Structure of persistent current in the presence of a magnetic flux and an electrostatic potential

Daisuke Takai and Kuniichi Ohta
Fundamental Research Laboratories, NEC Corporation, 34 Miyukigaoka, Tsukuba, Ibaraki 305, Japan
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We present a calculation of persistent current in an ideal misoscopic ring coupled to an energy and electron reservoir. In this system, elastic scattering and energy dissipation in the ring occur only at the junction coupled to the reservoir. We consider the case where a magnetic flux and an electrostatic potential are applied simultaneously. It is demonstrated that an electrostatic potential modulates the period, amplitude, and phase of the persistent current vs magnetic-flux oscillation. In some electrostatic potentials, we find an $h/2e$ oscillation in the persistent current vs magnetic-flux relation. The current density and density of states in such a system is also derived. The electrostatic potential causes resonance peaks of the current density and density of states vs magnetic-flux oscillations if the magnetic phase shift approaches $n\pi$. For $\varepsilon=0.5$ (maximum coupling between the ring and wire), we also derive an analytical expression for the peak height of the current density and density of states vs magnetic flux under constant electrostatic potential.

I. INTRODUCTION

During the past few years, there has been considerable interest in quantum phenomena in mesoscopic rings. The nature of the quantum oscillation induced by an external field (magnetic flux,[1,2] electromagnetic field, etc.) has been studied. Recently the electrostatic Aharonov-Bohm (AB) effect in a mesoscopic ring has been studied in relation to the Aharonov-Bohm flux effect, and the period of electrostatic AB oscillation and the effect of temperature investigated.[3] More recently, the effects of an electromagnetic field[4] and spin-orbit interaction[5] in a normal metal ring threaded by Aharonov-Bohm flux have been discussed. This paper investigates the influence of an electrostatic potential and a magnetic flux in an ideal one-dimensional normal mesoscopic ring present, by varying each in turn while holding the other constant. The model adopted in this paper is the extension of Ref. 2, where a normal conductor is coupled to an energy and particle reservoir. In this system, elastic scattering and energy dispersion in the ring occur only at the junction coupled to the reservoir. It is shown that the electrostatic potential modulates the period, amplitude, and phase of the persistent current vs magnetic-flux oscillation. This paper is organized as follows. In Sec. II, we give the description of the model and derive the persistent current in a ring as a function of magnetic flux and electrostatic potential. In Sec. III, we investigate the effect of varying magnetic flux on the persistent current, current density, and density of states under different constant electrostatic potentials. In Sec. IV we study the effects of varying the electrostatic potential with different constant magnetic fluxes. In Sec. V the effect of scattering by the edges of the gate electrode is discussed briefly. The main conclusions are summarized in Sec. VI.

II. DERIVATION OF PERSISTENT CURRENT

In this section, we derive the persistent current in the mesoscopic one-dimensional normal conductor ring as a function of the magnetic flux and electrostatic potential. The model we consider is shown in Fig. 1. The magnetic flux threads the ring, and electrostatic potential is applied to the semicircle of the ring. The ring is coupled via a one-dimensional lead to the reservoir that simulates its dissipation by exchanging energy and particle with it. In this model, electrons in the ring are a grand-canonical ensemble[6] with fixed Fermi energy, and obey a Fermi-Dirac distribution function. Elastic scattering arises only at the junction between the lead and the ring. The junction is described by the S matrix:

$$\begin{bmatrix}\alpha'\\ \beta'\\ \gamma'\end{bmatrix}=\begin{bmatrix}-(a+b) & \sqrt{\varepsilon} & \sqrt{\varepsilon}\\ \sqrt{\varepsilon} & a & b\\ \sqrt{\varepsilon} & b & a\end{bmatrix}\begin{bmatrix}\alpha\\ \beta\\ \gamma\end{bmatrix},\quad a=\tfrac{1}{2}(\sqrt{1-2\varepsilon}-1),\quad b=\tfrac{1}{2}(\sqrt{1-2\varepsilon}+1), \tag{1}$$

where ε is a coupling constant between the ring and the wire. For $\varepsilon=0$, the loop is perfectly isolated from the wire. α,β,γ and α',β',γ' are the amplitudes of incoming and outgoing waves, as shown in Fig. 1. The Hamiltonian in the loop is described by

$$H=1/2m(p-eA/c)^2+eV, \tag{2}$$

where m is the effective mass of an electron, p is the

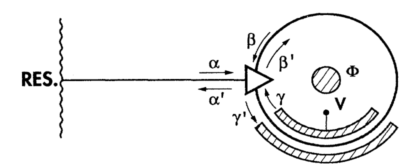

FIG. 1. A mesoscopic one-dimensional normal conductor ring coupled with an energy and electron reservoir via an ideal one-dimensional lead. Magnetic flux threads the ring and electrostatic potential is applied to the semicircle of the ring.

momentum operator, e is the electron charge, c is the velocity of light, and we take the vector potential A to be Φ/L. L is the path length of the loop, Φ is the magnetic flux that threads the ring, and V is the electrostatic potential applied to the semicircle of the ring. The wave function in the lead is written as

$$\psi=\alpha \exp(ikx)+\alpha'\exp(-ikx) , \tag{3}$$

where α is the amplitude of the wave that travels toward the reservoirs, α' is the amplitude of the wave traveling toward the junction, and x is the coordinate axis along the wire. For simplicity, we take the junction to be at $x=0$. If both a magnetic flux and an electrostatic potential are present, the wave function in the loop is of the form

$$\psi=\exp(i\theta y/L)\{\gamma'\exp[iky+i\zeta(y)] + \gamma \exp[-iky-i\zeta(y)]\} , \tag{4a}$$

$$\zeta(y)=2y\phi\eta(L/2-y)/L+\phi\eta(y-L/2) . \tag{4b}$$

Here the effects of a magnetic flux and an electrostatic potential are considered in terms of phase shift as in Ref. 7. γ and γ' are the amplitude of incoming and outgoing waves at the junction as shown in Fig. 1. y is the coordinate axis along the ring, and $\eta(y)$ is a step function, namely $\eta(y)=1$ for $y>0$, $\eta(y)=0$ for $y<0$, and $\eta(y)=\frac{1}{2}$ for $y=0$. $\zeta(y)$ is the phase shift due to the electrostatic potential that exists at $0<y<L/2$. θ and ϕ are the phase shifts due to a magnetic flux and an electrostatic potential, respectively, and are of the forms

$$\theta=\frac{e\Phi}{\hbar} , \tag{5}$$

$$\phi=\frac{\sqrt{2m}\,(\sqrt{E+eV}-\sqrt{E}\,)L}{2\hbar} , \tag{6}$$

where E is the electron energy in the ring. β and β' are the amplitudes of another set of incoming and outgoing waves at the junction as shown in Fig. 1. Inserting $y=L$ into Eq. (4a) yields two terms, the first being β and the second β'. Thus we have the following relation:

$$\gamma=\beta'\exp[i(-\theta+\phi+kL)] , \tag{7}$$

$$\beta=\gamma'\exp[i(\theta+\phi+kL)] , \tag{8}$$

Eliminating γ and γ' by Eqs. (7) and (8), and α' by Eq. (1), we can calculate the ratio β/α and β'/α. Then we find the current density $J(\theta,\phi,E)$ and density of states $\mathrm{DOS}(\theta,\phi,E)$ to be

$$J=2\varepsilon \sin(kL+\phi)\sin\theta/D(\theta,\phi,E) , \tag{9}$$

$$\mathrm{DOS}=2\varepsilon[1-\cos(kL+\phi)\cos\theta]/D(\theta,\phi,E) , \tag{10}$$

$$D(\theta,\phi,E)=(1-\varepsilon+\sqrt{1-2\varepsilon})[\cos(kL+\phi)-\cos\theta]^2 + (1-\varepsilon-\sqrt{1-2\varepsilon})\sin^2(kL+\phi) . \tag{10a}$$

Here we have used the relation $J=|\beta'/\alpha|^2-|\beta/\alpha|^2$ and the density of states (DOS) = $|\beta'/\alpha|^2+|\beta/\alpha|^2$. In the figures shown below, the current is normalized by $J_0=ev_F/L$, where $v_F=hk_F/m$ and k_F is the Fermi wave number. In the one-dimensional free-electron case, the density of states is given by $1/hv$, where v is the electron velocity. The current density $J=|\beta'/\alpha|^2-|\beta/\alpha|^2$ of Eq. (9) is actually the true current density $J=v(|\beta'/\alpha|^2-|\beta/\alpha|^2)$ multiplied by the factor $1/v$ in the state density. In the absence of electrostatic potential, our results reduce to Eqs. (6) and (7) of Ref. 2, and Eq. (7) of Ref. 8. The circulating current $I(\theta,\phi)$ in the loop at finite temperatures is given in terms of J of Eq. (9) by

$$I=e/h\int dE\, f(E)J(\phi,\theta,E) , \tag{11}$$

where $f(E)$ is the Fermi-Dirac distribution function.

Equation (9) indicates that at the weak-coupling limit ($\varepsilon\to0$) the numerator of the current density tends to $J=0$. To obtain a nonzero persistent current at this limit, the denominator of Eq. (9) must vanish as well. This condition yields the relation

$$\theta+\phi+kL=2n\pi . \tag{12}$$

In the absence of electrostatic potential, this equation reduces to the fluxoid quantization condition that is advocated by London. This equation can also be derived using the Bohr-Sommerfeld quantum condition

$$\oint p\, ds=nh . \tag{13}$$

In our model, the left-hand side of Eq. (13) is

$$\oint(\hbar k+e/cA)ds=\frac{[\sqrt{2m}\sqrt{(E+eV)}+\sqrt{2mE}\,]L}{2\hbar}+\frac{e}{c}\Phi. \tag{14}$$

Then, setting the right-hand side of Eq. (14) equal to nh, we get Eq. (12).

III. THE PERSISTENT CURRENT AS A FUNCTION OF MAGNETIC FLUX UNDER CONSTANT ELECTROSTATIC POTENTIAL

Under the basic formula described in Sec. II, we make numerical calculation for J, DOS, and I as a function of θ and ϕ. Unless otherwise, we assumed $\varepsilon=0.5$, $T=0.0$ K, and $kL=5\pi$ throughout this paper. The last equality means that at $T=0.0$ K, five states are occupied in the ring. Hereafter we write the wave number at the Fermi level as k.

A. J vs θ ϕ=const($\varepsilon=0.5, kL=5\pi$)

Now we investigate the effect of an electrostatic potential on the relation between the persistent current density and static magnetic flux threading the ring. Figure 2 shows the calculated results for the current density J in the ring as a function of θ for given ϕ's. For $\phi+kL\neq0$, the current density J exhibits positive and negative peaks in the vicinity of $\theta=(2n+1)\pi$. If $\phi+kL=n\pi$, Eq. (9) shows that the current density vanishes independent of θ. To obtain the positions of θ and the extremum values where the positive and negative peaks occur, we set [see Eq. (A1)]

$$dJ/d\theta=0 . \tag{15}$$

Equation (15) yields the polynomial equation for $x=\cos\theta$ of order 3:

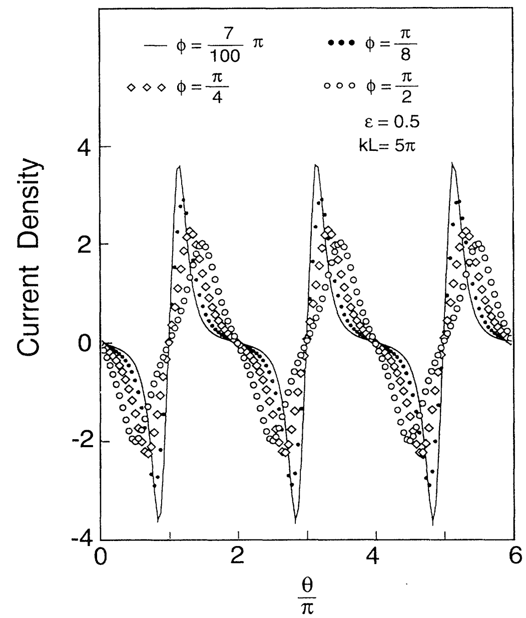

FIG. 2. The current density in the ring as a function of θ for various ϕ'($\varepsilon=0.5, kL=5\pi$).

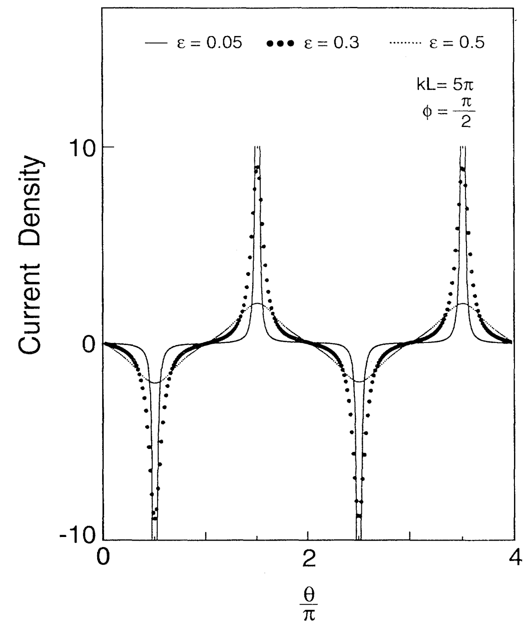

FIG. 3. ε dependence of the current density (current density vs $\theta, \phi=\pi/2$).

$$z^3-3z+2\cos\xi=0 \quad (\xi=\phi+kL, z=\cos\theta) \ . \tag{16}$$

There are in general three solutions for $z=\cos\theta$:[9]

$$z=\cos\theta=-2\cos\omega \quad [\omega=\xi/3,(2\pi\pm\xi)/3,\xi=kL+\phi] \ . \tag{17}$$

Since $-1\leq\cos\theta\leq+1$ must be satisfied, only one of the three z's gives the solutions for θ, and then θ has two solutions in every period of 2π. Inserting these solutions into Eq. (9), we find the peak height of current density J to be

$$J_{\text{peak}}=\frac{\pm2\sqrt{1-4\cos^2\omega}\sin\xi}{(1+4\cos^2\omega+4\cos\omega\cos\xi)} \ . \tag{18}$$

When $\cos\xi=1$ or -1, the two roots of the three solutions denoted by Eq. (17) coalesce to the root: $z=\cos\theta=1$ or -1, respectively. In the limit $\xi\rightarrow0$ or $\xi\rightarrow\pi-\delta$, the current density J diverges as

$$\lim_{\xi\rightarrow0} J \propto \lim_{\xi\rightarrow0}(1/\sqrt{\xi})\rightarrow\infty \ , \tag{19a}$$

$$\lim_{\xi\rightarrow\pi-\delta} J \propto \lim_{\xi\rightarrow\pi-\delta}(1/\sqrt{\delta})\rightarrow\infty \ . \tag{19b}$$

These divergences correspond to the peaks in the J vs θ relation in Fig. 2.

B. J vs θ, ϕ=const($\varepsilon\rightarrow0$)

Next we consider the ε dependence of the current density (see Fig. 3). For the limit of an isolated ring, namely $\varepsilon\rightarrow0$, the current density J asymptotically approaches a δ function as

$$\lim_{\epsilon\rightarrow0} J(\theta,\phi,E)\propto\delta(\theta-\phi-kL-n\pi) \ . \tag{20}$$

For small but finite ε, the peak height and width are proportional to $1/\varepsilon$ and ε, respectively.

C. DOS vs θ, ϕ=const($\varepsilon=0.5, kL=5\pi$)

We now consider the density of states (Fig. 4). Putting the derivative of the DOS of Eq. (10) with respect to θ to 0, we have [see Eq. (A2)]

$$\sin\theta=0 \ , \tag{21}$$

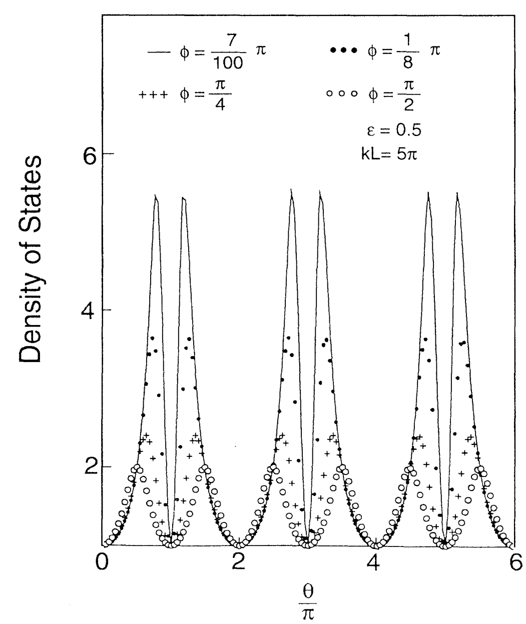

FIG. 4. The density of states in the ring as a function of θ for various ϕ'($\varepsilon=0.5, kL=5\pi$).

$$\cos\theta=\sec\xi-\tan\xi \quad \text{for } \sin\xi>0 \;,$$
$$\cos\theta=\sec\xi+\tan\xi \quad \text{for } \sin\xi<0 \quad (\xi=kL+\phi) \;. \tag{22}$$

Equation (21) shows that the DOS is minimum when $\theta=n\pi$ independently of electrostatic potential. Then substituting Eq. (22) into Eq. (10), we find the peak height of the DOS to be

$$\text{DOS}_{\text{peak}}=1+1/\sin(\xi) \;. \tag{23}$$

Equation (23) shows that the peak height of the density of states becomes minimum when $\xi=(2n+3/2)\pi$. Equation (23) is valid except near the divergent points $\xi=n\pi$.

D. I vs θ, ϕ=const($\varepsilon=0.5, kL=5\pi$)

Finally, we investigate the influence of electrostatic potential on the persistent current I driven by a magnetic flux. Figure 5 shows the calculated persistent current as a function of θ for various constant electrostatic potentials ($\varepsilon=0.5$). The electrostatic potential modulates the amplitude, phase, and frequency of the oscillation in the persistent current vs magnetic-flux relation. In particular, if ϕ is around $\pi/2$, the oscillation with a period of $h/2e$ can be seen. This double-frequency oscillation ($h/2e$) cannot be explained from the current density since the frequency of the oscillation in the current density J vs flux density relation is h/e for every electrostatic potential (see Fig. 2). Equation (9) indicates that the numerator of the current density J has opposite sign for $\phi+kL=(2n+1)\pi+\delta$ and $\phi+kL=(2n+1)\pi-\delta(\pi>>\delta>0)$. For this reason, as shown in Fig. 5, the I vs θ oscillation has the opposite phase. For $\phi=\pi/2$, the superposition of those Fourier components of opposite sign reveals the double frequency $h/2e$.

IV. THE PERSISTENT CURRENT AS A FUNCTION OF ELECTROSTATIC POTENTIAL UNDER CONSTANT MAGNETIC FLUX

In this section, we investigate the persistent current in the ring as a function of the electrostatic phase shift due to the electrostatic potential applied to the semicircle of the ring. In this case, the magnetic flux threading the ring is always fixed constant and taken as a parameter. In the absence of magnetic flux, there is no current flow in the ring irrespective of the electrostatic potential. However, in the presence of constant magnetic flux threading the ring, the persistent current oscillates as a function of ϕ, and the oscillation is modulated nearly periodically as a function of the magnetic flux.

A. J vs ϕ, θ=const ($\varepsilon=0.5, kL=5\pi$)

Figure 6 shows the effect of magnetic flux on the current density in the AB ring vs the electrostatic potential. For $\theta=(n+1/2)\pi$, current density J shows a sin curve since $J=2\sin(\phi+\pi)$. To obtain the position of ϕ and the extremum values of the current density J where their extrema occur, we set $dJ/d\phi=0$. This equation yields [see Eq. (A3)]

$$\phi=\text{arc}\sec[\tfrac{1}{2}(\cos\theta+\sec\theta)]-kL \;. \tag{24}$$

Substituting Eq. (24) into Eq. (9), we obtain the peak height of the current density J as

$$J_{\text{peak}}(\theta,E)=\pm2/\sin\theta \;. \tag{25}$$

Equation (25) shows that J_{peak} is minimum when $\theta=(n+1/2)\pi$. Note that Eq. (25) is valid except near the divergence point. Note that in the limit of $\theta=n\pi$,

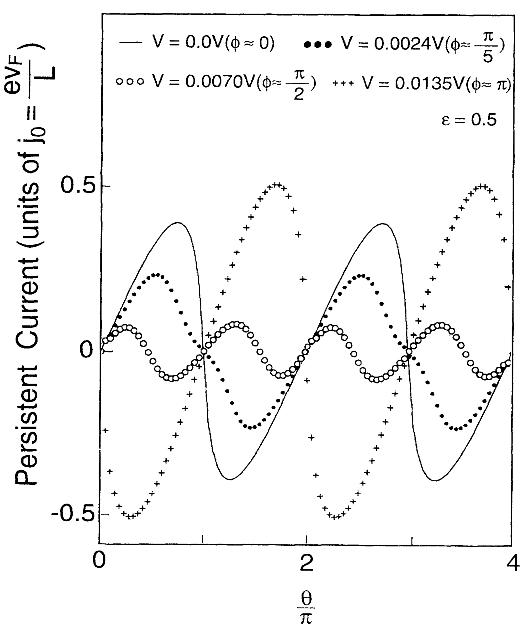

FIG. 5. Persistent current as a function of θ for various electrostatic potentials V.

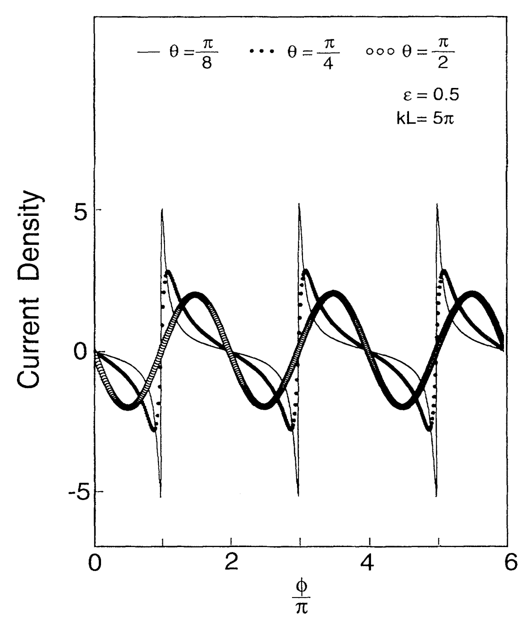

FIG. 6. The current density in the ring as a function of ϕ for various θ'($\varepsilon=0.5, kL=5\pi$).

the current density J diverges. In other words, the peak height of current density becomes sharp and high in this limit. + and − signs denote the positive and negative peaks.

B. J vs ϕ, θ=const($\varepsilon \to 0, kL = 5\pi$)

Figure 7 shows the calculated ε dependence of the current density. In the limit $\varepsilon \to 0$, as shown in Sec. II, a finite current density develops into a δ function.

C. DOS vs ϕ, θ=const($\varepsilon = 0.5, kL = 5\pi$)

Figure 8 shows the DOS as a function of ϕ for fixed θ. The peak position of the DOS is obtained by solving $d(\mathrm{DOS})/d\phi = 0$ [see Eq. (A4)]. This condition gives

$$\sin\xi = 0, \quad \cos\xi = \pm 1 \quad (\xi = kL + \phi) \; . \tag{26}$$

Substituting this equation into Eq. (10), we find the extremum values of the DOS to be

$$\begin{aligned} \mathrm{DOS} &= \mathrm{cosec}^2(\theta/2) \quad (\cos\xi = 1) \\ &= \sec^2(\theta/2) \quad (\cos\xi = -1) \; . \end{aligned} \tag{27}$$

Equation (27) means that the extremum value of the DOS is at a minimum when $\theta = n\pi$ (see Fig. 8). Note that Eq. (27) is valid except near the divergence point $\theta = n\pi$.

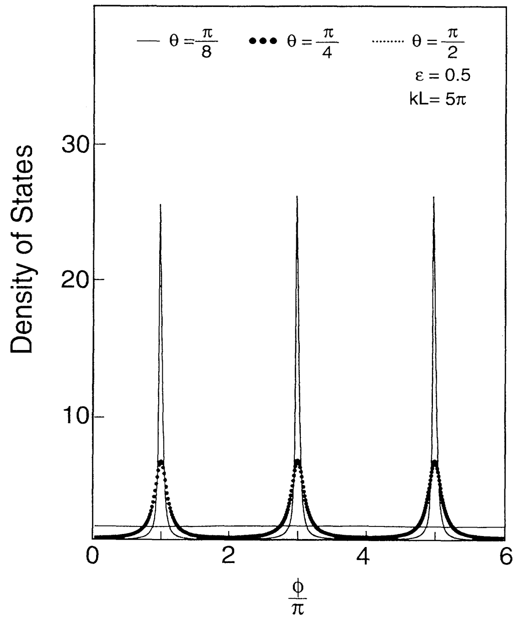

FIG. 8. The density of states in the ring as a function of ϕ for various θ'($\varepsilon = 0.5, kL = 5\pi$).

D. I vs V, θ=const($\varepsilon = 0.5, kL = 5\pi$)

Finally we investigate the effect of magnetic flux on the persistent current vs electrostatic potential relation. Figures 9 and 10 show the calculated persistent current as a function of ϕ for various magnetic fluxes. The magnetic flux modulates the amplitude and phase of the persistent current oscillation, but does not change the frequency since the magnetic phase shift θ is independent of the energy of the electron. As is predicted in the current density case, if θ reaches $n\pi$ the positive and negative peaks of the persistent current develop sharply.

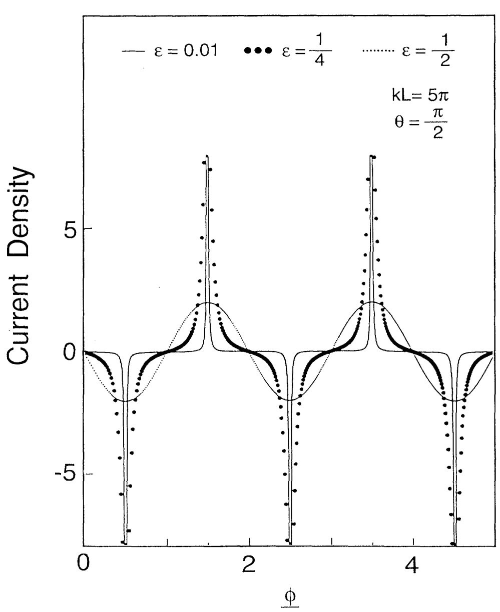

FIG. 7. ε dependence of current density (current density vs $\phi, \theta = \pi/2$).

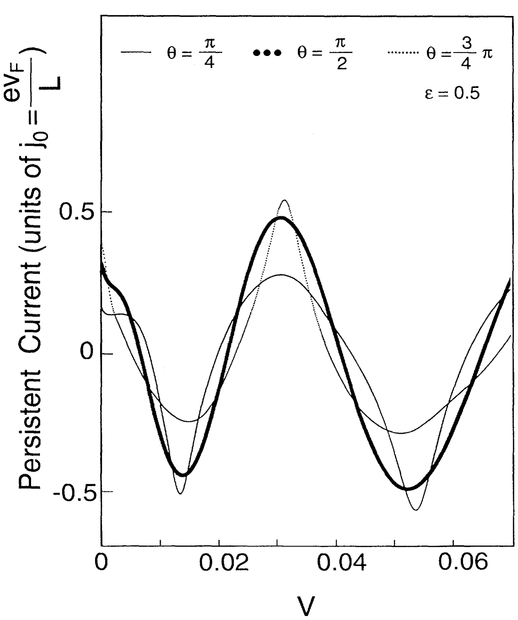

FIG. 9. Persistent current as a function of V for various θ'($0 < \theta < \pi$).

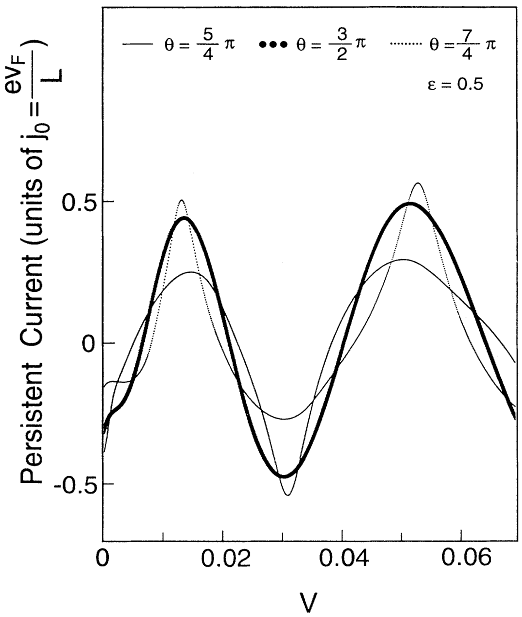

FIG. 10. Persistent current as a function of V for various $\theta'(\pi < \theta < 2\pi)$.

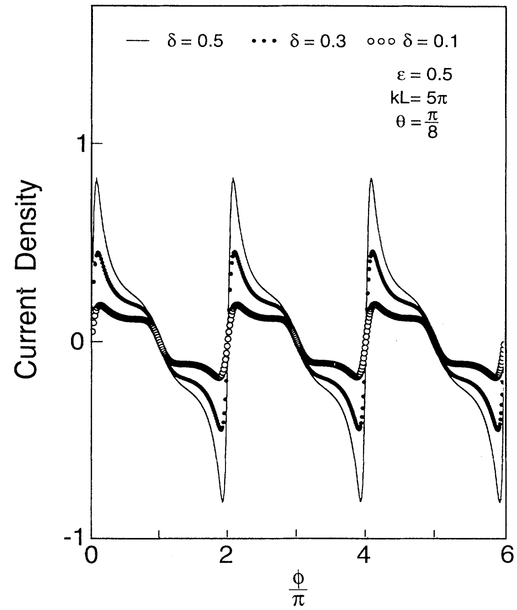

FIG. 12. The influence of scattering at the edge of the gate electrode (current density vs $\phi, \varepsilon=0.5, \theta=\pi/8, 0.1<\delta<0.5$).

V. THE INFLUENCE OF SCATTERING AT THE EDGE OF THE GATE ELECTRODE

In this section, we briefly discuss the effect of scattering due to the elastic scattering at the edge of the gate electrode to which the electrostatic potential is applied. For simplicity, we consider the situation where the elastic scattering occurs on one side of the gate electrode. To connect outgoing and incoming waves in this region, we choose the S matrix as follows:

$$\begin{bmatrix} \sqrt{\delta} & i\sqrt{1-\delta} \\ i\sqrt{1-\delta} & \sqrt{\delta} \end{bmatrix} . \tag{28}$$

Here $\sqrt{\delta}$ and $i\sqrt{1-\delta}$ denote the transmission and reflection amplitude that transmit and reflect into the gate electrode. Figure 11 shows that increasing the scattering on the gate electrode decreases the amplitude of the current density. For a small transmission in the gate electrode (see Fig. 12), the current density shows a clear stepwise change. This is the result of strong scattering at the edges of the gate electrode. We also extend our calculation to include the effect of scattering at both sides of the gate potential. The results are similar, particularly for the current density and density of states vs the magnetic flux under constant electrostatic potential. In this calculation, we assumed that the electrostatic potential varies slowly in the gate electrode. We have also investigated the situation where the electrostatic potential varies suddenly at the edge of the gate electrode, and found that both results are qualitatively similar.

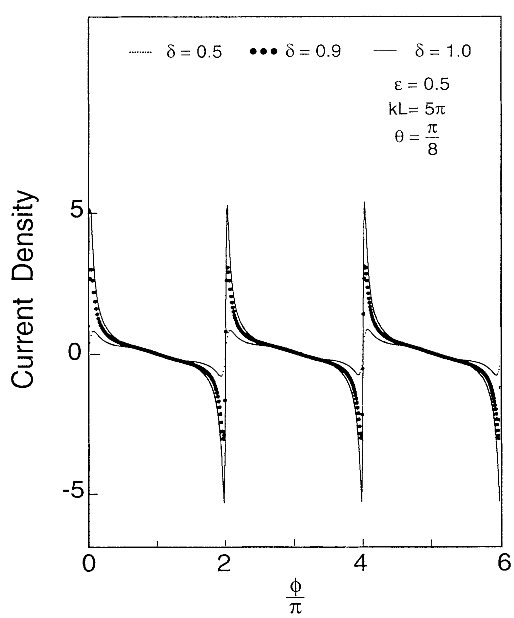

FIG. 11. The influence of scattering at the edge of the gate electrode (current density vs $\phi, \varepsilon=0.5, \theta=\pi/8, 0.5<\delta<1.0$).

VI. CONCLUSION

We calculated the persistent current, current density, and density of states associated with mesoscopic ring coupled to the energy and electron reservoir. We considered the case where the magnetic flux and electrostatic potential are applied simultaneously. The amplitude and phase of the oscillation in the AB current density vs magnetic-flux relation are modulated by the electrostatic potential, and the current density shows a sharp resonance peak where the electrostatic phase shift ϕ is about

$n\pi$. In particular, the double-frequency oscillation with a period $h/2e$ is predicted near $\phi=(n+\frac{1}{2})\pi$. The amplitude and phase of the AB oscillation in the persistent current and its current density vs electrostatic phase angle relation are modulated by the magnetic flux. The persistent current and current density show sharply resonant peaks where the magnetic phase θ is around $n\pi$. A fuller description of the effect of scattering at the edges of the gate electrode will be given in a future publication.

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APPENDIX

In this appendix, we present a detailed calculation of $dJ/d\theta$, $(d(\mathrm{DOS})/d\theta)$, and $dJ/d\phi(d(\mathrm{DOS})/d\phi)$. For simplicity, we consider the case only for $\varepsilon=0.5$ (strong coupling between ring and wire):

$$\frac{dJ}{d\theta}=\frac{2\sin(kL+\phi)[-\cos^3\theta+3\cos\theta-2\cos(kL+\phi)]}{F(\theta,\phi,E)^2}$$
$$F(\theta,\phi,E)=[1-2\cos(kL+\phi)\cos\theta+\cos^2\theta]^2\ . \tag{A1}$$

The condition $dJ/d\theta=0$ yields the extreme value of J. These solutions provide the peak height and minima of current density (J). $d(\mathrm{DOS})/d\theta, dJ/d\phi$ and $d(\mathrm{DOS})/d\phi$ are also calculated similarly and shown in Eqs. (A2), (A3), and (A4), respectively:

$$\frac{d(\mathrm{DOS})}{d\theta}=\frac{-2\sin\theta[\cos(kL+\phi)-2\cos\theta+\cos^2\theta\cos(kL+\phi)]}{F(\theta,\phi,E)^2}\ , \tag{A2}$$

$$\frac{dJ}{d\phi}=\frac{-2\sin\theta[2\cos\theta-\cos(kL+\phi)(1-\cos^2\theta)]}{F(\theta,\phi,E)^2}\ , \tag{A3}$$

$$\frac{d(\mathrm{DOS})}{d\phi}=\frac{-2\sin(kL+\phi)\cos\theta(1-\cos^2\theta)}{F(\theta,\phi,E)^2}\ . \tag{A4}$$

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