Exponentially narrow current dip for resonant-tunneling structure of three quantum dots

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We have found a resonance of an alternative type in the current-voltage curve of the non-one-dimensional resonant-tunneling structure with three quantum dots. It is an exponentially narrow dip with an exponentially small minimum that appears independently of the width of the energy spectrum of incident electrons. The structure considered is the simplest two-lead structure that has such a property.

In several recent papers[1–7] it was shown that the conductance of non-one-dimensional ballistic structures can have narrow backscattering resonances. For the resonant-tunneling structures (RTS's) these resonances can look much similar to the well-known Breit-Wigner resonances[8] (having, however, the overturned peak direction), their width being exponentially narrow and their minimum being exponentially small.[7] Thus, the non-one-dimensional RTS's have two types of transmission (conductance) resonances: (i) the conventional Breit-Wigner and (ii) the backscattering ones.

On the other hand, we know also that the RTS's can have exponentially narrow current Breit-Wigner resonances similar to the indicated conductance resonances of the first type. The simplest structure that exhibits such a property is a double quantum dot (well) structure (see, e.g., Ref. 9 and, for the model calculation in the Breit-Wigner approximation, Ref. 10). The narrow current resonance appears in such a structure when a level in the first quantum dot intersects, with changing the applied voltage, the level in the second quantum dot.

Though the existence of the first-type current resonances could be foreseen with use of the physically clear picture of level intersection, up to now it was not clear if there exist structures which have the current resonances of the second type, i.e., the exponentially narrow dips in the current-voltage curve, for the arbitrary width of energy spectrum of incident electrons.

This paper aims to show that the non-one-dimensional RTS with two (or more) leads can possess the second-type current resonances as well as the first ones. Recently, we have shown that such resonances appear for the three-lead double quantum dot RTS shown in Fig. 1(c) and unlike the second-type conductance resonances,[7] they cannot exist for the double quantum dot structure with two leads.[11] However, Ref. 11 remains unsolved whether or not there exists the conventional RTS with two leads that has current resonances of the second type. Here we demonstrate the second-type current resonance for the three quantum dot structure with two leads shown in Fig. 1(b).

Below we study the three quantum dot RTS assuming at first that it has several leads, as shown in Fig. 1(a). In order to simplify the consideration we propose that all the resonant levels of this structure correspond to the filled states of lead 1 and to the empty states of other leads (see left insets in Fig. 1). In this situation the Büttiker equations[12] for the current I_{1j} from lead 1 to lead j, $j=2,3,4$, take the simplest form:

$$I_{1j}=\frac{e}{\pi\hbar}\int_{-\infty}^{\infty} dE\, T_{1j}(E)\ , \qquad (1)$$

where the integration is performed over the energies of incident electrons. We define the transmission coefficients T_{1j} from lead 1 into lead j by the generalized Breit-Wigner formula.[13] Let $\Gamma_j^{(k)}$ be the partial width of level j connected with decay from quantum dot j into lead k, and let δ_{ij} be the flux overlap integral between quantum dots i and j (see Ref. 13 for more detailed definitions). It is proposed for simplicity that the only non-negligible decay widths are $\Gamma_j^{(k)}$ with $j=k=2,3$ and with $j=1$, $k=4$ [they correspond to the decay from quantum dots to the nearest leads, see Fig. 1(a)]. Then, as a particular case of results,[13] we find

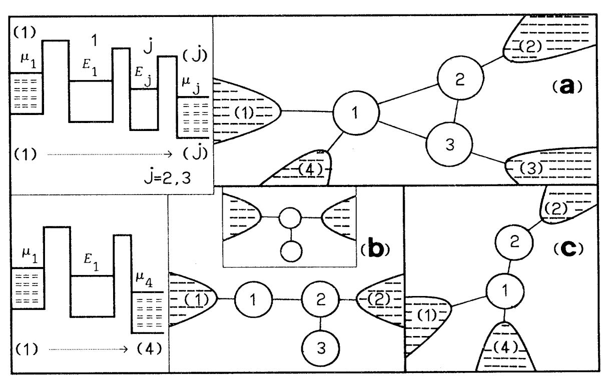

FIG. 1. Quantum dot RTS: (a) three quantum dot structure with four leads, (b) three quantum dot structure (in the inset: two quantum dot structure) with two leads, and (c) two quantum dot structure with three leads. The couplings between quantum dots and leads that are taken into account are denoted by straight segments. In the left insets: the sketches of energy diagrams along the paths connecting leads via quantum dots.

$$T_{12}(E)=\Gamma_1^{(1)}\Gamma_2^{(2)}|\delta_{12}(E-\varepsilon_3)-\delta_{12}\delta_{23}|^2/|D|^2 , \quad (2)$$

$$T_{14}(E)=\Gamma_1^{(1)}\Gamma_1^{(4)}|(E-\varepsilon_2)(E-\varepsilon_3)-\delta_{23}^2|^2/|D|^2 , \quad (3)$$

$$D=\det\begin{bmatrix}\varepsilon_1-E & -\delta_{12} & -\delta_{13}\\ -\delta_{12} & \varepsilon_2-E & -\delta_{23}\\ -\delta_{13} & -\delta_{23} & \varepsilon_3-E\end{bmatrix} ,$$

$$\varepsilon_j=E_j+\frac{i}{2}\Gamma_j , \quad \Gamma_1=\Gamma_1^{(1)}+\Gamma_1^{(4)} ,$$

$$\Gamma_2=\Gamma_2^{(2)} , \quad \Gamma_3=\Gamma_3^{(3)} ,$$

where Γ_j is the total width of level E_j in quantum dot j. The transmission coefficient $T_{13}(E)$ could be found from Eq. (2) with use of substitution $2\leftrightarrow3$. It is proposed that the level differences E_i-E_j [which are linear in applied voltages $V_{mn}=e(\mu_m-\mu_n)$ for V_{mn} small enough] and the level widths $\Gamma_j^{(k)}$ are small so that, for the energies giving the main contribution into the integral of Eq. (1), we can consider parameters δ_{ij} and $\Gamma_j^{(k)}$ as constants. Under this assumption we have calculated the integral of Eq. (1) with transmission coefficients (2) and (3) with the residue theory. The result of these cumbersome calculations is as follows:

$$I_{12}=\frac{e}{\hbar Q}\Gamma_1^{(1)}\Gamma_2^{(2)}\{|\delta_{12}\varepsilon_3+\delta_{13}\delta_{23}|^2C_0 -2\delta_{12}[\delta_{12}\mathrm{Re}(\varepsilon_3)+\delta_{13}\delta_{23}]C_1+\delta_{12}^2C_2\} , \quad (4)$$

$$\begin{aligned}I_{14}=\frac{e}{\hbar Q}\Gamma_1^{(1)}\Gamma_1^{(4)}\{&|\varepsilon_2\varepsilon_3-\delta_{23}^2|^2C_0\\ &-2\,\mathrm{Re}[(\varepsilon_2+\varepsilon_3)(\varepsilon_2^*\varepsilon_3^*-\delta_{23}^2)]C_1\\ &+[|\varepsilon_2+\varepsilon_3|^2+2\,\mathrm{Re}(\varepsilon_2\varepsilon_3)-2\delta_{23}^2]C_2\\ &-2\mathrm{Re}(\varepsilon_2+\varepsilon_3)C_3+C_4\} , \quad (5)\end{aligned}$$

with

$$\begin{aligned}C_0&=\mathrm{Im}(\alpha_1)[\mathrm{Im}(\alpha_3)+\mathrm{Im}(\alpha_1^*\alpha_2)]-[\mathrm{Im}(\alpha_2)]^2 ,\\ C_1&=\mathrm{Im}(\alpha_1)\mathrm{Im}(\alpha_1^*\alpha_3)-\mathrm{Im}(\alpha_2)\mathrm{Im}(\alpha_3) ,\\ C_2&=\mathrm{Im}(\alpha_1)\mathrm{Im}(\alpha_2^*\alpha_3)-[\mathrm{Im}(\alpha_3)]^2 ,\\ C_3&=\mathrm{Im}(\alpha_3)\mathrm{Im}(\alpha_1\alpha_3^*)+\mathrm{Im}(\alpha_2)\mathrm{Im}(\alpha_2^*\alpha_3) ,\\ C_4&=\mathrm{Im}(\alpha_3)\mathrm{Im}(\alpha_2^*\alpha_3)\\ &\quad+\mathrm{Im}(\alpha_1^*a_2)\mathrm{Im}(\alpha_2^*\alpha_3)-[\mathrm{Im}(\alpha_1\alpha_3^*)]^2 ,\end{aligned} \quad (6)$$

$$\begin{aligned}Q=&\tfrac{1}{4}\mathrm{Im}(\alpha_3^3)-\tfrac{3}{4}\mathrm{Im}(\alpha_3^*\alpha_3^2)-[\mathrm{Im}(\alpha_2)]^2\mathrm{Im}(\alpha_2^*\alpha_3)\\ &+\mathrm{Im}(\alpha_1)\mathrm{Im}(\alpha_2^*\alpha_3)\mathrm{Im}(\alpha_1^*\alpha_2)\\ &-2\,\mathrm{Im}(\alpha_1^*\alpha_2)[\mathrm{Im}(\alpha_3)]^2-3\,\mathrm{Im}(\alpha_2)\mathrm{Im}(\alpha_3)\mathrm{Im}(\alpha_1\alpha_3^*)\\ &-\mathrm{Im}(\alpha_1)[\mathrm{Im}(\alpha_1\alpha_3^*)]^2 ,\end{aligned}$$

where

$$\begin{aligned}\alpha_1&=\varepsilon_1+\varepsilon_2+\varepsilon_3 ,\\ \alpha_2&=\varepsilon_1\varepsilon_2+\varepsilon_1\varepsilon_3+\varepsilon_2\varepsilon_3-\delta_{12}^2-\delta_{13}^2-\delta_{23}^2 ,\\ \alpha_3&=\varepsilon_1\varepsilon_2\varepsilon_3-2\delta_{12}\delta_{13}\delta_{23}-\delta_{12}^2\varepsilon_3-\delta_{13}^2\varepsilon_2-\delta_{23}^2\varepsilon_1 .\end{aligned} \quad (7)$$

In Ref. 7 it was shown that the RTS of two quantum dots, like in the inset of Fig. 1(b), can have exponentially narrow second-type (backscattering) conductance resonances. Evidently, such a resonance could also show itself in the current-voltage curve if the current-carrying electrons were monoenergetic to a good accuracy with the energy varying by the applied voltage near the energy of the backscattering resonance. In order to ensure a monoenergetic electron flux falling on such a structure with exponential accuracy we add the third quantum dot in front of these two as shown in Fig. 1(b). That is a physical reason why we expect the appearance of the current resonance of the second type for the three quantum dot RTS of Fig. 1(b).

In order to verify the indicated presumption we set $\Gamma_3=\delta_{13}=0$ and assume that the values Γ_1 and $|E_1-E_3|$ are small compared with the other energy parameters of the structure. For the configuration of quantum dots shown in Fig. 1(b) we have $\Gamma_1^{(1)}=\Gamma_1$ and $\Gamma_2^{(2)}=\Gamma_2$. Then as a particular case of the equations found we obtain

$$I_{12}=\frac{2e}{\hbar}\Gamma_1\frac{(E_1-E_3)^2+(\Gamma_1/\Gamma_2)(\delta_{12}^2+\delta_{23}^2)}{(E_1-E_3)^2+(\Gamma_1/\Gamma_2)(\delta_{12}^2+\delta_{23}^2)^2/\delta_{12}^2} . \quad (8)$$

Consider the current I_{12} as a function of level difference E_1-E_3. It is not difficult to see from Eq. (8) that this function has a narrow current dip near the point $E_1=E_3$ for $\delta_{12}\ll\delta_{23}$. According to Eq. (8), the position of this dip is independent of the value of level E_2, and its width is

$$\gamma=2(\Gamma_1/\Gamma_2)^{1/2}(\delta_{23}^2/\delta_{12}) \quad (\delta_{12}\ll\delta_{23}) . \quad (9)$$

Equation (8) is valid only if Γ_1, $|E_1-E_3|\ll\delta_{23}$. Therefore, it defines the current I_{12} everywhere over the region of the dip if $\gamma\ll\delta_{23}$, i.e., if $(\Gamma_1/\Gamma_2)^{1/2}(\delta_{23}/\delta_{12})\ll1$. In Fig. 2 we plot the $I_{12}(E_1-E_3)$ dependence using Eqs. (4), (6), and (7) for $\Gamma_1/\Gamma_2=0.01$, 0.001, and 0.0001, and $\delta_{12}/\delta_{23}=0.1$. In good agreement with the exact calculations, Eq. (9) gives, for these cases, the width of the dip $\gamma=2\delta_{23}$, $0.66\delta_{23}$, and $0.2\delta_{23}$, respectively, and Eq. (8) shows that the minimum value of the current in the dip is then approximately $\delta_{23}^2/\delta_{12}^2=100$ times smaller than its value aside from the resonance.

Let us verify if the current vs $(E_1-E_3)/\delta_{23}$ curves in Fig. 2 are proportional to the conductance vs $(E_1-E_3)/\delta_{23}$ curves found for the device in Fig. 1(b) with quantum dot 1 taken away (in the latter case E_1 stands for the energy of incident electrons). Then, evidently, quantum dot 1 would play the role of energy filter and the physical mechanism of narrow current dip could be reduced to the mechanism of the backscattering conductance resonance studied earlier.[1-7] Consider the indicated two quantum dot structure with dots 2 and 3. Assuming, as for Eq. (8), $\Gamma_3=0$, we have for the conductance of this structure[13]

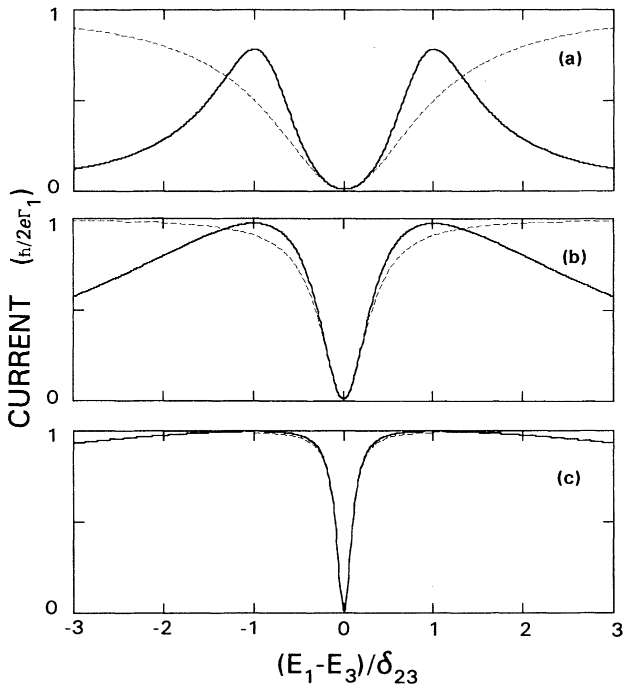

FIG. 2. Dimensionless current $I_{12}\hbar/2e\Gamma_1$ as a function of dimensionless level distance $(E_1-E_3)/\delta_{23}$ for the RTS shown in Fig. 1(b) found by Eqs. (4), (6), and (7) (solid lines) for $\delta_{12}/\delta_{23}=0.1$: (a) $\Gamma_1/\Gamma_2=0.01$, (b) $\Gamma_1/\Gamma_2=0.001$, and (c) $\Gamma_1/\Gamma_2=0.0001$. The dashed lines are the same dependencies found by Eq. (8).

$$G=\frac{e^2}{\pi\hbar}\frac{(E_1-E_3)^2\Gamma_2^{(1)}\Gamma_2^{(2)}}{\left|\left[E_1-E_2+\frac{i}{2}\Gamma_2\right](E_1-E_3)-\delta_{23}^2\right|^2} . \quad (10)$$

Though the minima of both I_{12} and G correspond to $E_1=E_3$, we see that Eqs. (8) and (10) are different in general. Proposing for simplicity $E_2=E_3$, we find that a narrow dip appears in G vs E_1-E_3 dependence for $\delta_{23}\ll\Gamma_2$. The width of this dip is

$$\tilde{\gamma}=\delta_{23}^2/\Gamma_2 . \quad (11)$$

The common feature of γ and $\tilde{\gamma}$ is that these values are proportional to δ_{23}^2. However, the dependence on the other parameters of the structures is different. In particular, the width γ, not like $\tilde{\gamma}$, vanishes as the coupling Γ_1 with lead 1 tends to zero. Also, notice the existence of conductance zero for $E_1=E_3$, that in no case appears for the current I_{12}. Thus, the current curves for the three quantum dots considered are not similar to the conductance curve of the two quantum dot structure shown in the inset of Fig. 1(b). Evidently, this is due to the interference phenomenon coming from additional multiple reflections of tunneling electron between dots 1 and 2. Hence, we have to conclude that the interference phenomenon that is responsible for the effect found is more complicated and it cannot be simply reduced to the effect of narrow backscattering conductance resonance for the two quantum dot structure.

In the case of the two quantum dot RTS with three leads shown in Fig. 1(c), Eqs. (4)–(7) become very simple and coincide with the ones obtained in Ref. 11. It was found[11] that the second-type resonance (exponentially narrow dip) can appear in this structure for the current I_{14} between leads 1 and 4. Simultaneously, current I_{12} has the first-type (Breit-Wigner) resonance. In the absence of lead 2 the dip of the current I_{14} disappears. Thus, interestingly, the RTS shown in Fig. 1(c) could serve as a sensitive Y-branch switch based on the effect discovered.[11]

In the present paper we disregarded the charging and the inelastic-scattering processes inside the RTS. These processes are essential for resonant-tunneling structures[14] and the study of their influence on the conductance and current resonances of the second type could be an important problem for the further consideration.

In summary, the current through the non-one-dimensional RTS, similar to the conductance, can have resonances of the second type, i.e., the exponentially narrow dips with an exponentially small minimum, in the current-voltage curve. The two-lead RTS can have current resonance of the second type starting from the three quantum dot structure. The result obtained makes clear that the existence of the current resonances of the second type, as of the first one, is a quite general feature of the non-one-dimensional RTS.

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