Exact and broken symmetries in a hydrodynamical description of chiral spin states

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Following the procedure recently proposed by Wiegmann, we improve his analysis and derive a complete effective Lagrangian describing long-wavelength fluctuations around hypothetical three-dimensional (3D) and 2D chiral states of Heisenberg spin systems. We study realizations of the previously proposed high symmetry group of the continuous theory [SO(3,1)×SU(2)×U(1) in 3D and SO(2,1)×$U_c(1)$×$U_s(1)$ in 2D] and observe that, in general, Lorentz symmetry is broken already in the bare Lagrangian, although it could be restored after a renormalization. In contrast to the conjectures made by Wiegmann, an additional non-Abelian SU(2) gauge symmetry expected in 3D (and its Abelian counterpart on 2D) are, in fact, destroyed due to a spontaneous parity violation in chiral ground states.

I. INTRODUCTION

Recent progress in the theory of strongly correlated fermion systems close to the Mott-Hubbard transition posed a challenging problem of a new classification of all possible spin states of a Mott insulator. Generalizing the Shubnikov theory of magnetic space groups a new scheme should include all spin disordered states with no local spin order $\langle \mathbf{S}\rangle = 0$. Among them there should be such states as spin nematics characterized by virtue of the two-spin correlation function $\langle S_i^a S_j^b\rangle$ (Ref. 1) as well as so-called scalar and tensor magnets having only three-point functions $\langle S_i^a S_j^b S_j^c\rangle$ nontrivial.[2]

A possible classification of various spin states in the framework of a hydrodynamical description was already discussed by Andreev and Marchenko.[3] However, such an extension certainly does not exhaust all the possibilities. Since 1987 numerous hypothetical spin states have been proposed within a mean-field treatment of the Hubbard or the t-J model. As a primary example this variety includes a so-called resonance valence bond (RVB) state first proposed by Anderson[4] which was realized as a kind of a quantum paramagnetic state.[5]

Another remarkable example is a principally new spin disordered state called the chiral spin state[6] which is generically coupled with the flux phase.[7] This state was originally discussed in the two-dimensional (2D) case and later it was generalized onto the 3D one.[8,9,10] It demonstrates a spontaneous violation of both space parity and time reversal symmetry. Instead of local spin ordering it is ordered in terms of local chiralities defined as mixed products of triples of adjacent spins. In a geometrical interpretation this variable is a measure of a solid angle subtended by all spins belonging to a given plaquette. Obviously, a chirality becomes an informative characteristic only for noncollinear spin structures.

Going further one can imagine an arbitrary inhomogeneous distribution of chiralities corresponding to various "flux liquids." It has been conjectured by Wiegmann that an adequate representation of those states can be built in terms of nonlocal variables corresponding to Wilson loops in QCD. These can be realized as traces of operators of cyclic permutations of some "probe" spin $\boldsymbol{\sigma}$ with the spins belonging to an arbitrary contour Γ:[5,10]

$$W = \mathrm{Tr} \prod_{\langle ij\rangle \in \Gamma} (1 + 2\boldsymbol{\sigma}\mathbf{S}_i). \tag{1}$$

It was also supposed in Ref. 10 that the phases of averages (1) can be understood as some Chern numbers of the spin wave function $\Psi(z_1, ..., z_N)$, where z_k are coordinates of up spins:

$$W = \sum_k^N \oint \epsilon_{\alpha\beta} \frac{\partial}{\partial z_k^\alpha} \Psi^\dagger(z_1, ..., z_N) \frac{\partial}{\partial z_k^\beta} \Psi(z_1, ..., z_N) d^2 S_k, \tag{2}$$

where integrals have to be taken over a bulk of a 2D system or a surface of a 3D one. To clarify this relationship, one can use a description of spin configurations in terms of a spinon representation of spin-one-half operators,

$$\mathbf{S}_i = \Psi_i^\dagger \frac{\boldsymbol{\sigma}}{2} \Psi_i, \quad \Psi_{i\sigma}^\dagger \Psi_{i\sigma} = 1, \tag{3}$$

and spinon bilinears $\Psi_{i\sigma}^\dagger \Psi_{j\sigma}$. It naturally arises in the context of Hubbard transformed Heisenberg Hamiltonians bilinear in spin operators:

$$H = \sum_{ij} J_{ij} \mathbf{S_i} \cdot \mathbf{S_j}, \tag{4}$$

where the matrix of coupling constants J_{ij} can include couplings between remote sites but it is assumed to decay fast enough with distance $|i-j|$ (we shall avoid a discussion of infinitely long-ranged couplings here). By virtue of the Hubbard transformation one can rewrite (4) in the form

$$H = \sum_{ij} \left(\Delta_{ij} \Psi_{i\sigma}^\dagger \Psi_{j\sigma} + \frac{|\Delta_{ij}|^2}{J_{ij}} \right) + \sum_i \lambda_i (\Psi_{i\sigma}^\dagger \Psi_{i\sigma} - 1), \tag{5}$$

where λ_i is a Lagrange multiplier field which implements the constraint (3). The variable Δ_{ij} stands for a scalar coupling of spinons on corresponding sites $\Delta_{ij} \equiv \Psi^\dagger_{i\sigma}\Psi_{j\sigma}$. The Hamiltonian (5) is locally symmetrical under a U(1) transformation

$$\Psi_\sigma \to \exp(i\phi_i)\Psi_\sigma, \quad \Delta_{ij} \to \exp(i\phi_i - i\phi_j)\Delta_{ij}. \tag{6}$$

Then a value of local spin chirality can be identified with a gauge invariant circulation of a phase of Δ_{ij} ("flux") around a plaquette

$$\Phi_P = \mathrm{Im} \prod_{\langle ij\rangle \in \partial P} \Delta_{ij}. \tag{7}$$

Presumably one could suppose that by finding out irreducible representations of the "loop group" of cyclic spin permutations one can enumerate all possible spin structures. However this problem is still far from a resolution. The best that was until now done is an attempt to describe possible ground states in terms of distributions of Δ_{ij} (modulo gauge transformations). Such an approach enables a conventional mean-field treatment where Δ_{ij} plays the role of the order parameter. Then a classification of mean-field solutions of the t-J or related models can proceed along the lines of the approach developed by Gor'kov and Volovik who found a symmetry classification of crystal groups for superconductors.[11] Such an attempt was undertaken in Ref. 12.

Unfortunately this consideration ignores the gauge symmetry (6). To restore the gauge invariance one should take into account all fluctuations of Δ_{ij} which do not change the energy of the system. Then one should consider a manifold of degenerate mean-field states as a physical ground state where an expectation value $\langle\Delta_{ij}\rangle$ is exactly zero.

It is evident that to describe physical states one has to construct a complete long-wavelength hydrodynamics for low-energy excitations around a given mean-field state. For both cases of the homogeneous RVB and the chiral state this problem was extensively discussed. A general belief is that for both states the fluctuations can be described by means of an Abelian gauge field coupled with chargeless spin-one-half fermions (spinons). This picture also enables a straightforward generalization onto a doped case which includes spinless charge carriers (holons). The space component of the gauge field A_{ij} arises from a phase of the order parameter $\Delta_{ij} = |\Delta_{ij}|\exp(iA_{ij})$, while a Lagrange multiplier imposing a constraint (3) becomes a temporal component A_0.[13,14]

Actually one can see that conventional derivations implicitly assume that Δ varies slowly in space and time. If so then one is allowed to perform an ordinary gradient expansion in terms of only one long-wavelength variable A_μ. This assumption seems to be correct in the simplest case of a ground state corresponding to a constant background $\Delta_{ij} = \Delta$. However it already does not necessarily take place for the background corresponding to any periodic modulation of Δ_{ij}. For instance, the simplest homogeneous chiral state characterized by a flux π through each plaquette of a 2D or a 3D lattice can be described as a distribution of Δ_{ij} with a wave vector $Q = (\frac{\pi}{2}, \frac{\pi}{2})$ [or $Q = (\frac{\pi}{2}, \frac{\pi}{2}, \frac{\pi}{2})$]. Moreover many other mean-field solutions studied in Ref. 12 have the same periodicity.

It was recently found by Wiegmann that a number of relevant fields appearing in a hydrodynamics of chiral states should be large enough.[10] He also proposed a new form of a complete long-wavelength theory of 2D and 3D chiral states. The mostly striking feature of those theories is a high symmetry group appearing in a continuous limit from a lattice Hamiltonian. Namely, it was stated in Ref. 10 that the symmetry group in 3D is SO(3,1)×SU(2)×U(1) while in 2D it lowers to SO(2,1)×$\mathrm{U}_c(1)$×$\mathrm{U}_s(1)$.

In the present paper we perform a more careful derivation of the effective Lagrangian sketched in Ref. 10 leaving apart the question about the microscopic nature of chiral states. It turns out that our results differ essentially from those of Ref. 10 and pose serious questions about the possibility of realization of the physical phenomenon of topological superconductivity discussed in Ref. 10 in the case of doped antiferromagnets.

II. CONTINUOUS GAUGE THEORY OF CHIRAL SPIN STATES

It should be mentioned that, in general, Hamiltonian (4) has an SU(2)-invariant "symplectic" structure corresponding to the symmetry between a spinor Ψ_α and a conjugated one $\epsilon_{\alpha\beta}\Psi^\dagger_\beta$. As a result it enables a continuous transformation of Δ_{ij} into another bilinear form $\eta_{ij} \equiv \epsilon_{\alpha\beta}\Psi_\alpha\Psi_\beta$ which means a local "symplectic" SU(2) symmetry of (4).[15] However we shall not discuss this additional symmetry because it has exclusively local ("onsite") nature and has nothing with any properties of distributions of Δ_{ij} in space. On the other hand any finite doping destroys this symmetry while a non-Abelian symmetry of Ref. 10 observed in a continuum limit was expected to persist under doping.

According to the definition of the chiral state proposed in Ref. 10 the operators of elementary translations along bonds of a 2D square (or 3D cubic) lattice anticommute when applied to the spin wave function

$$T_iT_j = -T_jT_i. \tag{8}$$

In a 3D case a consistency requires that translations along face diagonals of the cubic unit cell T_{ij} form an SU(2) algebra

$$T_{ij} = \epsilon_{ijk}\tau^k, \quad [\tau^i, \tau^j] = i\epsilon_{ijk}\tau^k. \tag{9}$$

In particular, Eq. (9) provides that the product of all three T_{ij} corresponding to 2π rotation around the main diagonal yields $T_{12}T_{23}T_{31} = C_3^3 = -1$.

On the basis of commutation relations (8) and (9) it was stated in Ref. 10 that the underlying structure corresponds to the so-called double space group. Such a construction was originally proposed by Bethe who considered electronic states in a crystal in presence of a spin-orbital interaction.[16] Physically double groups can be understood as remnants of the spinor nature of the electronic wave function in a continuous space. It preserves its double valuedness under the action of a finite group of discrete rotations belonging to one of several crystal point groups.

Remarkably the double valuedness of chiral state wave functions proposed in Ref. 10 has an orbital nature and it is indeed a specific feature of a corresponding distribution of Δ_{ij}.

However this exciting construction certainly deserves a more firm elaboration. First, a straightforward analysis of irreducible representations of the space group associated with a point double cubic group $\bar{O}_h$ (see, for instance, Ref. 17) reveals eight of those representations for the point Γ lying in the center of a Brillouin zone. The total number of the basic functions $\Psi_\alpha^{(n)}(\mathbf{k})$ $(\alpha = 1,...,8; n = 1,...,N_\alpha)$ (sum of dimensions of these representations $\sum_\alpha N_\alpha$) equals 18. For a momentum $\mathbf{k}$ having a general position in the Brillouin zone a stationary subgroup ("small" group of $\mathbf{k}$) is trivial and one obtains one 48-dimensional representation $\Psi_\alpha(\mathbf{k})$. Those functions could describe one-particle excitations in the 3D chiral flux phase in the same manner as corepresentations of Fedorov space groups intended to describe spin-one-half particles propagating in the Néel ordered antiferromagnetic background.[18]

At the same time in the 3D construction proposed in Ref. 10 an eight-dimensional representation arises quite naturally. The components of $\Psi_\alpha(\mathbf{k})$ $(\alpha = 1,...,8)$ correspond to values of the wave function taken at eight sites belonging to one unit cell. It was proposed in Ref. 10 to consider $\Psi_\alpha(\mathbf{k})$ as a reducible representation of a double point group of a cubic lattice $\bar{O}_h$. A correspondence between this treatment and a conventional representation theory of double space groups remains to be clarified.

Nevertheless the representation used in Ref. 10 is quite convenient. We shall primarily consider the case of a 3D cubic lattice and then present results for a 2D square one.

In 3D any translation is represented by an 8×8 matrix and can be written as a tensor product of three Pauli matrices. Each of these matrices acts in a two-dimensional subspace of two sites on the same link in a unit cell (see Fig. 1), the translation from one to the other site being given by σ_1. Within these prescriptions the translations (8) and (9) and T_{123} (along a space diagonal) can be represented by 8×8 matrices

$$\begin{aligned} T_1 &= \alpha_1 = \sigma_1 \otimes \sigma_3 \otimes \mathbf{1}, \\ T_2 &= \alpha_2 = \mathbf{1} \otimes \sigma_1 \otimes \sigma_3, \\ T_3 &= \alpha_3 = \sigma_3 \otimes \mathbf{1} \otimes \sigma_1, \\ T_{12} &= \tau^3 = \sigma_2 \otimes \sigma_1 \otimes \sigma_3, \\ T_{23} &= \tau^1 = \sigma_3 \otimes \sigma_2 \otimes \sigma_1, \\ T_{31} &= \tau^2 = \sigma_1 \otimes \sigma_3 \otimes \sigma_2, \\ T_{123} &= \alpha_1\alpha_2\alpha_3\beta = \sigma_1 \otimes \sigma_1 \otimes \sigma_1. \end{aligned} \tag{10}$$

The first three operators were identified in Ref. 10 with anticommuting 3D $\boldsymbol{\alpha}$ matrices appearing in the Dirac Hamiltonian $(H_D = \boldsymbol{\alpha}\mathbf{k} + \beta m)$. The matrix $\beta = \gamma_0$ can be defined as $\beta = \sigma_3 \otimes \sigma_3 \otimes \sigma_3$. Notice that the translation along a space diagonal coincides with a definition of a 3D parity operator $i\gamma_5\gamma_0 = \gamma_1\gamma_2\gamma_3$.

Mean-field Hamiltonian of the flux phase can be written in the form

$$H = \sum_{-\pi/2 < k_i \le \pi/2} \Psi^\dagger(\mathbf{k}) \left(|\Delta^0_{\rm NN}| \sum_i T_i \cos k_i + |\Delta^0_{\rm NNN}| \sum_{i<j} T_{ij} \cos(k_i + k_j) T_{123} \cos(k_1 + k_2 + k_3) + \hat{\lambda} \right) \Psi(\mathbf{k}), \tag{11}$$

where $\Psi(\mathbf{k})$ is an eight-component spinon wave function and matrix elements of $\hat{\lambda}$ have to be found together with $\Delta^{(0)}$ from an extremum condition for the energy (it actually shows that this matrix fluctuates around zero if one does not introduce different chemical potentials on some sublattices artificially). In rather general conditions the spectrum of the Hamiltonian (11) has so-called cone ("Dirac") points. In the vicinity of these points the spectrum of spinon excitations becomes that of a massive relativistic fermion.

In what follows we shall restrict our analysis onto such Hamiltonians. For example, choosing another [SO(6)-symmetrical] basis of translation operators[19] we observe that if (11) includes only nearest-neighbor, next-nearest-neighbor (face diagonal), and next-next-nearest-neighbor (space diagonal) couplings then the spectrum is given by the formula

$$E = \pm \sqrt{\sum_i |\Delta_{\rm NN}|^2 \cos^2 k_i + \sum_{i<j} |\Delta_{\rm NNN}|^2 \sin^2 k_i \sin^2 k_j + |\Delta_{\rm NNNN}|^2 \sin^2 k_1 \sin^2 k_2 \sin^2 k_3}. \tag{12}$$

The reduced Brillouin zone $(\frac{\pi}{2} < k \le \frac{\pi}{2})$ contains one Dirac point at $\mathbf{k} = (\frac{\pi}{2}, \frac{\pi}{2}, \frac{\pi}{2})$, so we obtain two degenerate species of four-component spin-one-half Dirac fermions Ψ^a_α $(\alpha = 1,...,4; a = 1,2)$. They can be treated as two components of some fictitious isospin SU(2) doublet. We also see that face and space diagonal couplings open a gap in the spinon spectrum.

It was first noticed in Ref. 10 that the operators (8) can be identified with usual Γ matrices while operators (9) are isospin generators. Presumably, this phenomenon of an occurrence of internal quantum numbers in a continuum limit could have fruitful applications in a modeling of continuous gauge field theories on a lattice (say, in Monte Carlo studies).

Now we intend to consider fluctuations around the mean-field configuration. It is easy to see that variations of Δ_{ij} can be expressed as a multiplication of T_i by an arbitrary matrix Λ which is a tensor product of matrices diagonal in all three two-dimensional subspaces. The basis of those operators includes eight components (four in 2D):

$$O_1 = \mathbf{1}\otimes\mathbf{1}\otimes\mathbf{1},\quad O_2 = \sigma_3\otimes\sigma_3\otimes\sigma_3,$$

$$O_3 = \sigma_3\otimes\mathbf{1}\otimes\mathbf{1},\quad O_4 = \mathbf{1}\otimes\sigma_3\otimes\mathbf{1},$$

$$O_5 = \mathbf{1}\otimes\mathbf{1}\otimes\sigma_3,\quad O_6 = \sigma_3\otimes\sigma_3\otimes\mathbf{1},$$

$$O_7 = \sigma_3\otimes\mathbf{1}\otimes\sigma_3,\quad O_8 = \mathbf{1}\otimes\sigma_3\otimes\sigma_3. \tag{13}$$

For example, an expansion of the variation Λ_1 over the basis (13) can be written in the form

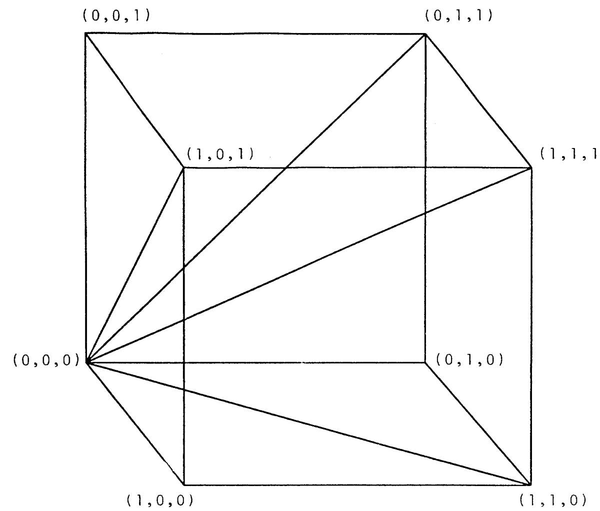

FIG. 1. Eight-site unit cell.

$$\Lambda_1 = \exp(iA_1 + i\tau^1\alpha_2\alpha_3\beta\Phi^1 + \tau^2\alpha_1\alpha_3\beta G^2_{03} + \tau^3\alpha_1\alpha_2\beta G^3_{02} + i\tau^3\alpha_1\alpha_2 B^3_2 + i\tau^2\alpha_1\alpha_3 B^2_3 + \tau^1\alpha_2\alpha_3\ {}^5B^1_0 + \beta F_{01}), \tag{14}$$

where a physical meaning of notations chosen for coefficient functions will be clarified below.

Since fast variations of Δ_{ij} are already accounted by the use of the eight-component representation one can suppose that all coefficient functions in (14) are varying slowly in space and time. Substituting (14) into a transformed Hamiltonian,

$$\begin{aligned} H = \sum_{\mathbf{k}} \Psi^\dagger(\mathbf{k}) \Bigg(&|\Delta^{(0)}_{\rm NN}| \sum_i \alpha_i\Lambda_i \exp(ik_i) + |\Delta^{(0)}_{\rm NNN}| \sum_{i<j} \epsilon_{ijk}\tau^k\Lambda_{ij}\exp(ik_i+ik_j) \\ &\times|\Delta^{(0)}_{\rm NNNN}|\alpha_1\alpha_2\alpha_3\beta\Lambda_{123}\exp(ik_1+ik_2+ik_3)\Bigg)\Psi(\mathbf{k}) + \text{H.c.} + \Psi^\dagger(\mathbf{k})\lambda^{(0)}\Lambda_0\Psi(\mathbf{k}) \\ &+\sum_i \frac{|\Delta^{(0)}_{\rm NN}|^2}{J_{\rm NN}}\text{Tr}(\Lambda_i\Lambda^\dagger{}_i) + \sum_{i<j}\frac{|\Delta^{(0)}_{\rm NNN}|^2}{J_{\rm NNN}}\text{Tr}(\Lambda_{ij}\Lambda^\dagger{}_{ij}) + \frac{|\Delta^{(0)}_{\rm NNNN}|^2}{J_{\rm NNNN}}\text{Tr}(\Lambda_{123}\Lambda^\dagger{}_{123}), \end{aligned} \tag{15}$$

and expanding it up to first-order terms one can see that for a hermiticity of the Hamiltonian all coefficient functions have to be real-valued. But then it appears that only part of the fields introduced in (14) corresponds to phase fluctuations of Δ_{ij} and fluctuations of the real-valued Lagrange multiplier λ_i. The remainder describe fluctuations of moduli $|\Delta_{ij}|$ and acquire contributions from the second term in (5). After a rescaling of the fields by factors $\Delta^{(0)}$ the resulting Lagrangian of the effective continuous theory can be written in the form

$$\begin{aligned} L = \bar\Psi(\ &\gamma_\mu(i\partial_\mu + A_\mu + \boldsymbol{\tau}\mathbf{B}_\mu + \gamma_5\ {}^5A_\mu + \gamma_5\boldsymbol{\tau}\ {}^5\mathbf{B}_\mu) + \boldsymbol{\tau}\boldsymbol{\Phi} + i\gamma_5(\phi + \boldsymbol{\tau}\boldsymbol{\chi}) + i[\gamma_\mu,\gamma_\nu]F_{\mu\nu} + i\boldsymbol{\tau}[\gamma_\mu,\gamma_\nu]\mathbf{G}_{\mu\nu} \\ &+\gamma_5(i[\gamma_\mu,\gamma_\nu]\ {}^5F_{\mu\nu} + i\boldsymbol{\tau} + [\gamma_\mu,\gamma_\nu]\ {}^5\mathbf{G}_{\mu\nu}) + m)\Psi + \frac{1}{J_{\rm NN}}\left(\sum_{i\neq j}(B^i_j)^2 + \sum_i[(\chi^i)^2 + (F_{i0})^2]\right) \\ &+\frac{1}{J_{\rm NNN}}\left(\sum_{i<j}(F_{ij})^2 + \sum_i[(\ {}^5A_i)^2 + (\Phi^i)^2 + (B^i_0)^2]\right) + \frac{1}{J_{\rm NNNN}}\left(\sum_i (B^i_i)^2 + (\ {}^5A_0)^2\right). \end{aligned} \tag{16}$$

This expression differs from the result obtained in Ref. 10. First of all one can see that all 64 basic structures $\{1,\gamma_5,\gamma_\mu,\gamma_5\gamma_\mu,[\gamma_\mu,\gamma_\nu],\boldsymbol{\tau},\boldsymbol{\tau}\gamma_5,\boldsymbol{\tau}\gamma_\mu,\boldsymbol{\tau}\gamma_5\gamma_\mu,\boldsymbol{\tau}[\gamma_\mu,\gamma_\nu]\}$ did appear in (16). Moreover the 28 fields which get masses already in the bare Lagrangian are $\{\mathbf{B}_\mu,\ {}^5A_\mu, F_{\mu\nu}, \boldsymbol{\chi}, \boldsymbol{\Phi}\}$. The multiplet of the remainder 36 massless fields (Goldstone modes) $\{\mathbf{A}_\mu,\ {}^5\mathbf{B}_\mu, \mathbf{G}_{\mu\nu}, m, \phi\}$ also differs from the result of Ref. 10 where the set of Goldstones included only 17 components $\{A_{\mu\nu}, \mathbf{B}_\mu, m\}$ from a total 44.

We obtain that at special values of bare couplings ($J_{\rm NN} = J_{\rm NNN} = J_{\rm NNNN}$) the mass terms in (16) turn out to be Lorentz invariant and then the whole Lagrangian does demonstrate this SO(3,1) invariance. We believe that the radiative corrections to (16) renormalize masses towards a symmetrical point and the long-wavelength theory is always Lorentz invariant.

Now we shall discuss other (internal) symmetries. A complete perturbative analysis of the effective Lagrangians (16) and (24) could be as hard as a study of the theory of strongly interacting hadrons. Instead one can draw important conclusions from a consideration of anomalies which constitute the basis of nonperturbative phenomena.[20]

In the Lagrangian (16) we accounted m as a separate field. It corresponds to a harmonics of the Lagrange multiplier λ_i which is constant within one eight-site unit cell. According to the mean-field spinon spectrum (12) this component acquires a nonzero vacuum expectation value $\langle m\rangle = (3|\Delta_{\rm NNN}|^2 + |\Delta_{\rm NNNN}|^2)^{1/2}$. It leads to a nonconservation of an axial "isospin" current ${}^5\mathbf{J}_\mu$:

$$D_\mu\, {}^5\mathbf{J}_\mu = i\langle m\rangle \bar{\Psi}\gamma_5 \boldsymbol{\tau}\Psi \tag{17}$$

which means that, in fact, the Lagrangian (16) is not invariant under the axial "isospin" transformations gauged by ${}^5\mathbf{B}_\mu$. Thus the expected "isospin" SU(2) symmetry appears to be incompatible with a gapful spinon spectrum and, consequently, the radiative corrections to (16) can provide ${}^5\mathbf{B}_\mu$ with a mass.

A simple analysis of the one-loop corrections shows that being not protected by a gauge symmetry ${}^5\mathbf{B}_\mu$ becomes massive together with other bare Goldstone fields $G_{\mu\nu}$ and ϕ while A_μ remains massless. Thus the only exact internal symmetry is an original U(1) corresponding to phase transformations (6).

Remarkably at $m = 0$ the theory (16) does obey the "isospin" axial SU(2) symmetry gauged by ${}^5\mathbf{B}_\mu$ and the highest possible symmetry group SO(3,1)×SU(2)×U(1) occurs. It is indeed nontrivial. Although on the classical level the current ${}^5\mathbf{J}_\mu$ is conserved, the gauge symmetries can nevertheless be broken by anomalies. According to general theory[20] massless Dirac fermions coupled with a vector Abelian field A_μ and an axial SU(2) field ${}^5\mathbf{B}_\mu$ exhibit the following anomalous current divergencies:

$$D_\mu\, {}^5\mathbf{J}_\mu = \frac{1}{4\pi}\epsilon_{\mu\nu\lambda\rho}\mathrm{Tr}\boldsymbol{\tau}\partial_\mu({}^5\hat{B}_\nu\partial_\lambda\, {}^5\hat{B}_\rho + \tfrac{1}{2}{}^5\hat{B}_\nu\, {}^5\hat{B}_\lambda\, {}^5\hat{B}_\rho), \tag{18}$$

$$D_\mu \mathbf{J}_\mu = \frac{1}{4\pi}\epsilon_{\mu\nu\lambda\rho}\partial_\mu A_\nu D_\lambda\, {}^5\mathbf{B}_\rho, \tag{19}$$

$$\partial_\mu\, {}^5J_\mu = \frac{1}{4\pi}\epsilon_{\mu\nu\lambda\rho}[\partial_\mu A_\nu \partial_\lambda A_\rho + \mathrm{Tr}\partial_\mu({}^5\hat{B}_\nu\partial_\lambda\, {}^5\hat{B}_\rho + \tfrac{1}{2}\, {}^5\hat{B}_\nu\, {}^5\hat{B}_\lambda\, {}^5\hat{B}_\rho)], \tag{20}$$

where Tr stands for a trace over "isospin" indices. The vector U(1) current J_μ is always conserved while both the axial U(1) current ${}^5J_\mu$ and a vector "isospin" current $\mathbf{J}_\mu$ are not. But in addition the rhs of (18) is identically equal to zero for a special case of an SU(2) group and the corresponding axial "isospin" symmetry turns out to be free of anomalies.

So only in the case of gapless spinons $m = 0$ could one obtain a large internal symmetry group $\mathrm{SU}_A(2)$×U(1) including axial "isospin" transformations which retain under renormalization. Otherwise the only relevant gauge symmetry is a conventional U(1) and only the corresponding gauge field A_μ is massless. We suppose that namely the latter is the case because the very fact of parity violation in the 3D chiral state means that a chiral invariance between "left" $\frac{1-\gamma_5}{2}\Psi$ and "right" $\frac{1+\gamma_5}{2}\Psi$ fermions is absent.

One more remark is that the spinon gap arises due to next-nearest-neighbor couplings which are themselves necessary to obtain complete field multiplets and to get a covariant Lagrangian (16). In addition we stress that the set of terms obtained in (16) exhausts all possibilities and cannot be enlarged by an inclusion of new spin couplings J_{ij} which preserve the "relativistic" SO(3,1) symmetry of the bare spinon spectrum and the Dirac point at $Q = (\frac{\pi}{2}, \frac{\pi}{2}, \frac{\pi}{2})$.

Performing an analogous procedure in 2D we define the planar translations as

$$\begin{aligned} T_1 &= \alpha_1 = \sigma_1 \otimes \sigma_3, \\ T_2 &= \alpha_2 = \mathbf{1} \otimes \sigma_1, \\ T_{12} &= \beta = i\alpha_1\alpha_2 = \sigma_1 \otimes \sigma_2. \end{aligned} \tag{21}$$

Suspecting a spin symmetry group SU(2) or its subgroup U(1) we shall also define $\boldsymbol{\tau}$ matrices as

$$\begin{aligned} \tau^1 &= \sigma_1 \otimes \mathbf{1}, \\ \tau^2 &= \sigma_3 \otimes \sigma_1, \\ \tau^3 &= -\sigma_2 \otimes \sigma_1. \end{aligned} \tag{22}$$

Now a unit cell includes four sites of a square lattice and the number of different operator structures which could appear in a continuous theory (as well as the number of fields) is equal to 16.

The four diagonal operators can be represented in the form

$$\begin{aligned} O_1 &= \mathbf{1} \otimes \mathbf{1}, \\ O_2 &= \sigma_3 \otimes \mathbf{1}, \\ O_3 &= \mathbf{1} \otimes \sigma_3, \\ O_4 &= \sigma_3 \otimes \sigma_3, \end{aligned} \tag{23}$$

again manifesting a possible internal "isospin" symmetry group SU(2) and a (2+1)-dimensional Lorentz group SO(2,1) additional to an intrinsic "charge" $\mathrm{U}_c(1)$ group of phase transformations (6).

An effective Lagrangian for four-component spinons Ψ^a_α ($\alpha = 1,2; a = 1,2$) coupled with all possible fluctuations of Δ_{ij} and λ_i has the form

$$L = \bar{\Psi}[\gamma_\mu(i\partial_\mu + A_\mu + \boldsymbol{\tau}\mathbf{B}_\mu) + m + \boldsymbol{\tau}\boldsymbol{\Phi}]\Psi + \frac{1}{J_{\rm NN}}\left(\sum_{i=1,2}(B^3_i)^2 + (\Phi^i)^2\right) + \frac{1}{J_{\rm NNN}}[(B^3_0)^2 + m^2]. \quad (24)$$

We obtain that an "isospin" group is, in fact, broken up to $\mathrm{U}_s(1)$ in agreement with the statement made in Ref. 10. Again an exact Lorentz symmetry occurs at $J_{\rm NN} = J_{\rm NNN}$. However the set of massless fields includes Φ^3 and $B^{1,2}_\mu$ instead of B^3_μ and m as proposed in Ref. 10.

Again we find a spinon gap m to be nonzero as a manifestation of 2D parity violation. Then calculating one-loop corrections one readily obtains a mass term for the field $\boldsymbol{\Phi}$, but $B^{1,2}_\mu$ are massless due to the remnant $\mathrm{U}_s(1)$.

Note that in the odd-dimensional case there is not any current nonconservation but instead the Lagrangian can be noninvariant itself with respect to "large" gauge transformations. It is, in fact, an appearance of Chern-Simons terms after integrating over fermions

$$\delta L = \frac{1}{4\pi}\epsilon_{\mu\nu\lambda}[A_\mu\partial_\nu A_\lambda + \mathrm{Tr}(\hat{B}_\mu\partial_\nu\hat{B}_\lambda + \tfrac{2}{3}\hat{B}_\mu\hat{B}_\nu\hat{B}_\lambda)]. \quad (25)$$

These terms provide both A_μ and $B^{1,2}_\mu$ with gauge invariant masses, so there are no physical gapless excitations in the 2D chiral state. However due to the topological nature of the Chern-Simons terms an instantaneous long-range Bohm-Aharonov interaction persists. It is then responsible for a well-known phenomenon of a transmutation of spinon statistics.[21]

III. DISCUSSION

Thus one can see that a complete hydrodynamical description of spin disordered chiral states with a simplest intrinsic periodicity with the wave vector $\mathbf{Q} = (\frac{\pi}{2}, \ldots, \frac{\pi}{2})$ reveals a whole zoo of relevant degrees of freedom. Moreover these variables are arranged into multiplets of Lorentz as well as some internal symmetry group which appears in the continuum limit. This fact has to be assigned to the complex nature of the order parameter Δ_{ij} describing chiral states. It can be compared with an appearance of new branches of spin waves in multisublattice antiferromagnets.

We notice that one can encounter a somewhat similar situation in the theory of He^3 (both A and B phases). This system has a large symmetry group of the order parameter and exhibits a rich spectrum of bosonic collective modes which includes an Abelian as well as non-Abelian SU(2) gauge fields and also gravity.[22]

Of course, an occurrence of a relativistic symmetry in an effective theory of low-energy antiferromagnetic excitations can be expected on general grounds.[3] However in contrast to the case of the He^3 hydrodynamics the internal symmetries of the 3D Lagrangian (16) exist only in the limit of zero spinon mass (spin gap). The reason is that the extra gauge field ${}^5\mathbf{B}_\mu$ is coupled with the axial current which does not conserve even on the classical level once a spin gap opens. Other massless fields present in the classical Lagrangian (16) are not associated with any gauge symmetry and receive their masses from radiative corrections. As a result, these fields mediate some short-range spinon interactions which add to the primarily important long-range interaction via the vector field A_μ gauging phase transformations (6). Nevertheless massive fields can manifest themselves in internal structure of soliton excitations if these are present in the field theory (16).

In the 2D case described by the Lagrangian (24) all excitations become gapful due to the generated Chern-Simons terms (25), although the instantaneous long-range Bohm-Aharonov interaction responsible for semionic statistics of spinons[21] persists. As an effect of an additional Abelian gauge symmetry provided by the complex vector field $B^\pm_\mu = B^1_\mu \pm iB^2_\mu$ one can expect that the spectrum of the theory (24) contains vortex excitations carrying two different topological charges equal to magnetic fluxes of A_μ and $B^\pm_\mu$. Physically these vortices correspond to different types of circular domain walls.

In conclusion, our results do not confirm a conjecture made in Ref. 10 that massless excitations include an additional vector SU(2) gauge field in 3D [U(1) in 2D] required for a scenario of topological superconductivity. It makes questionable a realization of this intriguing mechanism of superconductivity in the context of hypothetical chiral states of doped antiferromagnets. However it does not rule out the general possibility of finding this phenomenon in some other condensed matter system which can be effectively described in terms of unbroken local gauge symmetries additional to the conventional "RVB gauge symmetry" (6).

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