RAPID COMMUNICATIONS

PHYSICAL REVIEW B VOLUME 48, NUMBER 4 15 JULY 1993-II

Rapid Communications

Rapid Communications are intended for the accelerated publication of important new results and are therefore given priority treatment both in the editorial office and in production. A Rapid Communication in **Physical Review B** *should be no longer than four printed pages and must be accompanied by an abstract. Page proofs are sent to authors.*

Intrinsic electron-pumping mechanism in the oscillating-barrier turnstile

Julian D. White and Mathias Wagner*
Hitachi Cambridge Laboratory, Cavendish Laboratory, Madingley Road, Cambridge, CB3 OHE, United Kingdom
(Received 10 May 1993)

Coulomb-blockade-based oscillating-barrier turnstile devices have recently been proposed as possible future current standards. In the present work we study the sidebands in the tunneling probability generated by an oscillating barrier, and discuss how the presence of these sidebands will affect the accuracy of an oscillating-barrier turnstile device.

The realization of a current source that could be accurately synchronized to an external signal with an accuracy of 1 part in 10^8 or smaller would allow closure of the so-called metrological triangle, where the standard for voltage derived from the Josephson effect and the standard for resistance derived from the quantum Hall effect could be independently cross correlated.[1] Were such a closure to be realized, the accuracy of all three quantities would be simultaneously improved. The recent development of devices which can manipulate single electrons using the Coulomb blockade effect has brought this goal one step closer.[2]

The Coulomb blockade effect arises when an electron tries entering a small isolated region whose total capacitance to its environment, C_Σ, is so small that the charging energy $E_C = e^2/2C_\Sigma$ necessary for the electron to successfully enter this region is much larger than the thermal energy $k_B T$.[3] The region is isolated in the sense that the wave functions of electrons tend to be localized on the island due to the resistance R of the leads into and out of the region being much larger than the quantum Hall resistance R_K ($h/e^2 \approx 25.8$ kΩ). Under these conditions an electron cannot move into the region unless its potential is raised an amount E_c relative to the central region by an external voltage $V_c = E_C/e$. For voltages less than V_c no current can flow at zero temperature and the system is said to be in the Coulomb blockade regime.

To date, three device structures employing Coulomb blockade have been proposed and realized for achieving a current standard:[4] the *turnstile*,[5] the *pump*,[6] and the oscillating barrier turnstile.[7,8] Both the *turnstile* and the *pump* rely on tunnel barriers of fixed height and width, where the charges are transported through the devices by sequentially changing the potential of one-dimensional arrays of coupled islands with external oscillating voltages. In contrast, the oscillating-barrier turnstile (or quantum-dot turnstile), as shown in Fig. 1(a), relies on keeping the potential of a single intermediate island fixed, while varying the transparency of two tunnel barriers on either side. For the latter device we will discuss the effect of electrons excited by the oscillation of the barrier potential and show that to achieve high counting accuracy, a careful choice of bias voltage needs to be made in order to avoid this effect. For a detailed account of the oscillating barrier turnstile the reader is referred to Refs. 7 and 8, but what follows is a short summary in order that the origin and effects of the intrinsic charge pumping mecha-

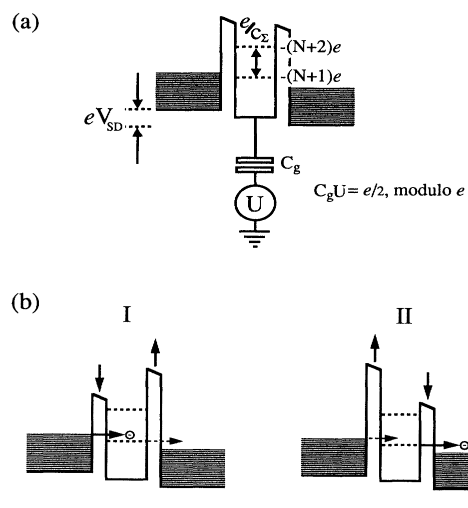

FIG. 1. The oscillating-barrier turnstile has a central island [(a)] which is electrically isolated from reservoirs on the left- and right-hand side with tunnel barriers, such that $R_l, R_r \gg R_K$. In order for the device to count electrons then as shown in (b)I, the left-hand barrier is lowered and the right-hand barrier is raised such that a single electron will tunnel into the island. In the next half cycle, (b)II, the left barrier is increased while the right is decreased such that an electron can tunnel out of the island. In this way, in an ideal device only one electron is transferred through the system during one clock cycle. The tunneling rate through the opaque barriers is assumed to be low enough, such that the accuracy is not unduly degraded.

nism can be more fully appreciated.

In this device the passage of the electrons is synchronized to an external frequency f by periodically modulating the height, and hence transparency, of the tunnel barriers at the entrance and exit of the island, with a phase shift of π radians between the left-hand and the right-hand barrier modulation [see Fig. 1(b)]. In this way, when the left-hand barrier is most transparent, the right-hand one is strongly opaque and vice versa. If initially there is no electrostatic energy penalty for one electron moving onto the island, then as long as electrons have sufficient time to tunnel through the transparent, say, left-hand barrier, only one electron is transported from the left-hand reservoir to the central region of the device. After the island has filled, the left-hand barrier is raised and the right-hand barrier is lowered, thus enabling an electron to tunnel from the central island to the right-hand reservoir, thereby emptying the island of the excess electron. During this time, further electrons are blocked from entering through the barrier because of its high impedance. As this is repeated, one electron is transported through the island during one period of the applied signal, thus generating a well-defined current $I_Q = ef$, where f is the clocking frequency. The accuracy of this process is limited by the following conditions: the impedance R of the barriers in their transparent state still has to be much larger than R_K in order for the Coulomb blockade to remain effective, yet it has to be small enough to ensure with the desired accuracy that the island gets charged and/or discharged during half of a clock cycle (i.e., the RC time constant $\tau_{RC} = RC_\Sigma$ has to be much smaller than $1/f$). On the other hand, the impedance of the barriers in their opaque state has to be made as large as possible to sufficiently reduce unwanted tunnel events.

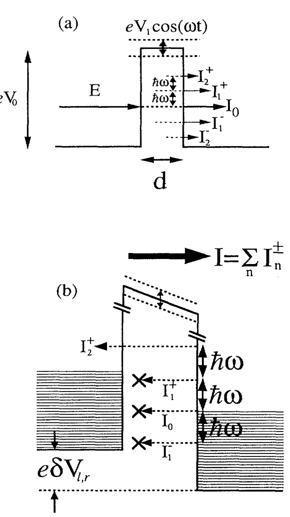

FIG. 2. (a) As electrons tunnel through an oscillating potential barrier, in the emerging stream, there is not only a stream at the original energy E, but also an infinite ladder of sidebands with energies at $E \pm n\hbar\omega$ (only the lowest four are shown for clarity). The higher-order modes have rapidly decaying amplitudes, relative to the unshifted I_0 band. (b) shows how the charge pumping mechanism will lead to electrons moving in the opposite direction to the applied voltage.

Another possible, and so far unconsidered source of inaccuracy is the oscillation of the barriers itself. It has been shown by Büttiker and Landauer[9] that for electrons of energy E, tunneling through a barrier whose height varies harmonically as $eV(x,t) = e[V_0(x) + V_1\cos(\omega t)]$, there is a finite probability that these electrons pick up or lose energy quanta $\hbar\omega$, which is similar in nature to photon-assisted tunneling in double-barrier systems.[10] As shown in Fig. 2(a), within the emerging stream of electrons on the far side of the barrier, in addition to electrons of energy E, there is a series of sidebands at $E \pm n\hbar\omega$, where n is a positive integer.

As will become clear in the following, the performance of oscillating-barrier turnstiles is affected by this charge pumping mechanism as it allows electrons on the island to tunnel back from the island to the source reservoir and electrons in the drain reservoir back to the island, both of which are undesired tunnel events. To analyze the effect in more detail, Fig. 2(b) shows a single barrier with a potential drop, where electrons originating at the Fermi energy on the right of the barrier can absorb $\pm n$ quanta from the barrier. Here, electrons that absorb a sufficient number of quanta can tunnel to the left-hand side; a classically forbidden process for $\omega, T = 0$. All electrons which lose excitation quanta are forbidden to tunnel in this direction. Likewise, electrons originating from the left-hand side of the barrier which lose a sufficient number of quanta are unable to tunnel. By analyzing the charging and/or discharging cycle of the central island, it is easily seen that to a very good approximation an excess electron stays on the island for one-half of the cycle, and furthermore, that for this period the transparency of the entrance barrier is higher than average, or at least equal to it. So instead of the original problem we can consider two simpler ones: To calculate the time-averaged probability of an electron tunneling from the island back to the source reservoir, we assume that the electron stays on the island for one *complete* cycle, and divide the final result by two. Similarly, to evaluate the probability of an electron tunneling from the drain reservoir back onto the empty island, we assume the island to be empty for a full cycle, and again divide the result by two. A conservative estimation for the error rate of the original oscillating-barrier turnstile device is then to first order given by the sum of these single error rates.

Both problems are virtually the same. A plane-wave incident on a single, oscillating barrier gets partially reflected and partially transmitted, the only difference being that in the first case this plane wave has its origin in the island whereas in the second case it comes from the drain reservoir. The *exact time-dependent* scattering states of an oscillating barrier can be obtained by matching the plane-wave solutions outside,

$$\psi_n^{l,r}(x,t) = [A_n^{l,r}\exp(ik_n x) + B_n^{l,r}\exp(-ik_n x)] \times \exp[-i(E + n\hbar\omega)t/\hbar] ,$$

with the exact, time-dependent solutions inside the barrier,

$$\psi_n(x,t)=\sum_l [A_l \exp(\kappa_l x)+B_l \exp(-\kappa_l x)] \times J_{n-l}(eV_l/\hbar\omega)\exp[-i(E+n\hbar\omega)t/\hbar] ,$$

where

$$\hbar k_n=\sqrt{2m^*(E+n\hbar\omega)},\ \hbar\kappa_l=\sqrt{2m^*(eV_0-E-l\hbar\omega)} ,$$

and J_i is the Bessel function of the first kind.[9,11] The boundary conditions employed are that the initial electron wave is incident with energy E, which constitutes the central $n=0$ band, and that on the far side of the barrier we have only transmitted waves. The intensities $I_n=|A_n^r|^2$ then give the *time-averaged* probabilities of finding the electron in sideband n at energy $E+n\hbar\omega$ after tunneling. Büttiker and Landauer have calculated to lowest order the relative intensity of the first sideband $n=\pm1$ in the opaque limit $\kappa_0 d \gg 1$ to be $I_1^{\pm}/I_0=(eV_1\tau/2\hbar)^2$, where $\tau=(m^*/\hbar\kappa_0)d$ is termed as the "tunneling time" through the barrier, and I_0 is the intensity of the central band.

The lowest-order result requires $eV_1 \ll eV_0-E$. However, to achieve a sufficient modulation of the barrier transparency, we need to investigate the range $eV_1\approx eV_0-E$. To this end we have numerically solved the exact $(4n+2)(4n+2)_{n\to\infty}$ transfer matrix by truncating this matrix at sideband $n=23$. The analysis of its general structure shows that the relative sideband intensities are primarily a function of $V_1\tau$ and $\kappa_0 d$. Figure 3 shows the relative sideband intensities as a function of $V_1\tau$, using data representative of an electrostatically defined tunnel barrier created in a two-dimensional electron gas by a surface Schottky gate.[12] The parameters used are $E=E_f=18.6$ meV, $eV_0=50$ meV, $m^*=0.067$, and $d=85$ nm, which gives $\tau=0.318$ meV^{-1} $=2.09\times10^{-13}$ s. To achieve a sufficiently high output current $I_Q=ef$, the clocking frequency f was chosen to be $f=1$ GHz, but the resulting sideband intensities are virtually unchanged for the whole range, from dc to 1 GHz. Moreover, even for clocking frequencies of 1 GHz, we found the difference in the intensities of all the calculated $\pm|n|$ sidebands to be negligible. This can be understood when considering that for $\omega\tau\ll1$ the incident electron stream sees a quasistatic barrier when traversing it. In this limit, the time-averaging procedure effectively carried out when calculating the *fluxes* in the sidebands can be replaced by an averaging over the different barrier heights over one period of a cycle—which does not depend on the frequency f but on the modulation amplitude V_1. The analytic first-order result of Büttiker and Landauer is indicated in Fig. 3 as a dashed line labeled BL. It agrees very well with the numerical result as long as $eV_1\tau/\hbar\ll1$, and it is worth noting that this range of validity is much larger than that claimed in Ref. 9 where it was assumed that the condition $eV_1\ll\hbar\omega\ll eV_0-E$ must hold. For large $eV_1\tau/\hbar$, the number of relevant sidebands increases linearly with $eV_1\tau/\hbar$ for a given threshold intensity. This fact can therefore be utilized for extrapolating the results of Fig. 3 to higher values of $eV_1\tau/\hbar$.

It is essential for the accurate operation of an oscillating-barrier turnstile that electrons cannot unintentionally enter or leave the island via the sidebands. One way of blocking an electron in the nth sideband is to apply a bias $\delta V_{l,r}$ across the oscillating barrier such that on the far side of this barrier the final states at energy $E+n\hbar\omega$ are filled. For a given sideband index n, this requires $e\delta V_{l,r}>n\hbar\omega$. An analysis of the rate equation for wanted and unwanted tunnel events then shows that the error rate for a current standard is approximately given by the ratio of the total flux in the *unblocked* channels to the total flux in all *unblocked* channels in the opposite direction. By using the data of Fig. 3, we find that in order to achieve an accuracy of one part in 10^8 with a modulation amplitude of $eV_1\tau/\hbar=5$, we need to block the lowest 12 sidebands, giving $e\delta V_{l,r}>12\hbar\omega$. This translates to $\delta V_{l,r}=48$ μV at $f=1$ GHz or $\delta V_{l,r}=0.48$ μV at $f=10$ MHz, and the total source-drain bias V_{SD} applied to the oscillating-barrier turnstile has to be at least twice that. It should be noted that the assumed modulation amplitude, corresponding to $eV_1=15.7$ meV for the parameters given, is still relatively small, and in practice it is likely that larger modulations would be needed.

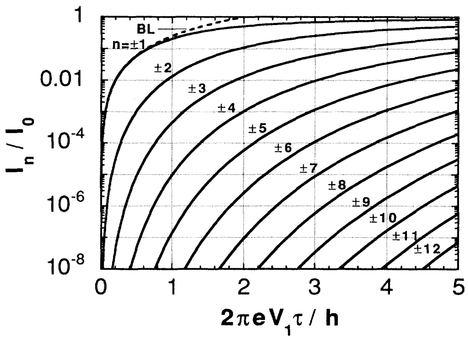

FIG. 3. The intensities of the sidebands in terms of the modulation strength $eV_1\tau/\hbar$ relative to the unshifted channel, for the order up to and including the $n=12$ sideband.

The conclusion is that in order to minimize the charge pumping effect, the source-drain bias V_{SD} has to be as large as possible. On the other hand, for the Coulomb blockade to be effective in the first place, we need to have $eV_{SD}\ll E_C$. Moreover, with increasing bias, cotunneling will increase drastically.[13] This leaves three options. First, one can try to reduce the intensity of the sidebands by suitably choosing the parameters $\kappa_0 d$ and $V_1\tau$. Though for constant $V_1\tau$ the sideband intensities increase slightly with decreasing $\kappa_0 d$, we find that for the required modulation amplitude $eV_1\approx eV_0-E\propto\kappa_0^2$ it is best to decrease $\kappa_0 d$ and hence increase the transparency as much as possible, while still satisfying the condition that the impedance of the barrier be much larger than R_K. The second option is to reduce the clocking frequency f. Though this approach is limited by the minimal acceptable current amplitude, some improvements are possible by operating many devices in parallel. Finally, the last option is to increase the Coulomb gap energy E_C, enabling the condition $\delta V_l+\delta V_r\ll E_C/e$ to be satisfied for higher applied voltages.

We would like to thank all members of the collaboration between the Hitachi Cambridge Laboratory and Microelectronics Research Centre of Cambridge University for help and advice during the course of this work. We would like to acknowledge D. Moss and K. Nakazato for critical proofreading of the manuscript. We are grateful to the Central Research Laboratories of Hitachi Limited for the hospitality during the preparation of this manuscript.

*Present address: Central Research Laboratory, Hitachi, Ltd., Kokubunji-Shi, Tokyo 185, Japan.

[1]B. N. Taylor, Phys. Today **42** (8), 23 (1989).

[2]For reviews of the field see D. V. Averin and K. K. Likharev, in *Quantum Effects in Small Disordered Systems,* edited by B. L. Altshuler, P. A. Lee, and R. A. Webb (Elsevier, Amsterdam, 1991), p. 173, and more recently the volume *Single Charge Tunneling: Coulomb Blockade Phenomena in Nanostructures,* edited by H. Grabert and M. H. Devoret (Plenum, New York, 1992).

[3]M. A. Kastner, Rev. Mod. Phys. **64**, 849 (1992); Phys. Today **46** (1), 24 (1993).

[4]Two proposed systems for producing an accurately defined current, which as yet have not been realized are to be found in Q. Niu, Phys. Rev. Lett. **64**, 1812 (1990); F. Guinea and N. García, *ibid.* **65**, 281 (1990).

[5]L. J. Geerligs, V. F. Anderegg, P. A. M. Holweg, J. Mooij, H. Pothier, D. Esteve, C. Urbina, and M. H. Devoret, Phys. Rev. Lett. **64**, 2691 (1990).

[6]H. Pothier, P. Lafarge, P. F. Orfila, C. Urbina, D. Esteve, and M. H. Devoret, Physica B **169**, 573 (1991); Europhys. Lett. **17**, 249 (1992).

[7]A. A. Odintsov, Appl. Phys. Lett. **58**, 2695 (1991).

[8]L. P. Kouwenhoven, A. T. Johnson, N. C. van der Vaart, C. J. P. M. Harmans, and C. T. Foxon, Phys. Rev. Lett. **67**, 1626 (1991); L. P. Kouwenhoven *et al.*, Z. Phys. B **85**, 367 (1991).

[9]M. Büttiker and R. Landauer, Phys. Rev. Lett. **49**, 1739 (1982); IBM J. Res. Dev. **30**, 451 (1986).

[10]W. Cai, T. F. Zheng, P. Hu, M. Lax, K. Shum, and R. R. Alfano, Phys. Rev. Lett. **65**, 104 (1990).

[11]P. K. Tien and J. P. Gordon, Phys. Rev. **129**, 647 (1963).

[12]S. J. Manion, L. D. Bell, W. J. Kaiser, P. D. Maker, and R. E. Muller, Appl. Phys. Lett. **59**, 213 (1991).

[13]D. V. Averin and A. A. Odintsov, Phys. Lett. A **140**, 251 (1989); D. V. Averin and Yu V. Nazarov, Phys. Rev. Lett. **65**, 2446 (1990); L. J. Geerligs, D. V. Averin, and J. E. Mooij, *ibid.* **65**, 3037 (1990); A. E. Hanna, M. T. Tuominen, and M. Tinkham, *ibid.* **68**, 3228 (1992).