Physical model for breakdown structures in solid dielectrics

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A quantitative physical model is presented for the breakdown of solids by void or channel discharges. It is found that fractal treelike structures only occur when the "fields" at the growth tips are allowed to fluctuate about their Laplace values. By treating the local-field enhancement factor as a white noise produced by the breakdown mechanism itself, it is shown that the amount of branching depends only on the range of fluctuations allowed.

I. INTRODUCTION

A. Breakdown structures in solids

Dielectric breakdown in solids may be caused by a number of alternative processes.[1] All such processes are electric-field driven mechanisms in which a positive feedback overcomes equilibrating factors at high fields causing a property such as the current density, temperature, or compressive strain to "run away" uncontrollably.[2,3] During breakdown the solid suffers irreversible damage of a form which supports a short circuit between the electrodes, i.e., an air channel, a conducting pathway, or a collapsed region of material. In an idealized homogeneous solid the damage would occur on a broad front, but in practice material inhomogeneity introduces regions favoring breakdown, the self-enhancing nature of which causes the ensuing damage to be restricted to a single narrow puncture between the electrodes. A major portion of high-voltage breakdowns observed during development tests on insulation systems[4] have structures of this type.

The fields which are applied to insulation systems in service are chosen to lie well below the theoretical onset values for the various breakdown mechanisms.[1,2] However the manufacturing process often allows defects to occur, e.g., sharp metallic asperities which raise the electric field locally, or voids filled with air whose breakdown strength is very much lower than that of the insulating solid. These features may lead to breakdown via the formation of branched hollow gas-filled tubular structures which have a "treelike" aspect, and are termed electrical trees.[3] Following the lead given by Niemeyer, Pietronero, and Wiesmann[5] in their work on single discharges in gases (Lichtenberg figures), these electrical trees have also been shown to be fractal objects,[3] i.e., $S \propto (L)^{d_f}$, where S is the "mass" of a structure of length L. In general electrical trees can take one of two basic forms, characterized as "branched" (fractal dimension in the range $d_f = 1.2-1.8$) or "bush" ($d_f \simeq 2.5$). It is known[6] that when grown under identical conditions branched trees propagate faster than bush trees (which may even cease growing) and are thus more dangerous to the insulation. It is therefore a matter of some importance to understand both what causes defect-induced breakdown in solids to assume the form of trees, and what the factors are that determine the tree's fractal dimension.

B. Comparison of electrical tree formation and stochastic models

The stochastic model of Niemeyer, Pietronero, and Wiesmann[5] associates the branched fractal propagation of a conducting discharge with a stepwise development in which the step (bond on a grid) to be added to the structure is chosen at random from all possible growth steps, each of which is assigned a failure probability proportional to E^{η}, where E is the local field along the bond.

While electrical trees have some things in common with this model, e.g., it is known that they propagate in steps,[7] they differ fundamentally in that they are a propagating damage structure rather than the advancing boundary of an injected charge fluid.[5,8] Gas discharges do take place during electrical tree formation but these are restricted to the tubular channels of the existing structure. The extension of the tree is not therefore given by the extension of the discharge beyond the preexisting structure. Instead each discharge, which has a finite duration ($\sim$20 ns), induces damage generating events in the insulation adjacent to its path, both to the side as well as to the front. Such damage does not give an instant extension to the tree. Tree extension is only accomplished when the damage *accumulates* until it attains the form of a tubular channel capable of sustaining a gas discharge. There is therefore a *minimum* length to each extension and experiment[3,7] shows that this is $\sim 4-10\ \mu$m. Channel formation typically requires damage from $\geq 10^3$ discharges in the preexisting tree channels (formation time ≥ 1 s).

Even if it is assumed[8] that channel-forming damage is the result of an attempt by a discharge to extend beyond the tree tip, the necessity of many discharges will inevitably alter the extension probability in electrical trees from that of a single discharge. This situation is to some extent reflected in the Tang[9] simulation rules, which require a given bond to be chosen a number of times (from a field-weighted distribution) before it is added to the structure. However the distribution of choices for all the bonds can only be regarded as the distribution of damage around the tree if the probability algorithm can be given

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a physical interpretation in terms of the field dependence of the damage. Wiesmann and Zeller[10] avoid this difficulty by postulating a threshold field below which discharge-induced events are not damaging, and above which they inevitably proceed to channel formation along the chosen bond. The accumulation of damage which is known to be required for electrical tree formation in ac fields is not allowed for in this model and it is probably more appropriate to breakdown by nanosecond impulses, where however it is not clear that the structures are fractals.[11] Neither of these two models therefore properly reflect the known physics of electrical tree formation.

The major drawback to the application of stochastic models to breakdown structures in solids is the absence of a physical derivation[8] for the parameter η. Such mechanistic arguments as have been proposed for the E^{η} probability law in gases[8] are based on the time required for the establishment of a filamentary projection of the discharge as a "conducting fluid" in a given region of local field, and are inappropriate in solids. Furthermore values other than $\eta=1$ have been given no real justification even in this case. The shape of the breakdown structure is however extremely sensitive to the value of η used, for example the pin-plane electrode system in a three-dimensional (3D)-lattice gives[10,12] bush structures ($d_f \approx 2.5$) for $\eta=1$ and branched structures only if η is increased to ~ 3. Stochastic models therefore leave unanswered questions concerning the physical and mechanistic origins of branched breakdown structures in solids.

C. Proposed quantitative approach

Here we present a quantitative physical model for breakdown structures in solids based on the known physical processes involved in electrical tree formation. Instead of advancing the structure by a stochastic selection of bonds the damage induced in each bond per unit time is calculated quantitatively and irreversibly accumulated until a critical level for channel formation is reached. Stochastic features are introduced into the model by treating factors, such as material strength and local-field enhancement, as random variables in space and/or time. As a result the appearance of various structures can be related to the physical, morphological, and mechanistic conditions prevailing.

II. DISCHARGE-AVALANCHE MECHANISM

The formation of electrical trees has an initiation stage which is dependent upon the type of initiating defect[1,3] (i.e., metallic asperity or preexisting void). However once initiated in the form of a tubule[7] they propagate in the same way. In an applied field a potential difference is produced between the tube ends because the tube walls are not conducting. When in an ac or ramped field it rises to the inception voltage[1,3,13] a gas discharge takes place within the tube, thereby reducing the potential between the tube ends (to the extinction voltage[3]) and increasing the fields in the insulation around it for the duration of the discharge ($\sim 10-100$ ns). It is known that ballistic bombardment of a surface by particles of similar kinetic energy (KE) to those of the tube discharge (i.e., ~ 10 eV) produces very little damage.[14] We therefore assume that the induced damage events take the form of electron avalanches which are known to cause breakdown in thin (~ 10 μm) films.[2,15] Here an electron in the solid is accelerated in the high field produced by the discharge to kinetic energies sufficient to ionize a molecule on its path thereby generating two electrons. The process is carried on in the form of a chain reaction until the electrons produced can no longer acquire sufficient KE for ionization and are thermalized and trapped.[2] The supply of initiating electrons is plentiful because of the gas discharge in the tube. The distance over which the avalanche takes place is limited for a number of reasons: (i) the required high fields are limited to the duration of the discharge (≤ 100 ns); (ii) the field reduces with distance from the tube tip and eventually will be too low to continue the process; (iii) the positive ions produced in the avalanche give a counter field which decelerates the electrons; and (iv) in solids trapping may remove electrons from the process. Because the damage eventually takes the form of a new tubule whose length is known to be[3,7] 4–10 μm, we restrict the range of an avalanche L_b to be $L_b = 10$ μm. We further assume that the amount of damage produced by the avalanche is proportional to the number of ionizations that occur. Such ionizations may lead to damage in a number of different ways: (a) direct breaking of chemical bonds during ionization; (b) mechanical deformation and fracture caused by electrostatic forces; and (c) indirect chemical bond scission utilizing energy released by subsequent charge recombination.[16]

The number of electrons n produced in an avalanche is determined from the incremental increase in number (dn) in the path-length interval x to $x+dx$, which is given by $dn=\alpha(E)n dx$. Here $\alpha(E)$ is the impact ionization coefficient defined as the number of ionizations per electron per unit path length.[2,3,13] Integration over x (up to L_b) gives $n=\exp(\alpha(E)L_b)$, and subtraction of the initiating electron gives the number of ionizations as

$$N_A=[\exp(\alpha(E)L_b)-1] \propto \text{damage per avalanche} . \tag{1}$$

Because of their easy availability a number of electrons n_0 may initiate avalanches simultaneously in a given region, and thus the number of ionizations per discharge $N_D=n_0 N_A$. The short duration of a discharge also allows a number, b, of discharges to occur on each half-cycle of the ac field (frequency f). Multiplication of N_D by b and the number of half-cycles in time t $(2ft)$ gives the number of ionizations N_t produced by time t, as

$$N_t=2ftbn_0[\exp(\alpha(E)L_b)-1] \propto \text{damage per region in time } t . \tag{2}$$

Note that because of charge recombination on opposing half-cycles, N_t is *not* the amount of positive charge present at t. The expression used for $\alpha(E)$,

$$\alpha(E)=(1/\lambda)\exp[-I_p/(e\lambda E)] \tag{3}$$

originates with the work of Seitz,[17] where λ is the meanfree-path length between collisions and the exponential factor is the probability that an accelerated electron will reach the ionization energy I_p without colliding. In solids the length λ and sometimes the ionization energy I_p are treated as adjustable parameters.[18] Here we have taken $I_p = 9.6$ eV as for polyesters and determined λ as ~ 60 nm from experimental data on electrical tree growth,[19] a value that is consistent with the separation of ionizable centers in polymeric insulation.[20]

Each discharge will initiate avalanches at several points along its path within the tree, and the damage accumulated in each of the avalanching regions, at time t, will be proportional to N_t as calculated from Eq. (2) with the field E determined from the local potential difference $\Delta\phi$ along the avalanche path, i.e., $E = \Delta\phi / L_b$. At some time the damage will become sufficient for a new tubule to form. We take this critical amount of damage to be a material dependent quantity equivalent to a critical number, N_c, of ionizations. By setting N_t, from Eq. (2), equal to N_c an expression may be obtained for the time to form a tubular channel t_{ch} under the assumption that the discharge-induced local field $(\Delta\phi / L_b)$ remains invariant during its formation,

$$(1/t_{\mathrm{ch}}) = 2f(bn_0/N_c)[\exp(\alpha(E)L_b) - 1] \ . \tag{4}$$

A comparison[19] with experimental data for polyesters gives $N_c/bn_0 = 10^8$. A rough estimate of N_c can be obtained by assuming that each ionization can only break one chemical bond in forming a new cylindrical tube of length 10 μm and radius 1 μm, giving $N_c \sim 10^{13}$ in regions of normal density. Bigger values may be expected in regions of high mechanical strength and smaller ones in low-density regions. Since typical tube discharges are,[3] 1–5 pC ($\sim 10^7$ electrons) consistency between the experimental and theoretical estimates of N_c can be obtained if the avalanches make use of only a very small fraction ($\leq 0.3\%$) of the available electrons.

During tree propagation local values of E will alter if a tree branch is added at a point other than the one under consideration, or if a modification to the discharge and space charge distribution occurs. In this case we treat Eq. (2) as giving the number of ionizations in the time interval Δt_q for which $E = E_q$ (a constant) and express the amount of damage accrued during the period as a fraction, f_q, of that required for tube formation,

$$f_q = \Delta t_q 2f(bn_0/N_c)[\exp(\alpha(E_q)L_b) - 1] \ . \tag{5}$$

Values of f_q in successive intervals q are added until the sum reaches unity at which point a new channel has been formed, i.e.,

$$\sum_1^{q_{\mathrm{ch}}} f_q = 1 \quad \text{defines } t_{\mathrm{ch}} = \sum_1^{q_{\mathrm{ch}}} \Delta t_q \ . \tag{6}$$

Equations (5) and (6) allow the damage and time to channel formation to be calculated at various points around a tree structure in terms of the parameters λ, N_c/bn_0, which have been determined from experiment, and the local field E_q, which must be computed.

III. COMPUTATION OF STRUCTURES

In the discharge-avalanche model, the stepwise development, the damage accumulation in many regions and the treatment of some factors as random variables forces computer computation of the breakdown structures. The grid chosen for the calculation must reflect the known physics of the mechanism. Thus the mesh size (bond length) is the *minimum* length of a tubular extension L_b, i.e., 10 μm and the electrode separation is typical of solid insulation, e.g., 1 mm in laboratory experiments. There are therefore 100 bonds across the system, and we have chosen a 100×100 grid, since the grid width does not influence the structures unless they approach close to the sides.[12,21] Note that solid insulation[22] is usually no more than 6 mm thick and so the largest grid appropriate to the problem would be one 600 bonds wide. Very big grids such as used in diffusion-limited aggregation simulations[23] are unphysical and incorrect for the problem addressed. They would tend to map out the path of an individual avalanche, which in a gas *is* an extension of the discharge. In solids, however, the *repetition* of avalanches, which is essential to tree extension, will take place in the same bond but with different overlaying paths. As a result the fine structure of an individual path becomes irrelevant. The only important feature is the total damage in the bond, which is dependent upon the potential difference[24] $\Delta\phi$ across it, since this determines the maximum kinetic energy available for damage production.

An important consequence of using the small grid imposed by the physical mechanism is that the breakdown structures computed will not reach a single definitive form.[12] Instead repeat calculations will yield a distribution of structures (e.g., lengths, dimensions, total "mass"). This is in fact what is observed in the experimental investigation of electrical trees,[25] where it leads to a probability description of the likelihood of insulation failure in a given time.[6,25] We have therefore made 50 repeat calculations (a typical experimental value) and analyzed the data in terms of the Weibull function commonly used in breakdown. An excellent fit was obtained. Here the cumulative probability $P(S)$ that the total number of bonds ($\equiv$mass) in the structure at failure is $\leq S$, is given by [3]

$$P(S) = 1 - \exp[-(S/S_c)^{\beta}] \ , \tag{7}$$

where S_c is the characteristic tree mass, and β the shape parameter. The bigger the β the narrower the distribution. Table I contains the results for a range of calculations.

The defect used in our calculations was a centrally placed void of length 50 μm. This choice allows us to work with a uniform applied field, and to set a value, 10 kV/mm, which is both close to that typically found and sufficient to cause the void to discharge.[1,3,22] The calculation proceeds by first using Laplace's equation $(\nabla^2\phi = 0)$ to determine the potential difference across all the bonds due to the tube discharge. Initially the potential difference along the discharge was taken to be zero, but this almost certainly is not the case for discharges

TABLE I. Parameters defining the cumulative distribution function of the structures [$P(S)$, Eq. (7)] obtained for the calculations stated. The Weibull distribution density ($\equiv dP(S)/dS$) is asymmetric with a standard deviation (SD) about the mean [$S_c\Gamma(1+1/\beta)$] of[3] $S_c[\Gamma(1+2/\beta)-\Gamma(1+1/\beta)^2]^{1/2}$, which gives a SD of $0.097S_c$ for $\beta=12$, $0.115S_c$ for $\beta=10$, and $0.2916S_c$ for $\beta=3.4$.

	Potential drop in tube discharges E_d			
	$E_d=0$		$E_d=4$ kV/mm	
Calculation	S_c	β	S_c	β
Frozen electrical disorder	604	11.7	243	3.4
Time-dependent field fluctuations (TDFF)	856	10.4	339	10.4
TDFF with one-order-of-magnitude spatial fluctuations in N_c/bn_0	875	12.0	355	9.2
TDFF with two-order-of-magnitude spatial fluctuations in N_c/bn_0	953	12.9	378	10.0

confined to narrow tubular channels.[26] To obtain a rough estimate of its value we assumed that the discharges lie on a negative-differential-resistance portion of the current-voltage (I-V) curve. Using the empirical relation[27] $E_d=k/I$, with $k=1$ W/m and $I\approx 5$ pC/20 ns $=0.25$ mA gives $E_d\approx 4$ kV/mm, which is close to the extinction field for pico-Coulomb discharges in air gaps of 0.05–1 mm length.[1,3,13]

Having calculated the fields ($E=\Delta\phi/L_b$) induced along each bond adjacent to a discharge every time that it occurs, the bond that fails in the shortest time is determined from Eqs. (5) and (6). This bond is then failed. The time interval required is calculated from Eq. (6) and used to derive the incremental fractions of channel-forming damage accumulated (f_q) in *all* the other adjacent bonds during this period. The field distribution is now calculated for the extended structure and the procedure repeated until the structure crosses the grid.

Unlike the stochastic models[5,9] the bond that fails is *not* the result of a random selection. It is instead the consequence of a deterministic accumulation of damage under specified local conditions of material strength ($\propto N_c$) and field E. Solid insulation invariably contains regions of different material strength and density, and this is accommodated in our model by treating N_c/bn_0 as a random variable. A value was assigned to each bond which was selected at random from a distribution chosen to reflect the known morphology of insulating solids. Most regions in these materials have a well-defined density and strength, but there is a long tail towards low density and strengths that may be orders of magnitude weaker. We therefore chose a Gaussian distribution in $\log_{10}(N_c/bn_0)$, truncated at the maximum value $N_c/bn_0=10^8$ and minimum value of zero, with a half-width corresponding to $N_c/bn_0=10^6$. Solid insulation may also possess electrical inhomogeneity (i.e., spatial variation in resistivity and permittivity), which leads to distributed local values for the bond impedances. Since the total potential difference along a path from the discharge to the electrode is divided up in proportion to the impedances of the bonds in the path, electrical inhomogeneity will cause the local field to vary about its calculated Laplace value. We have represented this effect by multiplying the calculated $\Delta\phi$ by a factor g, which is the ratio of the actual local impedance to the average value of a nominally uniform material. In order to retain simplicity and avoid biasing any particular direction, values for g assigned to each bond were randomly selected from a "top-hat" distribution of limited range [i.e., Prob(g)=const, $0.5\le g\le 2.5$; Prob(g)=0, otherwise] whose extrema correspond roughly to a local doubling of either the admittance or impedance.

The above features represent the effects of "frozen disorder" previously commented[8] upon as a possible origin for branching in solids. There is however another possible origin for local fluctuations about the calculated Laplace field. This is the breakdown mechanism itself, which as a result of the tube discharges will deposit space charge around the tree tips and on the tube walls. Such space charges will modify the local fields, in general reducing them through a smoothing of the periphery of the tree. They may also however prevent some sections of the tree from discharging simultaneously with the others. Consequently the Laplace calculation which considers the whole tree as a single conducting entity may overestimate the shielding of the tips and thus underestimate the local fields. A detailed calculation of these effects is a major undertaking and in order to gain an insight into their influence we have resorted to a common approximation for the field around a conducting (or polarizable) defect. This treats the defect-modified field as being given by the applied field multiplied by a factor dependent upon the defect shape and orientation.[22,28] The Laplace equation is first used to obtain the local fields appropriate to a discharge in the complete tree skeleton. These are then converted to an effective local field by multiplying by a factor g whose value represents the ratio between the actual local field enhancement and that of the tree skeleton. Thus extra shielding reduces the enhancement of the skeleton giving $g<1$, whereas $g>1$ obtains when only part of the skeleton discharges. As before we assign a value of g to each bond by random selection from a top-hat distribution with the range $0.5\le g\le 2.5$. The local field enhancements obtained in this way are well within the range expected for conducting inclusions, which may vary from 2 to 100 times the average field. Unlike the case of frozen electrical disorder we expect these mechanism-induced field modifications to vary in time. We have therefore reassigned g values to each bond after every extension of the structure. We have also investigated the effect of an *additional* variation in the local field by including an *extra* reassignment of g after fixed time intervals. A variety of times were used reflecting the different timescales on which space charge may be expected to rearrange.

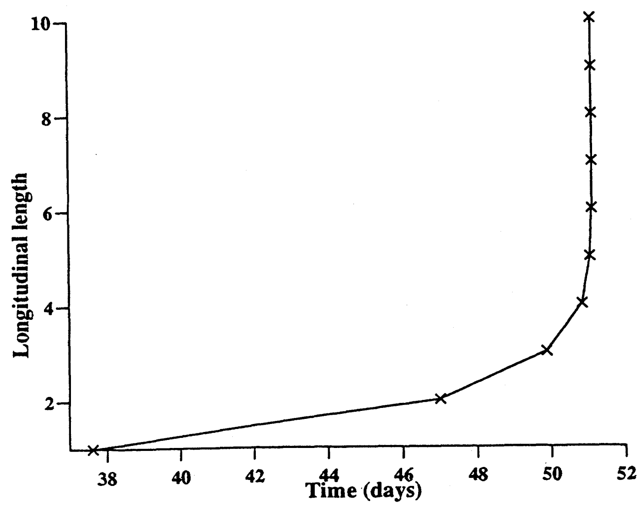

FIG. 1. The calculated propagation behavior of the *single puncture* breakdowns produced when spatial and temporal fluctuations of N_c/bn_0 and E were neglected (i.e., $N_c/bn_0=10^8$; $E=E$(Laplace). The length is quoted in units of $L_b=10$ μm.

IV. RESULTS

When material inhomogeneity and field fluctuations are neglected the structures obtained had the form of a straight puncture hole between the electrodes. Damage *was* accumulated in the lateral bonds but never reached more than 0.1% of the level required for channel formation. In spite of its step-by-step development the breakdown exhibited a runaway propagation, Fig. 1, during which the calculated channel formation time eventually reached unphysically small values considering the mechanical disruption involved. We therefore subsequently restricted t_{ch} to the range $t_{ch}\geq 1$ ns, in accord with the fastest measured time for tree propagation.[29]

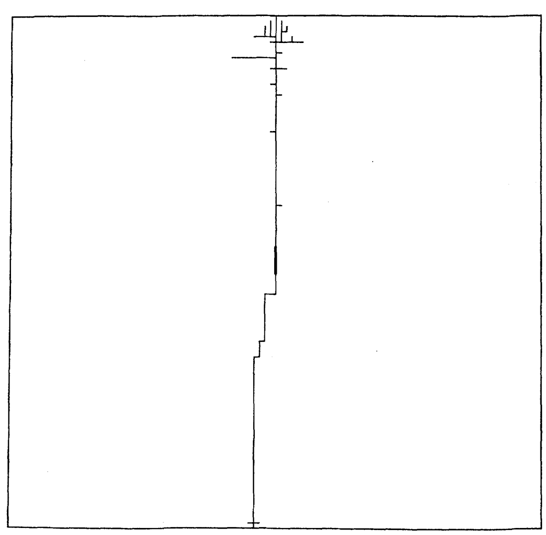

FIG. 2. Characteristic breakdown structures in the distribution produced when "frozen spatial disorder" in the material strength parameter N_c/bn_0 is allowed for. *See text for details of the N_c/bn_0 distribution used.* The centrally placed initiating void is shown as a thick line in this and the subsequent figures.

This restriction tended to allow a few lateral branches to form as the puncture approached close to an electrode giving a "thistlehead" to the structure. The introduction of *material strength* inhomogeneity tended to introduce weak paths in the solid which were followed by the breakdown structure, leading to kinking and a few short lateral branches, Fig. 2, though without changing the basic form from that of a single puncture. Reducing the half-width of the N_c/bn_0 distribution to $N_c/bn_0=10^7$ generally eliminated the kinking. Under these conditions, therefore, the stepwise damage-accumulation mechanism behaves as if it were almost a deterministic high-field runaway process.

When the local field is allowed to fluctuate about the Laplace value branched structures are formed. Figure 3 shows the characteristic structures obtained for the case of frozen electrical disorder, and Fig. 4 those for the time-varying (dynamic) mechanism-induced field fluctuations. In both cases widening the range allowed for the

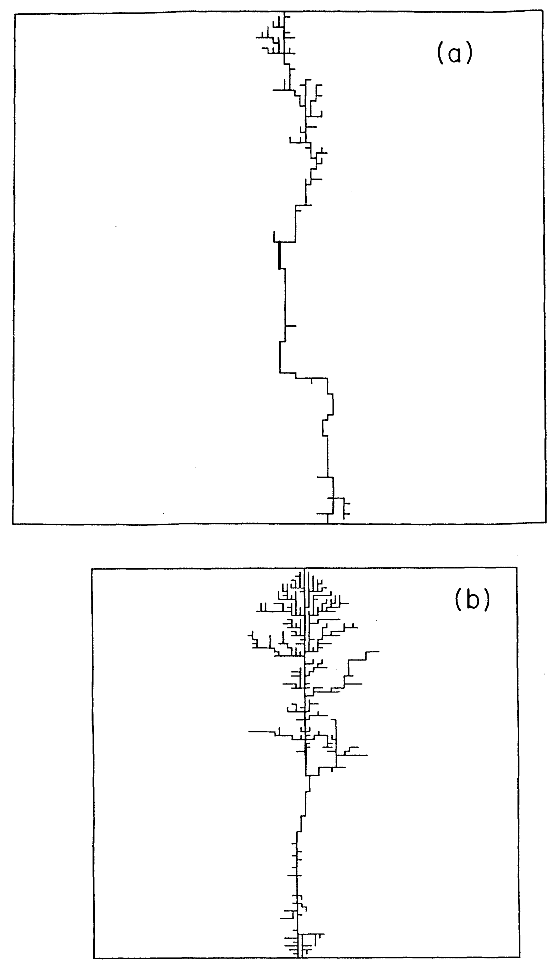

FIG. 3. Characteristic breakdown structures in the distribution produced when frozen electrical inhomogeneity is introduced into the calculation via randomly assigned local values of the field-enhancement factor g [i.e., E(local) = gE(Laplace)]. (a) g selected from a top-hat distribution with range $0.5\leq g\leq 2.5$; (b) range of g widened to $0.5\leq g\leq 3.5$.

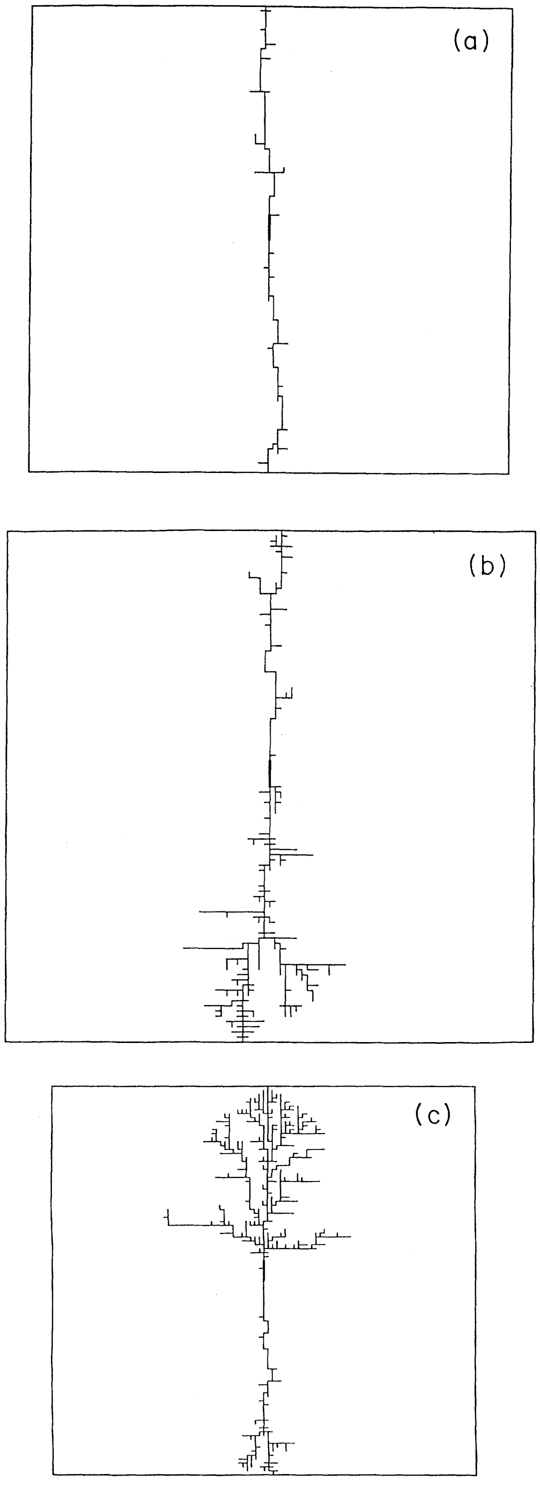

FIG. 4. Characteristic breakdown structures in the distribution produced when time-dependent field fluctuations are considered (local enhancement factors g altered after each tree extension). (a) allowed range of g $0.1 \le g \le 0.5$, potential drop in tube discharge = 0; (b) range of g $0.5 \le g \le 2.5$, potential drop in tube discharge $E_d = 4$ kV/mm, material strength inhomogeneity included; (c) range of g $0.5 \le g \le 3.5$, $E_d = 4$ kV/mm, material strength inhomogeneity included.

field-enhancement factor g increases the number of branches produced as the structure crosses the grid, compare Fig. 3(a) with 3(b), and Fig. 4(b) with 4(c). In contrast to the stochastic model,[10] reducing the potential difference within the tube discharge of the tree *increases* the amount of branch formation in addition to accelerating the propagation. Frozen electrical disorder differs from mechanism-induced field fluctuations in allowing the possibility of *permanent* high-field routes in the solid. As a result there is a tendency to form material-oriented kinked breakdown structures such as Fig. 3(a). This tendency is reflected in the wider Weibull distribution of structures (smaller β) obtained for this case in comparison to all others, see Table I.

The only structures that can be truly regarded as fractals are the ones where the field fluctuations are mechanism induced, and even here it is necessary for g to both enhance and reduce the Laplace field, compare Figs. 4(a) and 4(b). In this case the structures start off growing linearly but then tend asymptotically towards a fractal dimension d_f, Fig. 5. The characteristic tree of the distribution was found to increase in fractal dimension as the range of field fluctuations widened, i.e., $d_f \approx 1.4$ for $0.5 \le g \le 2.5$ and $d_f \approx 1.6$ for $0.5 \le g \le 3.5$, however d_f will also increase if E_d is reduced from 4 kV/mm. There is no evidence in our calculations for structures which crossover from $d_f > 1$ to $d_f = 1$ as the tree length increases, such as is found in some stochastic models.[30] The introduction of *additional* field fluctuations which vary on a fixed timescale does not alter the fractal dimension of the characteristic tree, or the width of the structure distribution ($\propto 1/\beta$). The width of the propagation time distribution does however widen considerably as the fluctuation timescale increases beyond ~ 1 min.

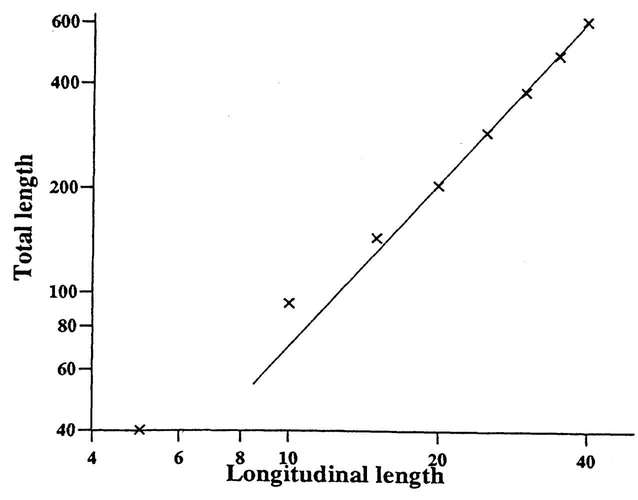

FIG. 5. A log-log plot of the total length (number of failed bonds) in the tree structure formed on one side of the void as a function of its longitudinal length measured from the void in units of $L_b (= 10\ \mu m)$. The data presented is taken from a calculated breakdown whose overall structure [S in Eq. (7)] has the characteristic value in the distribution of tree structures produced by the time-dependent field fluctuations. An asymptotic approach to a fractal dimension of $d_f = 1.4 \pm 0.05$ is found.

V. DISCUSSION

The *physical* conditions under which a known breakdown process in solids may be converted from a deterministic runaway mechanism giving single narrow punctures to one giving branched structures, has been identified by means of these calculations. It has been shown that spatial inhomogeneity in material strength is insufficient[10] and only causes the breakdown puncture to follow weak paths, examples of which can be found experimentally.[4] The necessary condition for the formation of branched structures is the existence of local-field fluctuations about the Laplace values calculated for the tree skeleton discharging as a whole. Even in this case the structures only take a fractal form when the fluctuations vary in *time* as well as space. If the fluctuations are caused by frozen local disorder, high-field routes may influence the structures obtained destroying any fractal arrangement. It is likely that both factors may contribute to experimental results leading to the wide distributions often found.[3,6]

The time-varying field fluctuations responsible for fractal breakdown structures in solids have been identified here with the breakdown mechanism itself, which generates nonuniformly distributed regions of trapped space charge (on tube walls and around tube tips), within which the tree skeleton is embedded. When discharges occur in the tree tubes the resulting Laplace fields in the solid will be either enhanced or reduced by the space charges (of both polarities). Space charge redistribution may occur both on tree extension or as a result of the discharge activity itself. By defining the local field via a randomly selected enhancement factor g, $[E(\text{local})=gE(\text{Laplace})]$, we are treating each region as a stress modifying defect[22,28] whose shape and orientation are determined by local factors such as charge deposition, diffusion, trapping and resistivity, which are known only within certain limits. The top-hat distribution from which g is selected treats all values of g within the defined range as equally probable, and hence g can be regarded as a white-noise variable of limited range.

This picture of the electrical tree and its formation in solids allows us to draw a parallel between the present physical model and an approach to single-discharge structures in gases which ascribes branching to charge-density fluctuations on the advancing charge boundary. In that case it was argued that if the local field is sufficiently enhanced by a random fluctuation then a filamentary projection can be stabilized.[3,10] A similar explanation has been proposed for viscous fingering in fluid displacement.[31,32] In both cases branch formation can be regarded as the result of a noise which causes the driving force of the mechanism to fluctuate, and in the process stabilizes structural variations from the symmetry (radial or linear) imposed by the applied field. Unlike the stochastic models however the effect of noise is here incorporated directly into the magnitude of the driving field of a quantitatively expressed physical mechanism. Instead of introducing a parameter η which cannot be assigned a value *a priori*, the noise here is quantifiable via its type (i.e., white-noise: top-hat distribution) and range. It can therefore be demonstrated that the range required to obtain branch type trees of a fractal dimension typically observed ($d_f=1.4-1.8$) is equivalent to a physically realistic range of field enhancements. For example if the local regions are treated as conducting ellipsoids, the tree skeleton with a ratio of major-to-minor axis of ~ 5 gives a field enhancement of ~ 10, and the range $0.5 \leq g \leq 2.5$ then corresponds to axis ratios varying between[28] ~ 2 and ~ 20, with that of a single unshielded branch being ~ 10. Widening the range of g leads to structures which are progressively more space filling and could be expected to form bush trees ($d_f \approx 2.5$) in 3D calculations. In the context of this model it is therefore possible to evaluate the effect of various factors upon breakdown through their influence on the range of field fluctuations that they allow. For example, the applied field, its frequency, the temperature and thermal history, may all alter the amount and location of space-charge trapping, and its diffusion, giving rise to the known changes in tree structures when these conditions are altered.[3,15]

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