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# Existence of spin-wave solitons in an antiferromagnetic film

A. D. Boardman
*Department of Physics, Joule Laboratory, University of Salford, Salford, M5 4WT, England*

S. A. Nikitov
*Institute of Radioengineering and Electronics, Russian Academy of Sciences, Mokhovaya Street, 11, 103907, Moscow, Russia*

N. A. Waby
*Department of Physics, Joule Laboratory, University of Salford, Salford, M5 4WT, England*
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The propagation of nonlinear dipole spin waves in a film consisting of a two-sublattice, uniaxial, antiferromagnetic material has been investigated. The external magnetic field is assumed to be parallel to the anisotropy axis of the antiferromagnetic film and is directed parallel, or perpendicular, to the film surface. The existence of envelope solitons of spin waves is predicted and conditions for their creation are discussed.

## I. INTRODUCTION

In the last few years envelope solitons of spin or magnetostatic waves in thin ferromagnetic films have attracted a significant degree of attention. The conditions for their existence have been discussed and experiments directed towards their observation have been reported.[1-3] In contrast, thin films of antiferromagnetic materials have only recently[4] been studied, because the main problem was to obtain high-quality films. This is now possible due to advanced technology based on molecular beam epitaxy. It is expected that many new things will now be possible even though the microwave properties of bulk antiferromagnetic media have been studied rigorously in the past. Indeed, even the spectra of linear dipole spin waves in antiferromagnetic plates has been predicted and calculated.[5-12] It is the nonlinear behavior of antiferromagnetics that is of interest in this paper and these have been investigated, theoretically, during a study of the transmission of electromagnetic waves through an antiferromagnetic plate.[13,14] In the latter investigations the nonlinear susceptibility and permeability tensors were calculated and bistable regimes of transmission were obtained. Further investigations of the nonlinear properties of antiferromagnetics seem to be called for: first of all, because of the interest in the fundamental properties of this type of material and, second, because of its possible applications.

The antiferromagnetic resonance frequencies and the frequencies of any excited spin waves belong to the infrared part of electromagnetic wave spectrum and these facts make the use of antiferromagnetic media in different applications very attractive.

In this paper we discuss the formation of spin-wave envelope solitons in thin antiferromagnetic films, for different combinations of external magnetic-field spin-wave propagation directions.

## II. GENERAL BRIGHT SOLITON CONDITIONS

The starting point for the investigation of nonlinear spin waves in an antiferromagnetic film is a coupled set of Maxwell and Landau-Lifshitz equations for the ac magnetization. In anticipation of amplitude-modulated pulse transmission, it is assumed that pulse envelope is a slowly varying complex function. This means that the potential function in frequency space $\omega$ can be written as[15]

$$\phi(\mathbf{r},\omega-\omega_0)=F(x,y)A(z,\omega-\omega_0)\exp(izk_0)\ , \tag{1}$$

where $F(x,y)$ is the modal field of the guided wave, $k_0$ is the carrier wave number, $\omega_0$ is the carrier frequency and, most importantly of all, the envelope $A(z,\omega-\omega_0)$ is a *slowly varying* function of the propagation distance. Inserting this function into the Landau-Lifshitz equation, coupled to Maxwell's equations, leads to more equations involving the nonlinear wave number $\tilde{k}(\omega)$, where $\tilde{k}(\omega)=k(\omega)+\Delta k(\omega)$, $k(\omega)$ is the linear wave number as a function of frequency $\omega$, and $\Delta k(\omega)$ is the nonlinear shift. After expanding $k(\omega)$ about $k_0$ (the carrier value) in a power series in $(\omega-\omega_0)$, the following familiar nonlinear Schrödinger equation arises[16,17]

$$i\frac{\partial A}{\partial z}=\tfrac{1}{2}\,\mathrm{sgn}(\beta_2)\beta_2\frac{\partial^2 A}{\partial T^2}-\gamma\,\mathrm{sgn}(\gamma)|A|^2A\ , \tag{2}$$

where $\beta_2=(\partial^2 k/\partial\omega^2)_{\omega_0}$ is the *linear* group-velocity dispersion parameter (evaluated at $\omega_0$), $\mathrm{sgn}(\beta_2)$ is the sign of $\beta_2$, $\gamma=\partial k/\partial|A|^2$ is the *nonlinear* coefficient and $\mathrm{sgn}(\gamma)$ is the sign of $\gamma$. It should be emphasized that $\beta_2$ is defined by the continuous-wave (cw) *linear* dispersion equation, whereas $\gamma$ is defined by the cw *nonlinear* dispersion equation. Also, the necessary condition for the formation of *bright* envelope solitons is the Lighthill criterion[18]

$$\beta_2\gamma<0\ . \tag{3}$$

 

Let us now consider different types of dipole spin waves propagating in an antiferromagnetic film.

### III. TANGENTIALLY MAGNETIZED ANTIFERROMAGNETIC FILMS

In this section an antiferromagnetic film, with thickness $d$, and an external magnetic field $\mathbf{H}_0$ parallel to the easy axis is considered. It is assumed that the antiferromagnet has two sublattices and that the easy axis is parallel to the film surface. The net magnetizations ($\mathbf{M}_0$) of each sublattice are equal to each other. For this case, surface and volume waves could propagate in the film but the calculation is restricted to waves propagating perpendicular to the external magnetic field, as surface spin waves.

The dispersion relation for surface spin waves has the following form:[9,10]

$$\exp(2kd)=[(\mu_1-1)^2-\mu_2^2]/[(\mu_1+1)^2-\mu_2^2] , \tag{4}$$

where $\mu_1,\mu_2$ are the magnetic permeability tensor components[5]

$$\begin{aligned} \mu_1&=1+\frac{R^2g^2(\omega_+\omega_- -\omega^2)}{(\omega_+^2-\omega^2)(\omega_-^2-\omega^2)} ,\\ \mu_2&=R^2g^2\frac{(\omega_- -\omega_+)\omega}{(\omega_+^2-\omega^2)(\omega_-^2-\omega^2)} ,\\ R^2&=8\pi M_0H_a , \end{aligned} \tag{5}$$

where $\omega_\pm=g(H_c\pm H_0)$, $H_c^2=H_a(2H_E+H_a)$, $H_a=(\beta-\beta_1)M_0$, $H_E=-\lambda M_0$, $\beta$ and $\beta_1$ are anisotropy constants, $\lambda$ is the exchange constant, and, finally, $g$ is a gyromagnetic ratio. There are two branches of surface waves. There is a lower branch, between the frequencies

$$\omega_1<\omega<g\left[\left[H_c^2+\frac{R^2}{2}\right]^{1/2}-H_0\right] .$$

This region has a positive $\beta_2$ [i.e., $\mathrm{sgn}(\beta_2)=+1$] and is called the normal dispersion region. For the upper branch, between frequencies

$$g\left[\left[H_c^2+\frac{R^2}{2}\right]^{1/2}+H_0\right]<\omega<\omega_2,$$

$$\beta_2<0 \quad [\mathrm{sgn}(\beta_2)=-1] ,$$

the region has anomalous dispersion. The boundaries of the frequency spectra are set by $\omega_1$ and $\omega_2$, where

$$\omega_{1,2}=\frac{1}{\sqrt{2}}\{2g^2(H_c^2+H_0^2)+g^2R^2 \mp[g^4R^4+8R^2g^4H_0^2+16g^4H_0^2H_c^2]^{1/2}\}^{1/2} . \tag{6}$$

If $H_0=0$ (note that the microwave field is not equal to 0) the waves propagate within the frequency regions

$$\begin{aligned} &gH_c<\omega<g\left[H_c^2+\frac{R^2}{2}\right]^{1/2} ,\\ &g\left[H_c^2+\frac{R^2}{2}\right]^{1/2}<\omega<g[H_c^2+R^2]^{1/2} . \end{aligned} \tag{7}$$

The schematic spectra for these waves are shown in Fig. 1(a).

For this last case ($H_0=0$) the group-velocity dispersion coefficients are the following:

Lower branch:

$$\begin{aligned} \beta_2&=+\frac{1}{V^3}\frac{g^2R^2d^2e^{-kd}}{2\omega_0}>0 ,\\ V&=\frac{g^2R^2de^{-kd}}{2\omega_0},\\ \omega_0&=\left[g^2H_c^2+g^2\frac{R^2}{2}(1-e^{-kd})\right]^{1/2} . \end{aligned} \tag{8}$$

Upper branch:

$$\begin{aligned} \beta_2&=-\frac{1}{V_1^3}\frac{g^2R^2d^2e^{-kd}}{2\omega_{01}}<0 ,\\ V_1&=\frac{g^2R^2de^{kd}}{2\omega_{01}} ,\\ \omega_{01}&=\left[g^2H_c^2+g^2\frac{R^2}{2}(1+e^{-kd})\right]^{1/2} . \end{aligned} \tag{9}$$

The nonlinear coefficients $\gamma$ [Eq. (2)] can be calculated in a number of ways. A more complicated way[13,14,19] than the simpler method of Zvezdin and Popkov[1] is not justified in terms of accuracy. The simple method, developed for ferromagnetic films, yields a value of the nonlinear coefficient $\gamma$ that is the correct order of magnitude for this coefficient at the homogeneous antiferromagnetic resonance. It is, therefore, correct to rather a high accuracy for thin films, when $kd\ll 1$.

The calculation of $\gamma$ can be performed by setting the uniaxial axis to coincide with the $z$ axis. For small deviations of the magnetization from the equilibrium state, the $z$ component of the magnetization then becomes

$$M_z\simeq M_0\left[1-\frac{|m_x|^2+|m_y|^2}{2M_0^2}\right] , \tag{10}$$

where $M_0$ is the static value and $m_{x,y}$ are the ac components.

In the limit $kd\ll 1$, $M_z\simeq M_0-M_0|A|^2$, and, after setting $H_0=0$ and using the inequalities (7), the nonlinear coefficient for the lower and upper branches are respectively,

lower branch:

$$\gamma\simeq+\frac{1}{V}\frac{gH_aH_E}{[2H_aH_E+H_a^2]^{1/2}} ; \tag{11}$$

upper branch:

$$\gamma\simeq+\frac{1}{V_1}\frac{gH_EH_a}{[2H_aH_E+H_a^2]^{1/2}}. \tag{12}$$

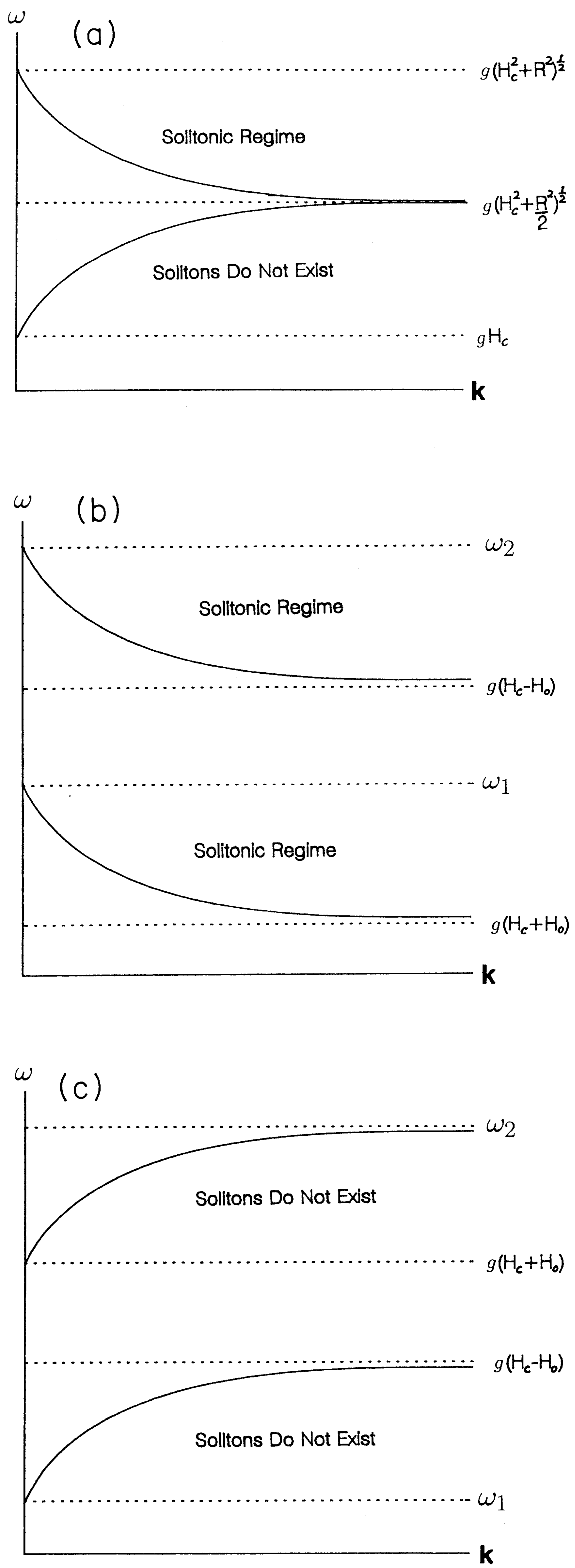


FIG. 1. Dispersion spectra $\omega(k)$ for the three types of magnetostatic spin waves in an antiferromagnetic film: (a) surface waves in a tangentially magnetized film; (b) volume waves in a tangentially magnetized film; (c) volume waves in a normally magnetized film.

Thus, the Lighthill criterion (3) is satisfied for the upper branch but is not satisfied for the lower branch. The dispersion equation for *volume spin waves* is

$$\tan\left[\frac{kd}{\sqrt{\mu_1}}\right]=\frac{2\sqrt{\mu_1}}{\mu_1-1}\ . \tag{13}$$

The spin-wave spectrum for $H_0\neq 0$ consists of two branches. The lower branch is

$$g(H_c+H_0)<\omega<\omega_1 \tag{14a}$$

and the upper branch is

$$g(H_c-H_0)<\omega<\omega_2\ , \tag{14b}$$

where $\omega_{1,2}$ are defined by Eq. (6). The schematic spectra for these spin-waves are shown in Fig. 1(b). In this case, however, the waves belonging to *both* branches possess anomalous dispersion.

If $H_0=0$ the only branch that exists with anomalous dispersion lies in the frequency region

$$gH_c<\omega<g[H_c^2+R^2]^{1/2}\ . \tag{15}$$

For this last example

$$\begin{aligned}\beta_2&=-\frac{1}{V_2^3}\frac{g^2R^2d^2}{4\sqrt{g^2H_c^2+g^2R^2}}<0,\\ V_2&=-\frac{g^2R^2d}{2\sqrt{g^2H_c^2+R^2}}\ .\end{aligned} \tag{16}$$

The nonlinear coefficient is given by Eq. (12), except for replacing $V_1$ by $V_2$. Thus, for these waves, the Lighthill criterion (3) is also fulfilled.

## IV. PERPENDICULARLY MAGNETIZED ANTIFERROMAGNETIC FILM

The two-sublattice antiferromagnetic film with uniaxial anisotropy and an easy axis perpendicular to the film surface will now be investigated. In this case, for a rather weak external magnetic field $(0<H_0<H_c)$, magnetizations of the sublattices are counter parallel to each other and are also perpendicular to the film surface.

The dispersion relation for dipole spin waves is the following:[6]

Symmetric modes:

$$\mathrm{cotan}\sqrt{\mu_1}kd=\sqrt{\mu_1}\ . \tag{17a}$$

Antisymmetric modes:

$$\tan\sqrt{\mu_1}kd=-\sqrt{\mu_1}\ . \tag{17b}$$

The spectrum, again, consists of two branches, for which the frequencies are defined as follows:

Lower branch:

$$\omega_1<\omega<g(H_c-H_0)\ . \tag{18a}$$

Upper branch:

$$g(H_c+H_0)<\omega<\omega_2\ ; \tag{18b}$$

where $\omega_{1,2}$ are also given by Eq. (6) and the spectra are shown in Fig. 1(c). Here, however, waves belonging to both branches have normal dispersion. In zero external field ($H_0=0$) only one branch of dipole spin waves exists, namely

$$gH_c < \omega < g[H_c^2+R^2]^{1/2} \tag{19}$$

with a group-velocity dispersion

$$\beta_2 = -\frac{1}{V_3}\frac{gH_c d^2}{4} < 0, \quad V_3 = \frac{gH_c d}{2} . \tag{20}$$

The nonlinear coefficient is

$$\gamma \simeq -\frac{1}{V_3}\frac{gH_E H_a}{H_c} < 0 . \tag{21}$$

Hence the Lighthill criterion is not satisfied.

It is worth noting that this situation is opposite to that with a normally magnetized *ferromagnetic* film. For the latter, the nonlinear coefficient is positive. This difference arises because of the influence of the demagnetizing field inside a ferromagnetic film. In a normally magnetized antiferromagnetic film with oppositely magnetized sublattices this demagnetizing field is equal 0. This is an important distinction that can change the sign of the nonlinear coefficient and as a consequence, break the fulfillment of the Lighthill criterion. Nevertheless, the Lighthill criterion can be fulfilled for the case of a normally magnetized antiferromagnetic film, if the external magnetic field is greater than $H_c$. Under this physical condition the preferred arrangement of the magnetic moments of the sublattices, with respect to the uniaxial axis, is symmetric. This produces a ferromagnetic magnetization vector, in the direction of the uniaxial axis. The analogous situation can appear for a uniaxial antiferromagnet with weak ferromagnetism. These cases will be considered in a forthcoming paper. In the meantime for the cases considered here the Lighthill criterion and, consequently, the necessary condition for the existence of envelope solitons is satisfied for the surface (upper branch) and volume (both branches) dipole spin waves in tangentially magnetized antiferromagnetic films, with the uniaxial axis parallel to the film surface.

The necessary and sufficient conditions for such envelope soliton formation must include the fact that the power of the spin wave is above a certain threshold, defined by solving the nonlinear Schrödinger equation exactly by the inverse scattering method.[20] This dimensionless power threshold is

$$|A|^2 = \frac{\beta_2}{T_0^2\gamma} \tag{22}$$

where $T_0$ is the width of the initial excited spin-wave pulse. $|A|^2$ can be related to the power of a dipole spin wave in a tangentially magnetized antiferromagnetic film through the result

$$P = \pi L d^2 M^2 \omega |A|^2 \tag{23}$$

where linear modal fields have been used (within a weakly nonlinear approximation), $L$ is the width of the film and $d$ is its thickness.

The substitution of (23) into (22) gives the desired threshold value for the creation of solitons of dipole spin waves. In order to arrive at an estimate, the following parameters of a typical antiferromagnet $MnF_2$ have been used:[4] $M_0=0.6$ kG, $H_a=8$ kOe, $H_E\simeq 530$ kOe, $d=10$ $\mu$m, $L=0.5$ cm, $\omega/2\pi=80$ GHz, $T_0=5\times10^{-8}$ s. These figures imply a power threshold, $P=5$ mW. This value can be very easily achieved in a conventional microwave or spin-wave experiment. Hence, the necessary and the sufficient condition for soliton formation can be readily fulfilled for dipole spin waves in antiferromagnetic films.

## V. CONCLUSIONS

The possibility of envelope soliton formation for types of dipole spin waves, propagating in two-sublattice uniaxial antiferromagnetic films, has been investigated. If the external magnetic field is parallel to the axis of anisotropy, and is in a plane of the film, surface and volume dipole spin waves propagate within the film. For an external magnetic film perpendicular to the film surface, and parallel to the anisotropy axis, volume dipole spin waves can exist in the antiferromagnetic film, for rather weak fields ($H_0<H_c$). It has been shown that, for the tangentially magnetized antiferromagnetic film, the necessary and sufficient conditions for envelope soliton formation are satisfied. Hence, the solitons can exist for the surface waves belonging to the optical (upper) branch of the spectrum and for volume waves belonging to both the optical (upper) and acoustic (lower) branches of the spectrum. It is also explained why solitons cannot exist for volume waves propagating in a normally magnetized antiferromagnetic film. An expression for the power required for soliton formation is derived and estimates are provided using the antiferromagnetic material $MnF_2$ as an example.


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