Classical optics in generalized Maxwell Chern-Simons theory

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We consider the propagation of electromagnetic waves in a two-dimensional polarizable medium endowed with Chern-Simons terms. The dispersion relation (refractive index) of the waves is computed and the existence of linear birefringence and anomalous dispersion is shown. When absorption is taken into account we find the classic signature of a Voigt effect. In the case where linearly polarized, three-dimensional waves pass through a two-dimensional plane we show that there is optical activity and compute the analog of Verdet's constant.

I. INTRODUCTION

The successful application of fractional statistics in the description of the fractional quantum Hall effect has led to the study of a large number of models involving anyons and the Chern-Simons interaction.[1,2] Motivated by this success some authors have considered effective field theories for high-temperature superconductors in two dimensions. Such models involve the Chern-Simons interaction coupled to matter in a planar system in which there is no residue of the third spacelike direction. The models describe what is usually referred to as the "anyon superconductor." The gauge field for such an interaction is not the electromagnetic field A_μ but a "statistical" gauge field a_μ which does not have a propagating F^2 term in the Lagrangian. Other authors have emphasized the importance of including electromagnetic interactions as well.[3,4]

One of the characteristic features of the Chern-Simons interaction is the breaking of parity and time reversal symmetries. This has prompted a number of optical experiments to determine whether such a phenomenon might be observed in the high-temperature superconducting materials.[5–8] The experiments use the fact that anyons would induce a Faraday effect (also called circular birefringence) resulting in optical rotation of plane polarized waves impinging normal to the layers. The anyon or Chern-Simons field generates an effective magnetic field which points in a particular direction in space, breaking P and T invariance. A theoretical model for this scenario has been studied by Wen and Zee.[9] They look at a system which describes a model superconductor in which two independent gauge invariances are broken by condensate effects. A more general discussion of P and T breaking effects in the relation to the experiments may be found in Refs. 10–13. More literature may be found in Refs. 14 and 15.

However, optical activity (circular birefringence) is not the only characteristic feature of wave propagation in a medium with anyons. P and T breaking effects should show up in wave modes which travel along the layers and not merely perpendicular to them. In particular one should have the Voigt effect which implies a splitting in the absorption spectrum due to the presence of an effective magnetic field. It is therefore of great interest to examine the propagation of waves in two-dimensional systems, including the effects of a Chern-Simons interaction.

In this paper we consider waves both in a two-dimensional system and in three dimensions impinging normally on a layered structure of two-dimensional systems. In the latter case our setup differs from that of Wen and Zee.[9] Rather than considering the model of an anyon superconductor we investigate the magneto-optical effects for a Maxwell field coupled to a classical polarizable medium with Chern-Simons interactions included. For the medium we use a standard model with a continuum of charged particles bound to their sites by harmonic oscillator forces. In the interest of generality we include two types of gauge field: an electromagnetic A_μ and a statistical gauge field a_μ and include the most general array of Chern-Simons terms for these. The arguments for including both flavors of gauge field have been presented a number of times[3,4] though few authors have considered a general array of Chern-Simons terms. One could think of these terms as effective interactions due to (microscopic) degrees of freedom which are not explicitly taken into account.

We derive the dispersion relation and absorption spectrum of the purely two-dimensional system and discuss the characteristics of wave propagation in this system. For the (3+1)-dimensional system in which a layered structure of (2+1)-dimensional systems is embedded, we derive the dispersion relation and examine the rotation of the polarization plane (circular birefringence) of this medium.

II. IN THE PLANE

We begin with a Lagrangian of general form:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{\alpha}{2}\varepsilon_{\mu\nu\lambda}A^\mu\partial^\nu A^\lambda + \beta\varepsilon_{\mu\nu\lambda}A^\mu\partial^\nu a^\lambda + \frac{\gamma}{2}\varepsilon_{\mu\nu\lambda}a^\mu\partial^\nu a^\lambda - J^\mu A_\mu - j^\mu a_\mu + \mathcal{L}_M \quad (1)$$

with α, β, and γ as Chern-Simons parameters. A_μ and a_μ are electromagnetic and statistical gauge fields, respectively, with $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, and $\mathcal{L}_M$ is a Lagrangian for matter. J^μ and j^μ are conserved currents. We assume that $\mathbf{J} = \mathbf{j}$ since the currents arise from a single species of particle, but that $J^0 \neq j^0$ in general. It is clearly important to identify which of the gauge fields various charges couple to. Models of the anyon superconductor take into account a neutralizing background charge density for the electromagnetic gauge field A_μ: the zeroth components of the currents do therefore differ in a general model. We assume that all the three Chern-Simons terms, in principle, can be generated by microscopic variables. With a change of variables, $a_\mu \to \tilde{a}_\mu$,

$$\tilde{a}_\mu = \left(1 - \frac{\beta}{\gamma}\right)^{-1} \left(a_\mu + \frac{\beta}{\gamma} A_\mu\right) , \tag{2}$$

the Lagrangian is kept in the same form, but with transformed parameters,

$$\tilde{\alpha} = \alpha - \frac{\beta^2}{\gamma}, \quad \tilde{\beta} = 0, \quad \tilde{\gamma} = \left(1 - \frac{\beta}{\gamma}\right)^2 \gamma , \tag{3}$$

and renormalized charge and current,

$$\tilde{J}^\mu = J^\mu - \frac{\beta}{\gamma} j^\mu, \quad \tilde{j}^\mu = \left(1 - \frac{\beta}{\gamma}\right) j^\mu . \tag{4}$$

We note that $\beta = \gamma$ is a special case, since the coupling between the gauge fields and currents then vanishes and the statistics field disappears, leaving only a free, massive Maxwell-Chern-Simons field A_μ. If $\alpha = \beta = \gamma$, this is simply a free, massless Maxwell field. However in the following we shall assume that we do not have this special situation. We work with the transformed parameters and fields, but use the original notation of Eq. (1). The only effect of the transformation then is to put the mixed Chern-Simons term to zero, i.e., $\beta = 0$.

With $\beta = 0$ the field equations are decoupled:

$$\partial_\nu F^{\mu\nu} - \frac{\alpha}{2} \varepsilon^{\mu\nu\lambda} F_{\nu\lambda} = -J^\mu , \tag{5}$$

$$\frac{\gamma}{2} \varepsilon^{\mu\nu\lambda} f_{\nu\lambda} = j^\mu \tag{6}$$

with $f_{\nu\lambda} = \partial_\nu a_\lambda - \partial_\lambda a_\nu$. Thus we have one Maxwell-Chern-Simons field of mass α and one pure Chern-Simons field, both coupled to essentially the same current. In addition to the field equations, the equation of motion for microscopic charges should be specified. Here we take the simplest classical model for a polarizable medium:[16]

$$m\ddot{s}^i + m\gamma_d \dot{s}^i + m\omega_0^2 s^i = -e[E^i + \mathcal{E}^i + \varepsilon_{ij}\dot{s}^j(B + b)] , \tag{7}$$

$\mathbf{s}$ is a displacement vector, γ_d is a damping factor, and ω_0 is the resonant frequency. E^i and B are the electric and magnetic parts of the field $F_{\mu\nu}$ and $\mathcal{E}$ and b the corresponding components of $f_{\mu\nu}$. We introduce parameters which distinguish between the dynamical or "optically active" charges and the static or "background" charges in the material. $-en_0$ is the (charge) density of the optical or moving charges. For the A_μ field, this density may be fully or partially neutralized by a background charge density en_b. For generality we also include a neutralizing background ρ_b for the anyonic charges coupling to a_μ, though this may be small compared to $-en_0$. The currents may be written

$$j^0 = \rho + \rho_b, \tag{8}$$

$$J^0 = \rho + n_b e, \tag{9}$$

$$\mathbf{J} = \mathbf{j} = \rho \dot{\mathbf{s}} . \tag{10}$$

Since the Chern-Simons (CS) field is nondynamical, it can be eliminated from the equations by means of Eq. (6), giving

$$\mathcal{E}^i = \varepsilon_{ij} \frac{j^j}{\gamma}, \quad b = -\frac{\rho + \rho_b}{\gamma} . \tag{11}$$

We approximate the charge density and current by continuous functions. For the CS field this corresponds to a mean field approximation. The effect of the CS field is to introduce an additional effective magnetic field, $-\rho_b/\gamma$. Note that the nonstatic part of the charge density ρ does not contribute to this effective field since it cancels in the contributions from $\mathcal{E}$ and b. The equation of motion for the charges becomes

$$m\ddot{s}^i + m\gamma_d \dot{s}^i + m\omega_0^2 s^i = -e\left[E^i + \varepsilon_{ij}\dot{s}^j \left(B - \frac{\rho_b}{\gamma}\right)\right] . \tag{12}$$

The limit $\gamma \to \infty$ corresponds to the situation with only the Maxwell-Chern-Simons field A^μ present, and the additional effective magnetic field then vanishes.

In order to obtain self-consistent wave solutions we assume the following form for the waves:

$$\begin{aligned} \mathbf{E} &= \mathbf{E_0} e^{i(kx - \omega t)} , \\ B &= B_0 e^{i(kx-\omega t)} + B_c , \\ \mathbf{J} &= \mathbf{J_0} e^{i(kx-\omega t)} , \\ J^0 &= \rho_0 e^{i(kx-\omega t)} + \rho_c , \end{aligned} \tag{13}$$

where $\rho_c = (n_b - n_0)e$ and propagation is along the x axis. The constant part of the magnetic field is related to the charge density by

$$B_c = -\frac{\rho_c}{\alpha} . \tag{14}$$

Thus for $\alpha \neq 0$ the average charge density is linked to the average magnetic field. However when $\alpha = 0$ (a massless Maxwell field) B_c is independent of ρ_c. In order for Eq. (13) to be correct we must then require that ρ_c also be zero. Following Ref. 16 we disregard the oscillating part of B in the particle equation, since it is small by a relativistic factor v/c. The approximation leads to linear equations for the oscillating fields.

Note that the effect of the Chern-Simons interaction is to render the waves massive and thus there is loss of transversality. Parallel and transverse components of the vectors may be defined by

$$E_{0\|} \equiv k_i E_0^i / k \ , \quad (15)$$

$$E_{0\perp} \equiv \varepsilon_{ij} k^i E_0^j / k. \quad (16)$$

After eliminating B, ρ, and $\mathbf{J}$ from the field equations, two equations are obtained with $E_{0\|}$ and $E_{0\perp}$ unknown. Nontrivial solutions of this matrix equation are obtained when the determinant of the matrix for the coefficients of $E_{0\perp}, E_{0\|}$ vanishes. This gives immediately the dispersion relation:

$$k^2 = \omega^2 \left\{ 1 - \frac{\alpha^2}{\omega^2} + \omega_p^2 \frac{(-\omega^2 - i\gamma_d\omega + \omega_0^2)(1 + \frac{\alpha^2}{\omega^2}) + \omega_p^2 - 2\alpha\omega_c}{(-\omega^2 - i\gamma_d\omega + \omega_0^2)^2 + \omega_p^2(-\omega^2 - i\gamma_d\omega + \omega_0^2) - \omega^2\omega_c^2} \right\}, \quad (17)$$

where $\omega_p^2 = Ne^2/m$ and $\omega_c = \frac{e}{m}(B_c - \rho_b/\gamma)$. These are the plasma frequency and cyclotron frequency, respectively. The mean refractive index is thus $n = k/\omega$. The solutions are further characterized by

$$E_{0\perp} = iAE_{0\|}. \quad (18)$$

where

$$A = \frac{\omega}{\alpha} \left\{ 1 + \frac{\frac{\omega_c}{\alpha}(\omega^2 - k^2 - \alpha^2) + \omega_p^2}{(-\omega^2 - i\gamma_d\omega + \omega_0^2) - \frac{\omega_c}{\alpha}(\omega^2 - k^2)} \right\}. \quad (19)$$

When we are not too close to the natural frequency ω_0, we have $A \simeq \frac{\omega}{\alpha}$ and the electrical field propagates like

$$\mathbf{E} = E_{0\|}[\mathbf{e}_x \cos(kx - \omega t) - A\mathbf{e}_y \sin(kx - \omega t)]. \quad (20)$$

Thus, the waves are elliptically polarized in the plane, and away from the resonances the two main axes of the ellipse are defined by the longitudinal and transverse directions of the wave propagation.

Let us now summarize the physical consequences of these solutions. Going back to the dispersion relation, we see that there are two resonances, which for $\gamma_d = 0$ are located at

$$\omega_\pm^2 = \omega_0^2 + \frac{\omega_p^2 + \omega_c^2}{2} \pm \sqrt{\omega_0^2\omega_c^2 + \frac{1}{4}(\omega_p^2 + \omega_c^2)^2} \ . \quad (21)$$

This implies two areas of anomalous dispersion (i.e., the wave number decreases as the frequency increases), which then is associated with absorption. When $\omega_0 \gg (\omega_p^2 + \omega_c^2)/\omega_c$, the anomalous dispersion and absorption are close to the resonances at

$$\omega_\pm \approx \omega_0 \pm \frac{1}{2}\omega_c \ . \quad (22)$$

See Figs. 1 and 2. This is the two-dimensional version of the Voigt effect.[17] Normally, in three-space dimensions, this effect is found when plane waves are sent perpendicular to a constant $\mathbf{B}$ field. Linear birefringence and absorption are known to result. When the plane containing $\mathbf{E}$ is perpendicular to $\mathbf{B}$, the dispersion relation and absorption curve look qualitatively like the figures. However in three dimensions, we may also send the plane waves with $\mathbf{E}$ in the same plane as $\mathbf{B}$ (but still $\mathbf{k} \perp \mathbf{B}$) and then we only have one area of anomalous dispersion, lying between the two others. Thus, in three dimensions, one has two different wave numbers, say $k_\|$ and $k_\perp$, for a given frequency, and an absorption triplet. In the present two-dimensional case, there is only a single wave number $k_\|$, but two absorption lines. Therefore the two-dimensional analog of the Voigt effect yields an absorption doublet in the presence of an effective magnetic field. Note that, for $\rho_b \neq 0$, this effect is present

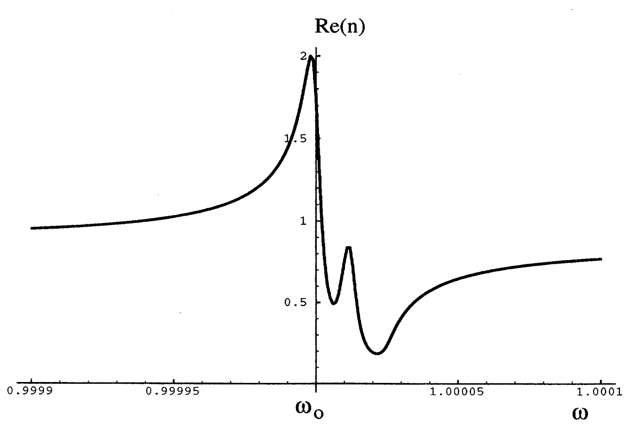

FIG. 1. Dispersion in the neighorhood of the resonant frequency. This is the real part of the complex refractive index, for $\alpha = 0.6$, $\gamma_d = 5 \times 10^{-6}$, $\omega_0 = 1$, $\omega_c = 0$, $\omega_p = 0.005$.

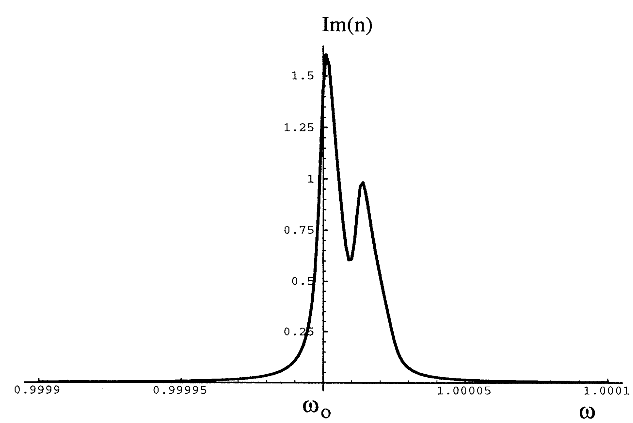

FIG. 2. Absorbtion in the neighborhood of the resonant frequency. This is the imaginary part of the complex refractive index for the same parameter values as in Fig. 1. Note that one peak is significantly higher than the other, as predicted in Eq. (18).

even with $B_c = 0$, due to the effect of the Chern-Simons interactions. On the other hand, when $B_c = \rho_b/\gamma$ there is a cancellation between the CS and the magnetic fields which gives $\omega_c = 0$. But even in this case there is a small splitting of the absorption line with resonances at

$$\omega_\pm = \begin{cases} \omega_0 \ , \\ \sqrt{\omega_0^2 + \omega_p^2} \ . \end{cases} \tag{23}$$

Only when $\alpha \to 0$ will the resonance at $\sqrt{\omega_0^2 + \omega_p^2}$ disappear, since the strength of this resonance is proportional to α^2. Thus, in this limit, with no P and T breaking effects, only the absorption line at ω_0 survives.

For a conducting medium, with a vanishingly small binding constant and frequency ω_0, we find only one resonance at

$$\omega = \sqrt{\omega_p^2 + \omega_c^2} \ . \tag{24}$$

The frequency of the second resonance tends to zero with ω_0, and also the strength of this resonance tends to zero, due to the prefactor ω^2 in Eq. (17).

From the discussion above we note the different effects of the two P and T breaking Chern-Simons terms in the Lagrangian, proportional to α and γ, respectively. The γ term introduces a shift in the resonance frequencies, like the effect of an external magnetic field. The α term, on the other hand, only affects the strength of the resonances. In the general case there are two absorption lines associated with the resonances, but in the P and T symmetric case, $\alpha = \omega_c = 0$, the strength of the second line goes to zero, and only the one at ω_0 survives.

III. FARADAY EFFECT IN THREE DIMENSIONS

We now consider the situation which is closest to the experiments referred to earlier, namely with waves impinging normally on a structure with two-dimensional layers. In a continuum description of the system, the effect of the layers is to constrain the currents to the two (x, y) directions in the layers and to include Chern-Simons terms involving only these two directions. The Lagrangian density then has the form (with indices now running from 0 to 3)

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{\alpha}{2}\varepsilon_{\mu\nu\lambda 3}A^\mu\partial^\nu A^\lambda + \frac{\gamma}{2}\varepsilon_{\mu\nu\lambda 3}a^\mu\partial^\nu a^\lambda - J^\mu A_\mu - j^\mu a_\mu + \mathcal{L}_M \ . \tag{25}$$

We consider waves propagating in the z direction, i.e, in the direction normal to the layers. We use the same equation as before (7) for the motion of the charges and disregard in this equation, once again, the oscillating part of the magnetic field. It is of some utility to introduce a new imaginary unit j which corresponds to the unit vector in the y direction in the plane. This leads to doubly complex numbers,[18] with $j^2 = i^2 = -1$ and $ij \neq -1$. We write $E = E^x + jE^y$, $S = s^x + js^y$, etc. Proof of optical rotation reduces to showing that the wave vector is a doubly complex number of the form $k = k_r - ij\,k_{ij}$. When k^2 (and thus k) is ij complex, we have for the electrical field

$$\begin{aligned} E &= E_0 e^{i(k_r z - \omega t)} e^{jk_{ij}z} \\ &= E_{0x} e^{i(k_r z - \omega t)}(\cos k_{ij}z + j\sin k_{ij}z). \end{aligned} \tag{26}$$

The combination ij ensures conservation of energy. (We do not consider a damping term γ_d in this section. Assuming wave solutions of the form $e^{i(kz-\omega t)}$, we find the dispersion relation to be

$$k^2 = \omega^2\left\{1 - ij\frac{\alpha}{\omega} + \frac{\omega_p^2}{-\omega^2 + \omega_0^2 + ij\omega\omega_c}\right\}, \tag{27}$$

which may be cast in an alternative form:

$$k^2 = \omega^2\left\{1 + \omega_p^2\frac{-\omega^2 + \omega_0^2}{(-\omega^2 + \omega_0^2)^2 - \omega^2\omega_c^2} - ij\left(\frac{\alpha}{\omega} + \omega_p^2\frac{\omega\omega_c}{(-\omega^2 + \omega_0^2)^2 - \omega^2\omega_c^2}\right)\right\} \tag{28}$$

and compared to $k = k_r - ijk_{ij}$.

It is most usual in the literature to decompose the system into contra-rotating circular polarizations.[16] In this formalism we have $E_y = \pm iE_x$, corresponding to two different wave numbers $k_\pm$. It is easy to get the dispersion relation in this form too; one simply writes $ij \to \mp 1$ in (27). The different wave numbers are connected by $k_\pm = k_r \pm k_{ij}$.

We briefly discuss some characteristic features of this expression. First, we note that the Faraday effect is present both for $\omega_c \neq 0$ and for $\alpha \neq 0$. In other words, both Chern-Simons terms introduce effects similar to that of an external magnetic field, as one would expect. The Faraday effect only disappears when $\alpha = \omega_c = 0$. Note that, in general this does not imply a vanishing external magnetic field, but rather a cancellation of the effects of the magnetic field and the γ term.

After propagating a length ℓ through the medium, the polarization plane is rotated by an angle $\chi = k_{ij}\ell$. For pure Maxwell theory $(\alpha \to 0, \gamma \to \infty)$ we recover for ω_c small, the standard expressions

$$\chi = \ell V B_c \ , \tag{29}$$

$$V \simeq \frac{Ne^3\omega^2}{n\,m^2(\omega_0^2 - \omega^2)} \ , \tag{30}$$

where $n = k_r/\omega$, the mean refractive index and V is Verdet's constant. A similar expression is valid for finite γ and vanishing α, but with B_c then replaced by the effective field $B_c - \rho_b/\gamma$. If on the other hand $\alpha \neq 0$, but

$\omega_c = \omega_0 = 0$ (a conducting medium with no magnetic field), we have the simple expression

$$k^2 = \omega^2 - \omega_p^2 - ij\alpha\omega \ . \tag{31}$$

When $\alpha\omega \ll |\omega^2 - \omega_p^2|$ this gives a rotation angle for a propagating wave

$$\chi \simeq \frac{\alpha\omega l}{4(\omega^2 - \omega_p^2)} \ . \tag{32}$$

We note that the dispersion relation (31) has the same form as that found by Wen and Zee for their model of the T and P breaking superconductor [ω_p and α then correspond, in their Eq. (14) to the parameters μ and β, respectively]. For small frequencies, $\omega \ll \omega_p$, there is an exponential damping of both oscillation modes. This corresponds to a situation with a total reflection of the waves at the boundary of the medium, with a corresponding expression for the rotation angle for the reflected wave as given by Wen and Zee.

IV. SUMMARY

We have examined the optical effects of Chern-Simons interactions in the case of a classical polarizable medium. For wave propagation in a (2+1)-dimensional system we find in general elliptically polarized waves. When the charges are bound to sites by harmonic oscillator forces, we find an absorption doublet in the frequency spectrum, similar to that of the Voigt effect for waves propagation in a magnetic field.

We have also determined the dispersion relation of waves in a layered (3+1)-dimensional system, with Chern-Simons interactions defined in the planes associated with the layered structure. As expected we find a Faraday rotation of the polarization plane for waves propagating in the direction normal to the planes. The effect is present even without a magnetic field.

Considered as a macroscopic model for a system of (microscopic) anyons there clearly are some approximations involved in this model. In particular, the "smearing out" of the charge density implicitly involves a mean field approximation to the Chern-Simons interaction. This explains that a pure CS field affects the charges even in a purely classical treatment.

Clearly, the model discussed is not a very realistic model for the systems normally associated with anyons, namely the quantum Hall system and the high-temperature superconductors. But due to the P and T breaking effects of the Chern-Simons interactions, we expect optical effects similar to what we have discussed in any layered structure with anyonic excitations. This is particularly so for the Faraday rotation of waves impinging normal to a layered anyonic structure. But even the two-dimensional effects which we have discussed may be of some relevance for such a layered system. The two-dimensional waves then would correspond to waves in three dimensions propagating along the planes, with the electric field polarized in the plane and the magnetic field polarized normal to the planes. In a three-dimensional structure, in addition to these waves, there would be waves traveling along the planes with the electric field normal to these. But these waves will propagate as free Maxwell waves when the currents of the medium are confined to the planes. This difference in velocities leads to linear birefringence.

We have clearly not exhausted classical optics in this letter. It is natural to try to extend the solutions here to cope with boundaries. What is the analog of Fresnel's equations for these solutions? Reflection and refraction are also, in principle, ways of detecting a Chern-Simons term. In view of the nature of the Chern-Simons field it is likely that an external magnetic field would have some interplay with the effect of a nonzero CS coefficient, as we have observed here, perhaps leading to some new type of experiment for determining the value of the coefficient. We shall return to this and other questions.[19]

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