Critical dynamics of one-dimensional long-range exchange models

Adriana G. Moreira, J. Kamphorst Leal da Silva, and F. C. Sá Barreto
Departamento de Física, Instituto de Ciências Exatas, Universidade Federal de Minas Gerais, Caixa Postal 702, 30161-970 Belo Horizonte, Brazil
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One-dimensional kinetic Ising models with long-range exchange of two spins with opposite signs are considered. The exchange probability is a power-law function of the distance between exchanged spins. Rigorous lower bounds for the dynamical exponent z are obtained from the initial response and the scaling hypothesis of the relaxation time. The dependence of the z exponent on the maximal exchange distance and on the exchange probability is studied. Nonuniversal behavior in the ferromagnetic model with two different alternating coupling constants is found. The z exponent is also obtained from the domain-wall argument.

I. INTRODUCTION

Kinetic Ising models[1] are the simplest models exhibiting nontrivial critical dynamical behavior. They can be described as an Ising model in contact with a heat bath, which is responsible for a stochastic dynamics. Essentially, we have two types of dynamics: (A) the dynamics without order-parameter conservation and (B) the dynamics with order-parameter conservation. The Ising model with Glauber dynamics[1] is used to describe systems, such as magnets, in which the order parameter relaxes to its equilibrium value. Systems with order-parameter conservation, such as phase separation in binary alloys, are described by Kawasaki dynamics,[2] in which two nearest-neighbor spins of opposite signs are exchanged. Note that the conservation law occurs on a local scale. It is well known that the systems with Kawasaki dynamics are in a dynamical universality class different from the one constituted by systems with Glauber dynamics, because of the existence of conservation laws.[3]

Recently, a long-range exchange model[4] has been introduced in order to describe systems in which the order-parameter conservation occurs in a global scale. The spinodal decomposition in a binary fluid mixture including convection flow of particles is an example of such systems.[5] In this system the growth law for the ordering process is different from that for a conserved order-parameter system described by the usual Kawasaki dynamics.[4–6] In fact, it has shown by numerical simulations that the global conservation law changes the dynamical exponent and it is very important for the determination of the dynamical universality class.[7] In order to investigate the effects of the conservation laws only in a global scale, a kinetic Ising model with the exchange probability being a power law of the distance between the exchanged spins has been proposed.[8] A generalized time-dependent Ginzburg-Landau equation has been derived by using a coarse-grained picture and, in particular, the critical behavior has been studied by the renormalization-group method.

In this work we consider the critical dynamics of the one-dimensional kinetic Ising model with a Kawasaki dynamics. The exchange probability between two spins with opposite signs is a function of the distance r between them, as already proposed.[8] However, we introduce a slight modification: The exchanged spins can be separated up to a maximal distance R. This problem is analyzed by means of the initial response,[9] and we obtain rigorous lower bounds for the critical dynamical exponent z. At high temperature, we solve exactly the equations of motion for the local order parameter. It is worth mentioning that one-dimensional problems are important in critical dynamics because they can be studied analytically without using the dynamical renormalization-group method, which presents some problems in this context.[10,11] We also consider the dynamical behavior of ferromagnetic models with two different alternating coupling constants. It is well known that this model has a nonuniversal z exponent depending on the ratio of the couplings.[12,13] However, one could expect a universal exponent when a spin can exchange with every one of the spins of the chain because now the system can choose the fastest way to relax. Moreover, we also obtain the z exponent by means of a generalization of the domain-wall argument (DWA).[14] Although the DWA is supposed to give an upper bound to the dynamical exponent, it turns out that it predicts the exact value of z in one dimension.[12] This paper is organized as follows. In the next section the dynamical model is defined and some equilibrium properties are discussed. The initial response is briefly discussed in Sec. III. In Sec. IV we present the high-temperature behavior, the lower bounds of the z exponent, and the evaluation of the z exponent by means of the DWA for the model with one constant coupling. We discuss the dynamical behavior of the alternating coupling constants model by using the initial response and the DWA argument in Sec. V. Finally, a brief discussion of the problem in d dimensions and our concluding remarks are presented in the last section.

II. LONG-RANGE EXCHANGE DYNAMICAL MODEL

We consider the one-dimensional ferromagnetic Ising model, defined by the following Hamiltonian:

$$-\beta H = \sum_{j=1}^{N} K_j s_j s_{j+1} \ , \qquad (2.1)$$

where $\beta = 1/k_B T$, $s_j = \pm 1$, and $\{K_j = J_j/k_B T\}$ are the reduced ferromagnetic couplings. Here we consider $J_j = J_1$ for j odd and $J_j = J_2$ for j even with $J_1 \geq J_2$. The static properties are easily obtained. The critical temperature is $T_c = 0$. The correlation length near $T = 0$ depends only on the weak interaction J_2, i.e.,

$$\xi \sim \exp[2K_2] \ . \qquad (2.2)$$

For further use, we note that some of the correlation functions are given by

$$\langle s_i s_j \rangle = \prod_{n=i}^{j-1} \tanh(K_n) \ , \qquad (2.3)$$

$$\langle s_i s_j s_k s_l \rangle = \prod_{n=i}^{j-1} \tanh(K_n) \prod_{r=k}^{l-1} \tanh(K_r) \ , \qquad (2.4)$$

where the sites considered satisfy the relation $l > k > j > i$. The Fourier transform $s_{\mathbf{q}}$ of the spins is defined by

$$s_{\mathbf{q}} = \frac{1}{\sqrt{N}} \sum_{m=1}^{N} s_m \exp(i\mathbf{q} \cdot \mathbf{a}m) \ , \qquad (2.5)$$

where $\mathbf{a}$ is the lattice spacing. Using (2.3) it is easy to show that the structure factor $S(\mathbf{q})$ is given by

$$S(\mathbf{q}) = \frac{(1 - \Gamma_1\Gamma_2)[1 + \Gamma_1\Gamma_2 + (\Gamma_1 + \Gamma_2)\cos(qa)]}{1 - 2\Gamma_1\Gamma_2\cos(2qa) + \Gamma_1^2\Gamma_2^2} \ , \qquad (2.6)$$

where $\Gamma_n = \tanh(K_n)$ with $n = 1, 2$. The fluctuation-dissipation theorem relates it to the static susceptibility $\chi_{\mathbf{q}}$ by $S(\mathbf{q}) = k_B T \chi_{\mathbf{q}}$. The structure factor diverges at the critical temperature for the critical wave vector $\mathbf{q}_c = 0$ as $S(0) \sim \xi$.

The Kawasaki dynamics with long-range exchange of spins is given by the following master equation:

$$\frac{\partial P(\{s\};t)}{\partial t} = \sum_{r=1}^{R} \sum_{j=1}^{N-r} [-w_{j,j+r}(s_j, s_{j+r}) P(\ldots, s_j, \ldots, s_{j+r} \ldots; t) + w_{j,j+r}(s_{j+r}, s_j) P(\ldots, s_{j+r}, \ldots s_j, \ldots; t)] \ , \qquad (2.7)$$

where $P(\{s\};t)$ is the probability density that the configuration $\{s_1, s_2 \ldots, s_N\}$ is realized at time t, $w_{j,j+r}(s_j, s_{j+r})$ is the transition rate of the $\{s_1, \ldots, s_j, \ldots s_{j+r} \ldots\}$ configuration to the $\{s_1 \ldots, s_{j+r} \ldots s_j, \ldots\}$ one, and R is the maximal distance of two exchanged spins. Note that only spins with opposite signs are exchanged (i.e., $w_{j,j+r}(s_j, s_{j+r}) = 0$ if $s_j = s_{j+r}$), implying that the total magnetization is constant.

In order to assure the equilibrium distribution at long times, the transition rates must obey the detailed balance condition, namely,

$$w_{j,j+r}(s_j, s_{j+r}) P_{\text{eq}}(\ldots, s_j, \ldots, s_{j+r} \ldots) = w_{j,j+r}(s_{j+r}, s_j) P_{\text{eq}}(\ldots, s_{j+r} \ldots s_j, \ldots) \ . \qquad (2.8)$$

Here $P_{\text{eq}}(\{s\})$ is the Gibbs probability of the $\{s\}$ configuration. This condition determines only partially the transition rates. So we can choose $w_{j,j+r}(s_j, s_{j+r})$ in several forms. In the next sections we will choose the standard transition rates.

Let us define the function $\langle s_k(t) \rangle$ as

$$\langle s_k(t) \rangle = \sum_{\{s\}} s_k P(s_1 \ldots s_k \ldots s_N; t) \ , \qquad (2.9)$$

in order to consider the time evolution of the local magnetization. The equation for the evolution of this object is obtained by multiplying both sides of Eq. (2.7) by s_k and summing over all configurations. After some standard algebra steps we obtain

$$\frac{\partial \langle s_k(t) \rangle}{\partial t} = -2 \sum_{r=1}^{R} \sum_{j=1}^{N-r} [\ \langle s_k w_{k,k+r}(s_k, s_{k+r}) \rangle \delta_{j,k} + \langle s_k w_{k-r,k}(s_{k-r}, s_k) \rangle \delta_{j+r,k}] \ . \qquad (2.10)$$

Unfortunately this equation cannot be solved exactly because correlation functions of higher order will appear in the right-hand side of the equation.

III. INITIAL RESPONSE RATE

In this section we establish a rigorous lower bound for the relaxation time from the initial response rate of the system. For the sake of completeness we discuss briefly the initial response method[9] and generalize it to long-range exchange models. Let us define a function ϕ by the relation $P(\{s\}, t) = P_{\text{eq}}(\{s\})\phi(\{s\}, t)$. Then the master equation (2.7) can be written as

$$\frac{\partial}{\partial t}\phi(\{s\}, t) \ = \ -L\phi(\{s\}, t) \ , \qquad (3.1)$$

where the L operator is defined by

$$L\phi(\{s\},t) = \sum_{r,j} w_{j,j+r}(s_j, s_{j+r}) \times [\phi(\dots s_j \dots s_{j+r} \dots, t) - \phi(\dots s_{j+r} \dots s_j \dots, t)] \ . \quad (3.2)$$

From the detailed balance condition (2.8) we can show that the Hermitian L operator has the properties $\langle f^* L g\rangle_{\text{eq}} = \langle g L f^*\rangle_{\text{eq}}$ and $\langle g^* L g\rangle_{\text{eq}} \geq 0$, where f and g are arbitrary functions of $\{s\}$. Thus the eigenvalues ν_i of L are real and non-negative. We consider the time-dependent autocorrelation function of g,

$$C_g(t) = \langle g^*[0]g[t]\rangle_{\text{eq}} - \langle g^*[0]g[\infty]\rangle_{\text{eq}}, \quad (3.3)$$

where $g[t] = \exp(-Lt)g$. It has a spectral representation of the form

$$C_g(t) = \int_0^\infty \varphi(\nu)\exp(-\nu t)d\nu, \quad (3.4)$$

with $\varphi(\nu) \geq 0$ for all ν. The *characteristic time* (τ_g) and the *initial relaxation rate* (ν_g) for the variable g are defined by

$$\tau_g = C_g(0)^{-1}\int_0^\infty C_g(t)dt \ , \quad (3.5)$$

$$\nu_g = -C_g(0)^{-1}\frac{d}{dt}C_g(t)\Big|_{t=0} \ . \quad (3.6)$$

We can write these equations in the spectral representation and apply the Schwartz inequality in order to obtain the following inequality:

$$\tau_g \geq \nu_g^{-1} \ . \quad (3.7)$$

If we put $g = s_{\mathbf{q}}$, τ_g is the *relaxation time of the system* for $\mathbf{q} \to 0$. In order to use inequality (3.7) we must evaluate the initial rate ($\nu_{s_{\mathbf{q}}}$). This is easily done and we find that

$$\tau_q \geq \frac{k_B T \chi_{\mathbf{q}}}{\frac{4}{N}\sum_{r=1}^{R}\sum_{j=1}^{N-r}[1-\cos(\mathbf{q}\cdot\mathbf{a}_r)]\langle w_{j,j+r}(s_j,s_{j+r})\rangle_{\text{eq}}} \ , \quad (3.8)$$

where $\chi_{\mathbf{q}}$ is the static susceptibility and $\mathbf{a}_r$ is the spacing vector between sites j and $j+r$. In the high-temperature limit this inequality becomes an equality (conventional theory).[3] The physical reason is that in this limit ($T \to \infty$) a spin behaves independently of the others. Thus, the initial relaxation time is equal to the asymptotic one. In order to obtain a lower bound to the z exponent we must evaluate $\langle w_{j,j+r}(s_j, s_{j+r})\rangle_{\text{eq}}$ and assume the scaling hypothesis[3] for the relaxation time

$$\tau_q = \xi^z f(\mathbf{q}\xi) \quad \text{for} \quad \mathbf{q} \to \mathbf{0} \ . \quad (3.9)$$

IV. ISOTROPIC MODEL

Let us consider the isotropic ferromagnetic model ($K_j = K$ for all j). The transition rates are the generalization of the Kawasaki ones for the long-range model. The exchange between two nearest-neighbor spins ($r = 1$) is described by the following transition rate:

$$w_{j,j+1}(s_j, s_{j+1}) = \frac{\alpha C}{4}(1 - s_j s_{j+1})\left[1 - \gamma\frac{s_j}{2}(s_{j-1} - s_{j+2})\right] \ . \quad (4.1)$$

Here $\gamma = \tanh(2K)$. For $r \geq 2$ the transition rates can be written as

$$w_{j,j+r}(s_j, s_{j+r}) = \frac{\alpha C}{4r^\mu}(1 - s_j s_{j+r})\{ 1 - c_1 s_j(s_{j-1} + s_{j+1} - s_{j+r-1} - s_{j+r+1}) + c_3 s_j[s_{j-1}s_{j+1}(s_{j+r-1} + s_{j+r+1}) - s_{j+r-1}s_{j+r+1}(s_{j-1} + s_{j+1})]\} \ , \quad (4.2)$$

where $c_1 = (1/8)[\tanh(4K) + 2\tanh(2K)]$ and $c_3 = (1/8)[\tanh(4K) - 2\tanh(2K)]$. In the last two equations, C is a normalization constant evaluated in such way that $2/\alpha$ is the relaxation time for a spin in high temperature. It is easy to show that

$$C^{-1} = \sum_{r=1}^{R}\frac{1}{r^\mu} \ . \quad (4.3)$$

The standard Kawasaki dynamics is obtained by setting $R = 1$ or by considering $R > 1$ and $\mu \to \infty$. When $R > 1$ and μ is finite we describe a model in which a spin can be exchanged with several others; in this case μ determines how the probability of exchange depends on the distance between the exchanged spins. The generalized Kawasaki model with exchange of spins separated by an arbitrary distance[7] is obtained by setting $R = N - 1$ and $\mu = 0$.

At high temperatures we have $c_3 = c_1 = \gamma = 0$. In this limit, the transition rates (4.1) and (4.2) are given by $w_{j,j+r}(s_j, s_{j+r}) = \alpha C(1 - s_j s_{j+r})/4r^\mu$. The equations for the evolution of $\langle s_k(t)\rangle$ [Eq. (2.10)] can now be exactly solved. We do it now for further comparison with the results of the initial response. For simplicity, let us consider the case with $\mu = 0$ on the *closed chain* (i.e., $s_{N+k} = s_k$). The equations for $R < N$ are given by

$$\frac{d\langle s_k(t)\rangle}{dt} = -\frac{\alpha C}{2}\left\{2R\langle s_k(t)\rangle - \sum_{r=1}^{R}[\langle s_{k+r}(t)\rangle + \langle s_{k-r}(t)\rangle]\right\}, \quad k = 1, \dots, N \ . \quad (4.4)$$

These equations can be solved by using the Fourier transforms defined in Eq. (2.5). We can write that

$$\frac{d\langle s_{\mathbf{q}}(t)\rangle}{dt} = -\frac{\alpha}{R}\langle s_{\mathbf{q}}(t)\rangle\left\{R - \sum_{n=1}^{R}\cos(qna)\right\} \quad . \tag{4.5}$$

Here the term in $\cos(qna)$ is the signature of the exchange between spins separated by a distance n. Therefore, we have that $\langle s_{\mathbf{q}}(t)\rangle = \langle s_{\mathbf{q}}(0)\rangle \exp[-t/\tau(q)]$, with the relaxation time given by

$$\tau(q) = \frac{R}{\alpha}\left[R - \sum_{n=1}^{R}\cos(qna)\right]^{-1},$$
$$\tau(q) \sim \frac{12}{\alpha q^2 a^2 (R+1)(2R+1)} \quad \text{for} \quad q \to 0 \quad . \tag{4.6}$$

Comparing this result with the next-nearest-neighbor exchange (set $R = 1$), we see that the system now relaxes faster. The relaxation time diverges as $q \to 0$ because the total magnetization is constant.

We consider now the case in which a spin can be arbitrarily exchanged with any other spin of the *open chain* (the result for the closed chain is the same). For $T \to \infty$ we have that

$$\frac{d\langle s_k(t)\rangle}{dt} = -\frac{\alpha C N}{2}\{\langle s_k(t)\rangle - m\} \quad , \tag{4.7}$$

where $m = \sum_{j=1}^{N}\langle s_j(t)\rangle/N$ is a constant independent of time. Without loss of generality we can choose $m = 0$ and obtain that $\langle s_k(t)\rangle = \langle s_k(0)\rangle \exp(-\alpha t/2)$. Each spin now relaxes in the same way with the smallest relaxation time, namely, $2/\alpha$. In terms of the Fourier transform we have that $\langle s_{\mathbf{q}}(t)\rangle = \langle s_{\mathbf{q}}(0)\rangle \exp[-t/\tau(q)]$, with $\tau(q)$ given by

$$\tau(q) = \frac{2}{\alpha} \quad \text{for} \quad q \neq 0 \quad , \tag{4.8}$$

$$\tau(0) \to \infty \quad . \tag{4.9}$$

Since one spin can be exchanged with any other spin of the lattice with the same rate, all the modes q relax in the same way with the fastest relaxation time, namely, $2/\alpha$. The $q = 0$ mode, however, is frozen $[\tau(0) \to \infty]$. Note that we cannot obtain the relaxation time of the $q = 0$ mode by evaluating the relaxation time of the mode q in the limit $q \to 0$ because $\tau(q)$ *is not continuous at* $q = 0$. It is worth mentioning that these exact solutions at high temperature are valid for any dimension d.

The high temperature and the critical behavior will now be analyzed by the initial response. In order to use the inequality (3.8) we must evaluate $\langle w_{j,j+r}(s_j, s_{j+r})\rangle_{\text{eq}}$, with the transition rates defined in Eqs. (4.1) and (4.2). Using (2.3) and (2.4) for the equilibrium correlation functions we obtain that

$$\langle w_{j,j+1}(s_j, s_{j+1})\rangle_{\text{eq}} = \frac{\alpha C}{4}[1 - \Gamma(1)][1 - \gamma\Gamma(1)] \quad , \tag{4.10}$$

$$\langle w_{j,j+r}(s_j, s_{j+r})\rangle_{\text{eq}} = \frac{\alpha C}{4r^{\mu}}\Big\{ 1 - \Gamma(r) - 2c_1[2\Gamma(1) - \Gamma(r+1) - \Gamma(r-1)] + 2c_3[\Gamma(r+1) + \Gamma(r-1) - 2\Gamma(3)]\Big\} \quad \text{for} \quad r \geq 2. \tag{4.11}$$

Here $\Gamma(m) = \tanh^m(K)$.

We consider first the high-temperature limit. When $T \to \infty$ we have that $\langle w_{j,j+r}(s_j, s_{j+r})\rangle_{\text{eq}} = \alpha C/4r^{\mu}$ for $r \geq 1$. In this limit, the inequality (3.8) becomes an equality, namely,

$$\tau_q = \frac{1}{N^{-1}\alpha C \sum_{r=1}^{R}\sum_{i=1}^{N-r} r^{-\mu}[1 - \cos(qar)]} \quad . \tag{4.12}$$

Here we have used that $k_B T \chi_{\mathbf{q}} = 1$. In the case of R finite and smaller than N, we expand the term $\cos(qar)$ near the critical wave vector $q_c = 0$ to obtain the following relaxation time:

$$\tau_q = \frac{2N\sum_{r=1}^{R} r^{-\mu}}{\alpha (qa)^2 \sum_{r=1}^{R} r^{2-\mu}(N - r)} \quad . \tag{4.13}$$

Note that for $\mu = 0$ and $N \gg 1$, this result agrees with the exact one (4.6). The case of the arbitrary exchanges ($R = N - 1$ and $\mu = 0$) must be carefully analyzed. Now we cannot expand the term $\cos(qar)$ for $q \to 0$ and we must evaluate the denominator of Eq. (4.12). We have that $\sum_{r=1}^{N}\cos(qar) = N\delta_{q,0}$ and $\sum_{r=1}^{N-1} r\cos(qar) = -N/2$. Here, $\delta_{q,0}$ is the Kronecker delta. After trivial steps we obtain a relaxation time

$$\tau_q = \frac{2}{\alpha} \quad \text{for} \quad q \neq 0 \quad , \tag{4.14}$$

which is the same as the exact one (4.8). Note that a wrong result has been obtained in Ref. 15 for this case because an expansion for $q \to 0$ has been made.

Now, let us consider the behavior near the critical temperature ($T_c = 0$). We obtain from (4.10) and (4.11) that $\langle w_{j,j+1}(s_j, s_{j+1})\rangle_{\text{eq}} \sim \alpha C \exp(-4K)$ and $\langle w_{j,j+r}(s_j, s_{j+r})\rangle_{\text{eq}} \sim 3\alpha C r^{-\mu}\exp(-4K)$. Then, inequality (3.8) can be written as

$$\tau_q \geq \frac{k_B T \chi_{\mathbf{q}} N \exp(4K)}{4\alpha C\Big\{3\sum_{r=1}^{R}[r^{-\mu}(N - r)\{1 - \cos(qar)\}] - 2(N-1)\{1 - \cos(qa)\}\Big\}} \quad . \tag{4.15}$$

The generalized Kawasaki model in which a spin can be exchanged with any other spin of the lattice with a probability independent of the distance ($\mu = 0$ and $R = N-1$) will now be studied. The full evaluation of the denominator of (4.15) must be done (as in the case of $T \to \infty$ and for $N \gg 1$); we obtain

$$\tau_q \geq \alpha^{-1}\xi^3 g(qa\xi) \ . \tag{4.16}$$

Here we have used that $C = 1/(N-1)$ and expressions (2.2) and (2.6), respectively, for the correlation length and $k_BT\chi_{\mathbf{q}}$. Note that the dependence of τ_q on q appears, for $q \to 0$, because of $\chi_{\mathbf{q}}$. If we assume the scaling hypothesis (3.9) for the relaxation time, we obtain that $z \geq 3$. Moreover, as the lower bound usually coincides with exact result in one dimension and the DWA furnishes the same exponent (as it will be shown later), $z = 3$ could be the exact value of the dynamic exponent. This result is in agreement with the relation found in Ref. 7 between the z exponent and the standard Kawasaki model exponent z_K, namely, $z = z_K - 2$. Reference 15 obtained a wrong exponent because of the expansion for $q \to 0$.

When $\mu = 0$ and $R \ll N$ we can consider the limit $q \to 0$ and expand $\cos(qar)$ up to second order in q. Now we have that

$$\tau_q \geq \frac{2k_BT\chi_{\mathbf{q}}NR\exp(4K)}{\alpha q^2a^2\{2NR(R+1)(2R+1) - 3R^2(R+1)^2 - 8(N-1)\}} \ . \tag{4.17}$$

If $R = 1$ we have the next-neighbor exchange model and it is very easy to obtain from the above equation that $z_k \geq 5$. Using the relations of Sec. II for the correlation length and $\chi_{\mathbf{q}}$ we obtain that

$$\tau_q \geq \frac{\xi^5}{\alpha R^2} g(qa\xi) \ , \tag{4.18}$$

in the case that $R \gg 1$. For bounded R we have the same exponent of the standard Kawasaki dynamics ($z \geq 5$). However if we consider the maximal distance of the exchanged spins to be proportional to the correlation length, i.e., $R \sim \xi$, Eq. (4.18) gives us $z \geq 3$, the same result obtained above for the model in which a spin can exchange with any other spin. This result has already been obtained in the literature.[15] Therefore for $\mu = 0$ we can conclude that if $R \ll \xi$ we are in the dynamical universality class of the standard Kawasaki model ($z_K = 5$). On the other hand, if $R \sim \xi$ or greater the dynamical exponent is $z = 3$.

We discuss now the case with $\mu \neq 0$ and $R \ll N$. We consider inequality (4.15) in the limit $q \to 0$ and expand the term $\cos(qar)$ up to second order in q. After some standard steps we obtain that

$$\tau_q \geq \frac{\xi^5 N \sum_{r=1}^{R} r^{-\mu}}{2\alpha\left\{3\sum_{r=1}^{R}[r^{2-\mu}(N-r)] - 2(N-1)\right\}} f(qa\xi) \ . \tag{4.19}$$

If $R \ll \xi$, the denominator of the equation is given by bN, where b is a constant. So we have that $z \geq 5$ independent of the value of μ. The most interesting cases occur when $R \sim \xi$. First we consider $\mu = 1$. Since $R \gg 1$, we find that $\sum_r r^{-1} \sim \ln(R)$ and $\sum_r r(N-r) \sim NR^2/2$. Then we obtain that $z \geq 3$ with logarithmic corrections. For $\mu = 2$ we have that $C \sim \pi^2$ and $\sum_r (N-r) \sim NR$. It is easy to obtain that $z \geq 4$. When $\mu = 3$ we have that $C \sim 1.2$ and $\sum_r r^{-1}(N-r) \sim N\ln(R)$. Using these two values we obtain that $z \geq 5$ with logarithmic corrections. For $\mu \geq 3$ we find that the z exponent is equal to the standard Kawasaki value $z_K = 5$. Therefore we can conclude that for $R \sim \xi$ and $\mu \geq 3$ we have that $z \geq 5$. The result $z = z_K$ for $\mu \geq 3$ agrees with the work of Hayakawa and Family.[8] For $R \sim \xi$ and $\mu \leq 1$ we find that $z \geq 3$. If $1 < \mu < 3$, the z exponent changes with μ. In particular for $\mu = 2$ we find $z \geq 4$. It means that $z = 4$ could be the exact value of the dynamical exponent for $\mu = 2$.

The dynamical critical behavior of long-range exchange models can be better understood by a simple physical argument based on the movement of domain walls (DWA) proposed for the ferromagnetic Ising model with the standard Kawasaki dynamics by Cordery, Sarker, and Toboshnik.[14] For asymptotic times near the critical temperature ($T_c = 0$) we have domains of aligned spins separated by sharp walls. The behaviour of the relaxation time is determined by the time it takes for a domain wall to move a typical distance ξ in the *fastest way.* The z exponent evaluated by the DWA is an upper bound to the exact exponent.[14]

Let us discuss first the exchanges of next neighbors with opposite spins. The DWA consists of the following three steps: (i) Two nearest-neighbor spins of opposite signs, situated on the domain wall, are exchanged with a slow rate w; (ii) the exchanged spin, e.g., the down one, moves through the domain of spins up and comes out at the other side; for a domain of size ξ this happens with probability $P_\xi \sim \xi^{-1}$; (iii) the whole domain has moved one step in a time $1/wP_\xi$; since the domain wall makes a random walk , the two previous steps must happen ξ^2 times. From (4.1) we evaluate that $w = w_{i,i+1}(s_i, s_{i+1}) \sim \alpha\xi^{-2}$. Then we have that

$$\tau \approx \frac{\xi^2}{P_\xi w} \approx \frac{\xi^5}{\alpha} \ , \tag{4.20}$$

which gives us $z = 5$. It is easy to see that this is the fastest way (perhaps the only way) of decay of a wall. Note that this value coincides with the lower bound of the initial response and with the exact value.[16]

When $R = N-1$ and $\mu = 0$ a spin can be exchanged with any other spin and then the system can choose a mechanism of relaxation that does not involve step (ii). The exchange can now occur between two spins situated in different domain walls which are separated at least by a distance of order ξ. The exchange rate is w. The number of spins in domain walls is of the order of N/ξ. Since a spin on a given domain wall can be exchanged with any other spin out of the total N/ξ, the rate at which this domain wall moves a step is wN/ξ. From (4.2) we obtain that $w = \alpha/N$. Since this must happen ξ^2 times, we obtain that

$$\tau \approx \frac{\xi^3}{\alpha} \quad . \tag{4.21}$$

Note that if the spin i exchanges with an inner domain spin, the diffusion mechanism increases the time of decay. So the above mechanism is the fastest one. Therefore we have that $z = 3$ and this value agrees with the lower bound of the initial response (4.16).

Let us consider the model with $\mu \neq 0$ and $R \sim \xi$. There is now a competition between the two mechanisms presented above: (a) the diffusion of a spin by next-neighbor exchange and (b) the exchange between spins on domain walls separated by a distance ξ. In case (a) the domain wall spends an average time $t_a = \xi^3/\alpha C$ to move a step. This time is obtained by considering steps (i) and (ii) of the mechanism of the standard Kawasaki discussion. Note that C depends on μ. In case (b) the domain wall spends an average time $t_b = \xi^\mu/\alpha C$ to move a step. This time is obtained by considering the transition rate (4.2) with the spins on two adjacent domain walls. The other mechanisms in which a spin in the domain wall exchanges with an inner domain spin happen always with a time either close to t_a or close to t_b depending on the distance between the spins in the first exchange. For $0 \leq \mu \leq 2$ we have that $t_b < t_a$, meaning that mechanism (b) is the fastest way of decay of a domain wall. On the other hand, for $\mu \geq 3$ we obtain that $t_a < t_b$ and the mechanism of the standard Kawasaki dynamics is the fastest way of relaxation. Therefore, the relaxation time will be given by

$$\tau \approx \xi^2 t_b \quad \text{for} \quad 0 \leq \mu \leq 2 \quad , \tag{4.22}$$

$$\tau \approx \xi^2 t_a \quad \text{for} \quad \mu \geq 3 \quad . \tag{4.23}$$

We obtain that $z = 5$ for $\mu \geq 3$, $z = 4$ for $\mu = 2$ and $z = 3$ when $0 \leq \mu \leq 1$. It is worth mentioning that even the logarithmic corrections are obtained for $\mu = 1$. All these exponents coincide with the lower bounds of the initial response, indicating that they could be the exact ones.

V. ALTERNATING BOND MODEL

We consider here the alternating ferromagnetic bond model ($K_{2n+1} = K_1$ and $K_{2n+2} = K_2$ for $n = 0, 1, \ldots$) with $K_1 \geq K_2$. The transition rates for $r = 1$ are the natural extension of the standard Kawasaki ones, namely,

$$w_{i,i+1}(s_i, s_{i+1}) = \frac{\alpha}{4}(1 - s_i s_{i+1}) \times \left[1 - \frac{1}{2}(\eta_i^+ s_{i+1} s_{i+2} + \eta_i^- s_{i-1} s_i)\right], \tag{5.1}$$

$$\eta_i^\pm = \tanh(K_{i-1} + K_{i+1}) \pm \tanh(K_{i+1} - K_{i-1}) \quad . \tag{5.2}$$

For $r \geq 2$ the transition rates are given by

$$\begin{aligned} w_{i,i+r}(s_i, s_{i+r}) = \frac{\alpha C}{4 r^\mu}(1 - s_i s_{i+r})\{\, 1 + s_i[&A_1 s_{i+r-1} + A_2 s_{i+r+1} - A_3 s_{i-1} - A_4 s_{i+1} \\ &+ B_1 s_{i-1} s_{i+1} s_{i+r-1} + B_2 s_{i-1} s_{i+1} s_{i+r+1} \\ &- B_3 s_{i+r-1} s_{i+r+1} s_{i+1} - B_4 s_{i+r-1} s_{i+r+1} s_{i-1}]\} \quad . \end{aligned} \tag{5.3}$$

Here A_1 has the following expression:

$$\begin{aligned} A_1 = \tfrac{1}{8}[\, &\tanh(K_{i+r-1} + K_{i+r} + K_{i-1} + K_i) + \tanh(K_{i+r-1} - K_{i+r} + K_{i-1} + K_i) \\ &+ \tanh(K_{i+r-1} + K_{i+r} - K_{i-1} + K_i) + \tanh(K_{i+r-1} + K_{i+r} + K_{i-1} - K_i) \\ &- \tanh(-K_{i+r-1} + K_{i+r} + K_{i-1} + K_i) + \tanh(K_{i+r-1} + K_{i+r} - K_{i-1} - K_i) \\ &+ \tanh(K_{i+r-1} - K_{i+r} + K_{i-1} - K_i) + \tanh(K_{i+r-1} - K_{i+r} - K_{i-1} + K_i)] \quad , \end{aligned} \tag{5.4}$$

and B_1 is given by

$$\begin{aligned} B_1 = \tfrac{1}{8}[\, &\tanh(K_{i+r-1} + K_{i+r} + K_{i-1} + K_i) - \tanh(-K_{i+r-1} + K_{i+r} + K_{i-1} + K_i) \\ &+ \tanh(K_{i+r-1} - K_{i+r} + K_{i-1} + K_i) - \tanh(K_{i+r-1} + K_{i+r} - K_{i-1} + K_i) \\ &- \tanh(K_{i+r-1} + K_{i+r} + K_{i-1} - K_i) + \tanh(-K_{i+r-1} + K_{i+r} + K_{i-1} - K_i) \\ &+ \tanh(K_{i+r-1} + K_{i+r} - K_{i-1} - K_i) - \tanh(K_{i+r-1} - K_{i+r} + K_{i-1} - K_i)] \quad . \end{aligned} \tag{5.5}$$

The other coefficients A_n are obtained from A_1. To obtain A_2 we change K_{i+r-1} and K_{i+r}, respectively, by K_{i+r} and K_{i+r-1}. If K_{i+r-1} and K_{i-1} are exchanged, we obtain A_3. The coefficient A_4 is obtained by exchange of K_{i+r-1} and K_i. In the same way, the B_n $(n > 1)$ are obtained from B_1. When we exchange K_{i+r-1} and K_{i+r} we have B_2. B_3 is obtained from the exchange of K_{i+r} and K_{i-1}. Finally we get B_4 if we change K_{i+r} and K_i by K_i and K_{i+r}, respectively. Note that in the limit of $J_1 = J_2$ we recover c_1 and c_3 defined in Eq. (4.2).

In order to use inequality (3.8) of the initial response, we must evaluate $\langle w_{i,i+r}(s_i, s_{i+r})\rangle_{\rm eq}$ with $w_{i,i+r}(s_i, s_{i+r})$ given by Eq. (5.1) or (5.3). In fact, because of the symmetry of the lattice, we must evaluate only the rates with i even and $i+r$ even, and the rate with i even and $i+r$ odd. For $r = 1$ we obtain that

$$\langle w_{i,i+1}(s_i, s_{i+1})\rangle_{\rm eq} = \frac{\alpha}{4}(1-v_2)(1-\gamma_1 v_1) \text{ for } i \text{ even} , \tag{5.6}$$

where $v_n = \tanh(K_n)$ and $\gamma_n = \tanh(2K_n)$. The average of the rate with i odd is obtained by exchanging K_1 and K_2 in this equation. The equilibrium average of the transition rates (5.3) for r even and i even can be written as

$$\begin{aligned}\langle w_{i,i+r}(s_i, s_{i+r})\rangle_{\rm eq} = \frac{C\alpha}{r^\mu}\{ & 1 - [v_1 v_2]^{r/2} \\ & + A_1 t(v_2, v_1) + A_2 t(v_1, v_2) \\ & + B_2 T(v_2, v_1) \\ & + B_1 T(v_1, v_2)\} ,\end{aligned} \tag{5.7}$$

where the functions t and T are given by

$$t(v_2, v_1) = [v_2 v_1]^{r/2}[v_1^{-1} + v_1] - 2v_1, \tag{5.8}$$

$$T(v_2, v_1) = t(v_2, v_1) + 2v_1 - 2v_1 v_2^2 . \tag{5.9}$$

The average $\langle w_{i,i+r}\rangle_{\rm eq}$ for i odd and $i+r$ odd can be obtained from (5.7) by the exchange of v_1 and v_2. For i even and $i+r$ odd we obtain that

$$\begin{aligned}\langle w_{i,i+r}(s_i, s_{i+r})\rangle_{\rm eq} = \frac{C\alpha}{r^\mu}\{ & 1 - [v_1^{\frac{r+1}{2}} v_2^{\frac{r-1}{2}}] + 2t_-(v_2, v_1) \\ & \times [A_1 + B_2 v_1^2] \\ & + 2t_+(v_1, v_2)A_2 + 2B_1 v_2^{-2} \\ & \times (t_+ - v_1 - v_1 v_2^4)\} , \\ t_\pm(v_2, v_1) = [v_1 v_2]^{\frac{r\pm1}{2}} - v_2 & .\end{aligned} \tag{5.10}$$

If we exchange v_1 and v_2, we have the equilibrium average for i odd and $i+r$ even.

Near the critical temperature $T_c = 0$ we can expand all these equilibrium averages. For the case $r = 1$ we find that averages (5.6) for odd and even i have the same behavior, namely $\langle w_0\rangle_{\rm eq} \sim \alpha C \exp[-2(K_1 + K_2)]$. If i and $i+r$ are even (odd), we find that $\langle w_1\rangle_{\rm eq} \sim 2\alpha C r^{-\mu} \exp[-2(K_1 + K_2)]$. When i is even (odd) and $i+r$ is odd (even) the average rate behaves as $\langle w_2\rangle_{\rm eq} \sim 4\alpha C r^{-\mu} \exp[-2(K_1 + K_2)]$.

For the case $R = 1$ we obtain the usual result of the literature.[12,13] A nonuniversal behavior is found, and we have that $z = 4 + J_1/J_2$. Here we are especially interested in the case $R = N-1$ and $\mu = 0$. In this case a spin can be exchanged with any other and, in principle, the system could choose the fastest way of relaxation in order to avoid the nonuniversal behavior. Near $T = 0$, inequality (3.8) can be written as

$$\tau_q \geq \frac{k_B T \chi_{\mathbf{q}} N(N-1)}{\alpha \exp[-2(K_1 + K_2)]\{D - 4(N-1)\cos(qa)\}} , \tag{5.11}$$

where D is given by

$$\begin{aligned} D = 8 & \sum_{n=1}^{\frac{N-2}{2}} (N - 2n)[1 - \cos(2nqa)] \\ & + 16\sum_{n=1}^{N/2}(N - 2n + 1)\{1 - \cos[qa(2n-1)]\} .\end{aligned} \tag{5.12}$$

Again we cannot expand the term in $\cos(qna)$ for small q. After the evaluation of these series and doing the others steps already discussed, we obtain

$$\tau_q \geq \frac{\xi^{2+\frac{J_1}{J_2}}}{\alpha} g(qa\xi) . \tag{5.13}$$

Therefore we have a nonuniversal behavior, characterized by a dynamical exponent $z = 2 + J_1/J_2$. This behavior can be understood in terms of the DWA as follows. Near $T = 0$ and for asymptotic times, we have several domains separated by sharp walls. But now the two spins of a domain wall can interact by either a J_2 bond or a J_1 one. The rate of exchange of the spins belonging to different domain walls is determined by these interactions. If the interaction between the spins of a wall is J_2 and in the other wall we have the same bond, the rate is very slow and is proportional to $\exp[-4(J_1 + J_2)]$. In the other cases the rates are constants. Then, in the beginning the system can choose the fastest way of relaxation and avoid the slow ones. After a finite time, however, only J_2 interactions appear between the spins of the walls determining the nonuniversal behavior. We have made some numerical simulations, not shown here, that have confirmed this picture.

VI. CONCLUDING REMARKS

Before we enter into the concluding remarks, let us briefly discuss the results for the d-dimensional lattices. The main result (3.8) of the initial response can also be used to derive an inequality for the z exponent valid for d-dimensional lattices. Now, $\mathbf{a}_r$ is a d-dimensional vector and r is a d-folded index. For $R = 1$ we have the Kawasaki inequality $z \geq 2 + (\gamma/\nu)$, where γ is the exponent of the susceptibility and ν is the exponent of the correlation length. This inequality is obtained as follows. For $d > 1$ we have that $\langle w\rangle_{\rm eq}$ will be finite near T_c.[9] Note that $\langle w\rangle_{\rm eq}$ is proportional to ξ^{-2} in $d = 1$ only because $T_c = 0$. Moreover, we have that $\langle w\rangle_{\rm eq}$ has several correlation functions which have 1 as an upper bound. Near T_c the susceptibility behaves as $k_B T \chi_{\mathbf{q}} \sim \xi^{\gamma/\nu} g(qa\xi)$. For small q, using that $\langle w\rangle_{\rm eq} \sim \alpha$ and writing the equations in the scaling variables $qa\xi$, we obtain that

$$\tau_q \geq \xi^{2+\frac{\gamma}{\nu}} f(qa\xi) . \tag{6.1}$$

Assuming the scaling hypothesis for the relaxation time we get $z \geq 2 + \gamma/\nu$ for $R = 1$. Let us consider $R = N-1$ and $\mu = 0$. Again $\langle w\rangle_{\rm eq}$ has several correlation functions which have for upper bounds the value 1. So we can assume that $\langle w\rangle_{\rm eq}$ is a constant. However, now we cannot expand the term $\cos(\mathbf{q}\cdot\mathbf{a}_r)$ for small q. When this term is evaluated we obtain terms proportional to $\delta_{q,0}$ and to N. Therefore we have

$$\tau_q \geq \xi^{\frac{\gamma}{\nu}} f(qa\xi) \ , \tag{6.2}$$

which gives us $z \geq \gamma/\nu$. This lower bound is the same as the one obtained for the Glauber dynamics.

In conclusion, we have described one-dimensional ferromagnetic Ising models with Kawasaki dynamics near T_c by using the initial response and the domain wall argument (DWA). The exchange transition rates between two opposite sign spins separated by a distance r is proportional to $r^{-\mu}$. The maximal distance of exchange between them is R. If R is much smaller than the equilibrium correlation length ξ, we obtain that the dynamical exponent z is equal to the standard Kawasaki exponent $z_K = 5$. When a spin can exchange with every other spin with a rate independent of the distance ($R = N - 1$ and $\mu = 0$), the order parameter is only conserved globally and we obtain that $z = 3$. It means that the system is in a dynamical universality class different from the standard Kawasaki dynamics. This result agrees with the hypothesis of Tamayo and Klein[7] that $z = z_K - 2$. For models in which the maximal distance of exchanges is proportional to the correlation length we obtain a z exponent depending on μ. For $0 \leq \mu \leq 1$ we have $z = 3$. For $\mu \geq 3$ we obtain the same exponent of the Kawasaki dynamics with exchanges of next-neighbor spins $z_K = 5$. For μ between 1 and 3, z changes with μ. In particular, we obtain $z = 4$ for $\mu = 2$.

We have also studied the one-dimensional ferromagnetic model with alternating interactions J_1 and J_2, with $J_1 \geq J_2$. For $R = N - 1$ and $\mu = 0$ we find a nonuniversal behavior, characterized by a dynamical exponent depending on the ratio of the couplings, namely, $z = 2 + J_1/J_2$.

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