Cubic models with random anisotropy

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A Monte Carlo algorithm has been used to study the cubic ferromagnet with a random uniaxial single-site anisotropy on simple cubic lattices, in the strong anisotropy limit. For both two-component and three-component spins the phase diagram contains a "domain-type" ferromagnetic phase for strong random anisotropy, in addition to the simple ferromagnetic phase which is seen for weak random anisotropy. For two-component spins both paramagnet-ferromagnet transitions seem to be second order. For three-component spins the magnetization probably disappears discontinuously, and there appears to be a thin layer of infinite susceptibility phase between the paramagnetic and ferromagnetic phases.

I. INTRODUCTION

The study of ferromagnetic models with random uniaxial anisotropy was begun twenty years ago by Harris, Plischke, and Zuckermann[1] (HPZ), who proposed the following Hamiltonian:

$$H_{\rm HPZ}=-J\sum_{\langle ij\rangle}\sum_{\alpha=1}^{m}S_i^\alpha S_j^\alpha-D\sum_i\left[\sum_{\alpha=1}^{m}(n_i^\alpha S_i^\alpha)^2-1\right], \tag{1}$$

where $J>0$ and each $\mathbf{S}_i$ is an m-component unit-length spin. The $\mathbf{n}_i$ are usually assumed to be uncorrelated random m-component unit vectors whose probability distributions are uniformly weighted over the m sphere. HPZ showed that a mean-field approximation for Eq. (1) gives a ferromagnetic (FM) phase at low temperatures. An isotropic distribution of the $\mathbf{n}_i$ is a reasonable model for an amorphous alloy. There has been a large amount of subsequent work, whose objective has been to improve upon this mean-field theory. Our current understanding of this model is that in a real three-dimensional amorphous alloy with $m=3$ spins even a small amount of random anisotropy will destabilize the FM phase.[2-4] The $m=2$ case is more subtle, and understanding it requires a study of the role of vortex lines.[4-6]

Renormalization-group calculations have shown[7-9] that for three-dimensional ferromagnets the isotropic Heisenberg critical fixed point remains stable in the presence of cubic crystalline anisotropy, when m is 2 or 3. There are many crystalline materials which have both a cubic lattice-type and some alloy disorder. Therefore, it is interesting to consider simultaneously the effects of both a uniform crystalline anisotropy and a random uniaxial anisotropy. Aharony[10] argued that the presence of a crystalline anisotropy should stabilize the ferromagnetic ground state of a three-dimensional system, but that the magnetization M must then vanish discontinuously. He also argued that there might be a spin-glass phase intervening between the FM and paramagnetic (PM) phases. This analysis was later extended by a number of authors.[11-14] The work we present here is, to the author's knowledge, the first use of Monte Carlo simulations to test these analytical predictions. We will see that the simulations largely confirm the predictions for $m=3$ systems, but for $m=2$ there is no evidence for a discontinuity of M.

It has recently been demonstrated[5,6] that, in the absence of any crystalline anisotropy, a three-dimensional $m=2$ random uniaxial anisotropy model with an isotropic probability distribution for the random axes displays a finite-temperature phase with an infinite magnetic susceptibility because the FM phase is only marginally unstable. Thus it is no real surprise to find that Aharony's original analysis is not adequate for the $m=2$ case.

Both Aharony[10] and, even earlier, Halperin and Varma[15] pointed out that the model considered here could also be relevant for understanding some phenomena which occur in structural phase transitions in many ferroelectric materials, and also in the A15 superconductors. It should also be noted that a similar model, with a nonspherical distribution of random uniaxial anisotropy, but no explicit crystalline anisotropy, was also considered by Aharony,[16] and later studied in detail at the mean-field level by Amit, Gutfreund, and Sompolinsky[17] and also Fischer and Zippelius.[18] A model of randomly interacting quadrupoles with cubic anisotropy was studied by Carmesin.[19]

II. CUBIC FERROMAGNETS

The Hamiltonian for a (nonrandom) m-component ferromagnet with a cubic crystal field is

$$H_{\rm cubic}=-J\sum_{\langle ij\rangle}\sum_{\alpha=1}^{m}S_i^\alpha S_j^\alpha-K\sum_i\left[\sum_{\alpha=1}^{m}(S_i^\alpha)^4-1\right]. \tag{2}$$

By calculating the leading term in a 4-ε expansion[20] for Eq. (2), it is straightforward to show that the cubic crystalline anisotropy term destabilizes the isotropic critical point in less than four dimensions for $m\geq 4$. More sophisticated numerical calculations[7-9] have demonstrated that in three dimensions the isotropic critical point remains stable for $m\leq 3$, as long as $|K/J|$ is not too large.

The $m=2$ case of Eq. (2) has a special symmetry that allows the $K<0$ case to be mapped onto the $K>0$ case, by merely rotating the coordinate system of the spins by $\pi/4$. For larger values of m this is not possible, and the two cases are distinct. In this work we will consider the limit $K \to \infty$. This limit constrains each spin to point in one of the $2m$ directions parallel or antiparallel to the m cubic axes.

This reduction to a model with a discrete set of states for the spin variables greatly simplifies the Monte Carlo simulation procedure. With the spins confined to the $2m$ cubic directions, the Hamiltonian can now be written in the simple form

$$H = -J \sum_{\langle ij \rangle} \sum_{\alpha=1}^{m} \sigma_i^\alpha \sigma_j^\alpha , \qquad (3)$$

where each component σ_i^α can now assume only the values 0 or ± 1. The normalization condition is still $\sum_{\alpha=1}^{m} (\sigma_i^\alpha)^2 = 1$, so each $\boldsymbol{\sigma}_i$ has exactly one nonzero component. As is well known,[21] the $m=2$ case of Eq. (3) can be exactly reduced to a sum of two independent Ising models. The critical point is half that of the usual Ising model; thus[22] $T_c/J = 2.255\,75$ for the simple cubic lattice. For larger m, Eq. (3) has a first-order phase transition between the FM and PM phases.

Standard heat-bath Monte Carlo simulations were performed for the $m=3$ case, on $L \times L \times L$ simple cubic lattices with periodic boundary conditions and sizes up to $L=48$. The temperature dependence of the energy E is displayed in Fig. 1. The ordering transition occurs at $T_c/J = 1.620^{+0.003}_{-0.008}$, where the quoted uncertainties delineate the region of metastability for $L=48$. The transition has a definite first-order character, with a jump in the magnetization per spin ΔM of about 0.57, and a latent heat per spin ΔE of about $0.54J$. Despite this, the thermodynamic variables show "pseudocritical" effects near T_c, especially in the FM phase. Figure 2 is a log-log plot which shows the temperature dependence of M. Extrapolation of the FM phase data gives an upper pseudocritical point T_u at $1.628J$. This plot yields an estimated value of the pseudocritical exponent of $\beta = 0.135$, which seems unreasonably small for a three-dimensional β. Since the accessible range of M is quite limited, it is difficult to make a meaningful estimate of the accuracy of these pseudocritical parameters.

The magnetic susceptibility χ_M in the PM phase, as calculated from the fluctuations in the magnetization, is shown in Fig. 3. Analysis of this PM phase data gives an estimate for the lower pseudocritical point T_* of $1.602J$, and an estimate for the exponent γ which is consistent with three-dimensional tricritical behavior ($\gamma = 1$, with log corrections), as might be expected. In the FM phase the anisotropy of χ_M is an increasing function of T, with the incipient instability of the FM phase apparently being caused by diverging fluctuations of the *longitudinal* component. From these χ_l data we extract an estimate of about $1.633J$ for T_u, and a value of γ' of about 1.30. Note that the estimate of T_u obtained from the χ_l data is somewhat larger than the one obtained from the data for M. The author believes that the value of γ' is controlled by the (unstable) $m=3$ cubic critical point,[8,9] rather than the Heisenberg critical point, where, as M goes to zero, the correlation length diverges and χ becomes isotropic.

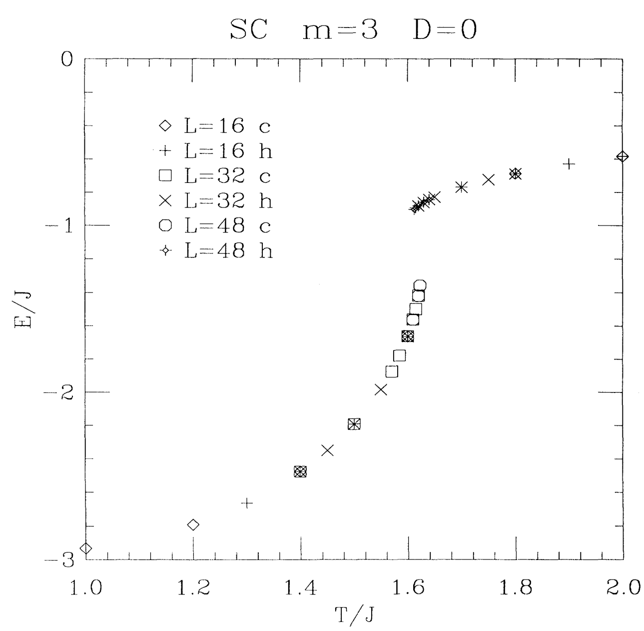

FIG. 1. Energy per spin vs temperature for the $m=3$ cubic ferromagnet with $K/J = \infty$, on simple cubic lattices. The symbols designated with a c show data from "cold start" runs, and the symbols designated with an h show data from "hot start" runs. The energy is discontinuous at T_c and there is a narrow region of metastability.

III. CUBIC FERROMAGNETS WITH RANDOM UNIAXIAL ANISOTROPY

It is interesting for a number of reasons to consider the effects of adding a random uniaxial anisotropy to our

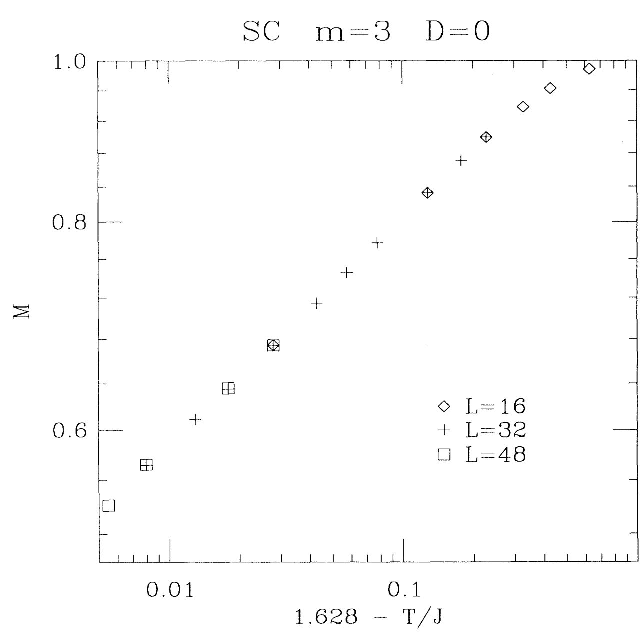

FIG. 2. Magnetization vs reduced temperature for the $m=3$ cubic ferromagnet with $K/J = \infty$ on simple cubic lattices. The axes are scaled logarithmically.

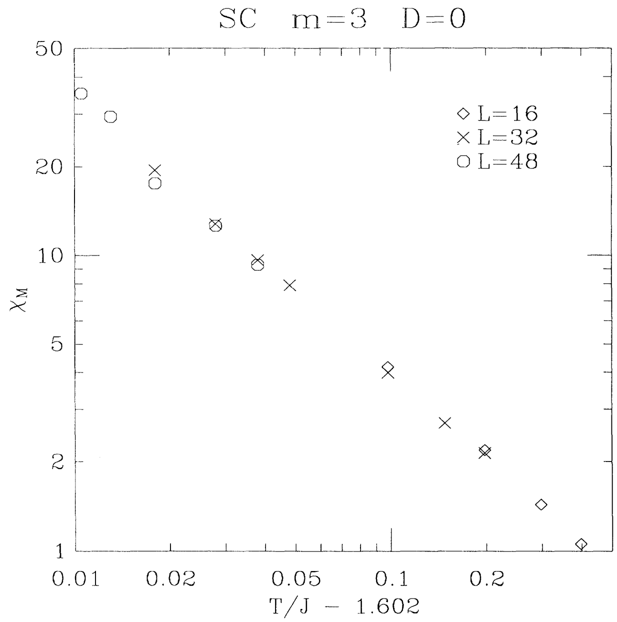

FIG. 3. Paramagnetic phase magnetic susceptibility vs reduced temperature for the $m=3$ cubic ferromagnetic with $K/J=\infty$ on simple cubic lattices. The axes are scaled logarithmically.

model for the cubic ferromagnet. All real cubic crystals contain substitutional impurities, whose presence may be intentional or unavoidable. Thus, understanding the behavior of point defects in crystals has long been an active area of research. Random anisotropy effects are often considered to be negligible compared to the simpler, and relatively well-understood, effects of random isotropic exchange. The importance of including the random anisotropy was emphasized by Mukamel and Grinstein.[12]

In this work we will use a random single-site anisotropy term. The effects of a symmetric (dipolar) random anisotropic exchange are similar, because a symmetric anisotropic exchange generates a single-site anisotropy under a renormalization-group rescaling. In addition, one might also consider an antisymmetric (Dzyaloshinsky-Moriya) exchange term, which can produce somewhat different effects. The cases for which one should obviously include random anisotropy are alloys containing non-S-state rate-earth ions.[23,24] Magnetic $3d$ transition-metal ions can create similar effects in systems possessing spin-density waves.[25,26]

The simple Hamiltonian which was used in the Monte Carlo simulations is

$$H=-J\sum_{\langle ij\rangle}\sum_{\alpha=1}^{m}\sigma_i^\alpha\sigma_j^\alpha-D\sum_i\left[\sum_{\alpha=1}^{m}\nu_i^\alpha\sigma_i^\alpha\right]^2, \qquad (4)$$

where each $\boldsymbol{\nu}_i$ is, for simplicity, fixed to point along one of the m spin axes. The $\boldsymbol{\nu}_i$ are independent identically distributed random vectors of unit length, with equal probability given to each spin axis. Simulations were run for $m=2$ and 3, using $D/J=1$, 2, and 3. The computer program takes advantage of the fact that some of the computation can be done with integer arithmetic when D/J is an integer. Equation (4) is a highly idealized representation of a real experimental system. We will see, however, that its behavior displays interesting features which, by the usual universality arguments, are likely to be observable in some real materials.

The model should also be useful for understanding structural phase transitions in "dirty displacive ferroelectrics."[15,27] Examples of such materials are doped $SrTiO_3$, which was studied by Bednorz and Müller,[28] and doped $KMnF_3$ which was studied by Gibaud and co-workers.[29] Off-stoichiometry A15 alloys such as V_3Si and Nb_3Ge (nominal compositions) also frequently display cubic-to-tetragonal structural transitions. All these examples correspond to the $m=3$ case of Eq. (4). The $m=2$ case may be useful for understanding the tetragonal-to-orthorhombic transitions in the highly anisotropic high-temperature superconducting materials, which typically have large concentrations of vacancies in their oxygen sublattices.

Since we have assigned equal weight to the probability that each $\boldsymbol{\nu}_i$ points along any of the axes, we know that in the infinite volume limit $(L\to\infty)$ a state which is fully aligned along any of the axes will have the same energy per spin. It is important to remember that, because of the statistical fluctuations, this is not exactly true for finite lattices. The energy differences per spin for fully aligned states pointing along different axes are thus of order $L^{-3/2}$ (in three dimensions). It is easy to show that when $D/J\le 2$ it is highly probable that the ground state for Eq. (4) of any lattice small enough to fit on a computer will be one of these $2m$ fully aligned states. Thus we should expect the data for the cases $D/J=1$ and 2 to have corrections to finite-size scaling due to these statistical fluctuations, and with this size dependence. We cannot eliminate this effect by averaging over different realizations of the random $\boldsymbol{\nu}_i$.

For nonrandom models such as Eq. (3) it is often possible to use techniques of finite-size scaling analysis to get rather precise answers from small lattices.[30] In random models such as Eq. (4) the data from such small systems are typically of little value for extracting the scaling behavior. This necessitates the use of large lattices. Another difficulty which must be faced for these finite random systems is that, primarily due to the differences in their ground-state energies caused by statistical fluctuations, there will be a distribution of "effective transition temperatures" for the lattices of a size L. If the width of this distribution is not narrow, it then becomes an additional source of error in the extrapolation procedure, unless one can somehow correct for the shifts in T_c.

For any nonzero value of D, if we look at an extremely large lattice, it is always possible to find rare local configurations of the $\boldsymbol{\nu}_i$ which cause the true ground state to differ from a fully aligned state. As long as these rare "nuggets"[31] are so distant from each other that they cannot interact effectively, they have no important consequences for the equilibrium properties of the model. As D/J is increased beyond 2, however, the nuggets rapidly become so common as to create a qualitative change in the character of the ground state.

A. The $m=2$ case

For the $m=2$ case of Eq. (4) interchanging the two components of each spin is equivalent to changing the sign of D. As we have already noted, for $D=0$ this model is equivalent to two decoupled pure Ising models. When $D=\infty$ each $\boldsymbol{\sigma}_i$ becomes constrained to point only along the axis favored by its $\mathbf{v}_i$. Thus in that case the model reduces to two 50% site-diluted Ising ferromagnets, which are again decoupled from each other. For any D between these extremes there is no decoupling, and we may anticipate that behavior of the model will become truly XY-like when the correlation length is large, i.e., near the critical line.

Because each site on the simple cubic lattice interacts with six nearest neighbors, for $D/J \geq 6$ one ground state of the spin system has each $\boldsymbol{\sigma}_i$ aligned parallel to its $\mathbf{v}_i$. Due to symmetry, there are always $2^m=4$ such equivalent ground states, even for a finite lattice. (This is different from the situation for small D/J, where exact symmetry only gives two equivalent states.) Where there is an isolated finite cluster of parallel $\mathbf{v}_i$, the $\boldsymbol{\sigma}_i$ of the cluster can be reversed without increasing the energy. The percolation concentration for site dilution,[32] p_c^s, on the simple cubic lattice is 0.311, which is substantially less than $\frac{1}{2}$. Thus, in the $m=2$ case isolated clusters of parallel $\mathbf{v}_i$ containing more than a few sites will be very rare.

As D/J is reduced below 6, isolated single sites become unstable and, as it is decreased still further, isolated pairs and then isolated triplets will also disappear from the ground states. For $D/J=3$ there will be no stable isolated clusters of $\boldsymbol{\sigma}_i$ on most "computer-sized" $m=2$ lattices. Each of the ground states of one of these finite lattices then maps onto the ground state of an Ising model in a bimodal random field.[33,34] Thus the ground states of Eq. (4) will, under these conditions, have the appearance of the "domain phase"[34] of the Ising model in a random field. This analogy is still useful for larger values of D/J. It follows that the order parameter for the low-temperature large D/J phase will be the direct product of $|M_x|$ and $|M_y|$, the magnetizations along the two spin axes. Therefore, we have two different kinds of FM phases, a [1,0] phase for small D/J and a [1,1] phase for large D/J. There must be some sort of a boundary between these two phases.

One might guess that between these two regimes, the one in which only one of the pair, $|M_x|$ and $|M_y|$ is positive, and the one in which $|M_x|=|M_y|>0$, there could exist another phase in which they were both positive, but unequal. The Monte Carlo simulations, however, indicate that such an intermediate phase is not stable. The results for the $m=2$ phase diagram on the simple cubic lattice are shown in Fig. 4. All of the transitions appear to be continuous, with no observable hysteresis. The data for $D/J=1$ are consistent with the hypothesis that the PM-[1,0] transition is a pure XY-type critical line. Given the uncertainties in the data, it would be an exaggeration to claim that this has been conclusively demonstrated. It is, however, expected *a priori* because in the presence of the infinite cubic crystal field the random uniaxial anisotropy becomes indistinguishable from, for example, a random six-fold anisotropy. In the latter case both the cubic crystal field and the random anisotropy are irrelevant, and pure XY-type critical transition is expected.[6]

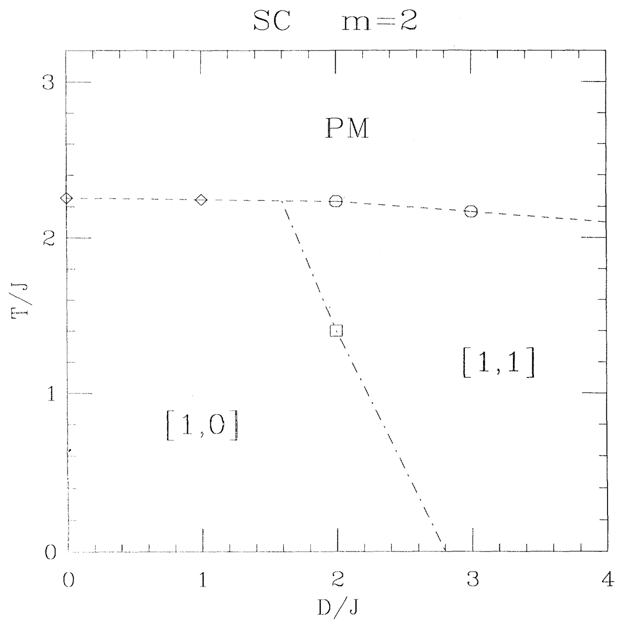

FIG. 4. Phase diagram of the $m=2$ cubic ferromagnetic with random uniaxial anisotropy on simple cubic lattices. The plotting symbols show actual data points. The dashed line indicates a second-order phase transition, and the dot-dashed line indicates a first-order phase transition.

For the PM-[1,1] transition we expect the analysis of Mukamel and Grinstein,[12] which predicts a new type of phase transition, to be applicable. The data for $D/J=2$ give values for the critical exponents which are close to those of the pure XY model. This can be understood as a finite-size crossover effect. The data for $D/J=3$, displayed in Figs. 5 and 6, indicate the presence of a new type of critical point, with the critical exponents $\beta=0.275\pm0.015$ and $\gamma=1.74\pm0.08$, with $T_c=2.166 \pm0.004$. Assuming the validity of the usual scaling relation, this gives $\alpha=-0.29\pm0.08$, which is a very reasonable value for the specific-heat exponent in this system. Before assuming that our exponents are actually correct within the quoted uncertainties, one should demonstrate that they are insensitive to the value of D/J which is used. For large values of D/J, however, one must deal with the crossover from the random exchange Ising critical point,[35] so this is difficult to do properly.

For $D/J=2$ we pass from the PM phase into the [1,1] phase and then into the [1,0] phase as the temperature is lowered. In a simple Landau-type mean-field theory this transition would be first order because of the change in symmetry of the order parameter. No first-order characteristics are seen in the $m=2$ simulation, however. As the [1,1] phase is highly inhomogeneous, there is no

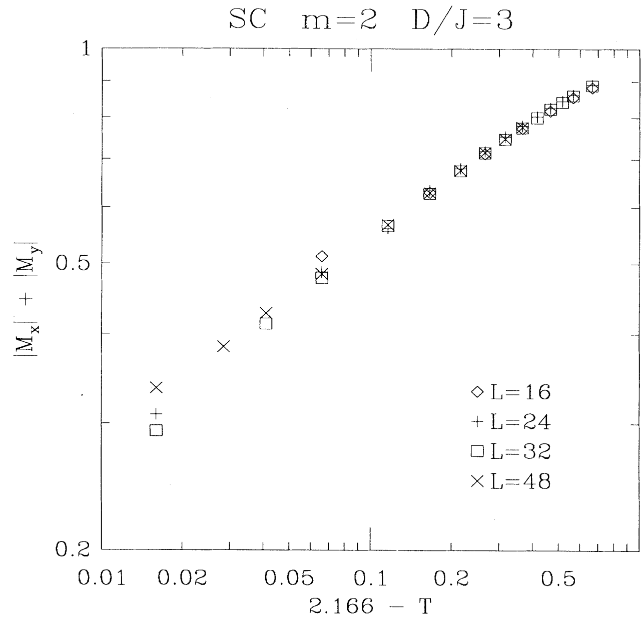

FIG. 5. Magnetization vs reduced temperature for the $m=2$ random anisotropy cubic ferromagnet with $D/J=3$ on simple cubic lattices. The axes are scaled logarithmically.

reason why the naive Landau theory must be applicable. It appears that the [1,1] phase should be treated as being made up of two interwoven infinite clusters; one of x-type spins and the other of y-type spins. This, of course, is the structure which exists for $D/J=\infty$. The two-point correlation function which distinguishes between the [1,0] and [1,1] phases is

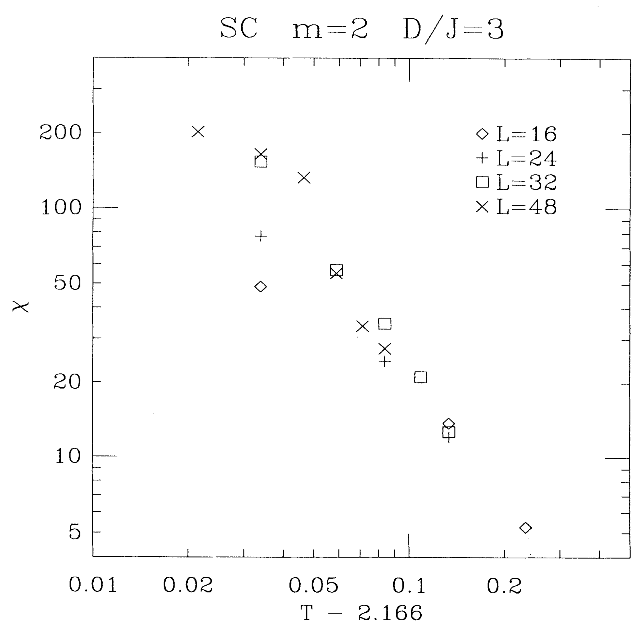

FIG. 6. Paramagnetic phase magnetic susceptibility vs reduced temperature for the $m=2$ random anisotropy cubic ferromagnet with $D/J=3$ on simple cubic lattices. The axes are scaled logarithmically.

$$\chi_Q(i,j)=\sum_{\alpha=1}^{m}[\langle\sigma_i^\alpha\sigma_j^\alpha\rangle^2-\langle\sigma_i^\alpha\rangle^2\langle\sigma_j^\alpha\rangle^2]\,, \tag{5}$$

where $\langle\ \rangle$ denotes a thermal expectation value. $\chi_Q(i,j)$ goes to zero as $|\mathbf{x}_i-\mathbf{x}_j|\to\infty$ in both the [1,0] and [1,1] phases. Thus we can use it to define the correlation length ξ_Q. Upon summing over i and j we obtain the susceptibility χ_Q, which is reminiscent of a spin-glass susceptibility. From the data for $D/J=2$, it appears that ξ_Q, and thus also χ_Q, grows rapidly upon approaching the [1,0]-to-[1,1] transition line. The existing data do not permit a more quantitative analysis, but the picture for $m=2$ is almost one of a standard second-order transition, which would be expected to obey the usual scaling relations. If we think of the [1,1] phase as the "ordered" phase and the [1,0] phase as the "disordered" phase, then the transition can be crudely approximated by a random exchange Ising critical point with an inverted temperature axis. In addition, however, we have the effect of a coupling to a polarizable medium,[36] which can destabilize the critical point. This would cause the transition to have a small first-order character, which is not apparent within the accuracy of the simulations reported here.

Thus we would expect the PM-[1,0]-[1,1] point (which was not observed directly) to be a tricritical point. It is conceivable that what we have called the [1,0]-to-[1,1] transition line is actually a wedge in which the order parameter rotates through intermediate directions between the [1,0] direction and the [1,1] direction. This would make the PM-[1,0]-[1,1] point a tetracritical point. Because of percolation effects, however, the rotation away from the [1,0] direction is unlikely to be continuous (except in a mean-field theory), and the author does not believe that such a wedge actually exists. Due to the similarity, mentioned above, between this problem and the random field Ising model,[33,34] the [1,0]-to-[1,1] transition should be first order at $T=0$, and probably for all T along the [1,0]-to-[1,1] transition line.[33]

B. The $m=3$ case

Although there is no longer an exact relation between $D>0$ and $D<0$ when $m=3$, the two cases should still remain qualitatively similar. Whether D is positive or negative, when one looks at the net anisotropy of a cluster of several sites one expects all three axes to be different, except for accidental degeneracies. Therefore, simulations were not performed for $D<0$. The phase diagram for $D\geq 0$ is shown in Fig. 7. The $m=3$ phase diagram is topologically similar to the $m=2$ case, with a [1,0,0] phase replacing the [1,0] phase, and a [1,1,1] phase replacing the [1,1] phase. For $D/J=2$, as the temperature is lowered we again pass from the PM phase into the [1,1,1] phase, and then into the [1,0,0] phase.

A closer inspection of the data reveals significant (but not unexpected) differences between $m=2$ and 3. The [1,0,0]-to-[1,1,1] transition is clearly first order, although the latent heat, ΔE, is only about $0.03J$. The PM-[1,0,0] transition for $D/J=1$ does not display any definitive first-order characteristics, although the ratio χ_l/χ_t is an increasing function of T below T_c, just as it was for the $D=0$ case. This ratio reaches a value of approximately

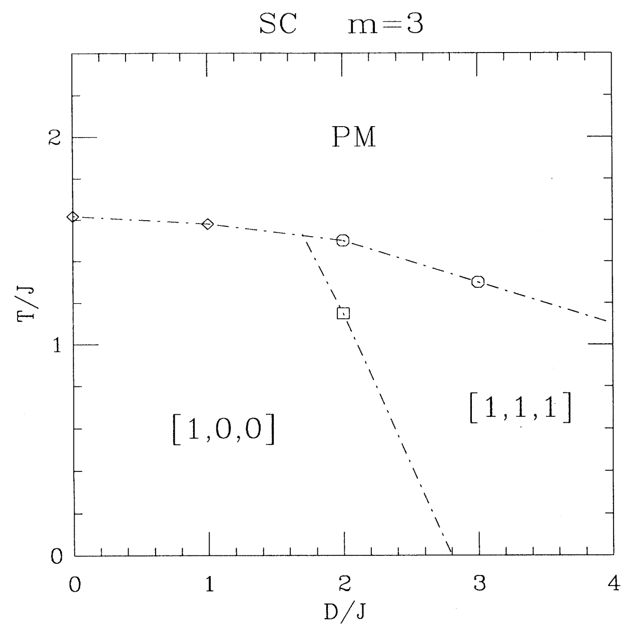

FIG. 7. Phase diagram of the $m=3$ cubic ferromagnet with random uniaxial anisotropy, on simple cubic lattices. The plotting symbols show actual data points, and the dot-dashed lines indicate first-order phase transitions.

4.5 at T_c, only about half the value reached in the $D=0$ case.

From the χ_M data for the PM phase, shown in Fig. 8, we obtain the estimates $T_*/J=1.580\pm0.003$, and $\gamma=1.32\pm0.08$. The magnetization probably jumps discontinuously to zero at $T_c/J=1.577\pm0.002$, although the discontinuity is not clearly visible in the Monte Carlo data. If we extrapolate the FM phase data using $T_u/J=1.580$, we find $\gamma'\approx\gamma$ and $\beta=0.29\pm0.02$. This result for β agrees well with the experimental estimates[29] for $KMnF_3$. There is no justification for assuming that $T_u=T_*$, however. The thermodynamic stability arguments of Mukamel and Grinstein[12] rule out the existence of a stable critical point in this system. If γ is the Heisenberg critical exponent, and γ' is the cubic critical exponent, their seeming equality is merely a numerical accident.[8] Since the cubic anisotropy is renormalization-group irrelevant for $m=3$ in three dimensions, other stability arguments[37,38] lead us to expect that the random anisotropy should cause ΔE to be zero at a PM-FM transition. A discontinuity in M is permitted by the general thermodynamic considerations, and required by the (nonrigorous) renormalization-group arguments.[10–14]

The PM-[1,1,1] transitions for $D/J=2$ and 3 are qualitatively similar to the $D/J=1$ transition. In all these cases ΔE is too small to resolve on an $L=48$ lattice. In Fig. 9 the specific-heat data obtained from numerically differentiating the energy of an $L=64$ lattice with $D/J=3$ are shown. A small bump at T_c, corresponding to a latent heat of at most $0.005J$, is barely visible. Even for $L=64$ a histogram of the energy fails to resolve two peaks. The magnetization data from lattices of various sizes with $D/J=3$ are displayed in Fig. 10. The magnetization tail for $T/J\geq1.325$ appears to be a finite size effect, but for $T/J\leq1.30$ the magnetization is essentially independent of L. The value of ΔM at this transition is about 0.20.

The behavior of the simple cubic lattice at T_c along the PM-FM line may be somewhat atypical, because on this lattice $p_c^S=0.311$, which is rather close to $\frac{1}{3}$. This causes the three infinite connected clusters of parallel spins in the [1,1,1] phase to be rather ramified. A consequence of

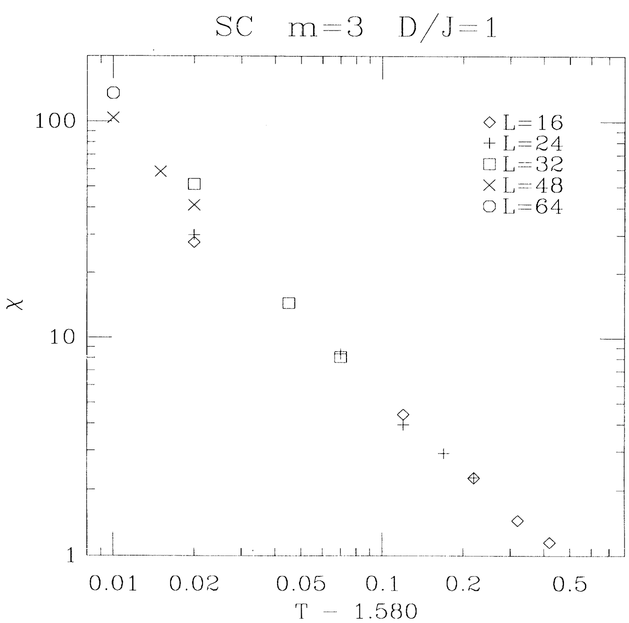

FIG. 8. Paramagnetic phase magnetic susceptibility vs reduced temperature for the $m=3$ random anisotropy cubic ferromagnet with $D/J=1$, on simple cubic lattices. The axes are scaled logarithmically.

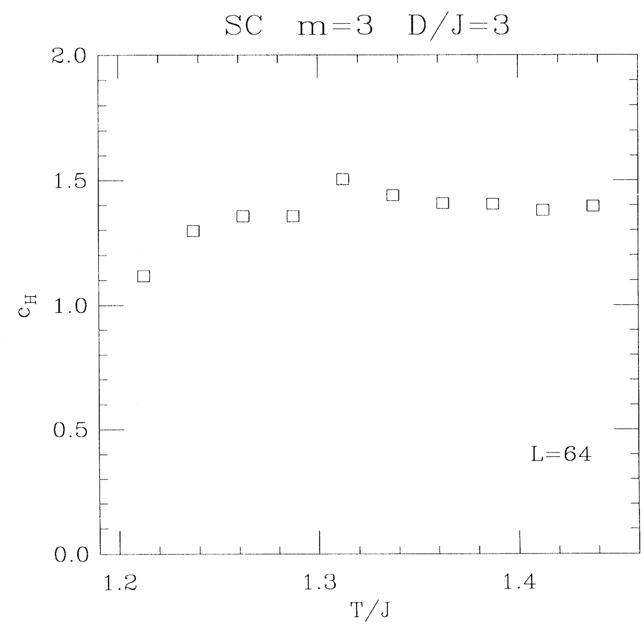

FIG. 9. Specific heat vs temperature for the $m=3$ random anisotropy cubic ferromagnet with $D/J=3$ on an $L=64$ simple cubic lattice.

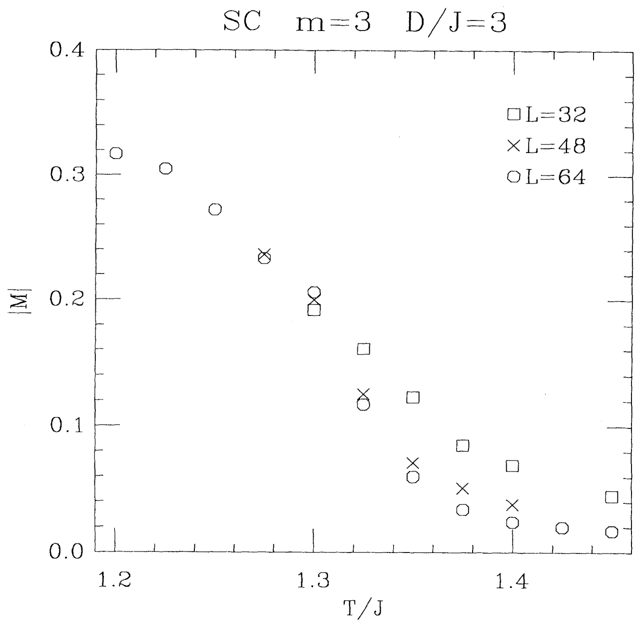

FIG. 10. Magnetization vs temperature for the $m=3$ random anisotropy cubic ferromagnet with $D/J=3$ on simple cubic lattices.

this was that it proved impossible in the simulations to form all three of the "percolating clusters" by simply cooling through the transition temperature, and then annealing. If simulations had been performed for $m=3$ on a face-centered cubic lattice, for which[32] $p_c^S=0.198$, it is likely that the [1,1,1] phase would nucleate spontaneously upon slow cooling through T_c.

Annealing a disordered state just below T_c was observed to produce a state in which only two percolating clusters had formed. It was then possible to take this "2-cluster" metastable state and form a percolating cluster along the third spin axis "by hand." This was done by forcing all of the $\boldsymbol{\sigma}_i$ lying in the unordered direction to become parallel. This part of the process necessarily lowers both the energy and the entropy, but the true quantity of interest is the net effect on the free energy. This state was then relaxed to equilibrium, keeping T fixed. Following this procedure, the "3-cluster" state was always observed to have a lower energy than its parent "2-cluster" state, and it remained stable. This is not a trivial observation, because the magnetizations of the two ordered components in the "2-cluster" state were larger than the magnetizations of individual components of the "3-cluster" state. The third cluster can only percolate on the simple cubic lattice by reducing the sizes of the other two clusters. Thus, at least on this lattice, the "2-cluster" state can be considered a distinct local minimum of the free energy, even when the "3-cluster" state is the global minimum.

For any given lattice structure, one could choose a value of m which is large enough so that $1/m$ is less than p_c^S. If $D/J>z$, where z is the number of neighbors of a site on the lattice in question, then there will not be any long-range order at $T=0$, and most likely not at any higher T either. It seems unlikely that for such a large value of m the "m-cluster" phase would be stable for any value of D/J. It is plausible that in at least some of these cases there will be truly stable phases at intermediate values of D/J analogous to the metastable "2-cluster" phase which was seen in the simulations for $m=3$ on the simple cubic lattice. It also seems possible that there may be a region of stable [1,1,0] phase for the simple cubic lattice, along the part of the [1,0,0]-[1,1,1] transition line.

The analysis of Aharony and Pytte[11] suggests another explanation for the observed nature of the PM-[1,1,1] transition. It may be that there is a narrow temperature range between the domain-FM and PM phases in which $\chi_M=\infty$, but $M=0$. The Monte Carlo data for $D/J=3$ are consistent with such a scenario in the range $1.30<T/J<1.36$, since in that range χ_M is limited by the sample size L. Such a phase, if it exists, may be controlled by the behavior of topological point defects,[39] and exhibit power-law decay of spin correlations. The latent heat associated with the disappearance of M might then be expected to be zero along the PM-[1,1,1] line, and the anomaly in the specific heat at T_c would be only a simple discontinuity.

Another reasonable hypothesis is that this behavior is related to the lower pseudocritical point T_*. If one thinks of the randomness as "smearing out" the first-order phase transition by causing fluctuations in the local value of T_c, then perhaps there are significant regions of space in which T_* becomes greater than T_c, causing χ_M to diverge even though T is too high to support a bulk magnetization. This is conceptually similar to the "Griffiths phase,"[40] which has been proposed to exist in the Ising spin glass.

IV. SUMMARY

In this work we have studied the effects of adding a random local anisotropy to a three-dimensional ferromagnet possessing a strong cubic crystal field which favors the cubic axes. For both the $m=2$ and 3 cases, we have found that at low temperatures there are two distinct types of ferromagnetic phases: a conventional ferromagnetic phase for weak random anisotropy, and a domain-type ferromagnetic phase for strong random anisotropy. For $m=2$ the PM-conventional FM transition appears to be pure XY type, while the PM-domain FM seems to be a new type of random critical point. In the $m=3$ case both PM-FM transitions seem to behave in the manner predicted by Aharony and Pytte,[11] with a discontinuous disappearance of the magnetization, and a thin region between the FM and PM phases in which the magnetic susceptibility is very large, and probably infinite.

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[1]R. Harris, M. Plischke, and M. J. Zuckermann, Phys. Rev. Lett. **31**, 160 (1973).
[2]R. A. Pelcovits, E. Pytte, and J. Rudnick, Phys. Rev. Lett. **40**, 476 (1978).
[3]C. Jayaprakash and S. Kirkpatrick, Phys. Rev. B **21**, 4072 (1980).
[4]R. Fisch and A. B. Harris, Phys. Rev. B **41**, 11 305 (1990).
[5]R. Fisch, Phys. Rev. Lett. **66**, 2041 (1991).
[6]R. Fisch, Phys. Rev. B **46**, 242 (1992).
[7]M. K. Grover, L. P. Kadanoff, and F. J. Wegner, Phys. Rev. B **6**, 311 (1972).
[8]E. Domany and E. K. Riedel, J. Appl. Phys. **50**, 1804 (1979).
[9]K. E. Newman and E. K. Riedel, Phys. Rev. B **25**, 264 (1982).
[10]A. Aharony, Solid State Commun. **28**, 667 (1978).
[11]A. Aharony and E. Pytte, Phys. Rev. Lett. **45**, 1583 (1980); see also Phys. Rev. B **27**, 5872 (1982).
[12]D. Mukamel and G. Grinstein, Phys. Rev. B **25**, 381 (1982).
[13]E. M. Chudnovsky and R. A. Serota, J. Phys. C **16**, 4181 (1983).
[14]Y. Y. Goldschmidt and A. Aharony, Phys. Rev. B **32**, 264 (1985).
[15]B. I. Halperin and C. M. Varma, Phys. Rev. B **14**, 4030 (1976).
[16]A. Aharony, Phys. Rev. B **12**, 1038 (1975).
[17]D. J. Amit, H. Gutfreund, and H. Sompolinsky, Phys. Rev. A **32**, 1007 (1985).
[18]K. H. Fischer and A. Zippelius, J. Phys. C **18**, L1139 (1986).
[19]H.-O. Carmesin, J. Phys. A **22**, 297 (1989).
[20]I. J. Ketley and D. J. Wallace, J. Phys. A **6**, 1667 (1973).
[21]D. D. Betts, Can. J. Phys. **42**, 1564 (1964).
[22]J. Adler, J. Phys. A **16**, 3585 (1983).
[23]P. Miles, M. A. A. Issa, K. N. R. Taylor, and G. J. Bowden, J. Phys. F **7**, 2421 (1977).
[24]P. M. Gehring, M. B. Salamon, A. del Moral, and J. I. Arnaudas, Phys. Rev. **41**, 9134 (1990).
[25]S. K. Burke and B. D. Rainford, J. Phys. F **13**, 441 (1983).
[26]J. W. Cable and Y. Tsunoda, J. Appl. Phys. **73**, 5454 (1993).
[27]G. Burns, Phys. Rev. B **13**, 215 (1976).
[28]J. G. Bednorz and K. A. Müller, Phys. Rev. Lett. **52**, 2289 (1984).
[29]U. J. Cox, A. Gibaud, and R. A. Cowley, Phys. Rev. Lett. **61**, 982 (1988); A. Gibaud, S. M. Shapiro, J. Nouet, and H. You, Phys. Rev. B **44**, 2437 (1991).
[30]J. Lee and J. M. Kosterlitz, Phys. Rev. B **43**, 3265 (1991).
[31]R. Fisch, Phys. Rev. B **46**, 11 310 (1992).
[32]D. Stauffer, Phys. Rep. **54**, 1 (1979).
[33]A. P. Young and M. Nauenberg, Phys. Rev. Lett. **54**, 2429 (1985).
[34]G. S. Grest, C. M. Soukoulis, and K. Levin, Phys. Rev. B **33**, 7659 (1986).
[35]I. O. Mayer, J. Phys. A **22**, 2815 (1989).
[36]P. Pfeuty and G. Toulouse, *Introduction to the Renormalization Group and to Critical Phenomena* (Wiley-Interscience, London, 1977), p. 150.
[37]M. Aizenman and J. Wehr, Commun. Math. Phys. **130**, 489 (1990).
[38]K. Hui and A. N. Berker, Phys. Rev. Lett. **62**, 2507 (1989).
[39]M.-h. Lau and C. Dasgupta, Phys. Rev. B **39**, 7212 (1989).
[40]M. Randeria, J. P. Sethna, and R. G. Palmer, Phys. Rev. Lett. **54**, 1321 (1985).