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Observation of a nondivergent Hall coefficient for a localized two-dimensional electron gas

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We have studied the Hall coefficient for a two-dimensional electron gas in a GaAs/Al_xGa_{1-x}As heterojunction characterized by variable range hopping. We find the value of the Hall coefficient remains approximately classical, $R_H=1/nec$, for diverging longitudinal resistivity over a broad range of loalization strength. The results show the behavior of a recently proposed Hall insulator in which $\sigma_{xx}\rightarrow 0$, $\sigma_{xy}\rightarrow 0$, and $\rho_{xx}\rightarrow\infty$ but $\rho_{xy}\rightarrow$const for $T\rightarrow 0$.

Considerable progress has been made in understanding the transport properties of localized electrons in the presence of a magnetic field.[1] In the variable-range-hopping regime, it is well established that the longitudinal resistivity diverges as $T\rightarrow 0$. However, the general behavior of another most rudimentary quantity in transport characterization, the Hall resistivity, remains unclear. Recently, there has been a great deal of theoretical and experimental interest in this subject.

Much of the recent work done on the Hall coefficient has focused on three-dimensional doped semiconductors. For three-dimensional samples it is well known[2] that there exists a metal-insulator transition (MIT). Magnetic-field-induced MIT has been studied for InSb and $Hg_{0.79}Cd_{0.21}Te$.[3] In these samples the Hall coefficient diverges at the MIT. Also the magnetic-field dependence of the Hall coefficient was studied for GeSb (Ref. 4) and InAs.[5] Near the MIT the Hall coefficient for these samples remained finite but still varied with magnetic field. The dependence of the Hall coefficient on density above the MIT was found to depend critically on density for Bi_xKr_{1-x},[6] GeSb,[7] and SiB.[8] The Hall coefficient for SiAs,[9] on the other hand, was found not to depend critically on density.

Theoretically, there has been some disagreement about the behavior of the Hall coefficient near the MIT. Shapiro and Abrahams[10] found for noninteracting electrons the Hall coefficient to be noncritical with density. On the other hand, Wang *et al.*[11] used a renormalization-group analysis to show that the Hall coefficient for noninteracting electrons should depend critically on density. For interacting electrons Altshuler and Aronov[12] demonstrated that the Hall coefficient varied critically with density.

In the presence of any disorder a two-dimensional system is strictly localized,[2] so there is no MIT. However, there is a transition from weak localization, $k_Fl\gg 1$, to strong localization, $k_Fl<1$, where k_F is the electron wave vector and l is the mean free path. The first calculation of the behavior of the Hall coefficient for a purely two-dimensional system was made by Fukuyama[13] and later by Altshuler *et al.*[14] Fukuyama studied the effects of disorder for the weak-field and *weak-localization* regime and found the Hall coefficient to be unaffected by localization. Altshuler *et al.* found that when electron-electron interactions are considered under the same conditions the Hall coefficient varied in proportion to the longitudinal resistivity, $\delta R_H/R_H=2\delta\rho_{xx}/\rho_{xx}$.

More recently, a calculation by Viehweger and Efetov[15] and one by Zhang, Kivelson, and Lee[16] showed that noninteracting electrons in the insulating regime should exhibit a finite Hall resistance at zero temperature. For this state, $\rho_{xx}\rightarrow\infty$ for $T\rightarrow 0$, but ρ_{xy} remains finite and approximately equal to B/nec. Hence, n is not a measure of density of *extended states.* This state was dubbed a "Hall insulator" by Zhang, Kivelson, and Lee and discussed in terms of the global phase diagram for the quantum Hall effect.[17] Experimentally, it has been shown[18,19] that the insulating state near $\nu=\frac{1}{5}$ displays the characteristics of the Hall insulator. However, this state is observed at high magnetic fields and is believed by some to be a highly correlated Wigner solid. An important point to the paper of Zhang, Kivelson, and Lee was that the Hall insulating state could be observed for vanishing magnetic fields where localization is caused purely by disorder.

We present a systematic study of the Hall resistivity and the Hall coefficient ($R_H=\rho_{xy}/B$) at low magnetic fields for a disorder-induced, strongly localized two-dimensional electron gas (2DEG). In this well-characterized system the degree of localization can be adjusted *in situ.* We find the Hall coefficient to remain finite and equal to B/nec for localization lengths down to 600 Å. Also the Hall coefficient is independent of temperature in the range of interest, but the longitudinal resistivity can change by more than an order of magnitude in the same temperature range.

The sample is a molecular-beam-epitaxy-grown GaAs/Al_xGa_{1-x}As heterojunction. As described elsewhere,[20] the sample does not have the conventional undoped spacer of Al_xGa_{1-x}As. This is done to increase the impurity potential experienced by the electron gas indicated by the relatively low mobility of $\sim 2.4\times 10^4$ $cm^2/V\,s$. To ensure a capacitive gate the sample has a large cap layer ($\sim$2000 Å) of undoped Al_xGa_{1-x}As. The sample was then patterned using photolithography to a

Hall bar geometry of size 2×4 mm^2 (see inset of Fig. 1). Homogeneity and probe alignment were checked by comparing resistivities for various contact configurations. The gate, which is used to adjust the localization strength, consists of a strip of aluminum evaporated onto the sample. Both the Hall and the longitudinal resistances were measured using a four-wire, lock-in technique at 3.2 Hz. The excitation current was carefully selected to avoid self-heating. The sample was placed in a commercial, ^{3}He cryostat with temperatures varied down to 300 mK. A magnetic field perpendicular to the sample of up to 12 T was supplied by a superconducting solenoid.

In Fig. 1 we show the logarithm of the longitudinal resistivity as a function of the gate voltage at $B=0$ T and at $T=300$ mK. A striking feature is the large change of resistivity with applied gate voltage. We attribute this to a reduction of screening when the density is reduced[20,21] and therefore the impurity potential at the electron gas is increased. When the gate voltage is varied beyond -1.30 V, the sample has a resistivity value above h/e^2, and the resistivity is highly dependent on temperature indicating strong localization. The density of the 2DEG is shown on the top scale. This density was found by both Shubnikov–de Haas oscillations for low gate voltages and by the minimum in resistivity at higher gate voltages.[20] As expected for a capacitive gate, this density scales linearly with applied gate voltage.

The graph in Fig. 2 shows the longitudinal resistivity as a function of gate voltage. A well-defined minimum occurs near $B=2.5$ T. We identify this minimum as filling factor $\nu=2$ where both spin-up and spin-down levels of the lowest Landau level are occupied. The magnetic field value where $\nu=2$ occurs gives a density of 1.2×10^{11} cm^{-2}.

The inset of Fig. 2 shows the longitudinal resistance versus T. The main feature of this graph is the resistivity's exponential dependence with decreasing temperature.

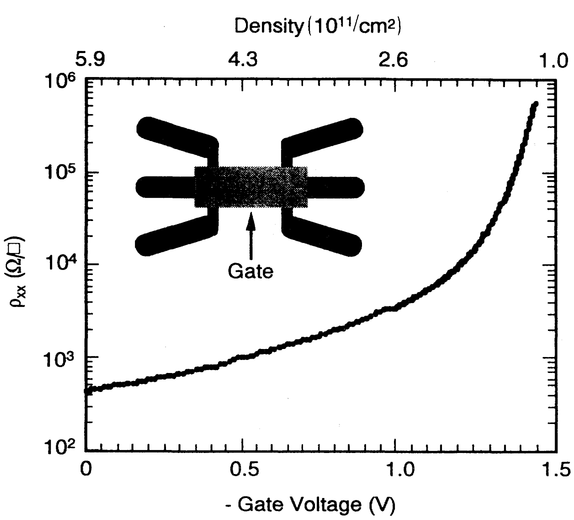

FIG. 1. Semilogarithmic plot of the longitudinal resistivity as a function of applied gate voltage for $B=0$ T and $T=300$ mK. The inset shows the sample configuration.

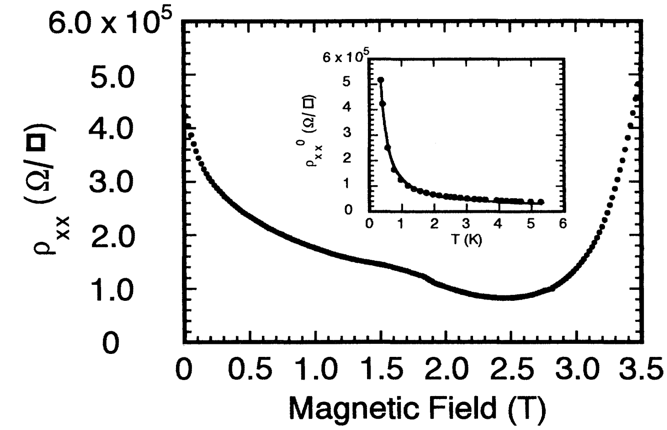

FIG. 2. The magnetic-field dependence of the longitudinal resistivity at -1.45 V and $T=300$ mK. The inset shows the temperature dependence of the longitudinal resistivity at the same gate voltage. The curve shows a fitting to $\rho_0\exp(T_0/T)^{1/3}$ giving a localization length of 600 Å.

The curve represents a fitting to $\rho_0\exp(T_0/T)^{1/3}$. A fit in this form was motivated by Mott's law for a system exhibiting variable range hopping $\rho\sim\exp(T_0/T)^{\nu}$.[22] We find that for all our 2DEG samples in the strongly localized regime least-square fitting gives $\nu=0.35\pm0.05$. This value is consistent with the theoretical exponent for a two-dimensional system, $\nu=\frac{1}{3}$. From the fitting the localization length ξ can be estimated using the theory of Mott, $k_BT_0=3.5/[g(\epsilon_F)\xi^2]$ where $g(e_F)$ is the density of states at the Fermi level. In our evaluation of the localization length we have assumed that the density of states is energy independent as in the free-electron case [$g(e)=m^*/\pi\hbar^2$ where $m^*=0.067m_e$ is the electron effective mass in GaAs]. In particular, the localization length for $V_G=-1.45$ V is approximately 600 Å.

The transverse resistivity as a function of magnetic field for the two perpendicular directions is shown in Fig. 3(b). A small degree of mixing with the longitudinal channel due to contact misalignment caused the Hall resistivity to have the same form as the longitudinal resistivity except scaled down by a factor of 100. There is a minimum in the Hall resistivity coincident with the longitudinal resistivity at $B\simeq2.5$ T. However, despite the mixing an upward trend with magnetic field in Hall resistivity is evident when the values for both magnetic-field directions are compared.

Figure 3(b) shows the Hall resistivity for $T=300$ mK and 4.2 K when the gate voltage is -1.45 V. The data at $T=300$ mK are obtained by taking the average of the data for the two directions of magnetic field shown in Fig. 3(a). This is done to eliminate mixing due to contact misalignment. After averaging, a well-defined Hall coefficient can be obtained for magnetic fields below 2 T. However, there is deviation in the Hall coefficient at higher fields. We attribute the source of this deviation to

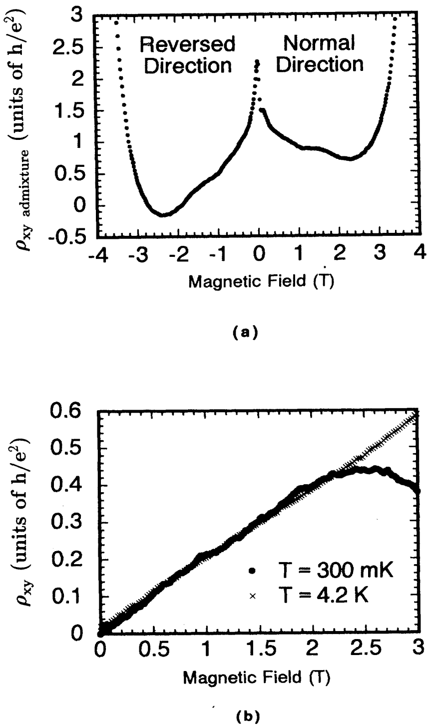

FIG. 3. Hall resistivity at a gate voltage of -1.45 V. (a) shows the transverse resistivity including admixture at $T=300$ mK before averaging. (b) shows the Hall resistivity for two temperatures $T=300$ mK (circle) and 4.2 K (box). The Hall resistivity for $T=300$ mK was found by averaging the two magnetic-field directions shown in (a).

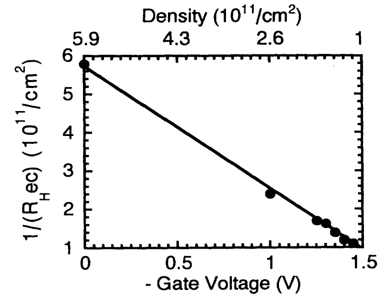

FIG. 4. The Hall coefficient written in terms of density as a function of gate voltage. The density indicated at the top of the plot was found from the position of the minimum in the longitudinal resistance. The densities follow a classical line drawn to help guide the eye.

averaging. The Hall resistance was found by starting at positive magnetic field and sweeping down to negative magnetic field. As the magnetic field is swept, the temperature varied slightly, and the longitudinal resistivity depends strongly on temperature for this gate voltage. Hence, the amount of mixing varied slightly at the far ends of the sweep, so averaging is no longer valid in this region. It is important to note, however, that the average value for the Hall resistivity at lower magnetic field is quite accurate because the temperature in this region is relatively constant. There is a small offset at zero magnetic field for $T=4.2$ K also associated with mixing with the longitudinal resistivity. For this temperature the longitudinal resistivity is an order of magnitude lower than at $T=300$ mK so mixing was slight and the two field directions were not averaged. The most striking feature of the figure is that the Hall coefficient is approximately independent of temperature for $T=300$ mK to 4.2 K. On the other hand, the longitudinal resistance changes by more than an order of magnitude over the same range. Also, if the classical formula for the Hall coefficient $R_H=1/nec$ is used, the density found from the position of the minimum corresponding to $\nu=2$ in the longitudinal resistivity and the Hall coefficient agree.

We show in Fig. 4 the Hall coefficient plotted as a function of gate voltage in terms of the density using the classical formula $R_H=1/nec$. We have included our findings for the Hall coefficient for all gate voltages studied. The data points in the figure represent localization lengths from 600 Å to lengths larger than the sample size. The Hall coefficient remains finite and changes classically over the entire range. Figure 1 indicates that over this same range the longitudinal resistance varies by more than three orders of magnitude.

Although, the data presented here are consistent with the characteristics of the theoretical proposed Hall insulating state,[15,16] there remains some question as to the applicability of these theories. First, both theories evaluated the conductivity tensor for $T=0$ and $\omega \rightarrow 0$. However, the sample was measured in the limit of $\omega=0$ and $T \rightarrow 0$. There is an important question as to whether these two limits commute. This issue has to be resolved by finite-frequency study. Second, as mentioned by Zhang, Kivelson, and Lee, strictly speaking their calculations apply only to the insulating regime in which $\xi \gg l_{\rm in}$, where $l_{\rm in}$ is the inelastic scattering length. Our result suggests, however, the Hall coefficient remains nondivergent even in the limit of $\xi < l_{\rm in}$ [the estimated $l_{\rm in}$ is greater than 4000 Å at $T=300$ mK (Ref. 23)].

It is worth mentioning that the temperature dependence of the resistivity at $B=0$ can be accurately described by $\rho \sim \exp(T_0/T)^\nu$ with $\nu \simeq \frac{1}{3}$ for the entire range of our studies. This fact implies that the electron-electron interaction is not a dominant effect. Otherwise a fitting with $\nu=\frac{1}{2}$ would be more accurate.[2] The effect of interaction on the Hall coefficient in the strongly correlated limit, i.e., in the very high magnetic-field regime, remains to be seen.

In conclusion, we have measured the Hall coefficient at low magnetic fields for a 2DEG in a gated sample of $GaAs/Al_xGa_{1-x}As$. We have shown that for this system the classical formula for the Hall coefficient $R_H=1/nec$ remains valid for localization lengths down to 600 Å and is independent of temperature over the range studied.

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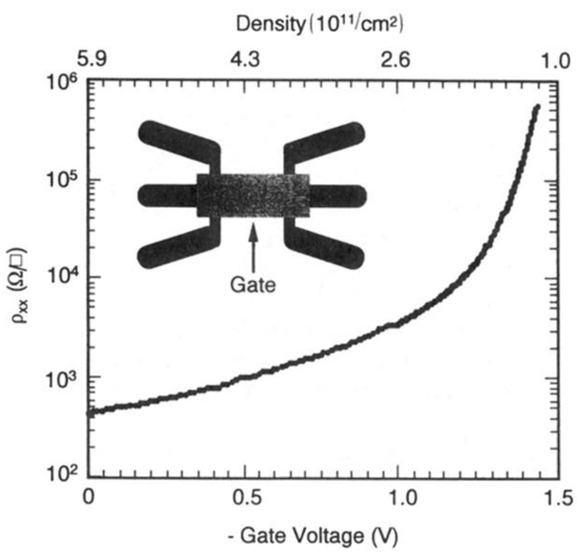

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