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Lattice scattering of a two-dimensional electron gas at low temperatures

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The acoustic scattering rate of the carriers in a two-dimensional electron gas is calculated here under the condition of low temperature when the approximations of the well-known traditional theory are not valid. The results of numerical calculations are presented for GaAs and Si. The results are interesting in that they are significantly different from those that follow from the traditional theory.

The advent of the metal-oxide-semiconductor field-effect transistor with easily controllable surface characteristics has made it important to study, from the solid-state device point of view, the electrical transport in the two-dimensional electron gas (2DEG) formed in the inversion and accumulation layers.[1–15] Because of advanced Si technology most of the studies made so far on the 2DEG are devoted to the Si-SiO_2 system. Work on compound semiconductor systems has also been undertaken in recent years.[15–18]

The electrical transport is determined by the dominant interactions which the carriers have with various static and dynamic lattice defects in the layer. It is now well known that under the prevailing circumstances with respect to lattice temperature, sample material, amount of impurity content, etc., the electrical transport characteristics of the 2DEG's are determined by one or more such mechanisms such as the electron-lattice scattering, the scattering due to surface roughness, and by the charged impurities near the oxide-semiconductor interface.[6–12] Of all these, the acoustic phonon scattering is know to be the most important mechanism in interpreting the available results of experiments on the transport characteristics at high lattice temperatures.[19] Useful results on the study of the transport at low lattice temperatures have also been reported. The 2DEG in GaAs has been realized by Störmer *et al.*[20] who employed a GaAs-$Ga_xAl_{1-x}As$ heterostructure and observed Shubnikov–de Haas oscillations around 4.2 K and reported the mobility values at the same temperatures. At such low temperatures though the impurity scattering is usually dominant if the impurity content is sufficiently large; the acoustic scattering also makes significant contribution in the transport process. This apart, since a range of lattice temperatures exist where the carriers in high-purity materials may interact dominantly with intravalley acoustic phonons, and the possibility of obtaining such materials of higher and higher purity being not beyond the scope of the advanced semiconductor technology, the study of the problem of electrical transport at low lattice temperatures has become interesting.[20–25]

The traditional theory of intravalley acoustic phonon scattering in the 2DEG is already developed and expressions for the corresponding relaxation times have been obtained under the condition of high lattice temperatures.[4,8] In the traditional theory the acoustic scattering is assumed to be elastic and the phonon energy is neglected in comparison to the carrier energy. As a consequence, the energy distribution of the phonons is assumed to be given by the equipartition law. These simplified assumptions indeed hold for relatively higher lattice temperatures of a few tens of degrees Kelvin or more.[22,23,26] However, at lower temperatures, the phonon energy becomes comparable to the carrier energy and hence can neither be neglected nor can the phonon distribution be approximated by the equipartition law. Under this condition the acoustic scattering becomes inelastic and a relaxation time can hardly be defined. As such, a wrong balance of electron energy may result if either the true phonon population or the finite phonon energy are not taken into account with sufficient precision. Actually, the small amount of energy exchanged in the collision between the carrier and the phonons now becomes important in ensuring the stationarity of the carrier-phonon system in the presence of any electric field.[21,22,26] A proper theory of the acoustic scattering in the 2DEG at low lattice temperatures is not yet available in the literature. Hence, it would be interesting not only from a purely theoretical point of view to develop the same without making the approximations of the traditional theory. The purpose of this paper is to develop the theory of the inelastic acoustic scattering in a 2DEG at low lattice temperatures giving due regard to the energy carried by an acoustic phonon and also to their true energy distribution. The result is then used to obtain the scattering rate and its dependence upon carrier energy, lattice temperature, impurity concentration, etc., in a 2DEG in GaAs and Si.

The conduction electrons in a 2DEG have classical free-electron motion in the x-y plane parallel to the oxide-semiconductor interface, and the motion in the z direction, perpendicular to the interface, is quantized. Even though the lattice wave is a three-dimensional wave, the scattering of the electrons will be confined to two dimensions.[8] Considering a nondegenerate 2DEG, the intravalley acoustic scattering rate $P_{ac}(\varepsilon_k)$ for an electron with an energy ε_k may be obtained from the perturbation theory in a similar way to the three-dimensional

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case[8,26] as

$$P_{ac}(\varepsilon_{\mathbf{k}})=\frac{2\pi}{\hbar}\int_{\theta=0}^{2\pi}\int_{q_1}^{q_2}\frac{s}{(2\pi)^2}[|\langle\mathbf{k}+\mathbf{q}|H'_{ac}|\mathbf{k}\rangle|^2\delta(\varepsilon_{\mathbf{k}+\mathbf{q}}-\varepsilon_{\mathbf{k}}-\hbar q u_l)+|\langle\mathbf{k}-\mathbf{q}|H'_{ac}|\mathbf{k}\rangle|^2\delta(\varepsilon_{\mathbf{k}-\mathbf{q}}-\varepsilon_{\mathbf{k}}+\hbar q u_l)]q\,d\theta\,dq \quad (1)$$

where the matrix element is given by

$$\langle\mathbf{k}\pm\mathbf{q}|H'_{ac}|\mathbf{k}\rangle=\left[\frac{\varepsilon_1^2\hbar q^2}{2sd\rho_v\omega_q}\right]^{1/2}\begin{bmatrix}n_q^{1/2}\\(n_q+1)^{1/2}\end{bmatrix}.$$

Here $\mathbf{k}$ and $\mathbf{q}$ are, respectively, the components of the electron and phonon wave vectors parallel to the surface. ε_1 is the effective deformation-potential constant which may assume a value larger than that for the bulk material for higher-order subbands.[8] In the above equation $\hbar=h/2\pi$, h being Planck's constant, s is the surface area of the layer, d is the width of the layer of lattice atoms with which the electrons can interact, ρ_v is the mass density, $\omega_q=u_l q$, u_l being the acoustic velocity, and n_q is the phonon population. The upper or lower sign (and n_q or n_q+1) must be taken, respectively, for the process of absorption and emission of phonons. q_1 and q_2 are, respectively, the lower and upper limits of q.

For spherical constant energy surfaces, the energy levels for electrons in a surface channel represented by a triangular potential well, can be expressed as[3,8]

$$\varepsilon_{\mathbf{k}}=\frac{\hbar^2k_x^2}{2m_1^*}+\frac{\hbar^2k_y^2}{2m_2^*}+\varepsilon_n=\frac{\hbar^2k^2}{2m_\parallel^*}+\frac{(e\hbar E_s)^{2/3}}{(2m_3^*)^{1/3}}\gamma_n , \quad (2)$$

where $m_\parallel^*=(m_1^*m_2^*)^{1/2}$ is the effective mass of the electron parallel to the surface; ε_n is the energy of the nth subband; E_s is the surface electric field related to the inversion-layer concentration N_i by $E_s=eN_i/\epsilon_{sc}$, e being the electronic charge and ϵ_{sc} the permittivity of the semiconductor; m_3^* is the effective mass perpendicular to the surface and γ_n are the roots of the equation $\mathrm{Ai}(-\gamma_n)=0$; $\mathrm{Ai}(-z)$ being the Airy function.

When the lattice temperature T is low and the phonon energy cannot be neglected in comparison to the carrier energy, one can perform the integration over θ in Eq. (1) to yield

$$P_{ac}(\varepsilon_{\mathbf{k}})=\frac{\varepsilon_1^2m_\parallel^*}{2\pi\hbar^2d\rho_vu_lk}\int_{q_1}^{q_2}\begin{bmatrix}n_q\\(n_q+1)\end{bmatrix}\times q\,dq/[1-(q/2k)^2(1-2m_\parallel^*u_l/\hbar q)^2]^{1/2} . \quad (3)$$

Now in order to carry out the integration over q, expressions for n_q and the limits q_1 and q_2 must be ascertained under the condition of low temperature of interest here. When the lattice temperature is higher than a few tens of degrees K, the energy distribution of the phonons is assumed in the traditional theory to be given by the equipartition approximation of the Bose-Einstein distribution function. But at lower lattice temperatures the equipartition approximation for the phonon distribution is hardly valid. A good approximation of the true n_q under this condition is given by the truncated Laurent expansion of the form[22]

$$n_q(x)\approx\begin{cases}1/x-\frac{1}{2}+x/12-x^3/720 & \text{for } x<\bar{x}\\ \exp(-x) & \text{for } x\geq\bar{x} ,\end{cases} \quad (4)$$

where $x=\hbar q u_l/k_BT$, k_B being the Boltzmann constant and $\bar{x}=3.5$. The upper and lower limits of q can be obtained from the energy-balance condition for any transition as a result of inelastic collision accompanied by either absorption or emission of a phonon. At low lattice temperatures the probability of multiphonon scattering process is rather small.[26] Since the phonon energy cannot be neglected in the present analysis, the range of the phonon wave vector involved in the absorption process comes out to be larger than the corresponding range for the emission process; whereas the traditional theory with elastic approximations assigns the same range 0–$2k$ for both the processes.

At low temperature there would be a good number of electrons in the energy range $\varepsilon\leq\varepsilon_s$ where $\varepsilon_s=m_\parallel^*u_l^2/2$ and these electrons can only absorb phonons while suffering collisions.[22] Thus for $\varepsilon\leq\varepsilon_s$ the integration limits are $q_1=2m_\parallel^*u_l/\hbar-2k$ and $q_2=2m_\parallel^*u_l/\hbar+2k$ when there would be only absorption. For $\varepsilon>\varepsilon_s$, $q_1=0$ and $q_2=2K+2m_\parallel^*u_l/\hbar$ in the case of absorption, and $q_1=0$ and $q_2=2k-2m_\parallel^*u_l/\hbar$ in the case of emission of a phonon. Now carrying out the integration[27] one can obtain the scattering rate for an electron in a nondegenerate 2DEG for relatively high surface fields and low-lattice temperature in the range[8] $\pi\hbar^2N_i/m_\parallel^*n_vk_B>T>8\varepsilon_s/\bar{x}k_B$, n_v being the number of equivalent valleys at the surface as

$$P_{ac}(\varepsilon_{\mathbf{k}})=\begin{cases}-\pi A_a a_4/\sqrt{\varepsilon_{\mathbf{k}}} , \quad 0<\varepsilon_{\mathbf{k}}\leq\varepsilon_s & (5a)\\[4pt] (A_a/\sqrt{\varepsilon_{\mathbf{k}}})\left[a_4(\theta_5-\pi/2-2\theta_3)+(b/2c\sqrt{-c})(\theta_5-\theta_3)+\sum_{i=0}^{3}a_ix_2'^i\sqrt{R(x_2')}\right.\\ \quad\left.+(1/c)\sqrt{R(x_2')}-(2a_0+1/c)\sqrt{a}\right] , \quad \varepsilon_s<\varepsilon_{\mathbf{k}}\leq\varepsilon_1 & (5b)\\[4pt] \frac{A_a}{\sqrt{\varepsilon_{\mathbf{k}}}}\left[a_4(\theta_4+\theta_5-2\theta_3)+(b/2c\sqrt{-c})(\theta_5-\theta_3)+\sum_{i=0}^{3}a_ix_2'^i\sqrt{R(x_2')}\right.\\ \quad\left.+\sum_{i=0}^{3}a_i\bar{x}_2^i\sqrt{R(\bar{x})}+(1/c)\sqrt{R(x_2')}-(2a_0+1/c)\sqrt{a}\right] , \quad \varepsilon_1<\varepsilon_{\mathbf{k}}\leq\varepsilon_2 & (5c)\\[4pt] \frac{A_a}{\sqrt{\varepsilon_{\mathbf{k}}}}\left[2a_4(\theta_4-\theta_3)+\frac{b}{2c\sqrt{-c}}(\theta_5-\theta_3)+2\sum_{i=0}^{3}a_i\bar{x}_2^i\sqrt{R(\bar{x})}+\frac{1}{c}\sqrt{R(x_2')}-\left[2a_0+\frac{1}{c}\right]\sqrt{a}\right] , \quad \varepsilon_2<\varepsilon_{\mathbf{k}}\leq\alpha , & (5d)\end{cases}$$

where

$$A_a=(\varepsilon_1^2 m_\parallel^{*3/2}/4\sqrt{2}\hbar^3)(1/\pi d\rho_v u_l)(k_B T/\sqrt{\varepsilon_s})^2\ ,\quad a_0=-[1/2c+b/16c^2+\tfrac{1}{720}(55ab/48c^3-35b^3/64c^4)]\ ,$$

$$a_1=1/24c-\tfrac{1}{720}(35b^2/96c^3-3a/8c^2)\ ,\quad a_2=7b/17\,280c^2\ ,\quad a_3=-1/2880c\ ,$$

$$a_4=-\frac{1}{\sqrt{-c}}\left[1+\frac{b}{4c}+\frac{1}{12}\left[\frac{3b^2}{8c^2}-\frac{a}{2c}\right]-\frac{1}{720}\left[\frac{35b^4}{128c^4}-\frac{15ab}{16c^3}+\frac{3a^2}{8c^2}\right]\right]\ ,\quad a=1-\frac{\varepsilon_s}{\varepsilon_{\mathbf{k}}}\ ,\quad b=\frac{k_B T}{2\varepsilon_{\mathbf{k}}}\ ,$$

$$c=-\frac{k_B^2T^2}{16\varepsilon_{\mathbf{k}}\varepsilon_s}\ ,\quad \theta_3=\sin^{-1}\left[\frac{b}{\sqrt{-\Delta}}\right]\ ,\quad \theta_4=\sin^{-1}\left[\frac{2c\bar{x}+b}{\sqrt{-\Delta}}\right]\ ,\quad \theta_5=\sin^{-1}\left[\frac{2cx_2'+b}{\sqrt{-\Delta}}\right]\ ,\quad \Delta=4ac-b^2\ ,$$

$$R(x)=a+bx+cx^2\ ,\quad x_2'=\frac{4\sqrt{\varepsilon_s}}{k_B T}(\sqrt{\varepsilon_{\mathbf{k}}}-\sqrt{\varepsilon_s})\ ,\quad \varepsilon_1=\left[\frac{\bar{x}k_B T}{4\sqrt{\varepsilon_s}}-\sqrt{\varepsilon_s}\right]^2\ ,\quad \varepsilon_2=\left[\frac{\bar{x}k_B T}{4\sqrt{\varepsilon_s}}+\sqrt{\varepsilon_s}\right]^2\ .$$

At high temperature, when the acoustic scattering may be considered elastic, one can neglect the phonon energy and assume the equipartition approximation for the phonon population to be valid. Carrying out the integration in Eq. (1), one now obtains a simple expression for the combined processes of emission and absorption probability:

$$P_{\rm ac}(\varepsilon_{\mathbf{k}})=4\pi A_a\sqrt{\varepsilon_s}/k_B T\ . \qquad (6)$$

It is seen that under this condition the scattering probability is independent of the carrier energy but increases linearly with the lattice temperature.

For an application of the above formulation, samples of GaAs and Si are considered with the parameter values given in Table I. It is assumed that the electrons occupy only the lowest subband when the layer thickness d is given by $[\hbar^2\epsilon_{\rm sc}/2m_3^* e^2 N_i]^{1/3}\gamma_0$. It is to be kept in mind that for GaAs, $m_\parallel^*=m_3^*=m^*$. In case of Si however, the six valleys are not always equivalent. If one considers (100) surface, $m_\parallel^*=m_t^*$ and $m_3^*=m_l^*$ in the two equivalent valleys and for the other four valleys $m_\parallel^*=(m_t^* m_l^*)^{1/2}$ and $m_3^*=m_t^*$.

The dependence of the scattering rate on the carrier energy as obtained for different lattice temperatures is shown in Figs. 1–3 using a logarithmic scale. Considering the (100) surface in Si one can see that for the four equivalent valleys the range of applicability of the above theory is rather limited and since $8\varepsilon_s/\bar{x}k_B\simeq 2.6$, the characteristics could not be obtained for $T=1$ K. It is seen from the figures that the scattering probability now indeed depends upon the carrier energy in a rather complex manner except for very low values of the energy $\varepsilon_{\mathbf{k}}\leq\varepsilon_s$ and there have been significant changes in both the qualitative and quantitative aspects of such dependence at low lattice temperatures in comparison to what follows from the traditional theory. Moreover, as expected the above effect is more significant the lower the lattice temperature. The discrepency of our results from those of the traditional theory arises because of the finite energy of the acoustic phonons. The scattering rate is now below the traditional result in an important range of energy. At lower temperatures a rather flat plateau in the energy dependence of the scattering rates is obtained around an energy equal to the average thermal energy of the carriers. At higher energies the scattering rate, like the three-dimensional case, assumes a simple dependence of the form $P_{\rm ac}(\varepsilon_{\mathbf{k}})\sim\sqrt{\varepsilon_{\mathbf{k}}}$. Such a dependence occurs at higher and higher energies the higher the lattice temperature. It may be further noted that in comparison to the

TABLE I. Material parameters of GaAs and Si.

Physical constant	GaAs	Si
Deformation potential ε_1 (eV)	12.0	9.8
Acoustic velocity, u_l (10^3 m s^{-1})	5.22	9.037
Density, ρ_v (10^3 kg m^{-3})	5.66	2.329
Static dielectric constant, $\epsilon_{\rm sc}$	13.1	11.9
Effective-mass ratio $\left[\frac{m_x^*}{m_0}\right]$	$\left[\frac{m^*}{m_0}\right]=0.072$	$\left[\frac{m_l^*}{m_0}\right]$ −longitudinal=0.96
		$\left[\frac{m_t^*}{m_0}\right]$ −transverse=0.19

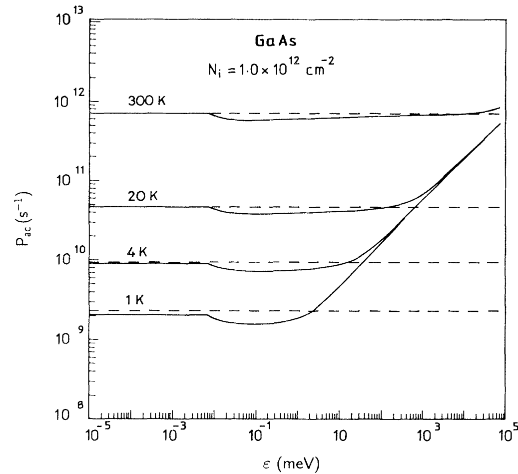

FIG. 1. Dependence of the acoustic scattering rate upon the carrier energy at different lattice temperatures for a 2DEG in GaAs. The solid curves are the results obtained from the present theory and the dashed ones are predicted from the traditional theory.

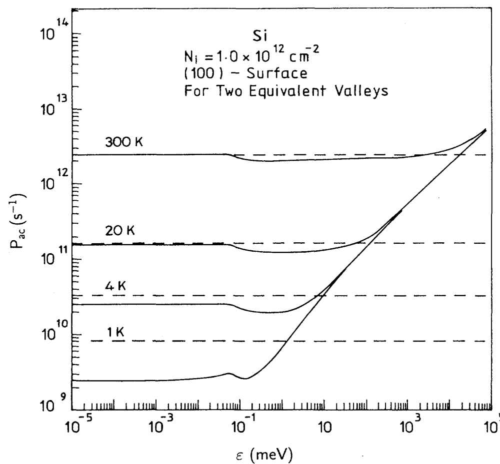

FIG. 2. Dependence of the acoustic scattering rate upon the carrier energy at different lattice temperatures for a 2DEG in Si. The solid curves are the results obtained from the present theory and the dashed one are predicted from the traditional theory.

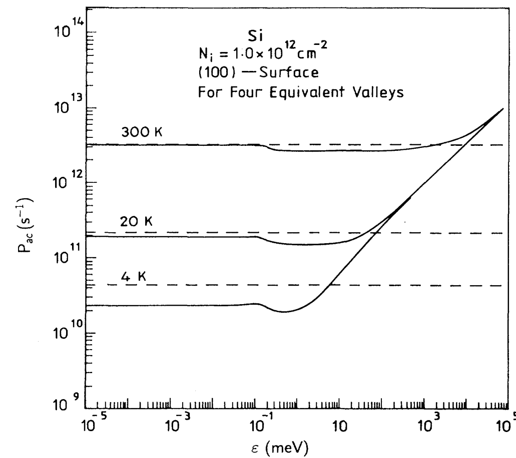

FIG. 3. Dependence of the acoustic scattering rate upon the carrier energy at different lattice temperatures for a 2DEG in Si. The solid curves are the results obtained from the present theory and the dashed one are predicted from the traditional theory.

results of the traditional theory, the dependence of the scattering probability on the concentration now remains unchanged, increasing slowly with the latter as $P_{ac}(\varepsilon_{\mathbf{k}}) \sim N_i^{1/3}$.

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[1]A. S. Grove *et al.* (unpublished).
[2]F. Stern and W. E. Howard, Phys. Rev. **163**, 816 (1967).
[3]D. Colman *et al.*, J. Appl. Phys. **39**, 1923 (1967).
[4]S. Kawaji, J. Phys. Soc. Jpn. **27**, 906 (1969).
[5]T. Sato *et al.*, Phys. Rev. B **4**, 1950 (1971).
[6]Y. C. Cheng and E. A. Sullivan, Surf. Sci. **34**, 717 (1973).
[7]C. T. Sah *et al.*, Surf. Sci. **32**, 561 (1972).
[8]C. Y. Wu and G. Thomas, Phys. Rev. B **9**, 1724 (1974).
[9]H. Ezawa *et al.*, Jpn. J. Appl. Phys. **13**, 126 (1974).
[10]T. H. Ning and C. T. Sah, Phys. Rev. B **6**, 4605 (1972); **9**, 527 (1974).
[11]M. E. Sjostrand and P. J. Stiles, Solid State Commun. **16**, 903 (1975).
[12]D. K. Ferry, Surf. Sci. **57**, 218 (1976); Phys. Rev. B **14**, 5364 (1976).
[13]*Electronic Properties of Two-Dimensional System,* Proceedings of the International Conference on the Electronic Properties of Two-Dimensional System, Berchtesgaden, Germany, 1977, edited by G. Dora and P. J. Stiles (North-Holland, Amsterdam, 1978).
[14]D. Roychoudhury and P. K. Basu, Phys. Rev. B **22**, 6325 (1980).
[15]P. K. Basu and B. R. Nag, Phys. Rev. B **22**, 4849 (1980).
[16]G. A. Antcliffe *et al.*, in *Physics of Semimetals and Narrow-Gap Semiconductors,* edited by D. L. Carter and R. T. Bate (Pergamon, Oxford, 1971).
[17]N. Kotera *et al.*, Phys. Rev. B **5**, 3065 (1972).
[18]A. Darr *et al.*, Solid State Commun. **17**, 455 (1975).
[19]F. F. Fang and A. B. Fowler, Phys. Rev. **169**, 619 (1968).
[20]H. L. Störmer *et al.*, Solid State Commun. **29**, 705 (1979).
[21]B. R. Nag, in *Electron Transport in Compound Semiconductors,* edited by M. Cardona, P. Fulde, and H. J. Queisser, Springer Series in Solid State Sciences Vol. II (Springer, New York, 1980).
[22]C. Canali *et al.*, Phys. Rev. B **12**, 2262 (1975).
[23]S. S. Paul and D. P. Bhattacharya, Phys. Rev. B **39**, 13 521 (1989).
[24]Peter J. Price, Solid State Commun. **51**, 607 (1984).
[25]H. L. Störmer *et al.*, Phys. Rev. B **41**, 1278 (1990).
[26]E. M. Conwell, in *Solid State Physics,* edited by F. Seitz and D. Turnbull (Academic, New York, 1967), Suppl. 9.
[27]I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series, and Products* (Academic, New York, 1965).