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Dissipative quantum systems modeled by a two-level-reservoir coupling

A. O. Caldeira
Departamento de Física do Estado Sólido e Ciência dos Materias, Instituto de Física Gleb Wataghin,
CP 6165, Universidade Estadual de Campinas, 13081-970 Campinas, São Paulo, Brazil

A. H. Castro Neto
Loomis Laboratory of Physics, University of Illinois at Urbana-Champaign, 1110 West Green Street, Urbana, Illinois 61801

T. Oliveira de Carvalho
Departamento de Física do Estado Sólido e Ciência dos Materias, Instituto de Física Gleb Wataghin,
CP 6165, Universidade Estadual de Campinas, 13081-970 Campinas, São Paulo, Brazil

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The coupling between a quantum dynamical system and a two-level-system reservoir is analyzed within the framework of the Feynman-Vernon theory. We stress the differences between this reservoir and the well-known bath of oscillators and show that, in order to obtain the Langevin equation for the system of interest in the high-temperature regime, we have to choose a spectral distribution function $J(\omega)$ that is finite for $\omega=0$.

In the past two decades, there has been great effort in bringing out a theoretical treatment of dissipation in quantum systems whose dynamics is described by the Langevin equation

$$m\ddot{x}+\eta\dot{x}+\frac{\partial V}{\partial x}=F(t)\ , \tag{1}$$

where $F(t)$ is a random force:

$$\langle F(t)\rangle=0\ . \tag{2}$$

One possible way to achieve that is by coupling the quantum system of interest to a reservoir.[1] Under very general conditions[2,3] we can choose the reservoir to be a set of decoupled harmonic oscillators, each of which has a different characteristic frequency ω_k. These frequencies are distributed in such a way that the spectral function $J(\omega)$ is given by

$$J(\omega)=\begin{cases}\eta\omega & \text{for } \omega<\Omega\ \text{,and}\\ 0 & \text{for } \omega>\Omega\end{cases} \tag{3}$$

in order that the two-time correlation of the random force becomes

$$\langle F(t)F(t')\rangle=\frac{1}{2\pi}\int e^{-i\omega(t-t')}\eta\hbar\omega\coth\left[\frac{\beta\hbar\omega}{2}\right]d\omega\ , \tag{4}$$

where $\beta=1/kT$ and a cutoff frequency for ω is assumed, say Ω. In the long-time and high-temperature limit

$$|t-t'|\gg\Omega^{-1}\ \text{ and }\ \beta\gg\hbar|t-t'|^{-1} \tag{5}$$

one obtains

$$\langle F(t)F(t')\rangle=2\eta kT\delta(t-t')\ , \tag{6}$$

which is the classical two-time correlation of the random force.

We propose here another kind of reservoir: a set of decoupled two-level systems (which may be understood as a projection onto the first two levels of the bath of oscillators) which is coupled to the system of interest by

$$\mathcal{H}=\mathcal{H}_S+\mathcal{H}_i+\mathcal{H}_b\ , \tag{7}$$

where $\mathcal{H}_S$ represents the Hamiltonian of the system we are interested in and

$$\mathcal{H}_i=-\sum_{k=1}^{N}J_kX\sigma_{x_k}\ , \tag{8}$$

$$\mathcal{H}_b=\sum_{k=1}^{N}\frac{\hbar\omega_k}{2}\sigma_{z_k}\ . \tag{9}$$

We assume that the system of interest and the bath are decoupled at $t=0$, so that the initial condition is given by

$$\rho(0)=e^{\beta\mathcal{H}_b}\tilde{\rho}(0)$$

as usual. We treat the interaction problem within the Feynman-Vernon theory, obtaining for the influence functional

$$\mathcal{F}[x,y]=Tr_b[e^{-\beta\mathcal{H}_b}A_{y'y}(0,t)A_{xx'}(t,0)]\ , \tag{10}$$

where we have defined

$$A_{y'y}(0,t)\equiv T\exp\left[+\frac{i}{\hbar}\int_0^t\tilde{\mathcal{H}}_i(x(\tau))d\tau\right]e^{+i\mathcal{H}_bt/\hbar} \tag{11}$$

and

$$A_{xx'}(t,0)\equiv e^{-i\mathcal{H}_bt/\hbar}T\exp\left[-\frac{i}{\hbar}\int_0^t\tilde{\mathcal{H}}_i(x(\tau))d\tau\right]\ , \tag{12}$$

where T denotes the time-ordered product and

$$\tilde{\mathcal{H}}_i\equiv e^{i\mathcal{H}_bt/\hbar}\mathcal{H}_i(x(\tau))e^{-i\mathcal{H}_bt/\hbar}$$

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is the interaction Hamiltonian in the interaction picture.

We calculate $A_{y'y}$ and $A_{xx'}$ up to second order in the interaction strength J_k, see Ref. 4

$$A_{y'y}(0,t)\approx\left\{1+\frac{i}{\hbar}\int_0^t d\tau\,\tilde{\mathcal{H}}_i(y(\tau))-\frac{1}{\hbar^2}\int_0^t d\tau\int_0^\tau d\sigma\,\tilde{\mathcal{H}}_i(y(\tau))\tilde{\mathcal{H}}_i(y(\sigma))\right\}\times e^{i\mathcal{H}_b t/\hbar} \tag{13}$$

and

$$A_{xx'}(t,0)\approx e^{-i\mathcal{H}_b t/\hbar}\times\left\{1+\frac{i}{\hbar}\int_0^t d\tau\,\tilde{\mathcal{H}}_i(x(\tau))-\frac{1}{\hbar^2}\int_0^t d\tau\int_0^\tau d\sigma\,\tilde{\mathcal{H}}_i(x(\tau))\tilde{\mathcal{H}}_i(x(\sigma))\right\}. \tag{14}$$

We evaluate the trace [see (10)] keeping the terms up to second order in $\mathcal{H}_i$, and this leads to

$$F[x,y]\approx 1-\frac{1}{\hbar^2}\int_0^t d\tau\int_0^\tau d\sigma\{\langle\tilde{\mathcal{H}}_i(y(\sigma))\tilde{\mathcal{H}}_i(y(\tau))\rangle+\langle\tilde{\mathcal{H}}_i(x(\tau))\tilde{\mathcal{H}}_i(x(\sigma))\rangle-\langle\tilde{\mathcal{H}}_i(y(\tau))\tilde{\mathcal{H}}_i(x(\sigma))\rangle-\langle\tilde{\mathcal{H}}_i(y(\sigma))\tilde{\mathcal{H}}_i(x(\tau))\rangle\}\;, \tag{15}$$

where the linear terms in $\langle\tilde{\mathcal{H}}_i\rangle$ vanish, since $\tilde{\mathcal{H}}_i$ has only zeros in its diagonal.

We can carry out the averages $\langle\;\rangle$ over the reservoir obtaining for the influence functional[6]

$$F[q,\xi]=\exp\left\{-\frac{1}{\hbar^2}\int_0^t d\tau\int_0^\tau d\sigma\sum_\alpha J_\alpha^2\left[\xi(\tau)\xi(\sigma)\cos\omega_\alpha(\tau-\sigma)-2iq(\tau)\xi(\sigma)\tanh\frac{\beta\hbar\omega_\alpha}{2}\sin\omega_\alpha(\tau-\sigma)\right]\right\}, \tag{16}$$

where

$$q=(x+y)/2\;,\quad \xi=x-y\;.$$

Taking a continuous distribution of two-level systems, we define as in Ref. 3,

$$J(\omega)=\sum_\alpha\frac{J_\alpha^2}{\hbar}\delta(\omega-\omega_\alpha) \tag{17}$$

and write the final form of the influence functional for the system of interest as

$$F[q,\xi]=\exp-\frac{1}{\hbar}\int_0^t d\tau\int_0^\tau d\sigma\int_0^\infty d\omega\,J(\omega)\left[\xi(\tau)\xi(\sigma)\cos\omega(\tau-\sigma)-2iq(\tau)\xi(\sigma)\tanh\frac{\beta\hbar\omega}{2}\sin\omega(\tau-\sigma)\right]. \tag{18}$$

Now we may compare this influence functional with that obtained in Ref. 3, for the bath of oscillators. If we take the same spectral function as in Refs. 2 and 3, that means $J(\omega)=\eta\omega\theta(\Omega-\omega)$, this new environment gives us a very different behavior from the previous model. As we easily see from (18) the imaginary part of the exponent of the influence functional becomes explicitly temperature dependent and its frequency dependence will not allow us to write a "memoryless" damping term as before. This only happens at very low temperatures ($\beta\rightarrow\infty$) when the damping term turns out to be exactly as before.

On the other hand, the real part of the exponent of the influence functional has no temperature dependence at all. The form of that term clearly shows that we cannot ever write a diffusion coefficient for this case. This term will always give a non-Markoffian diffusion process.

In order to recover the same behavior, as with the bath of oscillators, one should work with a new $J(\omega)$ given by

$$J(\omega)=\begin{cases}\eta\omega\coth\dfrac{\beta\hbar\omega}{2} & \text{for } \omega<\Omega\;,\text{ and}\\ 0 \text{ if } \omega>\Omega\;,\end{cases} \tag{19}$$

which is finite for $\omega\rightarrow 0$ and temperature dependent.

Although at the present stage our results are purely academic, we think this could be a reasonable starting point for dealing with the relaxation of the magnetization of a spin glass.[5]

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[1] R. P. Feynman and F. L. Vernon, Ann. Phys. **24**, 118 (1963).

[2] A. O. Caldeira and A. J. Leggett, Ann. Phys. **149**, 374 (1983).

[3] A. O. Caldeira and A. J. Leggett, Physica **121A**, 587 (1983).

[4] P. Hedegard and A. O. Caldeira, Phys. Scr. **35**, 609 (1987).

[5] A. O. Caldeira and T. O. Carvalho (unpublished).

[6] We have used the relation $1-x^2=e^{-x^2}$, since we are considering only terms up to second order in the interaction Hamiltonian.