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Thermal-noise-induced resistance and supercurrent correlation function in $YBa_2Cu_3O_{7-\delta}$ grain-boundary Josephson junctions

S. Schuster, R. Gross, B. Mayer, and R. P. Huebener
Physikalisches Institut, Lehrstuhl Experimentalphysik II, Universität Tübingen, Morgenstelle 14, D-7400 Tübingen, Germany
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Thermal noise is known to cause a finite resistance R_p of overdamped Josephson junctions even in the zero-current limit. By extending the treatment of Ambegaokar and Halperin the magnetic-field dependence of this resistance is analyzed for arbitrary current-phase relations. We show that the measurement of the magnetic-field dependence of R_p allows the determination of the magnetic-field dependence of the critical current in the temperature regime where thermal-noise rounding effects prevent the application of the usual methods. Applying this technique the correlation function of the supercurrent distribution of $YBa_2Cu_3O_{7-\delta}$ grain-boundary junctions could be determined with a spatial resolution of 130 nm. The correlation function indicates that the supercurrent distribution along the grain boundary is spatially nonuniform on a sub-μm scale.

In recent years the superconducting transport properties of grain boundaries in the high-temperature superconductors have been studied intensively.[1] Much of our present knowledge is based on the study of grain boundaries in $YBa_2Cu_3O_{7-\delta}$ where individual grain-boundary junctions (GBJ's) have been fabricated by growing epitaxial $YBa_2Cu_3O_{7-\delta}$ films on $SrTiO_3$ bicrystal substrates.[2,3] These GBJ's have been shown to behave as ideal overdamped Josephson junctions.[4] Despite the large research effort the nature of the electrical transport across grain boundaries in the cuprate superconductors is still discussed controversially. Here a key issue is the homogeneity of the current flow across grain boundaries. This question has been addressed recently by several experiments.[5,6] In this paper we present a new method for the determination of the supercurrent correlation function in $YBa_2Cu_3O_{7-\delta}$ GBJ's giving information on the homogeneity of the supercurrent flow across grain boundaries on a sub-μm scale.

In general, for Josephson junctions information on the spatial distribution of the critical current density $J_c(x,y)$ can be obtained by measuring the magnetic-field dependence of the integral critical current $I_c(H)$ of the junction. However, for junctions with spatial dimensions larger than the Josephson penetration depth λ_J the relation between the $I_c(H)$ dependencies and the supercurrent distribution usually is complex making it difficult to extract information on $J_c(x,y)$ from the measured $I_c(H)$ dependencies. On the other hand, such information can be obtained for small junctions where the relation is simple. As discussed in more detail below, in this limit $I_c(H)$ is given by the magnitude of the Fourier transform of the spatial variation of the critical current $\tilde{J}_c$ per unit length. Here, $\tilde{J}_c$ is the integral of the critical current density along the field direction.[7] Unfortunately, for small high-T_c GBJ's the coupling energy $E_J(T)=\hbar I_c(T)/2e$ often is of the same order as the thermal energy k_BT resulting in considerable thermal noise effects.[4] These effects prevent the simple measurement of $I_c(H)$ using a voltage criterion. Therefore, a new method for measuring the $I_c(H)$ dependence of small high-T_c junctions is required.

In order to make the influence of thermal noise effects more clear, in Fig. 1 we have plotted the temperature dependence of the Josephson penetration depth λ_J and the width W^* of a 100-nm-thick GBJ for which $E_J(T)=10k_BT$. The curves are calculated for $J_c(T=0)=10^5$ A/cm^2, which is typical for $YBa_2Cu_3O_{7-\delta}$ GBJ's.[1] Note that for $W<W^*$ thermal noise effects are important. For example, for a 10-μm-wide GBJ thermal noise effects have to be taken into account down to about $0.8T_c$. Below this temperature thermal noise is negligible, however, in this temperature regime a 10-μm-wide junction is already large ($W>\lambda_J$). For the theoretical analysis given below, $W<\lambda_J(T)$, $W^*(T)$ is required. Figure 1 shows that this requirement is satisfied for typical GBJ's over a wide temperature regime below T_c. In contrast, the condition $W>W^*(T)$ and $W<\lambda_J(T)$ is more difficult to satisfy. That is, for high-T_c GBJ's it is difficult to have small junctions with sufficiently high coupling energy required for the use of the straightforward technique of measurement of $I_c(H)$.

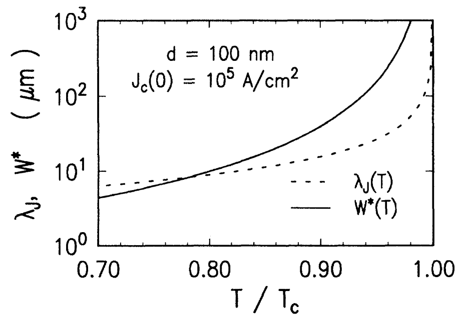

FIG. 1. Temperature dependence of the Josephson penetration depth λ_J and the width W^* at which $E_J(T)=10k_BT$ for a $YBa_2Cu_3O_{7-\delta}$ GBJ with thickness $d=100$ nm and $J_c(T=0)=10^5$ A/cm^2.

We note that the latter conditions can be satisfied over a larger temperature range than shown in Fig. 1, if the thickness of the GBJ's is increased.

A theoretical analysis of thermal effects on the characteristics of overdamped zero-dimensional Josephson junction has been given by Ambegaokar and Halperin (AH),[8] based on the resistively shunted junction (RSJ) model.[7] Solving the Smoluchowski equation (see, e.g., Ref. 9) in the stationary case they derived the I-$\langle V \rangle$ curves for a sinusoidal supercurrent-phase relation $I_s = I_c \sin(\phi)$. Here, $\langle V \rangle$ is the time-averaged voltage and ϕ the phase difference across the junction. They showed that the presence of thermal noise results in rounded I-$\langle V \rangle$ curves close to T_c. Furthermore, in the limit of zero current a finite resistance $R_p(T) = R_n[I_0(\hbar I_c(T)/2ek_B T)]^{-2}$ is obtained where R_n is the normal resistance of the junction, k_B is the Boltzmann constant, and I_0 the modified Bessel function.

The resistance $R_p(T)$ has been observed experimentally for $YBa_2Cu_3O_{7-\delta}$ GBJ's (Ref. 4) as a foot structure in the resistive transition. That is, a finite resistance $R_p(T)$ is observed down to temperatures well below the zero-resistance temperature of the superconducting electrodes forming the GBJ. Inverting the measured $R_p(T)$ dependence the temperature dependence of the critical current, $I_c(T)$, can be derived according to the above result of the AH theory.[4] Doing so, the question arises whether the foot in the measured $R_p(T)$ dependence is solely caused by thermally induced decoupling of the phase coherence across the GBJ as supposed in the AH theory or whether there is an additional effect due to a degraded region at the grain-boundary interface with locally reduced critical temperature. This issue can be clarified by measuring the magnetic-field dependence of R_p, since a foot structure solely caused by a degraded grain-boundary region is expected to have a negligible field dependence. For this we extend the AH model to the case of a spatially inhomogeneous overdamped Josephson junction in nonzero applied magnetic field. Our treatment, which is performed for arbitrary, single-valued current-phase relations, is applicable only to small Josephson junctions. Inverting the measured $R_p(H)$ dependencies now yields the $I_c(H)$ dependencies of the GBJ's despite thermal-noise effects.

Let us consider a GBJ formed by a superconducting film of width W and thickness d extending in the x-y plane as shown in Fig. 2 with an external magnetic field

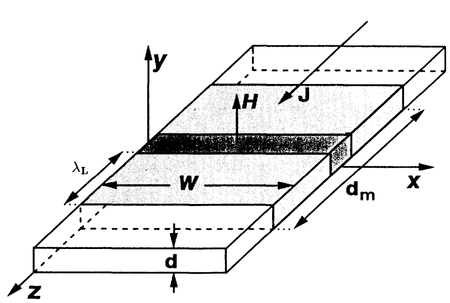

FIG. 2. Sketch of the geometrical configuration of a bicrystal grain-boundary junction.

H applied in the y direction. Although we are discussing the case of GBJ's in the following we note that our treatment applies for overdamped Josephson junctions in general. The spatial dimensions and the coupling energy of the junctions shall be small, i.e., $W, d \leq \lambda_J$ and $E_J \leq 10 k_B T$. As discussed above this requirement is easily satisfied by high-T_c GBJ's in a wide temperature regime below T_c. For small junctions self-field effects can be neglected and the phase difference function $\phi(x,t)$ assumes the simple form $\phi(x,t) = \phi(t) + qx$. Here $q = (2ed_m/\hbar c)H$ and $d_m = 2\lambda_L +$ barrier thickness $\simeq 2\lambda_L$ is the effective magnetic thickness of the junction, where λ_L is the a-b plane London penetration depth. With these assumptions the equations of motion of the phase difference are

$$\partial\phi/\partial t = (2e/\hbar)V \ , \tag{1}$$

$$C\frac{\partial V}{\partial t} = I - \int_0^W \tilde{J}_c(x)\sin[\phi(t)+qx]dx - \frac{V}{R_n} + \mathcal{L}(t) \ , \tag{2}$$

where C is the junction capacitance and $\tilde{J}(x)$ is the integral of the supercurrent density along the magnetic-field direction. For $H \| y$, $\tilde{J}(x)$ is the integral of the current density across the junction thickness d, which usually is small for high-T_c GBJ's (typically less than 300 nm). $\mathcal{L}(t)$ is a thermal noise current with the correlation function $\langle \mathcal{L}(t)\mathcal{L}(t+\tau)\rangle = (2k_B T/R_n)\delta(\tau)$. The integral can be written as $A(q)\sin[\phi(t)+B(q)]$, where $A(q)$ and $B(q)$ are the magnitude and phase of the integral

$$\int_{-\infty}^{\infty} \tilde{J}_c(x)e^{iqx}dx = A(q)e^{iB(q)} \ . \tag{3}$$

Hence the equations of motion have the same form as in the zero-dimensional case studied by AH. Thus all results of the AH theory can be transferred to our case by the simple replacement $I_c(T) \rightarrow A(q,T)$. In particular, for the foot resistance R_p we obtain

$$R_p(q,T) = R_n\{I_0[(\hbar/2ek_B T)A(q,T)]\}^{-2} \ . \tag{4}$$

R_p is expected to oscillate with increasing H in the same way as I_c. Since I_0 is a monotone function the $I_c(H)$ dependence can be obtained by inverting the measured $R_p(H)$ dependence.

We note that the phase shift B is equivalent to adding a constant phase ϕ_c in the Josephson equation $I_s = I_c \sin(\phi)$ and should not have any experimentally observable consequences. This can easily be checked by inserting $I_s = A\sin(\phi + B)$ in the analytically known form of the IV curves[8] and using $\int_0^p f(x)dx = \int_0^p f(x+c)dx$ with given c for a p-periodic function $f(x)$. We further note that the conditions allowing the reduction of the Fokker-Planck equation to the Smoluchowski equation remain unchanged, as only R_n and C are involved in fixing the damping.[8] Also it is not necessary to discuss a spatially varying $\mathcal{L}(t)$, since the noise current is only fixed by its equilibrium properties. Finally it should be mentioned that we consider only the $I \rightarrow 0$ limit where effects due to excess low-frequency noise (e.g., $1/f$ noise) can be neglected.

The above reasoning is not limited to a sinusoidal current phase relation. For the case of an arbitrary 2π-

periodic, single-valued current-phase relation $v(\phi)$ the supercurrent can be expressed as a sum of higher harmonics with coefficients c_n

$$I_s = I_c v(\phi) = I_c \sum_n c_n \sin(n\phi) \quad (5)$$

leading to the analytic result

$$R_p(q,T) = 4\pi^2 R_n [+]^{-1}[-]^{-1} , \quad (6)$$

where

$$[\pm] = \int_0^{2\pi} \prod_n \exp\left\{ \pm \frac{\hbar}{2ek_B T} \frac{c_n A(nq,T)}{n} \times \cos[n\phi + B(nq,T)] \right\} d\phi \ . \quad (7)$$

Note, that in this expression only the function $A(q,T)$ appears. Hence this result will be of importance for studies on $\tilde{J}_c(x)$ in weak links with known nonsinusoidal current-phase relation and could, furthermore, be useful in detecting deviations from the $\sin(\phi)$ dependence by means of transport measurements. In the following analysis we assume a sinusoidal current-phase relation for the investigated $YBa_2Cu_3O_{7-\delta}$ grain boundaries.

Our measurements have been performed with $YBa_2Cu_3O_{7-\delta}$ grain-boundary junctions fabricated on $SrTiO_3$ bicrystals as described elsewhere.[10] The resistance R_p was measured by applying a small ac current $I_{ac} \sin\omega t$ with amplitude I_{ac} small compared to I_c and detecting the resulting voltage using a lock-in amplifier. The temperature was held constant to ± 10 mK and the magnetic field, which was applied parallel to the grain boundary, was varied slowly. Perturbing external magnetic fields were carefully shielded using Cryoperm-10 shields.[11]

Figure 3 shows the $R_p(H)$ dependence of a 37-μm-wide $YBa_2Cu_3O_{7-\delta}$ GBJ at different temperatures. The observed fine structure was fully reproducible even after thermally cycling the sample. At the lower temperatures the symmetry and periodicity of the $R_p(H)$ dependence were the same as obtained from the $I_c(H)$ dependencies of the same GBJ in agreement with our theoretical expectations. With the monotone $R_p[A(q)]$ dependence [Eq. (4)] and the known R_n value, the measured R_p dependence can be inverted to yield $A(q)$ or, equivalently, $I_c(H)$. The result is shown in Fig. 4. The inversion procedure is most difficult for small R_p (large I_c) values since here the finite noise level of the experiment results in considerable inaccuracies of I_c. Furthermore, we note that the derived $I_c(H)$ dependencies do not reach zero periodically. This is caused by the fact that the measuring current is always finite in the experiment.

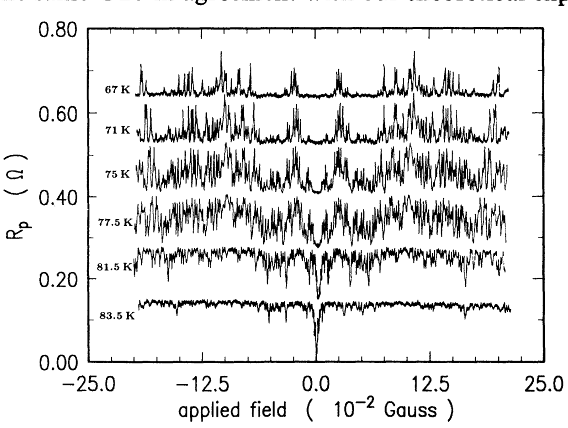

FIG. 3. $R_p(H)$ dependence of a 37-μm-wide $YBa_2Cu_3O_{7-\delta}$ GBJ at different temperatures. The upper curves are displaced vertically for clarity.

Our new method allows the determination of $I_c(H)$ by measuring $R_p(H)$ for small junctions $[W \leq \lambda_J(T)]$. As discussed above this condition is easily fulfilled at temperatures sufficiently close to T_c. For small junctions the relation between the magnetic-field dependence of the total supercurrent $I_c(H)$ or, equivalently, of $A(q)$ and the supercurrent distribution $\tilde{J}_c(x)$ is of the Fourier-transform type.[12] However, a direct reconstruction of $\tilde{J}_c(x)$ from $A(q)$ measurements is not possible due to the unknown phase factor $B(q)$ in the Fourier integral of Eq. (3). Several ways around this problem have been proposed (e.g., Refs. 7, 13, and 14). Of course, the most natural way to account for the loss of information on the phase $B(q)$ is to perform a statistical characterization of $\tilde{J}_c(x)$ by means of its correlation function $C(\delta) = \int_{-\infty}^{\infty} \tilde{J}_c(x)\tilde{J}_c(x+\delta)dx$. This was suggested by Zappe.[14]

The calculation of $C(\delta)$ from the $A(q)$ data which are obtained by inverting the measured $R_p(H)$ dependence is easily done on the basis of the Wiener-Khinchine relations (e.g., see Ref. 15)

$$C(\delta) = \int_{-\infty}^{+\infty} |A(q)|^2 e^{iq\delta} dq \ , \quad (8)$$

$$|A(q)|^2 = \frac{1}{2\pi} \int_{-\infty}^{+\infty} C(\delta) e^{-iq\delta} d\delta \ . \quad (9)$$

For example, for a regular array of supercurrent filaments a series of triangles with declining height is expected for $C(\delta)$. Their spacing is determined by the spacing of the filaments and the base of the triangles is equal to twice the diameter of the filaments. We note that such a simple result holds only for the case where q is not explicitly dependent on x. For GBJ's this is the case if the flux-focusing factor[16] is spatially constant. We further note that the flux-focusing factor should not change

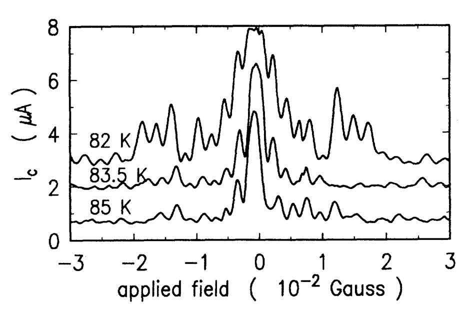

FIG. 4. $I_c(H)$ dependencies of a 37-μm-wide $YBa_2Cu_3O_{7-\delta}$ GBJ derived from the measured $R_p(H)$ dependencies. The upper curves are displaced vertically for clarity.

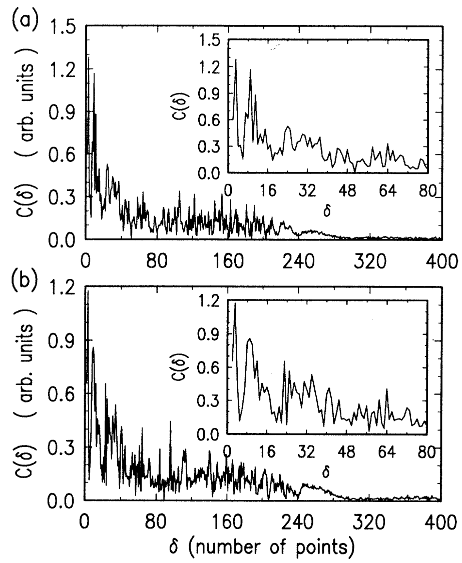

FIG. 5. Correlation function of the supercurrent distribution in a 37-μm-wide $YBa_2Cu_3O_{7-\delta}$ GBJ at $T=81$ K (a) and 82 K (b).

within the field range where $A(q)$ is determined. If the flux-focusing factor changes with the applied field rescaling of the applied field is required to give the correct field value present at the grain boundary. The spatial resolution obtained for the correlation function $C(\delta)$ is given by the number of maxima in the $A(q)$ dependence, i.e., by the range of the applied magnetic field. In principle, a spatial resolution in the 100-Å regime is possible by using fields of several tesla. However, since the change of the flux-focusing factor for fields above the lower critical field H_{c1} of the $YBa_2Cu_3O_{7-\delta}$ film is not precisely known, the value of H_{c1} is the limiting factor with respect to the spatial resolution achieved in a straightforward way.

A typical experimental result for the correlation function is shown in Fig. 5 for two independent sets of $A(q)$ data taken at 81 and 82 K using a 37-μm-wide $YBa_2Cu_3O_{7-\delta}$ GBJ. The correlation function deviates considerably from the linearly decreasing triangle shaped dependence expected for a homogeneous supercurrent distribution. It becomes zero for large arguments. Knowing that $\tilde{J}_c(x)$ and hence $C(\delta)$ is nonzero only from 0 to W, the argument where the correlation function becomes zero fixes the length scale and thereby calibrates the obtained spatial resolution. The latter is about 130 nm for the data shown in Fig. 5 and is limited only by the maximum field used in our measurements. The most peculiar feature emerging from these data is the oscillatory behavior of the correlation function for small distances. These oscillations are fully reproducible for different measurements (at similar temperatures) and are not noise effects or errors resulting from the $R_p[A(q)]$ inversion procedure. This was numerically checked using the known $A(q)$ dependencies of a homogeneous junction and an array of filaments. We point out that the resolution is already high enough to assure the significance of these oscillations. They suggest the existence of regions in the grain-boundary Josephson junction with reduced supercurrent. Such regions would, according to our present data, have a separation of less than 0.7 μm.

In summary we have extended the AH theory to the case of spatially inhomogeneous Josephson junctions and nonzero applied magnetic field. By measuring the magnetic-field dependence of the thermal-noise-induced resistance R_p the $I_c(H)$ dependence of the $YBa_2Cu_3O_{7-\delta}$ GBJ and the supercurrent correlation function were derived. For the latter a spatial resolution of about 130 nm was obtained. The measured correlation function shows that the supercurrent density across the grain boundary is spatially inhomogeneous on a sub-μm scale.

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