Surface paraconductivity induced by an external electric field

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The fluctuating properties of the surface superconducting layers created by an electric field perpendicular to the surface are investigated. Shifts of the critical temperature, heat capacity, and the conductivity above the critical temperature have been calculated for arbitrary relations between the screening and coherence lengths.

I. INTRODUCTION

For more than 30 years, there have been efforts to improve the properties of the superconductor by applying an electric field.[1] Indeed the external electric field which increases the density of charge carriers at the surface was expected to increase the critical temperature depending strongly on the charge density, as can be seen from simple BCS formulas. It seems that the shift of the critical temperature should be large. However, the results of the first experiments were disappointing.[2] Only for a very strong electric field ($E=10^8$ V/cm) was the shift of the critical temperature observed.[3]

Details of theoretical studies show the very important role played by the proximity effect in suppressing the surface superconductivity by electrons from the bulk.[4,5] Considering this theory, the relatively large effect has to be observed for superconductors with a low concentration of mobile charge carriers n (large screening length) and a short coherence length (in order to suppress the proximity effect). From this point of view the superconducting ceramics are the best samples in which to observe this phenomena. Indeed, these systems such as $BaPbBiO_3$, $YBa_2Cu_3O_7$, and $Bi_2Sr_2CaCu_2O_8$ have an intrinsic carrier concentration $n=(2-5)\times 10^{21}$ cm^{-3} which is relatively small. The coherence length in these structures is also extremely small. The shift of the critical temperature for monocrystal $BaPbBiO_3$ was observed ($\Delta T=0.3$ K) by Venevtsev and Bogatko.[6] This result was in a good agreement with theoretical prediction.[7] Field effect studies of several high-temperature systems have been carried out in the past few years by a number of groups. These studies included experiments in which an electric field was applied to a semiconductor $YBa_2Cu_3O_{7-\delta}$ in order to investigate the possibility of inducing superconductivity[8] as well as investigations of superconducting films.[9-11] In the first case, by applying electric field of only 3×10^6 at 160 K, the conductance of the semiconductor film was increased to about 0.1 e^2/h. In the latter case, the critical temperature T_c of $YBa_2Cu_3O_{7-\delta}$ films was shifted by as much as 0.1 K.[10,11]

Besides shifting T_c the electric field also modifies transport in the normal and superconducting states.[10] In particular, the electric field increases critical current. Another result of the external electric field is its effects on superconducting fluctuations. For usual superconductors these fluctuations do not play a significant role in thermodynamic properties because of a very large coherence length ξ_0.[12] On the other hand, the magnetic moment and the conductivity of the superconductor may be large enough at temperature $T>T_c$.[13,14] The temperature dependence of these functions is strongly dependent on the dimensionality of the system. In low-dimensional structures the fluctuation phenomena are strongly manifested. From this point of view the fluctuation phenomena in the surface layer are expected to exhibit very interesting features.

II. BASIC EQUATIONS

Let us consider the Ginzburg-Landau functional for anisotropic superconductors with an inhomogeneous constant of electron-electron interaction $g=g(x)$ in the form[7,5]

$$
\begin{aligned}
H_{\rm GL}&=N_0\int[a(x)|\Psi|^2+b|\Psi|^4+\xi_\mu^2|(i\partial_\mu-2eA_\mu/c)\Psi|^2+(B-H)^2/8\pi]d^3r\ ,\\
a(x)&=[T-T_c(x)]/T_c(x)\ ,\\
b&=7\zeta(3)/8\pi^2T_c^2\ ,\\
\xi_\mu^2&=\pi^2 v_\mu l_\mu/24T_c\ ,\\
T_{c0}&=1.14\Omega_D\exp(-1/g_0)\ ,\\
g_0&=N_0V_0\ ,\\
T_c(x)&=1.14\Omega_D\exp[-1/g(x)]\ ,\\
g&=N(x)V[N(x)]\ ,
\end{aligned}
\qquad (1)
$$

Ψ is the superconducting gap, A_μ is the magnetic vector potential, B and H are the induction and external magnetic field, respectively, N_0 is an unperturbed value of the density of states (DOS) at the Fermi level, v_μ and l_μ are the Fermi velocity and free path of the electron, $\mu=x,y,z$ (Fig. 1), Ω_D is the Debye frequency. (We assume that the surface of the layered structure coincides with the layers.) The coordinate-dependent critical temperature $T_c[N(x)]$ has a well-known form [see Ref. 5]:

$$T_c[N(x)]=T_c(N_0)\{1+\partial\ln T_c(N_0)/\partial\ln N_0[\delta N(x)/N_0]\}\ ,$$

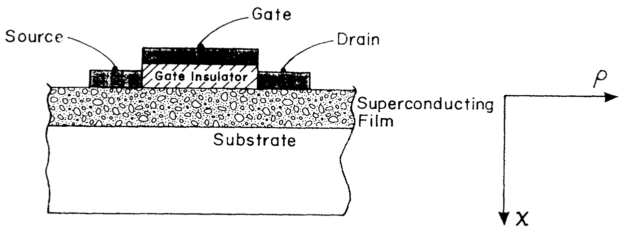

FIG. 1. The typical MOS structure to investigate field effect in superconductors (see Ref. 1).

$$\delta N(x)=N(x)-N_0 \,, \tag{2}$$

$$\gamma=[\partial \ln T_c(N_0)/\partial \ln N_0] \,.$$

(In the framework of the model with electric-field-independent value V,[5] we get for γ, $\gamma=1/g_0$.) In the framework of the Thomas-Fermi approximation $\delta N(x)$ has a well-known form (for zero electric potential Φ far from the surface inside the superconductor):

$$\delta N(x)/N_0=\kappa\exp(-x/l) \,, \quad \kappa=e\Phi_s/2E_f \,, \quad l^{-2}=\varepsilon/4\pi N_0 e^2 \,, \tag{3}$$

where ε is the dielectric susceptibility, Φ_s is the electric potential at the surface, E_f is the Fermi energy.

III. THE CRITICAL TEMPERATURE

In the vicinity of the critical temperature the order parameter Ψ is very small and one can neglect the fourth-order term in Eq. (1). As a result we get a Schrödinger-type equation for Ψ ($A=0$) (see Ref. 5):

$$-\xi_\perp^2 d^2\Psi/dx^2+[\tau-(\kappa\gamma)\exp(-x/l)]\Psi=0 \,, \tag{4}$$

where $\tau=T/T_c-1$. In order to obtain the shift of the critical temperature one has to obtain the minimum eigenvalue of Eq. (4) taking into account the boundary condition

$$(-i\partial-2eA/c)_n\Psi=0 \,. \tag{5}$$

The solution of Eq. (4) has a well-known form:[7]

$$\Psi_\nu(x)=J_\nu[\exp(-x/l)\alpha] \tag{6}$$

for which

$$\nu^2=4l^2\tau_\nu/\xi_\perp^2 \,,$$

$$\tau_\nu=(T_\nu-T_c)/T_c \,,$$

$$\alpha^2=(4l^2\kappa\gamma/\xi_\perp^2) \,.$$

The equation used to obtain all of the eigenvalues of the discrete part of the spectrum and satisfying the boundary condition (5) has the form

$$\nu_n/\alpha=J_{\nu_n+1}(a)/J_{\nu_n}(\alpha) \,. \tag{7}$$

The maximum value among T_n (T_0) is the critical temperature of the surface superconducting state creation. We get from (7) both for "weak electric field" ($\alpha \ll 1$) and for "strong electric field" ($\alpha \gg 1$) limits:

$$\Delta T/T_c=\begin{cases}(\kappa l\gamma/\xi_\perp)^2 \,, & \alpha \ll 1 \,,\\ (\kappa\gamma) \,, & \alpha \gg 1 \,.\end{cases} \tag{8}$$

The solution of Eq. (7) for all values of α is presented in Fig. 2. It is clear that at $\alpha=3.8$ the second eigenvalue appears. The wave function corresponding to the lowest eigenvalue determines the spatial behavior of the order parameter.[4,5] It means that the characteristic size of the surface superconducting state resulting from the electric-field action has the form

$$d=\begin{cases}\xi_\perp^2/kl\gamma \,, & \alpha \ll 1 \,,\\ (\xi_\perp^2 l/k\gamma^2)^{(1/3)} \,, & \alpha \gg 1 \,.\end{cases} \tag{9}$$

To use the Ginzburg-Landau theory one has to demand the condition $d \gg \xi_\perp$. This is the main restriction for our parameters.

IV. SMALL FLUCTUATIONS OF THE ORDER PARAMETER

Let us consider the free energy of our system at temperature $T > T_0$. In this region the mean value of the order parameter is zero and the Ginzburg-Landau functional is the effective Hamiltonian for fluctuations.[13] The free energy of these fluctuations may be represented in the form

$$F=-T\ln Z \,,$$

$$H_{\rm GL}\{\Psi\}=N_0\int\{a(x)|\Psi|^2+\xi_\mu^2|(i\partial_\mu-2eA_\mu/c)\Psi|^2\}d^3r \,, \tag{10}$$

$$Z=\int\exp(-H_{\rm GL}\{\Psi\}/T)D\Psi D\Psi^* \,.$$

Looking for the order parameter Ψ in the form

$$\Psi=\sum_\nu\int M_{\nu\mathbf{k}}f_\nu(x)\exp(i\mathbf{k}\cdot p)d^2\mathbf{k}/(2\pi)^2 \,, \tag{11}$$

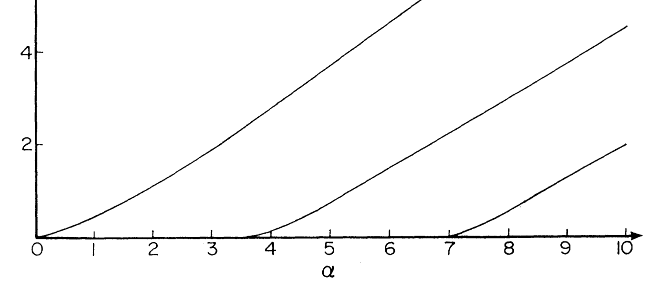

FIG. 2. The eigenvalues of the Schrödinger equation (4) vs parameter α.

where f_ν are the eigenfunctions of Eq. (4), $\boldsymbol{\rho}$ is a two-dimensional vector directed parallel to the surface. Using Eqs. (10) and (11) we get for the mean-square module of amplitude $M_{\nu\mathbf{k}}$

$$\langle |M_{\nu\mathbf{k}}|^2\rangle = T/[N_0 f_\nu^2(\xi_\parallel^2 k^2+\tau-\tau_\nu)S] \ , \tag{12}$$

where S is the area of the sample surface, $f_\nu^2=\int f_\nu^2(x)dx$.

Let us consider the heat capacity, for example. This function may be easily obtained from Eqs. (10) and (12) in the following way:

$$C(E)=T^{-1}\partial^2 F/\partial\tau^2 = (4\pi\xi_\parallel^2)^{-1}\left[\sum_{n=0}^{N(E)} 1/(\tau-\tau_n)+\sum_{\nu=0}^{\infty} 1/(\tau-\tau_\nu)\right] , \tag{13}$$

where $N(E)$ is the number of discrete levels in "potential well." (Here index ν denotes the continuous spectrum.) The second term in Eq. (13) appears to be due to the continuous part of the spectrum of Eq. (4). Its contribution to the sum can be calculated exactly. Taking into account the completeness of the system of eigenfunctions (see Ref. 12), both in the zero electric field ($E=0$) and also the system of eigenfunctions in the field, we conclude that the DOS at $\nu\to 0$ must change drastically. Indeed, as it was shown by Zel'dovitch and Rabinovitch,[12] the DOS at value $\nu=0$ must compensate the contribution of the discrete part of the spectrum.

The completeness of the system of eigenfunctions tells us the evident condition:

$$\int |\Psi_\nu(x)|^2\rho(\tau_\nu)d\tau_\nu=\int |\Psi_p(x)|^2\rho(\tau_p)d\tau_p \ , \tag{14}$$

where $\rho(\tau_\nu)$ and $\rho(\tau_p)$ are the DOS of Eq. (4) with and without potential energy, respectively. For a sufficiently weak potential ($\kappa \ll 1$) in Eq. (4) the wave function of the continuous part of the spectrum may be replaced by its quasiclassical value practically everywhere excluding extremely small ν. [It is clear from Eq. (5) that $p=\nu/2l$ is a momentum.] It means that the small ν in Eq. (14) must be taken into account exactly. Performing integration in Eq. (14) on x we represent the left-hand side of Eq. (14) in the form

$$\sum_{n=0}^{N(E)} |\Psi_\nu(x)|^2+\int_0^\varepsilon \rho(\tau_\nu)|\Psi_\nu(x)|^2 d\tau_\nu + \int_\varepsilon^\infty \rho(\tau_\nu)|\Psi_\nu(x)|^2 d\tau_\nu \ . \tag{15}$$

Here ε is a small parameter where the quasiclassical behavior of the wave function $\Psi_\nu(x)$ becomes incorrect.

In the third of the integrals one can substitute ν by momentum p and perform the integration. It is clear that the result is identical to those obtained from the integral on the right-hand side of Eq. (14) (in the lack of the potential). Two of the rest integrals in Eq. (15) must compensate each other if the DOS $\rho(\tau_\nu)$ at $\tau_\nu\to 0$ has the form

$$\rho(\tau_\nu)=-N(E)\delta(\tau_\nu) \ . \tag{16}$$

Substituting Eq. (16) in Eq. (13) we get

$$\sum_{\nu=0}^{\infty} 1/(\tau-\tau_\nu)=1/4\xi_\parallel\sqrt{\tau}-1/\tau \ . \tag{17}$$

Taking into account the well-known relation for the heat capacity in the absence of electric field $C(0)$,[13] $C(0)=1/16\pi^2\xi_\parallel^3$, we derive for the difference $\Delta C = C(E)-C(0)$,

$$\Delta C(E)=(4\pi\xi_\parallel^2)^{-1}\sum_{n=0}^{N(E)}[1/(\tau-\tau_n)-1/\tau] \ . \tag{18}$$

As is clear from Eq. (18) the heat capacity in the electric field has "jumps" at the electric-field values for which an additional discrete level appears. These values have been obtained from the solutions of Eq. (7) (see Fig. 2). In particular, $\kappa_1=3.8\xi_1^2/l^2\gamma$, $\kappa_2=7\xi_1^2/l^2\gamma$.

V. PARACONDUCTIVITY

The phenomena described above have to be manifested in all of the fluctuating phenomena. Let us consider the conductivity above the new critical temperature T_0. It is well known that the fluctuations of the order parameter above the critical temperature change the conductivity of the sample.[14,15] In our case a surface layer created by the electric field is more favorable for superconductivity than the matter in the bulk. In this layer the superconducting fluctuations of the order parameter may result in an increase of surface conductivity for current flowing along the surface at temperature $T>T_0$. In order to investigate this problem one has to use the equation for relaxation of the order parameter. The gauge-invariant equation of this type has a well-known form:

$$-\beta(\partial\Psi/\partial t+2ie\Phi_\parallel\Psi)=\int \delta H_{\rm GL}\{\Psi\}/\delta\Psi^* d^3r \ , \tag{19}$$

where $\Phi_\parallel=-\mathbf{E}\rho$, $\Phi_\parallel$ is a potential of the electric current flowing along the surface, β is the relaxation constant. We are interested in the change of the order parameter as a result of the electric field parallel to the surface ($E_\parallel$).

It is clear that this term has to be proportional to the electric field and as a result is not dependent on time. Substituting $H_{\rm GL}$ from Eq. (10) to Eq. (19) we get, in our case,

$$[2\beta ie\Phi_\parallel+a(x)]\Psi+\xi_\mu^2(-ih\partial_\mu-2eA_\mu/c)^2\Psi=0 \ . \tag{20}$$

Substituting the expression for Ψ from Eq. (11) to Eq. (20) and representing $M_{\nu\mathbf{k}}$ in the form $M_{\nu\mathbf{k}}=M_{\nu\mathbf{k}}(0)+M_{\nu\mathbf{k}}^{(1)}(\Phi_\parallel)$ we get, from Eq. (20),

$$2\beta eE_\parallel\partial M_{\nu\mathbf{k}}(0)/\partial\mathbf{k}+(\tau-\tau_n+\xi_\parallel^2k^2)M_{\nu\mathbf{k}}^{(1)}(\Phi_\parallel)=0 \ , \tag{21}$$

here $M_{\nu\mathbf{k}}(0)$ and $M_{\nu\mathbf{k}}^{(1)}(\Phi_\parallel)$ are the coefficients in the absence of electric potential along the surface and the first perturbated term, respectively.

Using the Ginzburg-Landau current in the form

$$J_\parallel=-ie\xi_\parallel^2N_0\langle\Psi\partial\Psi^*-\Psi^*\partial\Psi\rangle \ , \tag{22}$$

and substituting Ψ in the form (11) we get as a result

$$J=\int d^2\mathbf{k}/(2\pi)^2\sum_n e\mathbf{k}\xi_\parallel^2 N_0\langle |M_{\nu\mathbf{k}}|^2\rangle \ , \tag{23}$$

where

$$J=\int_0^\infty J_\parallel(x)dx \ ,$$
$$J=\int d^2\mathbf{k}/(2\pi)^2\sum_n e\mathbf{k}\xi_\parallel^2 N_0\langle M_{\nu\mathbf{k}}(0)M_{\nu\mathbf{k}}^{*(1)}(\Phi_\parallel)+\text{c.c.}\rangle \ . \tag{24}$$

Substituting $M_{\nu\mathbf{k}}^{(1)}(\Phi_\parallel)$ from Eq. (21) we get

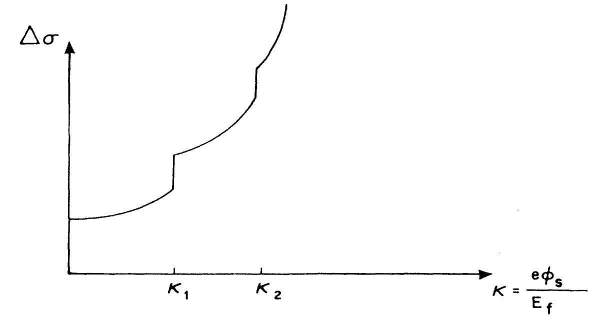

FIG. 3. The paraconductivity of the superconductor as a function of the electric field above a new critical temperature T_0.

$$J=-2\beta e^2\xi_\parallel^2 N_0\int d^2\mathbf{k}/(2\pi)^2\sum_n(\tau-\tau_\nu+\xi_\parallel^2 k^2)^{-1}\mathbf{k}(\mathbf{E}\cdot\partial/\partial\mathbf{k})\langle |M_{\nu\mathbf{k}}(0)|^2\rangle \ . \tag{25}$$

Taking into account $\langle |M_{\nu\mathbf{k}}|^2\rangle$ from Eq. (12) we have as a result

$$J=\sigma_\parallel E_\parallel \ ,$$
$$\sigma_\parallel=2\beta e^2 T\xi_\parallel^4\int k^2 d^2\mathbf{k}/(2\pi)^2\sum_\nu(\tau-\tau_\nu+\xi^2k^2)^{-3} \ . \tag{26}$$

Performing the integration we have for $\Delta\sigma=\sigma(E)-\sigma(0)$, in the manner described earlier [see Eqs. (16)–(18)],

$$\Delta\sigma_\parallel=(\beta e^2 T/\pi)\sum_{n=0}^{N(E)}[(\tau-\tau_n)^{-1}-\tau^{-1}] \ . \tag{27}$$

The paraconductivity depending on the external electric field has the set of jumps when the new eigenvalues appear (Fig. 3). From the formal point of view the sum in this expression is the same as that considered earlier [see Eq. (16)].

VI. CONCLUSION

Thus, in an external electric field both heat capacity and paraconductivity are a steplike function of the external electric fields. The origin of these steps is absolutely clear. Indeed, at certain electric-field values [see (18) and Fig. 2] new degrees of freedom for the fluctuating order parameter are opened. It results in jumps in all fluctuating above-the-critical-temperature thermodynamical functions. This effect is very sensitive to the type of materials. Let us consider three different superconducting systems, namely, tin, $YBa_2Cu_3O_7$, and $Bi_2Ba_2Ca_2Cu_3O_8$.

For all of them we have the parameters $l=1$ A, $\xi=1000$ A (tin), $l=5$ A, $\xi_\perp=5$ A (YBaCuO), and $l=5$ A, $\xi_\perp=3$ A (BiBaSrCuO).[16] For $\gamma=\frac{1}{3}$ and $\kappa=0.1$ we obtain

$$\Delta T/Tc=10^{-8}\ (\text{tin}) \ ,$$
$$\Delta T/T_c=10^{-1}\ (\text{YBaCuO}) \ , \ (Bi_2Sr_2CaCu_2O_8) \ .$$

The value of the ratio l/ξ is more dramatic for fluctuation phenomena. Indeed, for the small ratio l/ξ in sums (16) and (25) there remains only the first term. In this case both the heat capacity and paraconductivity have two-dimensional behavior $[(T-T_0)^{-1}]$ above the critical temperature T_0. The value of the electric field has to be large enough to create additional degrees of freedom which cannot be realized in conventional superconductors. Let us estimate the shift of charge density (κ) induced by the electric field at the surface which is required to observe the jumps in the fluctuating phenomena.

For the first jump we get

$$\kappa_1=0.25\ (\text{YBaCuO}) \ , \quad \kappa_1=0.1\ (\text{BiSrCaCuO}) \ .$$

These shifts of the electron density are achieved in the present experimental situation.

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[1]J. Mannhart, Mod. Phys. Lett. B **6**, 551 (1992).
[2]R. E. Glover III and M. D. Sherill, Phys. Rev. Lett. **5**, 248 (1960).
[3]H. Stadler, Phys. Rev. Lett. **14**, 979 (1965).
[4]V. M. Nabutovsky and B. Ya. Shapiro, Solid State Commun. **40**, 303 (1981).
[5]V. M. Nabutovsky and B. Ya. Shapiro, J. Low Temp. Phys. **49**, 461 (1982).
[6]Yu. V. Venevtsev and V. V. Bogatko, Fiz. Tverd Tela (Leningrad) **29**, 1654 (1987) [Sov. Phys. Solid State **29**, 1654 (1987)].
[7]B. Ya. Shapiro, Phys. Lett. **105A**, 374 (1984); Solid State Commun. **53**, 673 (1985).
[8]A. Levy, P. Falck, M. A. Kastner, W. J. Gallagher, A. Gupta,

and A. W. Kleinsasser, J. Appl. Phys. **69**, 4439 (1991).
[9]A. T. Fiory, A. H. Hebard, R. H. Eick, P. W. Mankievich, R. E. Howard, and M. L. O'Malley, Phys. Rev. Lett. **65**, 3441 (1990).
[10]J. Mannhart, D. G. Schom, J. G. Bednortz, and K. A. Muller, Phys. Rev. Lett. **67**, 2099 (1991).
[11]X. X. Xi, C. Doughty, A. Walkenhorst, C. Kwon, Q. Li, and T. Venkatesan, Phys. Rev. Lett. **68**, 1240 (1992).
[12]Ya. B. Zel'dovich and E. M. Rabinovitch, Zh. Eksp. Teor. Fiz. **10**, 1296 (1960) [Sov. Phys. JETP **37**, 924 (1960)].
[13]L. D. Landau and I. M. Lifshitz, *Statistical Physics* (Pergamon, New York, 1974).
[14]L. G. Aslamazov and A. I. Larkin, Phys. Lett. **26A**, 238 (1968).
[15]V. V. Shmidt, JETP Lett. **3**, 141 (1966).
[16]D. R. Harshman and A. P. Mills, Jr., Phys. Rev. B **45**, 10 684 (1992).