Various appearances of Rabi oscillations for 2π-pulse excitation in a semiconductor

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The number of complete Rabi oscillations of the electron-hole density induced by square optical pulses with area-2π is studied using the coherent part of the semiconductor Bloch equations. It is shown that the frequency of the density oscillations depends strongly on the exciton binding frequency, detuning, and the Rabi frequency of the light pulse. The well known doubling of the density oscillation frequency is only observed if the Rabi frequency and exciton binding frequency are approximately of the same order of magnitude. However for strong and weak excitation the semiconductor exhibits properties of atomic systems.

I. INTRODUCTION

Several coherent optical effects in semiconductors are observed experimentally by investigating semiconductor microstructures with short light pulses and described theoretically by solving the semiconductor Bloch equations. Examples are transient oscillations in differential transmission spectra, the optical Stark effect, the semiconductor photon echo, as well as coherent pulse breakup.[1–17] Another coherent effect observed theoretically is temporal oscillation of the electron-hole density (Rabi oscillations) during the presence of an ultrashort light pulse in the sample. These oscillations have been studied by several authors for resonant excitation of semiconductors such as GaAs and CdSe.[18–22] In these studies the temporal behavior of the density was shown to exhibit almost twice as many complete Rabi oscillations of the density as the number of complete Rabi oscillations known from the interaction of light with atomic systems. In the latter case, the number of complete Rabi oscillations depends on the area Θ of the light pulse which is defined by

$$\Theta = \int_{-\infty}^{+\infty} \Omega(t)dt. \tag{1}$$

Here the Rabi frequency $\Omega(t)$ of the light pulse is given by the dipole transition moment d_{cv} and the slowly varying envelope of the electric field $E(t)$:

$$\Omega(t) = \frac{d_{cv}E(t)}{\hbar} \tag{2}$$

if the total electric field is given by $\mathcal{E}(t) = \frac{1}{2}E(t)e^{i\omega_L t}$+c.c. For noninteracting two-level systems only one complete Rabi oscillation is expected if the area of the pulse is 2π.[23] However, pulses with the area of 2π induce almost two complete Rabi oscillations in the semiconductor electron-hole density.[19,21,22] This difference in comparison to one complete Rabi oscillation in the two-level system electron density was explained by the renormalization of the optical field due to Coulomb coupling of electrons and holes.[19] The question arises of whether or not the often observed doubling is a systematic effect. In the previous studies of complete Rabi oscillation[18–22] in semiconductors, light pulses with intensities of several GW/cm^2 and pulse durations in the femtosecond domain are used to excite the semiconductor. Hence the maximum Rabi frequency Ω of the light pulses is more or less in the order of the exciton binding frequency of the investigated materials. For different values of this ratio of exciton binding frequency and Rabi frequency drastic changes in the number of complete Rabi oscillations may occur even for fixed light pulse area of 2π [Eq. (1)]. The reason for this is given by competing effects of three characteristic frequencies which govern the interaction of light pulses with the semiconductor and especially by the so called exchange effects which are characteristic for the semiconductor material. These three frequencies are the maximum Rabi frequency $\Omega = \max\{\Omega(t)\}$ of the light pulse, the exciton binding frequency $\omega_x = \frac{e^2}{2\hbar a_0}$, and the detuning δ between the carrier frequency of the laser pulse and the exciton resonance. Here a_0 is the Bohr radius of the exciton. The aim of this paper is to show the strong dependence of the number of complete Rabi oscillations on the ratio of Rabi frequency and exciton binding frequency (and the detuning as well) for light pulse excitation with area 2π. The parameter regime $(\Omega, \omega_x, \delta)$ which causes the often observed doubling of the number of complete Rabi oscillations in a semiconductor is investigated.

II. BASIC EQUATIONS

The interaction of short light pulses with semiconductors is described by the coherent part of the semiconductor Bloch equations for a two band semiconductor as derived in Refs. 6–9. These equations determine the temporal dynamics of the polarization functions P_q and the electron and hole distribution functions f_q under the influence of the optical field $\mathcal{E}(t)$ with the carrier frequency ω_L:

$$\frac{\partial P_q}{\partial t} = i\left(\epsilon_q - \omega_L - \frac{1}{\hbar}\sum_{q'} V_{q-q'}(f^e_{q'} + f^h_{q'})\right) P_q$$
$$-i\Omega_q(1 - f^e_q - f^h_q), \tag{3}$$
$$\frac{\partial f^{e/h}_q}{\partial t} = i(\Omega^\star_q P_q - \Omega_q P^\star_q).$$

Here the renormalized field Ω_q is the sum of the optical Rabi frequency Ω and Coulomb corrections:

$$\Omega_q = \frac{\Omega}{2} + \frac{1}{\hbar}\sum_{q'} V_{q-q'} P_{q'},$$
$$\epsilon_q = \omega_{\text{gap}} + \frac{\hbar q^2}{2m_{\text{eff}}}. \tag{4}$$

These equations contain the Coulomb interaction $V_q = \hbar\omega_x \frac{8\pi a_0}{q^2}$ between electrons and holes in the time dependent Hartree-Fock approximation which leads to a correction of the source term $\Omega/2$ in Eq. (4) and to the band gap renormalization Eq. (3). The total electron-hole density is given by the sum over all distribution functions f_q:

$$n_2(t) = 2\sum_q f_q. \tag{5}$$

The factor of 2 accounts for the spin. It was shown that the quantity $n_2(t)$ undergoes coherent temporal oscillations (complete Rabi oscillations) for short pulse excitation of the semiconductor.[18–22] Because we are interested in the dependence of the number of those complete Rabi oscillations on the ratio between exciton binding frequency (or detuning of the exciton resonance with respect to the laser frequency) and Rabi frequency we introduce the dimensionless quantities α and β:

$$\alpha = \omega_x/\Omega, \qquad \beta = \delta/\Omega,$$
$$\delta = \omega_{\text{gap}} - \omega_x - \omega_L. \tag{6}$$

The parameter δ is a measure of the detuning of the laser frequency ω_L from the fundamental exciton resonance, which is obtained by a diagonalization of the linearized Eqs. (3) and (4) in the weak excitation limit.[24] The exciton resonance position is one binding energy $\hbar\omega_x$ below the band gap $\hbar\omega_{\text{gap}}$. We keep the area Eq. (1) of the pulse 2π and investigate the number of complete Rabi oscillations for different α and β. The choice of a 2π area has the advantage that the comparison with the atomic case (one complete Rabi oscillation for exact resonance) is very simple.[23]

III. ANALYTICAL CONSIDERATIONS FOR LIMITING CASES

To investigate the temporal behavior of the electron-hole density $n_2(t)$ for arbitrary excitation conditions one has to solve Eqs. (3) numerically.[18–22] However, to get some analytical insight and to understand the expected competition between Rabi frequency and exciton binding frequency we discuss two asymptotic cases.

(i) $\alpha \ll 1, \beta \ll 1$. In this case the Rabi frequency is supposed to be large compared with the exciton binding frequency and the detuning. Therefore it is expected that the dynamics of the system is determined mainly by the Rabi frequency Ω and not by the Coulomb interaction terms which scale like ω_x. Let us assume for a moment that the contribution of the Coulomb terms is zero ($V_q \to 0$). Under this condition we may solve Eqs. (3) exactly. Afterwards it is possible to determine the conditions under which this approximation is valid by calculating the Coulomb contributions with the obtained solution and estimating the contribution to the full solution of Eqs. (3). Provided the Coulomb terms in Eqs. (3) are zero the solution for the total electron-hole density is given by[23]

$$n_2(t) = 2\sum_q \frac{\Omega^2}{\Omega^2 + \delta_q^2}\sin^2\left(\sqrt{\Omega^2 + \delta_q^2}\frac{t}{2}\right),$$
$$\delta_q = \epsilon_q - \omega_L \tag{7}$$

if the Rabi frequency is approximated by a constant value $\Omega(t) = \Omega$ for the duration of the pulse. With this solution and the corresponding solution for the polarization as well it is now possible to estimate the contribution of the Coulomb terms in the full semiconductor Bloch equations. To give an example of this procedure let us estimate the contribution of the band gap renormalization on the right-hand side of the polarization equation (3) for the limit $\alpha \to 0$:

$$\left|2\sum_{q'} V_{q-q'} f_{q'} P_q\right| \le 2\sum_{q'} V_{q-q'} f_{q'}. \tag{8}$$

For the calculation of the Coulomb sum we use here the angle averaged Coulomb potential for a bulk semiconductor which was used in several studies of Rabi flopping,[18,19,21,24]

$$2\sum_{q'} V_{q-q'} f_{q'} \le \omega_x \frac{4}{\pi}\int_0^\infty d(q'a_0)\frac{q'}{q}\ln\frac{|q'+q|}{|q'-q|}\frac{1}{1+(\delta_{q'}/\Omega)^2}. \tag{9}$$

After a substitution of the integration variable we find for the ratio of band gap renormalization and Rabi frequency

$$\left|\frac{2\sum_{q'} V_{q-q'} f_{q'} P_q}{\Omega}\right| \le \frac{4}{\pi}\alpha q a_0 \int_0^\infty dx x \ln\frac{|1+x|}{|1-x|}\frac{1}{1+\alpha^2(qa_0x)^4}. \tag{10}$$

An expansion for $\alpha \to 0$ yields

$$\frac{4}{\pi}\alpha q a_0 \int_0^\infty dx x \ln\frac{|1+x|}{|1-x|}\frac{1}{1+\alpha^2(qa_0x)^4} \approx \frac{8}{\pi}\int_0^\infty \frac{dy}{1+(qa_0)^2y^4}\sqrt{\alpha q a_0} = 2\sqrt{2\alpha}. \tag{11}$$

Hence

$$\left|\frac{2\sum_{q'} V_{q-q'} f_{q'} P_q}{\Omega}\right| \leq 2\sqrt{2\alpha} \to 0 \quad \text{for} \quad \alpha \to 0. \tag{12}$$

It can be seen from Eq. (12) that the influence of the Coulomb contribution to Eqs. (3) vanishes if α approaches zero. Therefore it is possible to neglect the Coulomb coupling for sufficient small ratio of exciton binding energy and Rabi frequency in a first order approximation. Equations (3) exhibit in the limit $\alpha \to 0$ the same structure as an inhomogeneously broadened two-level system. For very large Ω the number of complete Rabi oscillations is expected to be almost the same as in the atomic case, i.e., one complete Rabi oscillation for 2π-pulse excitation.

(ii) $\alpha \gg 1, \beta \gg 1$. In this case the equations can be treated in linear response theory with respect to small Rabi frequencies. The detuning is always chosen to be positive, i.e., the carrier frequency of the light pulse is centered slightly below the exciton resonance ($\beta > 0$). The reason we did not choose exact resonance ($\beta \equiv 0$) is that an analytic perturbation theory which we apply for the linear response regime in this section is not applicable in case (ii). For the exact resonance of the laser frequency and the first exciton the linear response is discussed in Sec. V in terms of a simplified set of equations. Here we make the ansatz that the polarization functions and the distribution functions may be expanded in terms of $1/\alpha$. It is useful to discuss the linear response case in the low excitation coherent regime proposed in Ref. 9. The corresponding equations read

$$\left(\frac{\partial}{\partial t} - i\Delta_\lambda\right) P_\lambda = -\frac{i}{2} w_\lambda \Omega, \qquad \frac{\partial}{\partial t} w_\lambda = -i\left(\Omega^\star P_\lambda - \Omega P_\lambda^\star\right). \tag{13}$$

Here w_λ and P_λ are the inversion and the polarization of the state λ. The set of states λ consists of the bound (quantum number n) and the continuous states (quantum number q). The energy dispersion of the system is given by

$$\Delta_\lambda(n) = \omega_{\text{gap}} - \frac{\omega_x}{n^2} - \omega_L, \qquad \Delta_\lambda(q) = \omega_{\text{gap}} + \omega_x(qa_0)^2 - \omega_L. \tag{14}$$

The total macroscopic quantities such as the polarization are calculated by taking into account the Coulomb enhancement factor:[24]

$$\sum_\lambda P_\lambda \to \frac{1}{\pi a_0^3}\left\{\sum_n \frac{1}{n^3}P_n + \frac{1}{2}\int dx \frac{x e^{\pi/x}}{\sinh \pi/x} P(x)\right\}. \tag{15}$$

Here the Coulomb enhancement factor for a bulk semiconductor is applied. As already mentioned, a linear response theory for vanishing detuning is not applicable due to the large 2π area of the light pulse. Therefore we restrict ourselves here to the case of nonvanishing detuning whereas vanishing detuning is discussed in Sec. V. In lowest order of the expansion in terms of the Rabi frequency $(1/\alpha)$ one finds for the total electron-hole density

$$n_2(t) = 2\sum_q f_q = \frac{2}{a_0^3\pi}\sum_\lambda \left(\frac{\Omega}{\Delta_\lambda}\right)^2 \sin^2\left(\frac{\Delta_\lambda t}{2}\right). \tag{16}$$

Taking only the leading term, i.e., the first exciton, into account we find that the solution shows oscillations with the finite detuning $\delta \equiv \Delta_\lambda(n=1)$ as defined in Eq. (6) between the exciton resonance and carrier frequency of the laser pulse.

$$n_2(t) \approx \frac{2}{a_0^3\pi}\left(\frac{\Omega}{\delta}\right)^2 \sin^2\left(\frac{\delta t}{2}\right). \tag{17}$$

Furthermore, from Eq. (17) it can be seen that linear response theory with respect to Ω is not applicable for the case $\delta = 0$.

The discussion of (i) and (ii) shows two limiting cases of the competition of Rabi frequency and exciton binding frequency/detuning in the dynamics of the system. In the case of large α the oscillations of the electron density are determined by the finite detuning of the laser frequency and the exciton resonance whereas for small α the Rabi frequency dominates the dynamics. The interesting result is that in both limits the semiconductor is shown to behave like a two-level atom. However, the many body effects in Eqs. (3) are important for the intermediate case where the Rabi frequency reaches the order of magnitude of the exciton binding frequency ($\alpha \approx 1$). In this parameter regime complicated electron-hole density oscillations are expected due to the competition and interference effects of Rabi frequency and exciton binding frequency. The case $\alpha \approx 1$ leads to the well known doubling of the number of complete Rabi oscillations. This special coupling regime was already investigated in Refs. 18-22. The parameter regime between the asymptotic cases described in this section and the case $\alpha \approx 1$ can presently only be explored numerically.

IV. NUMERICAL RESULTS

In order to obtain results for the intermediate case, i.e., α in the order of unity, one has to carry out numerical calculations. For the numerical calculations we use here a semiconductor model with a quasi-one-dimensional long-range Coulomb-type interaction and consider N atoms on a ring with periodic boundary conditions.[25]

$$U_{|m-m'|} = \begin{cases} U_0, & m = m' \\ U\frac{\pi/N}{\sin\left(\frac{\pi}{N}|m'-m|\right)}, & |m'-m| \geq 1. \end{cases} \quad (18)$$

The denominator of Eq. (18) is the chord distance of two lattice points on the ring. It is necessary to introduce a cutoff $U_0 > 2U$ for the on-site Coulomb repulsion. The model interaction has the periodicity of the ring and for finite N produces a finite number of bound electron-hole pair states, which are the common Wannier excitons. The dispersion of the Bloch states of the two band model is given in a tight-binding approximation ($q \in$ BZ) (where BZ denotes Brillouin zone) by

$$\epsilon_q = \tilde{\Delta} - B\cos(q), \quad (19)$$

with a bandwidth parameter B and energy separation $\tilde{\Delta}$ of the atomic states. Instead of the Coulomb interaction given in a bulk semiconductor we choose this special model for two reasons.

(1) First, we could obtain from the proposed one-dimensional (1D) model a simplified set of three differential equations for the total electron-hole density and the total polarization which approximates the solution of the set of integro-differential equations (3) in a certain parameter range very well. Therefore we would like to compare our numerical results of this section with the simplified model which is decribed in Sec. V.

(2) Secondly, all qualitative features given by the 3D model with angle averaged Coulomb potential described in Refs. 18 and 19 are conserved by the 1D model used here. This concerns the asymptotic cases discussed in Sec. IV as well.

We believe therefore that the analytic approach to the nonlinear frequency of the electron-hole density oscillations given in Sec. V is an advantage of the 1D model, especially to study the case $\beta = 0$. The optical pulse $\Omega(t)$ in our numerical calculations is always chosen to be a square pulse with an area of 2π, i.e., the number of complete Rabi oscillations expected without Coulomb interaction is one. As shown in Figs. 1 and 2, we confirm by our numerical calculations the discussed asymptotic behavior for large and small values of α for a nonvanishing detuning (Sec. III). Figure 1 shows the electron-hole density n_2 as a function of the time $t\Omega/2\pi$ for $\alpha = 20$ ($\alpha = 100$), i.e. for the case of large α which corresponds to case (ii) of Sec. III. We reproduce clearly the electron-hole density oscillations with the frequency of the detuning $\delta = 10\Omega$ ($\delta = 50\Omega$). The absolute value of the amplitude is determined by the ratio $(\Omega/\delta)^2$. On the other hand, the Rabi oscillations for the atomiclike case $\alpha = 0.01$ are reproduced in Fig. 2. Because of our choice of the area of 2π we get exactly one complete Rabi oscillation, i.e., two-level-atomlike behavior. The intermediate case where α is in the order of unity is investigated in Fig. 2 as well. The curves for $\alpha = 1$ and $\alpha = 2$ in Fig. 2 show the deviation of the oscillations compared to one complete Rabi oscillation ($\alpha = 0.01$). For $\alpha \approx 2$ we obtain two complete Rabi oscillations. The complicated oscillations given in this intermediate case are determined by the competition of Rabi frequency, detuning, and exciton

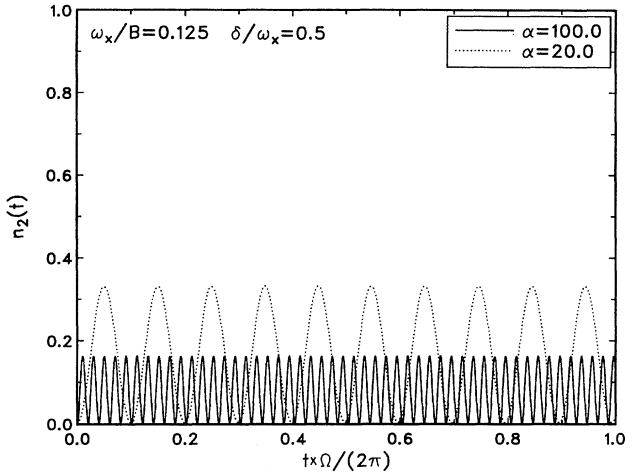

FIG. 1. Temporal Rabi oscillations of the electron-hole density for $\alpha = 100$ (solid line) and $\alpha = 20$ (fine dotted line) for $\beta = 50$ calculated as a solution of Eqs. (3). The oscillation frequency is determined by the detuning leading to nearly 50 (10) complete Rabi oscillations for $\alpha = 100$ ($\alpha = 20$) (fine dotted line) as predicted by linear response analysis. The scaling factors are 1.2×10^3 and 10^2, respectively.

binding frequency in Eqs. (3). A 3D plot for $\delta/\omega_x = 0.5$ summarizes the results in Fig. 3. There we have chosen a constant excitation 2π pulse and plotted the temporal electron-hole pair density evolution for α in the relevant parameter regime $0.1 < \alpha < 10$. From this figure it can be seen that the doubling of the number of complete Rabi oscillations occurs approximately over a wide range of α. This is the parameter regime studied in Refs. 18–22.

Now we would like to study the case of vanishing detuning $\delta = 0$. The oscillations of the electron-hole density

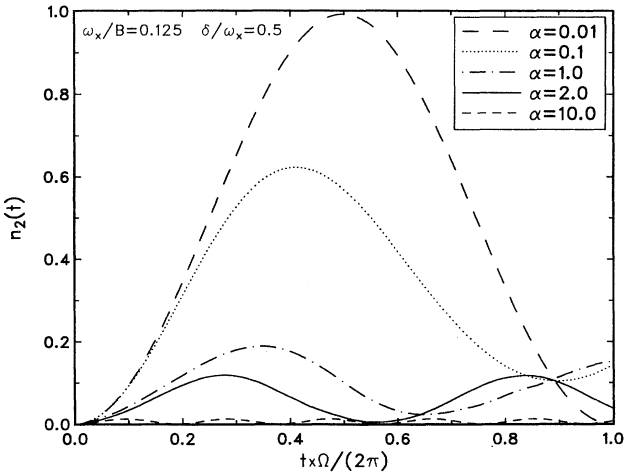

FIG. 2. Temporal Rabi oscillations of the electron-hole density for $\alpha = 0.01$ to 10.0 calculated as solutions of Eqs. (3). The oscillation frequency is determined by the Rabi frequency of the light pulse for α=0.01. The interplay between Rabi frequency and exciton frequency causes different oscillation patterns for $\alpha \approx 1$. The doubling of the number of complete Rabi oscillations is reached at $\alpha = 2$ (solid line).

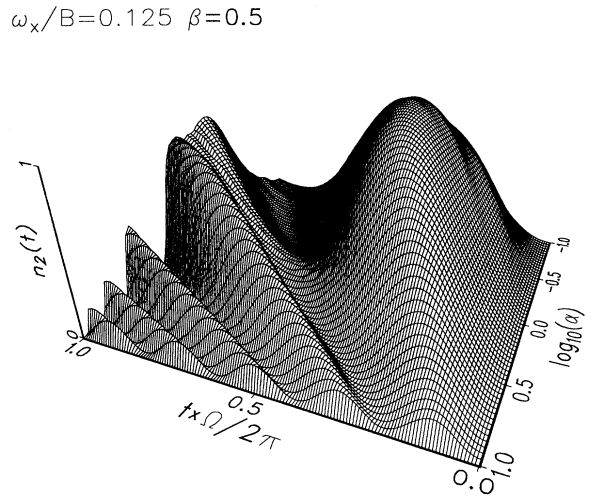

FIG. 3. Complete Rabi oscillation characteristics from $\alpha = 0.1$ to $\alpha = 10$ for the detuning $\beta = 0.5$ and constant excitation for the model system Eq. (19). The logarithmic scaling of the α axis indicates the large regime where the well known doubling of the complete Rabi oscillation may be observable for the physical relevant femtosecond pulse experiments. The density $n_2(t)$ is linearly scaled up to 10 at $\alpha = 10$ because of the effective amplitude decrease.

are depicted for various values of α ($\beta = 0$) in Fig. 4. The expected one complete Rabi oscillation is reproduced in the case of $\alpha = 0.01$. The situation changes in the case of large α. For $\alpha = 100$ the number of complete Rabi oscillations does not increase as drastically as in the case of nonvanishing detuning (Fig. 1). This is not surprising because the dominant frequency in the system is now the Rabi frequency due to the vanishing detuning. Hence the oscillation frequency (for the given case $\alpha \gg 1, \beta = 0$ denoted by ω_{NL}) is expected to scale somehow with the

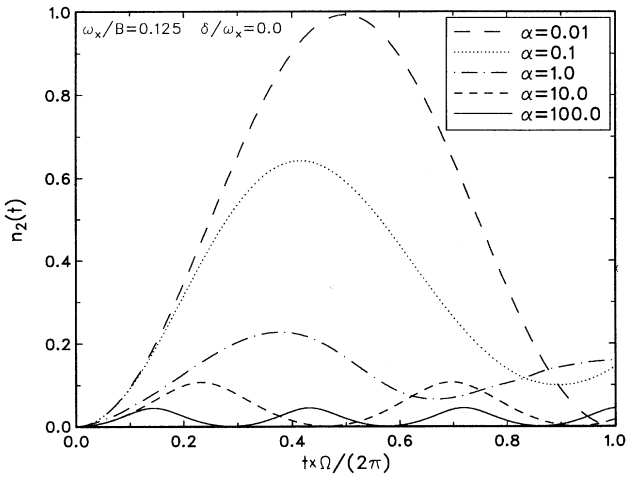

FIG. 4. Temporal Rabi oscillations of the electron-hole density for $\alpha = 0.01$ to 100.0 and β=0 calculated as solution of Eqs. (3). In contrast to Fig. 2 the oscillation frequency is determined mainly by the Rabi frequency of the light pulse for large values of α, which can clearly be seen at $\alpha = 100$.

Rabi frequency. From Eq. (17) it can be seen that in the case of vanishing detuning the linear response theory as proposed in Sec. II breaks down for $\alpha \gg 1$. Nevertheless an explanation of the scaling behavior of the nonlinear frequency ω_{NL} may be given by the already mentioned simplified model proposed in Sec. V.

V. SIMPLIFIED DYNAMICAL SYSTEM

In this section we would like to show that the dynamics of the full semiconductor Bloch equations (3) can be approximated for certain parameter ranges very well by a simplified system of equations which contain the macroscopic quantities $P(t) = \sum_q P_q$ and $n_2(t)$ only. These equations contain a nonlinearity comparable with the band gap renormalization of the semiconductor Bloch equations which leads to a dynamical detuning, which is related to the parameters of the model described in Sec. IV. Introducing the common Bloch vector description of the semiconductor Bloch equations with

$$\begin{aligned}\mathbf{S} = &[P(t) + P^{\star}(t)]\,\mathbf{e}_1 + i\,[P(t) - P^{\star}(t)]\,\mathbf{e}_2 \\ &+ [2n_2(t) - 1]\,\mathbf{e}_3,\end{aligned} \tag{20}$$

we propose a simplified dynamical system, which produces the essential features of the full microscopic description for the parameter range $\alpha, \beta \geq 1$. The dynamics of the macroscopic expectation values is approximated in the case of constant Ω by

$$\begin{aligned}\dot{S}_1 &= -\left[\delta + J_1(1 + S_3)\right] S_2, \\ \dot{S}_2 &= \left[\delta + J_2(1 + S_3)\right] S_1 - J_3 S_3, \\ \dot{S}_3 &= \Omega S_2.\end{aligned} \tag{21}$$

The S_i are the components of the vector $\mathbf{S}$ introduced in Eq. (20). Equations (21) contain only macroscopic variables, the control parameters Ω, δ [cf. Eq. (6)] and a phenomenological parameter vector $\mathbf{J} = (J_1, J_2, J_3)$. We will refer to the set of Eqs. (21) as the $\mathbf{J}$ model. We expect that the nonlinear nontrivial dynamics of the macroscopic Bloch vector $\mathbf{S}(t)$ may in principle be derived from the complete set of microscopic equations (3) and (4).

In the limit of strongly localized electrons and holes, we can estimate the parameters of the $\mathbf{J}$ model. We find

$$\begin{aligned}J_1 = J_2 &\equiv J_{\perp}, \\ J_3 &= \Omega.\end{aligned} \tag{22}$$

In this limit $J_{\perp}$ is given by the bare atomic energy separation where the intrinsic semiconductor ground state and the exciton ground state are degenerate.[26] It can be shown that Eqs. (21) possess a critical transition point at the exciton resonance $\delta = 0$. This is in agreement with the behavior of the full semiconductor Bloch equations. Similar equations occur in the treatment of dense two level media,[27–29] where the nonlinearity stems from the instantaneous dipole-dipole interaction.

In contrast for the usual bulk system with quasifree electrons interacting via Coulomb interaction, the param-

eter $J_\perp$ has no real physical meaning and must therefore be deduced from the full semiconductor Bloch equations. In the limit of $J_\perp \ll 1$ the dynamics of **S** is therefore asymptotically represented by the **J** model.

We would like to study the parameter regime for which Eqs. (21) are expected to be a suitable approximation of Eqs. (3). Figure 5 shows the difference between the solution of the full semiconductor Bloch equations and the solution of the corresponding **J** model. We have chosen the case of exact resonance and $\alpha = 10$ for the parameters of Fig. 4 for a 2π-pulse excitation. The agreement is satisfying, i.e., the very complicated set of integro-differential equations for the one particle quantities for each quantum number q is reduced to a simple set of coupled differential equations for the macroscopic quantities as total density and polarization. The approximation obviously becomes worse if $J_\perp = 0.33$ (lower density amplitude in Fig. 5). In this case the estimate (22) is insufficient and one has to at least determine all three coefficients in Eq. (21). For large α the dynamics is governed by a strong collective nonlinear motion and the **J** model is supposed to become asymptotically exact.

If we assume the validity of the **J** model in the limit $\alpha \to \infty$ it is possible to deduce the scaling behavior at zero resonance of the maximum density amplitude $a \equiv \max\{n_2(t)\}$ and the nonlinear frequency $\omega_{\rm NL}$ from the **J** model with $J_1 = J_2 = J_\perp$ and $J_3 = \Omega$ analytically. This was not possible in Sec. III on the basis of the full set of Eqs. (3) and (4). From an inspection of Eqs. (21') it is seen that the relation

$$\ddot{n_2} = -\frac{\partial V(n_2)}{\partial n_2}, \tag{23}$$

with

$$V(n_2) = \frac{n_2}{2}\left\{\Omega^2(n_2 - 1) + J_\perp^2 n_2^3\right\} \tag{24}$$

holds. The dynamical behavior of the macroscopic density is mapped onto a classical quartic oscillator. With $n_2(t = 0) = 0$ it is straightforward to derive the asymptotic behavior ($\Omega \to 0$) for the amplitude a determined by $V(a) = 0$ and the period of one oscillation integrating Eq.(23). We find

$$\begin{aligned} a &\longrightarrow \left(\frac{\Omega}{J_\perp}\right)^{2/3}, \\ \omega_{\rm NL} &\longrightarrow \frac{\Omega}{\sqrt{a}}\left(\int_0^1 \frac{dy/\pi}{\sqrt{y}\sqrt{1-y^3}}\right)^{-1}. \end{aligned} \tag{25}$$

The analytical results show how the number of complete Rabi oscillations scales with the Rabi frequency Ω independently of the precise values of $J_\perp$. The total number of complete Rabi oscillations diverges slowly at zero detuning whereas for finite δ it was shown analytically in Sec. III that a constant number of complete Rabi oscillations is reached depending on the value of β. This explains the slowly increase of the number of complete Rabi oscillations in Fig. 4 which is not described by a linear response treatment. The ratio of the nonlinear frequency $\omega_{\rm NL}$ and the amplitude a is independent of the excitation strength Ω for $\Omega \to 0$:

$$\lim_{\Omega \to 0} \frac{\omega_{\rm NL}}{a} \approx 1.29 J_\perp. \tag{26}$$

Figure 6 shows the scaling behavior of the density amplitude and the nonlinear frequency $\omega_{\rm NL}$ for the full semiconductor Bloch equations for different values of α (+). We find again a good agreement of the results obtained

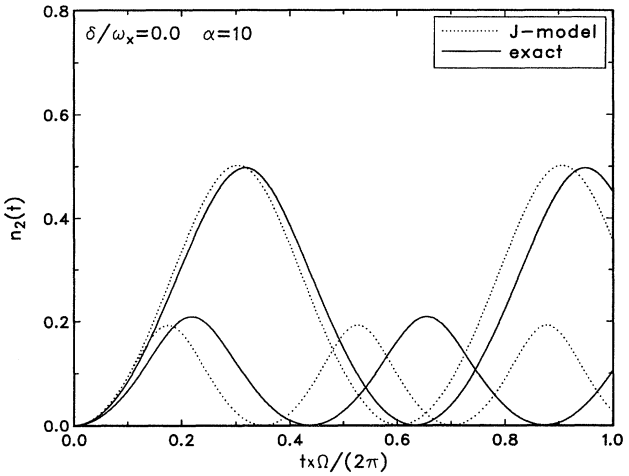

FIG. 5. Comparison of the solutions of the full semiconductor Bloch equations (solid line) with the **J**-model dynamics of Eq. (22) (fine dotted line). The value of α is fixed at $\alpha = 10$ and $B = 0.8$. For $U_0 = 1.6$ the approximative solutions show an excellent agreement with the exact results over the plotted time domain ($J_\perp = 0.19$). For $U_0 = 0.8$ (lower density amplitude) the approximation becomes less satisfying for the estimated value of $J_\perp = 0.33$, which is probably too large for a suitable expansion parameter.

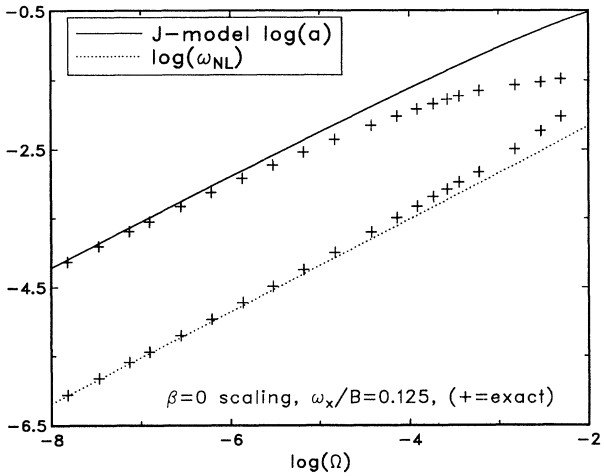

FIG. 6. Scaling of the nonlinear density amplitude a and the nonlinear frequency $\omega_{\rm NL}$ at exact resonance $\delta = 0$. The exact solution approaches the **J**-model-predicted behavior with a slope of 2/3. The fine dotted line is the nonlinear frequency calculated from Eq. (22) with the fixed asymptotic parameter $J_\perp = 0.11$. All logarithms are base e.

with Eqs. (3) and (4) and the analytical results predicted by the simple **J** model. The solid line is the result for a (dotted for ω_{NL}) predicted by the **J** model with the extrapolated asymptotic parameter $J_{\perp} \approx 0.11$. Despite this success it would be highly desirable to obtain analytical microscopic expressions for the **J**-model parameters for arbitrary values of α in order to decrease the numerical effort in solving the semiconductor Bloch equations.

VI. CONCLUSION

In conclusion, we have shown that the character of the coherent dynamics of the electron-hole density is mainly determined by the ratio of the Rabi frequency of the external light field and the exciton binding frequency of the semiconductor. In the case where the Rabi frequency of the light pulse is much smaller than the exciton frequency of the semiconductor the density oscillations are determined by the detuning of laser frequency and exciton resonance only. In the opposite case the number of oscillations is given by the well known dynamics of the atomic case (density oscillation frequency is equal to the Rabi frequency). Hence the semiconductor shows properties comparable to atomic systems, because exchange effects are not important in both asymptotic limits.

If the values of Rabi frequency and exciton binding frequency are of the same order of magnitude, both frequencies compete with each other and cause complicated oscillation patterns. The doubling of the density oscillation frequency compared to the atomic case is shown to dominate the dynamics if semiconductors such as CdSe and GaAs are excited with femtosecond pulses of an area of several π. A simplified model which may substitute the semiconductor Bloch equations for certain cases was proposed.

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