous germanium[1] (see Fig. 1). This was unsatisfactory, possibly due to the different superconducting layers having different transition temperatures, or disorder playing a significant role in the material, which was not allowed for in the theoretical calcu-

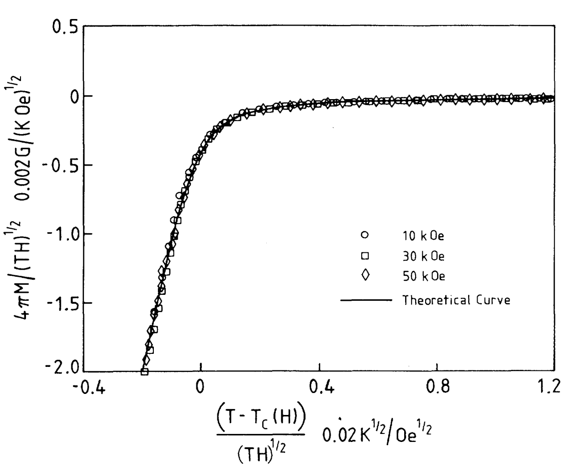


FIG. 2. Magnetization data for Bi 2:2:2:3 and [5,4] Padé-Borel Landau-Ginzburg magnetization curve, the integral of the curve used in the previous figure. The fit leads to a value of $\kappa \simeq 140$.

lations. We have now compared the magnetization data for Bi 2:2:2:3 ($Bi_2Sr_2Ca_2Cu_3O_{10}$), as measured by Li *et al.*,[4] with the theoretical results. This was much more successful. In fact we obtain such a good fit that the theoretical curve is hard to distinguish from the experimental data (see Fig. 2). The scaled experimental data is plotted as $M/(TH)^{1/2}$ versus $t=[T-T_c(H)]/(TH)^{1/2}$, where $t$ is related to $\alpha_T$ by

$$t=[(\beta k_B m_{ab}^2 \mu_0)/(e\hbar^3 \pi L_z)]^{1/2}\alpha_T/B' \ .$$

The constants in the functions for the magnetization and the temperature are well known apart from the system parameters $\kappa$ and $B'$ the slope of the $B_{c2}$ line. We therefore found the best fit by allowing ourselves freedom of scaling on the $x$ and $y$ axis and from there estimated $\kappa$ and $B'$. The value of $\kappa \simeq 140$ is similar to that of 170, calculated by Li *et al.* following the variational procedure of Hao and Clem.[5] The value of $B' \simeq 5$ T/K is different to that used in the scaling plot by Li *et al.* of 3.44 T/K. However, they state that the plot obtained is fairly insensitive to a fairly large range of $B'$ (Clearly it would be more satisfactory if one could replot the data using different values of $B'$ until the value inputted and that calculated from fitting the theoretical curve are consistent.)

Tešanović *et al.*[13] have also fitted this Bi 2:2:2:3 data using their nonperturbative Landau-Ginzburg approach. They also obtain a good fit but at the expense of an additional parameter.

## IV. THREE-DIMENSIONAL COMPARISON

In the three-dimensional case we compare the Landau-Ginzburg theory results with two very different superconductors—the conventional isotropic superconductor niobium and the high-$T_c$ superconductor $YBa_2Cu_3O_{7-\delta}$. For niobium we used the specific-heat data of Farrant and Gough,[7] and for $YBa_aCu_3O_{7-\delta}$ the magnetization data of Welp *et al.*[8] Niobium provided us with no problems, the data was already presented in terms of our temperature variable $\alpha_T$ so we could just superpose our theoretical curves (see Fig. 3). We found that the best fit was produced by the [5,4] Padé approximant, constrained to meet the mean-field value at low temperatures. The discrepancy at high temperatures is thought to be an artifact of the sample. (Farrant and Gough suggest that surface damage of the crystal causes the surface to go normal at a higher temperature than the bulk of the sample.) Overall, we found that the isotropic Landau-Ginzburg theory provides a reasonable fit to the specific-heat data for niobium.

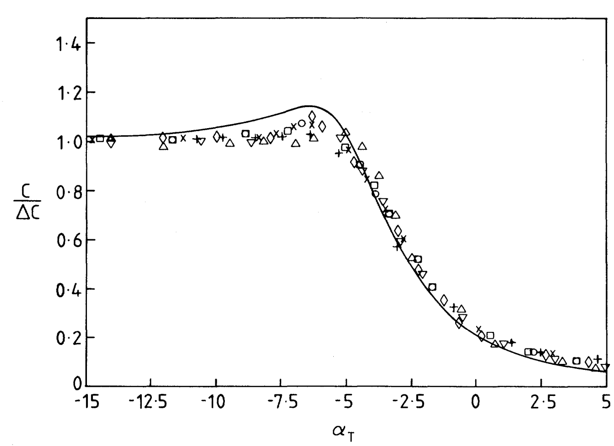


FIG. 3. Specific-heat data for niobium and three-dimensional [5,4] Padé approximant from Landau-Ginzburg theory. Deviations at high temperatures are thought to be due to surface damage of the sample.

We then considered the magnetization data for $YBa_2Cu_3O_{7-\delta}$ as measured by Welp *et al.*[8] The theoretical curve required is just the integral of the curve which we used successfully to fit the specific heat of niobium. In the case of niobium, we were left to choose which order of approximant gave the best fit, but in contrast, with $YBa_2Cu_3O_{7-\delta}$ we could not obtain a fit over any sizable temperature range at all. We allowed ourselves a scaling parameter on both the $x$ and $y$ axis, as we had done with Bi-2:2:2:3 previously. However, unlike Bi-2:2:2:3 this did not lead to the determination of the system parameters, as we were unable to find an adequate fit. Although anisotropic Landau-Ginzburg theory seemed a logical extension, it did not yield any improvements as it just allows for a rescaling of the $x$ axis, which we were already allowing to vary arbitrarily anyway.

The introduction of Gaussian disorder, using the calculation of Fujita, Hikami, and Larkin[22] did not provide a suitable correction to the theoretical curve either. We were left with the possibility that the disorder present was important but not well approximated by assuming it to be Gaussian or that the three-dimensional continuum model was not appropriate.

The latter seemed a plausible explanation: $YBa_2Cu_3O_{7-\delta}$ is a layered structure and we had been assuming that the correlation length would be sufficiently large that the system could be treated as three dimensional. The Lawrence-Doniach model is appropriate for such a layered structure: It is a Landau-Ginzburg-type model in which the (infinitely thin) layers perpendicular to the $c$ axis interact via Josephson coupling. The free energy for this model has been calculated to leading order by Ullah and Dorsey.[14] The resulting equation for the magnetization is

$$M=-\left[\frac{k_B T e^2 \mu_0^2 H\sqrt{2m_c}}{2m_{ab}\pi\hbar}\right]\frac{1}{\tilde{\alpha}^{1/2}[1+(m_c L_z^2/2\hbar^2)\tilde{\alpha}]^{1/2}} \ . \tag{4.1}$$

At high temperatures this reduces to the ordinary two-dimensional Landau-Ginzburg theory, with the form $M \propto 1/\tilde{\alpha}$. (Compare this with the three-dimensional form $M \propto 1/\tilde{\alpha}^{1/2}$.) We found that by using this two-dimensional expression we could fit the experimental data for $\alpha_T$ far better than we could with the three-dimensional one. This can be seen clearly in Fig. 4 where we have superposed on the experimental data the best three-dimensional fit using the series expansion and also the leading-order two-dimensional expression. The implication is that a high temperature $YBa_2Cu_3O_{7-\delta}$ behaves

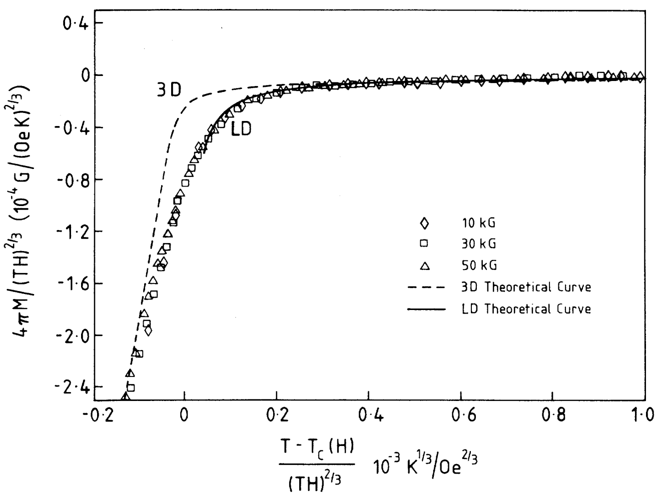


FIG. 4. Magnetization data for $YBa_2Cu_3O_{7-\delta}$. At high temperatures we demonstrate that the Lawrence-Doniach model provides the best fit to the data, that is $M \propto 1/\tilde{\alpha}$, cf. the three-dimensional form $M \propto 1/\tilde{\alpha}^{1/2}$. At low temperatures we show that it is possible to obtain a fit over a small range of temperatures using the three-dimensional [5,4] Padé approximant.

two dimensionally, in that the layers are isolated with the correlation length less than the interlayer spacing. As we are considering the three-dimensional scaling plot there should be some residual field dependence at high temperatures, of the form $(HT)^{-1/6}$. However, at high temperatures it is not possible to see any field dependence in the available data. This is not surprising as the magnitude of the discrepancy would be less than that of the symbols used.

Physically, the two-dimensional behavior of $YBa_2Cu_3O_{7-\delta}$ at high temperatures means that the correlation length in the $c$ direction must be less than the spacing between the superconducting copper-oxide planes, so that the layers are acting independently. At high temperatures an estimate of the correlation length can be found using $\xi_c \hbar/\sqrt{2m_c\tilde{\alpha}}$, which gives $\xi_c$ of the order of a few Angstrom. This is less than the interlayer spacing of $\sim 10$ Å.

We were then left with the problem of understanding what happens as the temperature is reduced. In the vicinity of the $H_{c2}$ line, (i.e., $\alpha_T \simeq 0$), neither the first-order Lawrence-Doniach approximation nor the three-dimensional Landau-Ginzburg theory can be made to fit. However, it is possible that a higher-order Lawrence-Doniach approximation would yield a fit. At low temperatures we can obtain a poor fit to the data using the three-dimensional series. It should be stressed that we are not saying that this convincingly shows that the system is three dimensional at low temperatures. However, we thought it amusing to calculate the system parameters, $\kappa_{ab}$ and the mass anisotropy ratio, $m_c/m_{ab}$ associated with this fit. This is in the light of the broad range of values encountered when trying to establish where the melting curve lay on the magnetization plots (see below). The values obtained using the [5,4] Padé approximant curve were of the order of $\kappa \simeq 65$ and $m_c/m_{ab} \simeq 1$. The value of $\kappa$ is well within the range quoted,[23–26] although the value of $m_c/m_{ab}$ is low, but is very sensitive to the fit chosen. The method used previously for estimating the correlation length predicts $\xi \sim 40$ Å, that is 3–4 layer spacings, at $\alpha_T \sim -10$, but the formula will only be a very poor approximation at these temperatures.

It is clear from just looking at the experimental data that we cannot obtain a good fit using the three-dimensional curve because the gradient of the experimental magnetization data is constant over a large temperature range, at low temperatures, whereas the gradient changes in the theoretical curve, as consistent with the specific-heat curve, (simple differential) which has a peak. This appears to be at odds with recent experimental data of Inderhees *et al.*[12] They have measured the specific heat of $YBa_2Cu_3O_{7-\delta}$ in an applied magnetic field, and it has a distinctive peak as the temperature is varied. They find that the simplest form of the three-dimensional scaling result does not fit the data. This is in agreement with our results which indicate that at high temperatures the two-dimensional form should be used and that near $H_{c2}$ the layered structure must be considered. However, since the scaling result works well in niobium, we believe that the failure to scale in $YBa_2Cu_3O_{7-\delta}$ is much more likely to be due to the layered structure than a need to use the crossover form postulated in Refs. 12 and 21.

To summarize, our analysis shows that at high temperatures $YBa_2Cu_3O_{7-\delta}$ is best fitted by the two-dimensional Landau-Ginzburg theory. As the temperature is reduced the correlation length grows and it is necessary to consider the interaction between layers. At low temperatures the system can be considered a continuum, but it is not certain yet whether this temperature is above or below the melting temperature.

## V. MELTING TEMPERATURE

It is widely believed that in the high-$T_c$ superconductors the flux lattice will melt at a temperature $T_M$ below the mean-field transition temperature $T_{c2}$. The resulting phase above $T_M$ would be a vortex liquid dominated by thermal fluctuation. Experimental investigations of $YBa_2Cu_3O_{7-\delta}$ have included many studies of the resistivity versus current in different applied magnetic fields. It is thought that one of the kinks in the data is associated with the melting temperature, see Worthington *et al.*[10] for details. From this data a melting curve of field versus reduced temperature can be plotted. We thought it would be interesting to get an estimate of where the melting temperature fitted on the scaling plots for the specific heat and magnetization. As the temperature $\alpha_T$ is universal this would also suggest where a melting transition in niobium might occur. Moreover, it might reveal whether the transition occurred in the region well approximated by the LLL.

We started with the resistivity data of Worthington *et al.*,[10] which suggests the relation $H \propto (T_c - T)^{3/2}$ for the melting curve. We have estimated the constant of proportionality by fitting to their data and obtain the relation $\mu_0 H = 0.155(T_c - T)^{3/2}$, with $T_c \sim 93.5$ K and $H$ measured in A/m. This is similar to the relation derived by Kwok *et al.*[25] They have $T_c \simeq 93.5$ K and $\mu_0 H = 103(1-T/T_c)^{1.41}$, which can also be written as $\mu_0 H = 0.174(T_c - T)^{1.41}$. We are interested in an estimate and so have only used the data of Worthington *et al.* to give us a value of $T_M$. The corresponding values of $\alpha_{T_M}$ were evaluated by combining the relation for $\alpha_T$ in terms of $\alpha_h$ with the linear approximation for the upper critical field line $T_c(H) = T_c + H/H'$.

The accuracy of the approximation now relies on the value of $\kappa_{ab}$ and the ratio of the effective masses $m_c/m_{ab}$ (a measure of the anisotropy of the system). These are quantities that are deduced from experiments, and are not universally agreed upon. We have considered the value $m_c/m_{ab} = 59$ of Farrell *et al.*[26] and $\kappa_{ab} = 55$ and also those of Welp *et al.*[24] and Krusin-Elbaum *et al.*,[23] $\kappa_{ab} = 85$ and $m_c/m_{ab} = 29$. These produce melting temperatures of the order of $\alpha_T \simeq -8.5$ and $\alpha_T \simeq -6$, respectively, although the actual values are slightly field dependent.

## VI. DISCUSSION

This paper has considered specific examples of the application of Landau-Ginzburg theory to superconductors which are either quasi-two or three dimensional. From these studies we are led to some interesting assertions for high-temperature superconductors in general.

We consider first the quasi-two-dimensional case, where the theoretical Landau-Ginzburg magnetization is found to fit the experimental data well. This could be used in future to calculate the system parameter $\kappa$. The method would be to scale the data with a value of $B$, the slope of the $H_{c2}$ line, consistent with that produced by the best fit of the theoretical data to the experimental results. This would completely determine the value of $\kappa$. The advantage of this procedure is that $\kappa$ would be derived from a theory which is known to be valid in the region in which it is applied. This should be an improvement on calculating $\kappa$ using mean-field theory expressions. However, as mean-field theory is no longer valid, the $\kappa$ we calculate will be a system parameter but not necessarily the ratio $\lambda/\xi$.

In three dimensions for high-temperature superconductors near the $H_{c2}$ line, the layered structure cannot be accurately treated as a continuum. Calculation of the Lawrence-Doniach model to beyond leading order is the obvious next step, although this will not necessarily be successful as it is still a fairly crude approximation to the actual structure of the high-temperature superconductors.

Finally, the scaling of recent experimental data of Welp[27] appears to work down to the temperatures at which it has been suggested that the vortex lattice melts. This is important because the data scaling indicates that the lowest-Landau-level approximation is still valid in fields above 5 T at these temperatures. However, many theories, for instance Nelson and Seung,[28] have started with the assumption that it is reasonable to treat melting as happening within the London regime.

## ACKNOWLEDGMENTS

One of us (N.K.W.) acknowledges the S.E.R.C. for financial support. We would also like to thank U. Welp and Q. Li and colleagues for the use of their data.

[1] N. K. Wilkin and M. A. Moore, Phys. Rev. B **47**, 957 (1993).
[2] G. J. Ruggeri and D. J. Thouless, J. Phys. F **6**, 2063 (1976).
[3] E. Brézin, A. Fujita, and S. Hikata, Phys. Rev. Lett. **65**, 1949 (1990); **65**, 2921 (E) (1990).
[4] Q. Li, M. Suenaga, T. Hikata, and K. Sato, Phys. Rev. B **46**, 5857 (1992).
[5] Z. Hao and J. R. Clem, Phys. Rev. Lett. **67**, 2371 (1991).
[6] J. S. Urbach, W. R. White, M. R. Beasley, and A. Kapitulnik, Phys. Rev. Lett. **69**, 2407 (1992).
[7] S. P. Farrant and C. E. Gough, Phys. Rev. Lett. **34**, 943 (1975).
[8] U. Welp, S. Fleshler, W. K. Kwok, R. A. Klemm, V. M. Vinokur, J. Downey, B. Veal, and G. W. Crabtree, Phys. Rev. Lett. **67**, 3180 (1991).
[9] W. E. Lawrence and S. Doniach, in *Proceedings of the Twelfth International Conference on Low-Temperature Physics,* edited by E. Kanda (Academic, Kyoto, 1971), p. 361.
[10] T. K. Worthington, M. P. A. Fisher, D. A. Huse, J. Toner, A. D. Marwick, T. Zabel, C. A. Feild, and F. Holtzberg, Phys. Rev. B **46**, 11 854 (1992).
[11] S. Hikami and A. Fujita, Phys. Rev. B **41**, 6379 (1990).
[12] S. E. Inderhees, M. B. Salamon, J. P. Rice, and D. M. Ginsberg, Phys. Rev. B **47**, 1053 (1993).
[13] Z. Tešanović, L. Xing, L. Bulaevskii, and M. Suenaga, Phys. Rev. Lett. **69**, 3563 (1992).
[14] S. Ullah and A. T. Dorsey, Phys. Rev. B **44**, 262 (1991).
[15] A. J. Bray, Phys. Rev. B **9**, 4752 (1974).
[16] J. A. O'Neill and M. A. Moore, Phys. Rev. Lett. **69**, 2582 (1992); Phys. Rev. B **48**, 374 (1993).
[17] R. Ikeda and T. Tsuneto (unpublished).
[18] R. E. Prange, Phys. Rev. B **1**, 2349 (1970).
[19] J. Kurijärvi, V. Ambegaokar, and G. Eilenberger, Phys. Rev. B **5**, 868 (1972).
[20] P. A. Lee and M. G. Payne, Phys. Rev. B **5**, 923 (1972).
[21] M. B. Salamon, W. Lee, K. Ghiron, J. Shi, N. Overend, and M. A. Howson (unpublished).
[22] A. Fujita, S. Hikami, and A. I. Larkin, Physica C **185**, 1883 (1991).
[23] L. Krusin-Elbaum, R. L. Greene, F. Holzberg, A. P. Malozemoff, and Y. Yeshurun, Phys. Rev. Lett. **62**, 217 (1989).
[24] U. Welp, W. K. Kwok, G. W. Crabtree, K. G. Vandervoort, and J. Z. Liu, Phys. Rev. Lett. **62**, 1908 (1989).
[25] W. K. Kwok, S. Fleshler, U. Welp, V. M. Vinokur, J. Downey, G. W. Crabtree, and M. M. Miller, Phys. Rev. Lett. **69**, 3370 (1992).
[26] D. E. Farrell, J. P. Rice, D. M. Ginsberg, and J. Z. Liu, Phys. Rev. Lett. **64**, 1573 (1990).
[27] U. Welp (private communication).
[28] D. R. Nelson and H. S. Seung, Phys. Rev. B **39**, 9153 (1989).