Transverse and longitudinal susceptibilities of a Heisenberg ferromagnet with dipolar forces below T_c

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The frequency dependencies of the uniform transverse and longitudinal susceptibilities of Heisenberg ferromagnets with a dipole-dipole interaction are studied in different temperature regions. Within a broad range of parameters the major contribution to the longitudinal component of the susceptibility, as well as to the dispersion of the fluctuational corrections to the transverse one is found to be the process containing a couple of spin waves in the intermediate state. The ferromagnetic-resonance (FMR) linewidth, which at zero temperature exists due to the quantum fluctuations is calculated. Deviations of the FMR line shape from a Lorentzian are shown to be rather significant at sufficiently high temperatures. The results obtained within the framework of the developed perturbation theory based on the approach of Vaks, Larkin, and Pikin are generalized for the critical dynamics region. The range of validity of the employed approach is discussed. It is shown that dipole-dipole forces cannot be considered as a perturbation not only in the close vicinity of the Curie point but also at any temperature at low enough frequencies. In particular, the longitudinal susceptibility reveals the infrared discontinuity in the zero-frequency limit.

I. INTRODUCTION

The energy of the dipole-dipole interaction between atomic magnetic moments in ferromagnets is usually 2 or 3 orders of magnitude smaller than the exchange interaction energy for neighboring atoms. However it is well known that the importance of the dipolar forces increases at long distances owing to their long-range character.[1] As a result the spectrum of long-wavelength spin fluctuations is significantly modified by the dipolar forces. Moreover, the internal magnetic field in a sample differs from the external field and depends on the shape of a sample. Finally, the dipolar interaction in contrast to the exchange one does not conserve the total spin of the system providing the mechanism for the uniform relaxation of the magnetization fluctuations.

This relaxation mechanism in the spin subsystem is of special interest among other influences of dipolar forces mentioned above. This is due to the fact that sometimes just the processes of internal spin-spin relaxation bring a major contribution to the absorption of the alternating external electromagnetic field.

The ferromagnetic-resonance (FMR) line broadening caused by these processes has been studied in a number of theoretical papers (see, e.g., Refs. 2–4). However, the picture of behavior of the transverse susceptibility as a function of field and temperature is still not complete, especially for weak fields. Moreover, to the best of our knowledge no studies of its frequency dependence have been undertaken yet. Also, despite the significant experimental interest,[5–7] the dynamical behavior of the longitudinal susceptibility has not been studied theoretically at all while the static case has been analyzed in Refs. 8 and 9.

In the present paper the frequency dependence of the uniform transverse and longitudinal susceptibilities in zero external magnetic field is examined. In order to cover a broad temperature range we have used the perturbation theory approach developed by Vaks, Larkin, and Pikin in Refs. 10 and 11. This approach provides us a regular way to account for the fluctuational corrections to the results of mean-field approximation (MFA) assuming that the effective range of the exchange interaction r_0 (measured in the interspin distances) is large enough. We also assume that the energy ω_0 characterizing the dipolar interaction is much smaller than the exchange energy V_0.

Thus, we built up the biparametrical perturbation theory which made it possible to calculate the dynamical susceptibilities within the unambiguously defined range of parameters where the decay of the uniform modes into a couple of virtual spin waves is the major process of their relaxation. The obtained results can obviously be reproduced at temperatures T much lower then the Curie temperature T_c using the conventional spin-wave theory. On the contrary, this theory is not appropriate at $T \sim T_c$, while our approach is quite suitable to introduce the scaling hypothesis and thus to generalize the results into the range of critical dynamics.

The outline of this paper is as follows. In Sec. II we formulate the main principles of the perturbation theory which will be applied to the analysis of the dynamical susceptibility tensor. In Sec. III we examine the behavior of the transverse susceptibility in three temperature regions, i.e., at zero temperature, in the intermediate tem-

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perature region, and in the vicinity of the Curie point. In this section we concentrate on the study of the following problems: (i) the residual FMR linewidth and the resonance frequency shift at zero temperature which appear owing to the zero-point oscillations induced by the dipolar forces, (ii) the deviation of the FMR line shape from the Lorentzian arises from the frequency dispersion of the fluctuational corrections, and (iii) the possibility to generalize our results beyond the mean-field temperature region into the critical one. At last, in Sec. IV we calculate the dynamical longitudinal susceptibility which in our approximation reveals the infrared discontinuity in the zero-frequency limit, and discuss the problem related to this feature.

II. HEISENBERG FERROMAGNET WITH DIPOLAR FORCES

Following Ref. 2, let us represent the Hamiltonian of a Heisenberg ferromagnet with dipolar interaction in the form $\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_{\rm int}$, where $\mathcal{H}_0$ corresponds to the mean-field approximation

$$\mathcal{H}_0 = -\sum_{\mathbf{r}} S^\mu_{\mathbf{r}} \left[g\mu_0 H^e_\mu + \sum_{\mathbf{r}'\neq\mathbf{r}} \langle S^\nu_{\mathbf{r}'}\rangle\, V_{\mu\nu}(\mathbf{r}-\mathbf{r}') \right] \qquad (1)$$

and $\mathcal{H}_{\rm int}$ describes the interaction between fluctuations

$$\mathcal{H}_{\rm int} = -\frac{1}{2}\sum_{\mathbf{r}\neq\mathbf{r}'} (S^\mu_{\mathbf{r}} - \langle S^\mu_{\mathbf{r}}\rangle)\,(S^\nu_{\mathbf{r}'} - \langle S^\nu_{\mathbf{r}'}\rangle)\; V_{\mu\nu}(\mathbf{r}-\mathbf{r}'), \qquad (2)$$

$$V_{\mu\nu}(\mathbf{r}-\mathbf{r}') = V(\mathbf{r}-\mathbf{r}')\,\delta_{\mu\nu} + (g\mu_0)^2\partial_\mu\partial_\nu\,|\,\mathbf{r}-\mathbf{r}'\,|^{-1}, \qquad (3)$$

where $\mathbf{S}_{\mathbf{r}}$ is the spin of the atom located in the lattice site with coordinate $\mathbf{r}$, $g\mu_0$ is the effective magnetic moment of the atom, $\mathbf{H}^e$ is the external magnetic field, $\langle\, \mathbf{S}_{\mathbf{r}}\,\rangle$ is the local mean value of the atomic spin, $V(\mathbf{r})$ is the exchange integral, $\partial_\mu = \partial/\partial r_\mu$, $\mu,\nu = x,y,z$. In Eq. (1) we have neglected an unimportant constant which is independent on the spin operators $\mathbf{S}_{\mathbf{r}}$.

Let us consider an ellipsoidal sample with the principal axes coinciding with the coordinate axes. We suppose that the external field is directed along the z axis. Then $\langle\, \mathbf{S}\,\rangle$ does not depend on $\mathbf{r}$, and the internal field H^i in the sample is uniform. This field is related to the external field and the magnetization M by the expression $H^i = H^e - 4\pi N_z M$, where N_z is the demagnetizing factor in the field direction.

The Fourier transform of the interaction potential $V_{\mu\nu}(\mathbf{r})$ is given by

$$V_{\mu\nu}(\mathbf{q}) = \left[V_{\mathbf{q}} + \frac{\omega_0}{3} \right] \delta_{\mu\nu} - \omega_0\, n_\mu n_\nu, \quad \mathbf{q}\neq \mathbf{0}, \qquad (4)$$

where $V_{\mathbf{q}} = \sum_{\mathbf{r}} V(\mathbf{r})\, e^{-i\mathbf{q}\cdot\mathbf{r}}$, $\omega_0 = 4\pi(g\mu_0)^2 v_c^{-1}$ is the characteristic energy of dipolar interaction, v_c is the magnetic unit-cell volume, and $n_\mu = q_\mu q^{-1}$; at $\mathbf{q} = 0$ the tensor $n_\mu n_\nu$ is transformed into the tensor of demagnetizing factors $N_{\mu\nu}$. In our geometry the latter has the diagonal form $N_{\mu\nu} \equiv N_\mu = {\rm diag}[N_x, N_y, N_z]$, ${\rm Tr}\, N_\mu = 1$, and is defined from the system of equations: $(g\mu_0)^2 \sum_{\mathbf{r}} \partial^2_\mu r^{-1} = \omega_0(1/3 - N_\mu)$.

Let us define the temperature Green function as follows:

$$G_{\mu\nu}(\mathbf{q},\omega_n) = \int_0^\beta d\tau\, e^{-i\omega_n\tau} \sum_{(\mathbf{r}-\mathbf{r}')} e^{-i\,\mathbf{q}\cdot(\mathbf{r}-\mathbf{r}')} \langle\, \hat{T}\, [S^\mu_{\mathbf{r}}(\tau) - \langle S^\mu\rangle]\, [S^\nu_{\mathbf{r}'}(0) - \langle S^\nu\rangle\,]\,\rangle. \qquad (5)$$

Here ω_n is the Matsubara frequency, $\beta = T^{-1}$, $\hat{T}$ is the τ-arrangement operator, and the brackets denote the thermodynamic averaging.

It is well known, that the analytic continuation of the temperature Green function to the real frequencies $i\,\omega_n \to \omega + i\delta$ coincides with the retarded Green function $G_{\mu\nu}(\mathbf{q},\omega)$. This function is related to the magnetic susceptibility of the material $\chi_{\mu\nu}(\mathbf{q},\omega)$ describing the linear response of the system to the alternating external magnetic field

$$\chi_{\mu\nu}(\mathbf{q},\omega) = \frac{\omega_0}{4\pi}\, G_{\mu\nu}(\mathbf{q},\omega). \qquad (6)$$

The function $G_{\mu\nu}(\mathbf{q},\omega_n)$ is given by the Larkin equation[10]

$$G_{\mu\nu}(\mathbf{q},\omega_n) = \Sigma_{\mu\nu}(\mathbf{q},\omega_n) + \Sigma_{\mu\sigma}(\mathbf{q},\omega_n)\, V_{\sigma\rho}(\mathbf{q})\, G_{\rho\nu}(\mathbf{q},\omega_n), \qquad (7)$$

where $\Sigma_{\mu\nu}(\mathbf{q},\omega_n)$ is the irreducible part of $G_{\mu\nu}(\mathbf{q},\omega_n)$.

For the convenience of further consideration let us introduce the circular spin components $S^\pm = 1/\sqrt{2}\,(S^x \pm iS^y)$. Then in the MFA only the following components of the tensor $\Sigma_{\mu\nu}(\mathbf{q},\omega_n)$ remain nonzero:

$$\Sigma^0_{-+}(\mathbf{q},\omega_n) = \Sigma^0_{+-}(\mathbf{q},-\omega_n) = b\,K(\omega_n),$$

$$\Sigma^0_{zz}(\mathbf{q},\omega_n) = \delta_{no}\beta b',$$

$$K(\omega_n) = [\,y/\beta - i\omega_n\,]^{-1},$$

$$b\,(y) = SB_s(Sy) = \langle S^z\rangle_0,$$

$$y = \beta\left[\left(V_0 + \frac{\omega_0}{3}\right)\langle S^z\rangle + g\mu_0 H^i\right], \qquad (8)$$

where $B_s(x)$ is the Brillouin function for the spin S and $\langle S_z\rangle_0$ is the mean value of an atomic spin in the MFA.

Substituting (8) into Eq. (7), for the transverse components of the Green function of MFA $G^0_{\mu\nu}(\mathbf{q},\omega_n)$ in the case $\omega_n \neq 0$ one has[2]

$$G^0_{-+}(\mathbf{q},\omega_n) = b\left(\varepsilon^0_{\mathbf{q}} + \frac{\alpha_{\mathbf{q}}}{2} + i\omega_n\right)[\varepsilon^2_{\mathbf{q}} - (i\omega_n)^2]^{-1},$$

$$G^0_{++}(\mathbf{q},\omega_n) = -b\,\frac{\alpha_{\mathbf{q}}}{2}\,e^{-2i\varphi_{\mathbf{q}}}[\varepsilon^2_{\mathbf{q}} - (i\omega_n)^2]^{-1},$$

$$G^0_{-+}(\mathbf{q},\omega_n) = G^0_{+-}(\mathbf{q},-\omega_n),$$

$$G^0_{++}(\mathbf{q},\omega_n) = [\,G^0_{--}(\mathbf{q},\omega_n)\,]^*. \tag{9}$$

Here $\varepsilon^0_{\mathbf{q}} = b\,(V_0 - V_{\mathbf{q}}) + g\mu_0 H^i$, $\alpha_{\mathbf{q}} = \omega_d \sin^2\vartheta_{\mathbf{q}}$, $\omega_d = \omega_0\langle S_z\rangle_0$, $\varepsilon_{\mathbf{q}} = [\,\varepsilon^0_{\mathbf{q}}\,(\,\varepsilon^0_{\mathbf{q}} + \alpha_{\mathbf{q}}\,)\,]^{1/2}$, $\vartheta_{\mathbf{q}}$ and $\varphi_{\mathbf{q}}$ are the polar and the azimuth angles of vector $\mathbf{q}$, respectively; $b\,(V_0 - V_{\mathbf{q}}) \simeq Dq^2$ at $\mathbf{q} \to 0$ and D is the spin-wave stiffness. We should point out that the other components of the tensor $G^0_{\mu\nu}(\mathbf{q},\omega_n)$ in the case $\omega_n \neq 0$ are equal to zero. The poles of the transverse components of the retarded Green function determine the well-known spin-wave spectrum[1,2]

$$\omega = \pm\varepsilon_{\mathbf{q}} = \pm\left\{(Dq^2 + g\mu_0 H^i) \times (Dq^2 + g\mu_0 H^i + \omega_d \sin^2\vartheta_{\mathbf{q}})\right\}^{1/2}. \tag{10}$$

Solving Eq. (7) in the case $\mathbf{q} = \mathbf{0}$, for the uniform spin precession frequency ε_0 one obtains[1,2]

$$\omega = \pm\varepsilon_0 = \pm\{(\omega_d N_x + g\mu_0 H^i)\,(\omega_d N_y + g\mu_0 H^i)\}^{1/2}. \tag{11}$$

In the present paper the uniform retarded Green functions $G_{\mu\nu}(0,\omega) = G_{\mu\nu}(\omega)$ are studied accounting for the first fluctuational corrections to the irreducible parts $\Sigma^1_{\mu\nu}(\omega_n)$. The diagrams corresponding to the functions $\Sigma^1_{\mu\nu}(\omega_n)$ are shown in Fig. 1 , where the circles represent the single-cell blocks of the diagram technique described in Refs. 10 and 11. In particular, the circles in Figs. 1(a) and 1(b) correspond to the Fourier transform of the average with the Hamiltonian $\mathcal{H}_0$ of three and four spin projection operators $\langle\,\hat{T}\,S^\mu(\tau_1)S^\sigma(\tau_2)S^\lambda(\tau_3)\,\rangle_0$ and $\langle\,\hat{T}\,S^\mu(\tau_1)S^\sigma(\tau_2)S^\rho(\tau_3)S^\nu(\tau_4)\,\rangle_0$, respectively. The wavy lines represent the effective interaction $R_{\mu\nu}(\mathbf{q},\omega_n)$ given by

$$R_{\mu\nu}(\mathbf{q},\omega_n) = V_{\mu\nu}(\mathbf{q}) + V_{\mu\sigma}(\mathbf{q})\,G^0_{\sigma\rho}(\mathbf{q},\omega_n)\,V_{\rho\nu}(\mathbf{q}). \tag{12}$$

Because of the tensor character of the Larkin equation in the presence of dipolar forces all Green functions in the MFA [and all components of the tensor $R_{\mu\nu}(\mathbf{q},\omega_n)$] have a static part proportional to $\delta_{n0}b'$. Moreover, the analytic expressions for the single-cell blocks contain terms proportional to $\delta_{n0}b^{[m]}$, where $b^{[m]} = d^m b/dy^m$. Hence, there are two types of fluctuational corrections. The first type is related to the scattering of spin waves by each other and the second one proportional to $b^{[m]}$ is connected with the scattering of spin waves by the static spin fluctuations. At the low temperatures the second type of corrections are exponentially small as compared with the spin-wave corrections whose temperature dependence is described by the power law, and their role goes up when the temperature increases. In the following we restrict ourselves mostly by the study of the spin-wave corrections, neglecting the terms proportional to $b^{[m]}$.

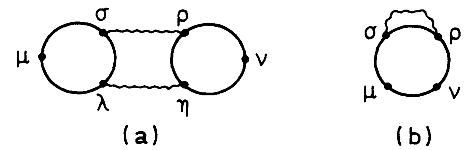

FIG. 1. Diagrams correspond to the first fluctuational corrections for the irreducible parts $\Sigma^1_{\mu\nu}(\omega_n)$.

Besides, hereinafter we shall consider a sample having the shape of an infinite cylinder magnetized along its axis. Such a sample can obviously be magnetized by any weak external field. The results presented below are applicable in practice for the case of a prolate spheroid in the field range $4\pi N_z M \ll H^e \ll 4\pi M$. For nearly isotropic ferromagnets in the considered range of H^e the external field suppresses the domain structure but at the same time this field is so weak that $H^i = H^e - 4\pi N_z M \simeq H^e \ll 4\pi M$. Then in the first approximation one can neglect the influence of the magnetic field on the spin dynamics. As it can be seen from the comparison of our results with the results of Ref. 2, the different processes determine the uniform relaxation in weak ($H^e \ll 4\pi M$) and strong ($H^e \gtrsim 4\pi M$) magnetic fields.

III. TRANSVERSE SUSCEPTIBILITY

In this section the transverse susceptibility $\chi_{-+}(\omega)$ will be analyzed by taking into account the decay process of the uniform precession mode into two spin waves in the intermediate state. First of all we should mention that the first fluctuational corrections $\Sigma^1_{++}(\omega_n)$, $\Sigma^1_{--}(\omega_n)$, $\Sigma^1_{z+}(\omega_n)$, $\Sigma^1_{z-}(\omega_n)$, $\Sigma^1_{+z}(\omega_n)$, $\Sigma^1_{-z}(\omega_n)$ are equal to zero. For the infinite cylindrical sample magnetized along its axis $N_x = N_y = 1/2$, $N_z = 0$, and thus in the uniform limit only the following components of tensor $V_{\mu\nu}$ differ from zero: $V_{-+} = V_{+-} = V_0 - \omega_0/6$, $V_{zz} = V_0 + \omega_0/3$. As a result the solution of the Larkin equation for $G_{-+}(\omega_n)$ has the form

$$G_{-+}(\omega_n) = \Sigma_{-+}(\omega_n)\,[1 - V_{+-}\,\Sigma_{-+}(\omega_m)]^{-1}, \tag{13}$$

where $\Sigma_{-+}(\omega_n) = \Sigma^0_{-+}(\omega_n) + \Sigma^1_{-+}(\omega_n)$. After summing over the intermediate frequencies and after analytic continuation the function $\Sigma_{-+}(\omega)$ can be written as

$$\Sigma_{-+}(\omega) = \langle S_z\rangle K(\omega) + bP(\omega + i\delta)K^2(\omega), \tag{14}$$

$$P(\omega+i\delta) = (2b)^{-1}\sum_{\mathbf{q}}\left\{\alpha_{\mathbf{q}}\varepsilon^0_{\mathbf{q}} - \beta_{\mathbf{q}}\left(2\varepsilon^0_{\mathbf{q}} + \alpha_{\mathbf{q}}\right) + \alpha_{\mathbf{q}}\beta_{\mathbf{q}}\varepsilon^0_{\mathbf{q}}\left(2\varepsilon^0_{\mathbf{q}} + \alpha_{\mathbf{q}} + \omega\right)\left[\varepsilon^2_{\mathbf{q}} - (\omega + i\delta)^2/4\right]^{-1}\right\}\varepsilon^{-1}_{\mathbf{q}}\left(n_{\mathbf{q}} + \frac{1}{2}\right), \tag{15}$$

where $\langle S_z \rangle$ is the mean value of the atomic spin projection including the first fluctuational correction, $\beta_{\mathbf{q}} = \omega_d \cos^2 \vartheta_{\mathbf{q}}$, $n_{\mathbf{q}} = [\exp(\beta\varepsilon_{\mathbf{q}}) - 1]^{-1}$. In Eq. (15) we have neglected the terms proportional to b' and b''. This is correct for the temperatures not very close to T_c (see Sec. III C).

In zero external magnetic field the uniform precession frequency in the MFA is equal to $\varepsilon_0 = \omega_d/2$ for the chosen sample shape. Considering the fluctuational corrections in the hydrodynamic frequency range $|\omega| \ll b V_0$, we can neglect the poles of the function $K(\omega)$. Then, accounting for the condition $\omega_0 \ll V_0$, it is convenient to represent the expression for the transverse susceptibility $\chi_{-+}(\omega)$ in the form

$$\chi_{-+}(\omega) = (2\pi)^{-1}\varepsilon\,[\,\varepsilon + \Delta\varepsilon(\omega) - \omega - i\gamma(\omega)\,]^{-1},$$
$$\Delta\varepsilon(\omega) = -\mathrm{Re}\, P(\omega + i\delta),\ \gamma(\omega) = \mathrm{Im}\, P(\omega + i\delta),$$
$$\varepsilon = \frac{\omega_0}{2}\,\langle S_z \rangle. \tag{16}$$

The FMR line shape is determined by the frequency dependence of the imaginary parts of the transverse components of the magnetic susceptibility tensor $\chi_{\mu\nu}(\omega)$, $\mu, \nu = +, -$. Below we shall refer to the expression $L(\omega) = \mathrm{Im}\,\chi_{-+}(\omega)$ given by

$$L(\omega) = (2\pi)^{-1}\varepsilon\,\gamma(\omega)\,[\,(\varepsilon + \Delta\varepsilon(\omega) - \omega)^2 + \gamma^2(\omega)\,]^{-1} \tag{17}$$

as either the "resonant curve" or the "FMR line shape."

Thus, the consideration of decay processes for the precession mode has led to (i) a renormalization of the magnetization in the expression for frequency of the uniform spin precession; (ii) the appearance of the uniform spin precession damping $\gamma(\omega)$, and (iii) a dynamical shift of the precession frequency $\Delta\varepsilon(\omega)$. The fluctuational renormalization of the magnetization in the Heisenberg ferromagnet with dipolar interaction for temperature not very close to T_c has been well studied (see, e.g., Ref. 1), and we concentrate on the detailed analysis of the effects connected with the presence of $\gamma(\omega)$ and $\Delta\varepsilon(\omega)$.

Fortunately, the imaginary part of the integral in Eq. (15) can easily be calculated and the result for $\gamma(\omega)$ has the form

$$\gamma(\omega) = \omega_0\,(q_d/r_0)^3\,\coth|\omega|/4T\,[\mathrm{sgn}\omega\, f_1\,(|\omega/\omega_d|) + f_2\,(|\omega/\omega_d|)\,],$$

$$f_1(x) = \frac{3\sqrt{3}}{2048\,\pi}\,x^{-1}\left[-\frac{\sqrt{2x}}{3}(27x^3 + 15x^2 - 5x + 15) + 8x^3\left(\arctan\frac{1-x}{\sqrt{2x}} + \frac{\pi}{2}\right) + (9x^4 + 6x^2 + 5)\ln\frac{1 + \sqrt{2x} + x}{\sqrt{1 + x^2}}\right],$$
$$f_2(x) = \frac{3\sqrt{3}}{256\,\pi}\left[-2x^3\left(\arctan\frac{1-x}{\sqrt{2x}} + \frac{\pi}{2}\right) - (1 - x^2)\ln\frac{1 + \sqrt{2x} + x}{\sqrt{1 + x^2}} + \frac{\sqrt{2x}}{3}(6x^2 - x + 3)\right], \tag{18}$$

where $q_d = (2\omega_0/V_0)^{1/2}$ has the sense of the characteristic dipolar momentum and we remind that r_0 is the effective range of the exchange interaction. However, Eq. (18) seems to be rather complicated, and we shall analyze its asymptotic behavior as a function of ω at low and high temperatures, respectively (see also Fig. 2).

The real part of the integral in Eq. (15) cannot be calculated in the general form. Therefore, in the subsequent sections it will be analyzed in the limiting cases. Here we just would like to mention that, similar to $\gamma(\omega)$, the function $\Delta\varepsilon(\omega)$ can also conveniently be decomposed in even and odd functions of ω:

$$\Delta\varepsilon(\omega) \sim [\,g_1\,(|\omega/\omega_d|) + \mathrm{sgn}\,\omega\, g_2\,(|\omega/\omega_d|)\,]. \tag{19}$$

A. Transverse susceptibility at zero temperature

As is evident from Eq. (15), the Larmor precession damping exists at zero temperature only due to the zero-point oscillations which are induced by the dipolar forces. In this case it follows from Eq. (18) that

$$\gamma(\omega) = \frac{3\sqrt{3}\,\omega_0}{4}\left(\frac{q_d}{r_0}\right)^3\begin{cases} 1/64\ \mathrm{sgn}\,\omega\,(\omega/\omega_d)^2, & |\omega| \ll \omega_d, \\ (\sqrt{2}/30\,\pi)\,(\omega/\omega_d)^{1/2}, & |\omega| \gg \omega_d,\ \omega > 0, \\ -(\sqrt{2}/315\,\pi)\,|\omega/\omega_d|^{-3/2}, & |\omega| \gg \omega_d,\ \omega < 0. \end{cases} \tag{20}$$

The analysis of the asymptotic behavior of $\Delta\varepsilon(\omega)$ at $T = 0$ gives

$$\Delta\varepsilon(\omega) = \frac{3\sqrt{3}\,\omega_0}{8\,\pi}\left(\frac{q_d}{r_0}\right)^3\begin{cases} -1/16\ \omega/\omega_d, & |\omega| \ll \omega_d, \\ -3/32, & |\omega| \gg \omega_d,\ \omega > 0, \\ \sqrt{2}/15\ |\omega/\omega_d|^{1/2}, & |\omega| \gg \omega_d,\ \omega < 0. \end{cases} \tag{21}$$

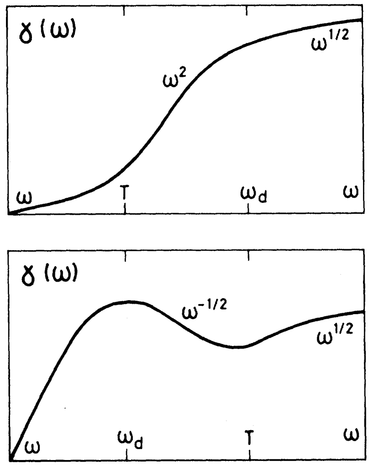

FIG. 2. Frequency dependence of the Larmor precession damping at a low (a) and high (b) temperatures, respectively.

We see that $\gamma(\omega)$ and $\Delta\varepsilon(\omega)$ alter their signs at $\omega = 0$; in the limit $\omega \to 0$ $\gamma(\omega)$ and $\Delta\varepsilon(\omega)$ decrease linearly and quadratically, respectively. In the high-frequency region two obstacles must be mentioned. First, we should note the difference in the behavior of both the damping $\gamma(\omega)$ and the shift $\Delta\varepsilon(\omega)$ at positive and negative values of ω: for example, $\gamma(\omega) \sim \omega^{1/2}$ at $\omega > 0$ and $\gamma(\omega) \sim -|\omega|^{-3/2}$ at $\omega < 0$. This is connected with the fact that the damping $\gamma(\omega)$ is a sum of two terms, the first of which is an odd function of ω and the second is an even one. In the case $\omega < 0$ the mutual cancellation of several terms of asymptotic expansions for functions f_1 and f_2 occurs while at $\omega > 0$ they have the same sign. A similar situation takes place for the shift $\Delta\varepsilon(\omega)$, but in this case the terms of the asymptotic expansions for g_1 and g_2 cancel each other at $\omega > 0$.

Secondly, it follows from Eqs. (20) and (21) that the fluctuational corrections increase infinitely with increased frequency: $\gamma(\omega) \sim \omega^{1/2}$ at $\omega > 0$ and $\Delta\varepsilon(\omega) \sim |\omega|^{1/2}$ at $\omega < 0$. However it is pertinent to remind one, that we have used the hydrodynamic approach here. Thus, Eq. (16) is valid only if $|\omega| \ll b V_0$. Within this range the corrections at $T = 0$ are always small compared to the resonance frequency ε_0 in the MFA.

In accordance with Eqs. (16), (20), and (21) the response function in the considered range of parameters reveals a rather sharp maximum at $\omega \simeq \varepsilon_0$. Therefore, in the vicinity of the resonance $|\varepsilon_0 - \omega| \lesssim \gamma(\omega)$, $\Delta\varepsilon(\omega)$ with the relative error of the order of $\max[\gamma(\varepsilon_0), \Delta\varepsilon(\varepsilon_0)]\,\varepsilon_0^{-1}$, one can neglect in Eq. (16) the dispersion of $\gamma(\omega)$ and $\varepsilon(\omega)$ and approximately set

$$\gamma(\omega) \approx \gamma(\varepsilon_0) \approx 0.16\,\omega_0 \left(\frac{\sqrt{3}\,q_d}{2\pi r_0}\right)^3 ,$$
$$\Delta\varepsilon(\omega) \approx \Delta\varepsilon(\varepsilon_0) \approx -0.32\,\omega_0 \left(\frac{\sqrt{3}\,q_d}{2\pi r_0}\right)^3 . \qquad (22)$$

This leads to a replacement of the actual resonance curve by the Lorentz function, which is usually applied in the treatment of experimental data on the FMR. At low temperatures the Lorentzian approximation seems to be quite reasonable, at least not very far away from the resonance position. Nevertheless, the actual FMR line shape deviates from the Lorentzian both close to and far from the resonance. In the latter case for $L(\omega)$ one has

$$L(\omega) = \frac{3\sqrt{3}\,\omega_0}{8\pi\varepsilon}\left(\frac{q_d}{r_0}\right)^3 \begin{cases} 1/64\ \mathrm{sgn}\,\omega\,(\omega/\omega_d)^2, & \omega_d \gg |\omega|, \\ (\sqrt{2}/30\pi)\,(\varepsilon/\omega)^2\,(\omega/\omega_d)^{1/2}, & \omega_d \ll |\omega|,\ \omega > 0, \\ -(\sqrt{2}/315\pi)\,(\varepsilon/\omega)^2\,|\omega/\omega_d|^{-3/2}, & \omega_d \ll |\omega|,\ \omega < 0. \end{cases} \qquad (23)$$

We see if $\omega \to 0$, then $L(\omega)$ tends to zero proportionally to ω^2 and changes its sign at $\omega = 0$ while the Lorentz function is always positive and finite at $\omega = 0$. In the high-frequency region the Lorentz function decreases as ω^{-2}, but $L(\omega) \sim \omega^{-3/2}$ at $\omega > 0$ and $L(\omega) \sim -|\omega|^{-7/2}$ at $\omega < 0$.

Thus the dipolar forces yield the finite width of the FMR line at zero temperature. Near its maximum the resonance curve may be approximated by the Lorentz function. However, due to the frequency dispersion of fluctuational corrections, there are significant deviations from the Lorentzian shape in the wings. We note that the results obtained at zero temperature are valid in the region $T \ll \omega, \omega_d$, as well.

B. Transverse susceptibility in the intermediate-temperature region

If the condition $T \gg \omega, \omega_d$ is satisfied, we can approximately set $(n_{\mathbf{q}} + 1/2) \approx (\beta\varepsilon_{\mathbf{q}})^{-1}$ in Eq. (15); hence in this temperature region the damping $\gamma(\omega)$ is written as

$$\gamma(\omega) = T\,b^{-1}\,(q_d/r_0)^3\,F(\omega/\omega_d), \qquad (24)$$

where $F(x) = 4|x|^{-1}[\mathrm{sgn}\,\omega\,f_1(|x|) + f_2(|x|)]$, and $f_{1,2}$ are defined by the same equation (18) as above. Therefore, in this case the asymptotic behavior of $\gamma(\omega)$ can easily be obtained from Eq. (20) accounting for the additional factor $4T/|\omega|$ in Eq. (24). It allows us to draw the qualitative behavior of the damping for both cases $T \ll \omega_d$ and $T \gg \omega_d$ as shown in Fig. 2.

In contrast to the case $T = 0$, at $T \gg \omega_d$ the shift $\Delta\varepsilon(\omega)$ can also be calculated in the whole range of $\omega \ll T$, this gives us the analytic expression for the transverse susceptibility. Thus, in addition to Eq. (19) for $\Delta\varepsilon(\omega)$, one has the following equation:

$$\Delta\varepsilon(\omega) = \frac{T}{b}\left(\frac{q_d}{r_0}\right)^3 \left[g_1\left(\left|\frac{\omega}{\omega_d}\right|\right) + \operatorname{sgn}\omega\, g_2\left(\left|\frac{\omega}{\omega_d}\right|\right)\right],$$

$$g_1(x) = \frac{3\sqrt{3}}{512\pi}\,x^{-2}\left[\frac{\sqrt{2x}}{3}\,(27x^3 - 15x^2 - 5x - 15) + \pi\,(8x^2+5) - (9x^4 + 6x^2 + 5)\left(\arctan\frac{1-x}{\sqrt{2x}} + \frac{\pi}{2}\right) + 8x^3 \ln\frac{1+\sqrt{2x}+x}{\sqrt{1+x^2}}\right],$$

$$g_2(x) = \frac{3\sqrt{3}}{64\pi}\,x^{-1}\left[(1-x^2)\left(\arctan\frac{1-x}{\sqrt{2x}} + \frac{\pi}{2}\right) - \pi + \frac{\sqrt{2x}}{3}\,(6x^2+x+3) - 2x^3\ln\frac{1+\sqrt{2x}+x}{\sqrt{1+x^2}}\right]. \tag{25}$$

In the limiting cases of low and high frequencies it follows from Eq. (25), that

$$\Delta\varepsilon(\omega) = \frac{3\sqrt{3}}{64}\frac{T}{b}\left(\frac{q_d}{r_0}\right)^3 \begin{cases} 1/4, & |\omega| \ll \omega_d, \\ 1, & \omega_d \ll |\omega| \ll \omega_m, \end{cases} \tag{26}$$

which shows only weak dependence on ω; here $\omega_m = \min[T, b V_0]$. It should be noted, however, that in accordance with Eqs. (21) and (25) the dynamical shift $\Delta\varepsilon(\omega)$ is usually small compared to the renormalization of the resonance frequency due to the fluctuations of the magnetization.

Similar to the case $T = 0$, in the frequency range $|\varepsilon_0 - \omega| \lesssim \gamma(\omega)$, $\Delta\varepsilon(\omega)$ the transverse susceptibility can be approximated by the Lorentz function with the parameters $\gamma(\omega) \approx \gamma(\varepsilon_0)$ and $\Delta\varepsilon(\omega) \approx \Delta\varepsilon(\varepsilon_0)$:

$$\gamma(\varepsilon_0) \approx 1.3\,T\,b^{-1}\left(\frac{\sqrt{3}\,q_d}{2\pi r_0}\right)^3,$$
$$\Delta\varepsilon(\varepsilon_0) \approx 1.2\,T\,b^{-1}\left(\frac{\sqrt{3}\,q_d}{2\pi r_0}\right)^3. \tag{27}$$

In contrast to the zero-temperature case, the relative error of the approximation $\max[\gamma(\varepsilon_0), \Delta\varepsilon(\varepsilon_0)]\,\varepsilon_0^{-1}$ depends now on temperature: it is proportional to T if $\tau = |T_c - T|T_c^{-1} \sim 1$ and proportional to τ^{-1} if $\tau \ll 1$. In other words, the differences between transverse susceptibility and its Lorentz approximation increases with increasing temperature and they may amount to 30–40 % at a sufficiently high temperatures.

This is illustrated in Fig. 3. In this figure the typical resonant curve $L(\omega)$ with Eqs. (24) and (25) taken into account is denoted by the solid line; the dashed line denotes the Lorentz function with the same value of the reduced resonant damping $\gamma(\varepsilon_0)\,\varepsilon_0^{-1}$. We see that the dispersion of the fluctuational corrections leads to (i) the asymmetry of the resonance line near its maximum, (ii) an additional broadening of the FMR line, and (iii) a lowering of its maximum value.

An even more dramatic deviation of the actual FMR line shape from the Lorentzian occurs beyond the resonant region of ω. Indeed, the asymptotic behavior of $L(\omega)$ can readily be obtained from Eq. (23) by accounting an extra factor $4T/|\omega|$. This, apparently, changes the decay of $L(\omega)$ at high frequencies. In particular, at $\omega_d \ll \omega \ll \omega_m$ one has $L(\omega) \sim \omega^{-5/2}$, i.e., the decay of the true function is steeper than the Lorentz one, while at $T = 0$ $L(\omega)$ is proportional to $\omega^{-3/2}$.

Therefore, the experimental data should be evaluated using Eqs. (24), (25), and (17). As a result a set of parameters, for example, ε_0, ε, and $\gamma(\varepsilon_0)$, can be extracted. On the other hand, if the Lorentz approximation is nevertheless used, one can extract from the data also three parameters, i.e., the resonance frequency ω_r, the width of the resonance line γ_r, and the normalizing factor χ_0. In general, for a well-defined resonance $\varepsilon \simeq \omega_r$ and the temperature dependence of the position of the maximum of the resonance is mostly determined by the temperature dependence of the magnetization. At $T \gg \omega_d$ both parameters $\gamma(\varepsilon_0)$ and γ_r show a linear dependence upon T. At $T \ll \omega_d$ the parameter $\gamma(\varepsilon_0)$ is temperature independent, whereas the width of the effective Lorentzian contains a term proportional to T^2 appearing only because of the nonlorentzian behavior of $L(\omega)$.

In summary we conclude that the dispersion of fluctuational corrections leads to the fact that the transverse susceptibility significantly deviates from the Lorentz function both apart and (at sufficiently high temperatures) near the resonance.

C. Transverse susceptibility close to the Curie point

It is well known[10,11] that the mean-field theory (MFT) in the exchange approximation is applicable when $\tau \gg r_0^{-6}$. However, our results could not be valid in the whole MFT range $\tau \gg r_0^{-6}$, if the condition $q_d r_0^3 \gtrsim$

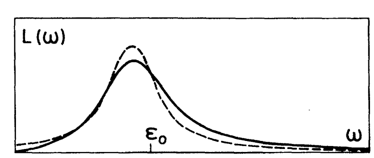

FIG. 3. Resonant curve $L(\omega)$ (solid line) and its Lorentz approximation (dashed line). The difference in behavior is connected with the dispersion of fluctuational corrections.

1 takes place. Indeed, inserting $b = \langle S^z\rangle_0 \sim \tau^{1/2}$ into Eq. (24), we can estimate the reduced resonant damping as $\gamma(\varepsilon_0)\,\varepsilon_0^{-1} \sim (r_0^3\tau^{1/2})^{-1}\,(q_d\,\tau^{-1/2})$, where we take into account that $T \sim T_c \sim V_0$, $q_d \sim (\omega_0/V_0)^{1/2}$, $F(\varepsilon_0/\omega_d) \sim 1$, and $\varepsilon \sim \omega_d \sim \omega_0\, b$. On the other hand,[2] the contribution to the reduced damping from the process of spin-wave scattering by the longitudinal fluctuations is proportional to $(r_0^3\tau^{1/2})^{-1}\,(q_d\,\tau^{-1/2})^3$. At low temperatures $\tau \gtrsim q_d^2$ this contribution is small whereas at $\tau \ll q_d^2$ it becomes the most important. At the same time, at low enough $\tau \sim \tau_d = (q_d/r_0)^{3/2}$ the relative damping appears to be of the order of unity. This means that in the temperature range $\tau < \tau_d$ the spin-wave approach is not valid anymore.

By definition $r_0 = (V_2/V_0)^{1/2}$, where $V_2 = v_c^{-2/3}\sum_{\mathbf{r}} r^2 V(\mathbf{r})$ is the second moment of the exchange integral. Therefore, similar to the case $T > T_c$ (see, e.g., Ref. 12) the characteristic dipolar temperature is defined as follows:

$$\tau_d = (\omega_0/V_2)^{3/4}. \qquad (28)$$

It is necessary to point out that the fluctuational contribution to the damping is the leading term in the quite narrow range $\tau_d < \tau \ll q_d^2$ and only if $\tau_d \gtrsim (V_0/V_2)^3$. It is interesting to note that the latter restriction can be rewritten in the form $\tau_d \gtrsim Gi$, where Gi is the Ginzburg parameter[13] of the phenomenological theory of phase transitions. Thus, within the range $\tau_d > \tau \gtrsim Gi$, where the MFT is still formally valid, we are not able to describe the uniform transverse susceptibility in the frame of perturbation theory.

In the opposite case, $Gi \gg \tau_d$, two temperature regions can be distinguished. In the first one the MFT is correct throughout the region $1 \gg \tau \gg Gi$, and the decay of the uniform precession mode into two spin waves is the major relaxation process. When the temperature increases, the range of the spin-wave approach validity is shrunken. However, the main contribution to the resonance shift and to the uniform precession damping comes from low-wavelength excitations, i.e., spin waves even in the temperature region $Gi \gg \tau$. Thus, the results of previous subsections can readily be generalized by means of the replacement of the mean-field exponents by the critical ones for the relevant quantities. In particular, the expression for $\gamma(\omega)$ can be written in the form [see Eqs. (24) and (18)]

$$\gamma(\omega) = A\,T_c\,\langle S^z\rangle^{-1}\,(\omega_d/D)^{3/2}\,F(\omega/\omega_d), \qquad (29)$$

where $A \sim 1$, and we assume that $\langle S^z\rangle \sim \tau^{\beta}$, $D = D_0\,\tau^{\beta'}$, $\omega_d = \omega_0\,\langle S^z\rangle$, $\beta \simeq \beta' \simeq 1/3$. This expression is convenient to represent via the inverse correlation length $\kappa(\tau) = r_c^{-1}(\tau) \sim a^{-1}\tau^{\nu}$, $a \sim v_c^{1/3}$, $\nu \simeq 2/3$, and renormalized characteristic dipolar momentum $q_0 \sim (\omega_0/T_c)^{1/2}$:

$$\gamma(\omega) = A_1\,T_c\,\kappa^{z/\nu}(q_0/\kappa)^3 F(\omega/\omega_d), \qquad (30)$$

where $A_1 \sim 1$ and $z \simeq 5/3$ is the dynamic critical exponent. Expression for $\Delta\varepsilon(\omega)$ may also be rewritten in the same way.

We see that $\gamma(\omega)$ written in form (30) has a critical dimension in accordance with the dynamic scaling theory,[14] and the factor $\Omega(\kappa) = T_c\kappa^{5/2}$ has the sense of the characteristic energy of critical fluctuations. In general, the scaling function F is the function of two dimensionless parameters, ω/ω_d and $\omega/\Omega(\kappa)$. However, the latter one is considered to be small in Eq. (30).

The resonance frequency ε in Eq. (16) is represented in the scaling notations as follows:

$$\varepsilon = B\,T_c\,\kappa^{2+\beta/\nu}\,(q_0/\kappa)^2 \simeq B\,\Omega(\kappa)\,(q_0/\kappa)^2, \qquad (31)$$

where $B \sim 1$. From Eqs. (30) and (31) it is clearly seen that $\varepsilon \sim \gamma \sim \Delta\varepsilon$ at $\kappa \sim q_0$ or at $\tau \sim \tau_d$, and the approximation used above is valid as $\kappa > q_0$. The range $\tau > \tau_d$ in complete analogy to the case of $T > T_c$ can be characterized as a range of the exchange critical dynamics, whereas the region $\tau < \tau_d$ corresponds to the dipolar one. The dipolar dynamics in the range $\tau > \tau_d$ above the Curie temperature has been considered in Ref. 15 and generalized for the EPR in Ref. 16. It was shown that the main relaxation process in this case is the decay of the uniform precession mode into the spin diffusion modes. This results in a precession mode damping proportional to $\Omega\,(\kappa)\,(q_0/\kappa)^4$, which is by factor (q_0/κ) less then in our case, when the spin diffusion in the intermediate state at $T > T_c$ is replaced by the spin waves at $T < T_c$.

As the boundary of the exchange region τ_d is approached, other relaxation processes (for example, spin-wave scattering by the longitudinal fluctuations, decays for three and more spin waves, etc.) become of the same importance as the considered one. Moreover, at this boundary $\omega_d \sim \Omega(\kappa)$ and the scaling function $F[\omega/\omega_d,\, \omega/\Omega(\kappa)]$ should be strongly renormalized.

Within the dipolar region $\tau < \tau_d$ the dipole-dipole interaction cannot be accounted for as a perturbation neither at $T > T_c$ nor at $T < T_c$. Above T_c the problem is reviewed in Ref. 12 in detail. Obviously, below T_c the situation should be similar in many respects, but the relevant problems are beyond the scope of this study. Here we just mention that at $\tau \ll \tau_d$ the resonance frequency of FMR is expected to be of the same order as its width.

IV. LONGITUDINAL SUSCEPTIBILITY

In this section the longitudinal retarded Green function $G_{zz}(\omega)$ is examined in the same way as it has been done in the previous sections. This function is related to the longitudinal susceptibility $\chi_{zz}(\omega)$ by the expression $\chi_{zz}(\omega) = \omega_0/4\pi\,G_{zz}(\omega)$. It is well known that the longitudinal Green function in the MFA is purely static, whereas its frequency dependence is related to the fluctuational corrections. The contribution of the first order in r_0^{-3} to the temperature Green function $G_{zz}(\omega_n)$ has the form

$$G^1_{zz}(\omega_n) = T\sum_{\mathbf{q}}\sum_{n_1}\left\{G^0_{-+}(\mathbf{q},\omega_{n_1})\,G^0_{-+}(\mathbf{q},\omega_{n_1}+\omega_n) + G^0_{++}(\mathbf{q},\omega_{n_1})\,G^0_{--}(\mathbf{q},\omega_{n_1}+\omega_n)\right\}, \qquad (32)$$

where functions $G^0_{\mu\nu}(\mathbf{q},\omega_n)$ are defined by Eq. (9). As above, we neglect the terms which are proportional to $b^{[m]}$. Expression (32) describes the process of the decay of the longitudinal fluctuation into two spin waves in the intermediate state. After the summation over the intermediate frequencies and the analytic continuation $i\omega_n \to \omega + i\delta$ we obtain

$$G^1_{zz}(\omega) = \sum_{\mathbf{q}} \left(\frac{\alpha_{\mathbf{q}}}{2}\right)^2 \varepsilon_{\mathbf{q}}^{-1} \left(n_{\mathbf{q}} + \frac{1}{2}\right) \left[\varepsilon_{\mathbf{q}}^2 - \left(\frac{\omega + i\delta}{2}\right)^2\right]^{-1} . \tag{33}$$

The integral over $\mathbf{q}$ in Eq. (33) can readily be evaluated in the absence of the internal magnetic field and for temperatures high enough, i.e., at $T \gg \omega, \omega_d$. Hence, substitution of $(n_{\mathbf{q}} + 1/2)$ by $(\beta\varepsilon_{\mathbf{q}})^{-1}$ in (33) gives

$$\begin{aligned} G^1_{zz}(\omega) &= \frac{T}{\omega_d^2}\left(\frac{q_d}{r_0}\right)^3 \left[p_1\left(\left|\frac{\omega}{\omega_d}\right|\right) + i\,\mathrm{sgn}\,\omega\, p_2\left(\left|\frac{\omega}{\omega_d}\right|\right)\right], \\ p_1(x) &= \frac{3\sqrt{3}}{64\pi}\, x^{-2}\left\{-2x\ln\frac{1+\sqrt{2x}+x}{\sqrt{1+x^2}} - 3\pi + 3\,(1+x)\left[(1-x)\left(\arctan\frac{1-x}{\sqrt{2x}} + \frac{\pi}{2}\right) + \sqrt{2x}\right]\right\}, \\ p_2(x) &= \frac{3\sqrt{3}}{64\pi}\, x^{-2}\left\{2x\left(\arctan\frac{1-x}{\sqrt{2x}} + \frac{\pi}{2}\right) + 3\,(1-x)\left[(1+x)\ln\frac{1+\sqrt{2x}+x}{\sqrt{1+x^2}} - \sqrt{2x}\right]\right\}. \end{aligned} \tag{34}$$

The corresponding analytical expression for the imaginary part of $G^1_{zz}(\omega)$ which is valid in both high- and low-temperature ranges $T > \omega_d$ and $T < \omega_d$ follows from Eq. (34) if one multiplies this expression by the factor $\omega/4T \coth \omega/4T$.

In the limiting cases of low (high) frequencies from (34) we find

$$G^1_{zz}(\omega) = \frac{3\sqrt{3}}{64}\, T\,\omega_d^{-2}\left(\frac{q_d}{r_0}\right)^3 \begin{cases} -3 + 2i\,\omega_d/\omega, & |\omega| \ll \omega_d, \\ (\omega_d/\omega)^2\left[-3 + i\,\mathrm{sgn}\,\omega\,(\sqrt{2}\,64/15\pi)\,|\omega_d/\omega|^{1/2}\right], & \omega_d \ll |\omega|. \end{cases} \tag{35}$$

At $T = 0$ $G^1_{zz}(\omega)$ does not vanish because of zero-point oscillations and at low and high frequencies it can be written as follows:

$$G^1_{zz}(\omega) = \frac{3\sqrt{3}}{4\pi}\,\omega_d^{-1}\left(\frac{q_d}{r_0}\right)^3 \begin{cases} 1/16\,[\,\ln|\omega_d/\omega| + i\,\mathrm{sgn}\,\omega\,\pi/2\,], & |\omega| \ll \omega_d, \\ \sqrt{2}/15\,|\omega_d/\omega|^{3/2}\,[-1 + i\,\mathrm{sgn}\,\omega\,], & |\omega| \gg \omega_d. \end{cases} \tag{36}$$

Thus, $G^1_{zz}(\omega)$ decreases fast enough at high frequencies $|\omega| \gg \omega_d$ both at $T \gg \omega_d$ and at $T = 0$. At the same time there are essential differences between these two cases at low frequencies. For instance, in the case $T \gg \omega_d$, $\omega = 0$ the real part of $G^1_{zz}(\omega)$ is finite and increases as temperature goes up, while the imaginary part of $G^1_{zz}(\omega)$ connected with the absorption of the alternating external magnetic field increases as ω^{-1} when $\omega \to 0$. On the other hand, in the case $T = 0$, $\omega = 0$ the imaginary part is finite, while the real part logarithmically diverges when $\omega \to 0$. It is well known, however, that the susceptibility should not show singularities in the limit $\omega \to 0$, therefore the formulas of this section are inapplicable in the region of low enough frequencies. The condition that $4\pi\,|\chi_{zz}| \lesssim 1$ is in fact the criterion of their applicability. This condition may be rewritten for the case of finite temperatures and low frequencies in the form $|\omega| \gtrsim \gamma(\varepsilon_0)$. In other words, if the damping of spin waves in the intermediate state is not accounted for the results are valid just for frequencies which are higher than the characteristic damping. It seems to be quite reasonable that taking the damping into consideration can remove the divergence in the limit of $\omega \to 0$ for the biparticle process. At the same time this procedure forces us to go beyond the frame of perturbation theory in parameter r_0^{-3} because it presupposes the selective resuming of the infinite series of terms of different orders in r_0^{-3}. To prove the validity of this procedure, it is necessary to analyze the general structure of the diagram expansion, i.e, to estimate the contribution to G_{zz} from the processes containing multispin-wave intermediate states. This is a problem which we do not address in the present paper.

Similar to the transverse Green function (see Sec. III C), the expression for the longitudinal one can be generalized for the temperature region of critical dynamics $\tau \ll Gi$. Using the scaling notations, we can rewrite Eq. (34) in the form

$$\begin{aligned} G^1_{zz}(\omega) = C\,(T_c\kappa^2)^{-1}\,(\kappa/q_0)\,[\,p_1\,(|\,\omega/\omega_d\,|) \\ + i\,\mathrm{sgn}\,\omega\; p_2\,(|\,\omega/\omega_d\,|)\,], \end{aligned} \tag{37}$$

where $C \sim 1$ and the quantities q_0, κ, ω_d are defined in Sec. III C. We see that the critical dimension of G^1_{zz} is in accordance with the dynamic scaling theory.[14] Considering only the high-frequency ($\omega \gtrsim \omega_d$) behavior of the longitudinal susceptibility we find that within the whole exchange critical range, $\tau > \tau_d$, the inequality $\chi_{-+} \gtrsim \chi_{zz}$ is valid. At the boundary of the dipolar critical range $\tau \sim \tau_d$ the processes with multispin-wave intermediate states become of the same importance as the considered decay into a couple of spin waves. At the same time it is necessary to take into account the spin-wave damping. Hence, our results are valid only as $\tau > \tau_d$.

In conclusion of this section we would like to note that in Ref. 17 where, in particular, the staggered susceptibility of antiferromagnets has been examined theoretically, an anomalous behavior of the corresponding quantities at $\mathbf{q} = 0$, $\omega \to 0$ has been found. It allows us to suppose that a similar situation is quite general and takes place for all cases in which the total spin of the system is not conserved.

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