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Nonlinear viscous motion of vortices in Josephson contacts

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Nonlinear dynamics of vortices in current-carrying long Josephson contacts is considered for the cases of weak ($\lambda_J \gg \lambda$) and strong ($\lambda_J \ll \lambda$) couplings, where λ_J and λ are the Josephson and London penetration depths, respectively. The first case concerns the Josephson vortices described by the sine-Gordon equation, whereas the case $\lambda_J \ll \lambda$ corresponds to Abrikosov-like vortices with highly anisotropic Josephson cores which are described by an integral equation for the phase difference φ within the framework of a nonlocal Josephson electrodynamics. At $\lambda_J \ll \lambda$, an exact solution for the moving vortex in the overdamped regime is obtained, the fluxon velocity $v(j)$ and the voltage-current characteristic $V(j)$ are calculated. It is shown that the lack of the Lorentz invariance of the integral equation for φ in the nonlocal regime leads to specific features of the vortex dynamics as compared to the Josephson vortices. The results obtained are employed for the description of nonlinear viscous motion of magnetic flux along planar crystalline defects in superconductors. It is shown that any percolating network of planar defects can considerably reduce the critical current, change the field dependence of the flux-flow resistivity, and result in a nonlinear $V(j)$ in the flux-flow regime.

I. INTRODUCTION

Vortex dynamics and pinning in superconductors is of great interest due to both a variety of physical phenomena in the system of strongly interacting vortex lines and the importance for applications. For instance, the short coherence length ξ in high-T_c superconductors gives rise to a strong influence of crystalline defects on properties of vortex structures, which manifests itself in a magnetic granularity caused by a superconducting decoupling of crystalline grains.[1] As a result, a vortex structure can consist of two different types of vortices, namely, intragrain Abrikosov (A) fluxons having normal cores of radius $\sim\xi$ and the Josephson (J) vortices localized on the grain boundaries. Unlike A vortices, J vortices have no normal core and are phase kinks of length λ_J much larger than the London penetration depth λ. Because of their larger sizes, J vortices get pinned much more weakly than A fluxons, which ultimately causes the superconducting decoupling if the grain boundaries form a percolative network along which the magnetic flux can move through a superconductor at much smaller critical current I_c than that determined by bulk pinning.[2–5]

Planar crystalline defects can be considered as Josephson weak links as long as their critical current densities j_c are much less than the depairing current density j_d. There are two characteristic cases for which the properties of vortices in Josephson contacts can be qualitatively different. The first one corresponds to incoherent planar defects, such as high-angle grain boundaries with large contact resistance R and small j_c as compared to the intragrain critical current density j_{cb}. These defects give rise to J vortices having sizes λ_J much larger than λ. The second case concerns coherent planar defects with low R (twins, stacking faults, low-angle grain boundaries, etc.) which do not cause strong lattice distortions, but can lead to a moderate local decrease of j_d to some value j_c which can be larger than j_{cb}. Such defects may be treated as Josephson junctions in the strong-coupling limit for which λ_J becomes smaller than λ. They give rise to neither J vortices nor the pronounced magnetic granularity, but rather play the role of "hidden" weak links which can qualitatively change the normal core of A fluxons. As a result, there arise Abrikosov vortices with highly anisotropic Josephson cores much larger than ξ, which corresponds to a crossover between A and J vortices.[7] Hereafter such vortices are called AJ vortices. Figure 1 shows how an A vortex successively turns into AJ and J vortices when decreasing j_c from $j_c = j_d$ to $j_c \ll j_d$ (see below). In this paper, I consider the dynamics of J and AJ vortices in long Josephson junctions as a model of nonlinear flux flow along a network of crystalline defects in superconductors.

The dynamics of J vortices is described by the sine-Gordon equation for the phase difference $\varphi(x,t)$,

$$\ddot{\varphi} + \eta\dot{\varphi} = \varphi'' - \sin\varphi + \beta \ , \tag{1}$$

where the prime and overdot denote derivatives over the dimensionless time $\tau = t\omega_J$ and coordinate $\zeta = y/\lambda_J$, respectively, $\lambda_J = (c\phi_0/16\pi^2 j_c\lambda)^{1/2}$ is the Josephson magnetic penetration depth, $\omega_J = (2ej_c/\hbar C)^{1/2}$ is the Josephson plasma frequency, $\eta = 1/\omega_J RC$ is a dimensionless damping constant, j_c is the critical current density across the contact, R and C are its specific resistance and capacitance, respectively, ϕ_0 is the flux quantum, c is the speed of light, $-e$ is the electron charge, and $\beta = j/j_c$ is the dimensionless transport current density which is assumed to be uniformly distributed over the contact.[6]

Equation (1) is valid as long as the phase $\varphi(y)$ changes slowly over lengths $\sim\lambda$, which results in the local relationship $H = (\phi_0/4\pi\lambda)d\varphi/dy$ between the magnetic field

$H(y)$ and the phase gradient $d\varphi/dy$.[6] If, however, $\varphi(y)$ varies over lengths much shorter than λ, the relationship between H and φ', as well as the equation for φ become nonlocal.[7] Such a situation occurs in the strong-coupling limit for which the Josephson penetration depth $\lambda_J(j_c)$ becomes smaller than λ, which is equivalent to

$$j_c > j_l = \frac{c\phi_0}{16\pi^2\lambda^3} \sim \frac{j_d}{\kappa} . \tag{2}$$

Here $j_d = c\phi_0/12\sqrt{3}\pi^2\lambda^2\xi$ is the depairing current density and $\kappa=\lambda/\xi$ is the Ginzburg-Landau parameter. Note that in extreme type-II superconductors ($\kappa \gg 1$) the inequality $\lambda_J \ll \lambda$ holds in a wide region of $j_c (j_d/\kappa \ll j_c \ll j_d)$. Similar nonlocal regimes can arise in the weak-coupling limit ($j_c \ll j_l$) as well if H exceeds the bulk lower critical field H_{c1b}. In this case the period of the Josephson vortex structure is smaller than λ and Eq. (1) becomes unadequate.

The equations for $\varphi(y,t)$ and $H(x,y)$ which generalize

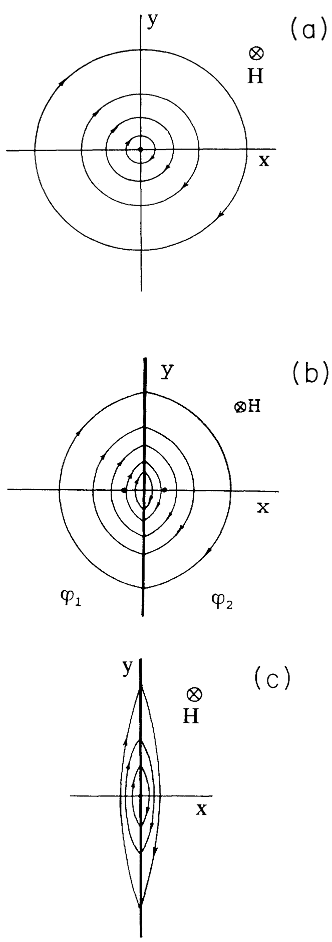

FIG. 1. Current lines in (a) A, (b) AJ, and (c) J vortices. Current lines in (b) are two sets of arcs centered at the point $x=-L$ for the half-plane $x>0$ and $x=-L$ for the half-plane $x<0$. The points $x=-L$ and $x=L$ indicate positions of fictitious A fluxons which determine $H(x,y)$ at $x>0$ and $x<0$, respectively (Ref. 7).

Eq. (1) to the case of $\varphi(y)$ changing over lengths much shorter than λ can be written as[7]

$$\ddot{\varphi}+\eta\dot{\varphi} = \frac{l}{\pi}\int_{-\infty}^{\infty} K_0\left[\frac{|x-u|}{\lambda}\right]\frac{\partial^2\varphi}{\partial u^2}du - \sin\varphi + \beta , \tag{3}$$

$$H(x,y) = \frac{\phi_0}{4\pi^2\lambda^2}\int_{-\infty}^{\infty} K_0\left[\frac{[x^2+(y-u)^2]^{1/2}}{\lambda}\right]\frac{\partial\varphi}{\partial u}du . \tag{4}$$

Here $K_0(x)$ is a modified Bessel function and the x and y axes are directed perpendicular and parallel to the plane of the contact, respectively. The characteristic length l is given by

$$l = \frac{\lambda_J^2}{\lambda} = \frac{3\sqrt{3}}{4}\frac{j_d}{j_c}\xi . \tag{5}$$

The integro-differential equation (3) reduces to Eq. (1) if $\varphi(y)$ slowly changes over the length $\sim\lambda$; thereby, $K_0(u)$ can be replaced by $\pi\delta(u)$. Otherwise, Eqs. (3) and (4) describe a nonlocal Josephson electrodynamics for which the phase $\varphi(y)$ changes over a length much shorter than that of $H(x,y)$. Here Eqs. (3) and (4) take into account only space variations of the phase of the order parameter $\Psi=\Delta\exp(i\varphi)$, assuming the superconducting gap Δ to be uniform, and $l \gg \xi$. This is valid if $j_c \ll j_d$; therefore, the contact can still be considered as a weak link which can carry the Josephson current density $j_c \sin\varphi$. It is the uniformity of Δ which enables one to generalize the Josephson electrodynamics to the nonlocal case, regardless of particular microscopic mechanisms. Here we do not consider the influence of anisotropy, tangential components of Ohmic currents which give rise to a term proportional to $\dot{\varphi}''$ in Eq. (1),[6] and also retardiation effects which can arise for fast variations of $\varphi(t)$.[8]

Depending on the relationship between j_c and j_l, Eqs. (3) and (4) can describe two qualitatively different types of vortices shown in Fig. 1. At $j_c \ll j_l$, Eq. (3) reduces to the sine-Gordon equation describing the J vortex in which both the phase $\varphi(y)$ and the field $H(y)$ vary over the same length $\sim\lambda_J$. As j_c increases above j_l, the Josephson penetration depth λ_J becomes shorter than λ, and so the phase $\varphi(y)$ and the field $H(y)$ in the vortex begin changing over differential spatial scales; i.e., the superconducting screening currents decay over the length λ, whereas the phase $\varphi(y)$ changes over the smaller length $l=\lambda_J^2/\lambda$.[7] In this case the J vortex turns into an AJ vortex with highly anisotropic Josephson core which is a phase kink of length $l \sim j_d\xi/j_c \gg \xi$ along the contact and of width ξ in the transversal direction. Unlike the A vortex, the AJ vortex has no normal core which appears only at $j_c \simeq j_d$ due to the suppression of $\Delta(r)$ in its center. To describe the normal core structure at $j_c \sim j_d$, one has to invoke both Eq. (3) and an equation for the gap Δ. Nevertheless, Eq. (4) with $\partial\varphi/\partial u = 2\pi\delta(u)$ gives the correct field distribution $H(y)=(\phi_0/2\pi\lambda^2)K_0(y/\lambda)$ in an A fluxon in the London approximation which assumes a pointlike core.[9]

The AJ vortices arise at $j_l < j_c < j_d$ when the nonlinear integral equation (3) cannot be reduced to the sine-

Gordon form. An exact static solution of Eqs. (3) and (4) which describes a single AJ vortex has been obtained in Ref. 7. In this paper, I consider the dynamics of AJ vortices which turns out to be different from that of J fluxons. This is due to the fact that, unlike the sine-Gordon equation (1), a more general integral equation (3) is no longer Lorentz invariant in the case of zero dissipation ($\eta=0$), with the Swihart velocity $c_s=\omega_J\lambda_J=c/(8\pi\lambda C)^{1/2}$ being the maximum speed propagation of electromagnetic waves along the contact.[6] As a result, the dynamic single-vortex solutions of Eq. (3) which describe AJ vortices moving with a constant velocity v at $\eta=0$ cannot be obtained from the static solution by the Lorentz transformation. Another distinctive feature of AJ vortices is the existence of a highly anisotropic phase core which turns into the normal core only at $j_c \sim j_d$.

The Josephson core manifests itself in a reduced pinning force along any planar crystalline defect with $j_c \ll j_d$ (twins, stacking faults, grain boundaries, etc.) which can become channels for preferential flux motion.[7] For instance, for the most effective core pinning (see, e.g., Ref. 10), the elementary pinning force $f_d \sim \alpha(\phi_0/4\pi\lambda)^2/l \sim \alpha\phi_0 j_c/c$ is inversely proportional to the core size $l \sim \xi j_d/j_c$ in the optimum case of a pinning potential varying over scales $\sim l$ (here α is the volume fraction occupied by pins). As a result, the longitudinal pinning force f_b along any defect with $j_c \ll j_d$ can be much smaller than the intragrain f_b due to the substantial difference in the core sizes of AJ and A vortices ($l \sim \xi j_d/j_c$ and ξ, respectively). Hence it follows that any percolative network of crystalline defects can strongly deform the normal cores of A fluxons, giving rise to weakly pinned J or AJ vortices which can move along such a dissipative network much more easily than intragrain A fluxons. As a first approximation, one can therefore neglect the pinning of AJ vortices and consider their viscous motion along the percolative network of "hidden" weak links, which, as will be shown below, results in a nonlinear current-voltage (I-V) characteristics in the flux-flow regime.

The paper is organized as follows. In Sec. II some general properties of moving J vortices needed for the further comparison to those of AJ vortices are considered. It is shown that qualitative features of the $v(j,\eta)$ dependence can be obtained by a dimensional analysis, regardless of specific form of single-vortex solutions of Eq. (1). In Sec. III the dynamics of AJ fluxons is analyzed. An exact solution which describes the moving AJ fluxon at $\eta \gg 1$ is obtained; the $v(j,\eta)$ dependences are calculated. On the basis of this solution, a self-consistent approximate description of the general case of arbitrary η is proposed. Section IV is devoted to nonlinear flux-flow regimes caused by viscous motion of J and AJ vortices under the action of the Lorentz force. Nonlinear voltage-current (V-I) characteristics and flux-flow resistivities R_f for both types of vortices are calculated. Planar defects are shown to change the field dependence of $R_f(H)$ from the linear Bardeen-Stephen law $R_f \propto H$ for A fluxons to $R_f \propto H^{1/2}$ for AJ vortices. In Sec. V manifestations of the peculiarities of J and AJ vortices in resistive and magnetic properties of superconductors are discussed.

II. J VORTEX

In the case of zero dissipation and driving force ($\beta=\eta=0$), Eq. (1) has the well-known solution $\varphi=\varphi(y-vt)$, which describes a moving J fluxon,

$$\varphi(y,t)=4\tan^{-1}\exp\left[\frac{y-vt}{\lambda_J\sqrt{1-v^2/c_s^2}}\right], \tag{6}$$

where the velocity v can take any value from $-c_s$ to c_s and the phase $\varphi(y)$ changes over the length $L=\lambda_J(1-v^2/c_s^2)^{1/2}$, which decreases as v increases. Because of such a Lorentz contraction, the vortex size $L(v)$ becomes comparable to λ as v approaches c_s, which results in a dynamic crossover between J and AJ vortices at $v>v_c \sim [1-(\lambda/\lambda_J)^2]^{1/2}c_s$. Therefore, in the narrow domain $v_c<v<c_s$, Eqs. (1) and (6) become inadequate, and the moving vortices are described by the more general Eq. (3). We shall not discuss here the case $v \geq v_c$ in more detail, assuming that $L(v) \gg \lambda$.

For nonzero β and η, the fluxon velocity $v(\beta,\eta)$ is determined by the balance of the Lorentz and viscous forces. Although the function $v(\beta,\eta)$ cannot be calculated analytically for arbitrary β and η, one can obtain an explicit dependence of v upon η by rescaling the coordinate

$$\xi \to (y-vt)/\lambda_J(1-v^2/c_s^2)^{1/2}.$$

Then Eq. (1) takes the form

$$\varphi''+f\varphi'-\sin\varphi+\beta=0, \tag{7}$$

where $f=\eta v(1-v^2/c_s^2)^{-1/2}$. As seen from Eq. (7), the single-vortex solution $\varphi(\xi)$, which satisfies the boundary conditions $\varphi(\infty)=2\pi$, $\varphi(-\infty)=0$, and $\varphi'(\pm\infty)=0$, exists at a certain eigenvalue f which depends only on β. Hence the velocity $v(\beta,\eta)$ can be expressed in terms of $f(\beta)$ as

$$v(\beta,\eta)=-\frac{c_s f}{\sqrt{\eta^2+f^2}}, \tag{8}$$

where the universal function $f(\beta)$, which gives the dependence of v on β in the overdamped regime $\eta \gg 1$, has been calculated numerically by Buttiker and Landauer.[11] As shown in Ref. 11, the value of $f(\beta)$ monotonically increases from 0 at $\beta=0$ to 1.19 at $\beta=1$, the function $f(\beta)$ being linear at small β and having a downward curvature ($d^2f/d\beta^2>0$). Hence it follows that the dependence $v(j)$ given by Eq. (8) also has a downward curvature at $\eta>\eta_c$ and an upward curvature at $\eta<\eta_c$, where $\eta_c \sim 1$. For instance, at small η formula (8) allows one to get an explicit dependence of v on β. In this case $v(\beta)$ sharply increases from 0 to c_s at small β and then remains constant as β increases. Therefore the details of $f(\beta)$ at $\beta \sim 1$ do not affect the dependence $v(\beta)$, which is mostly determined by the behavior of $f(\beta)$ at small β, where $f(\beta)=\pi\beta/4$.[11-14] Then Eq. (8) becomes

$$v=-\frac{c_s}{\sqrt{1+(4\eta/\pi\beta)^2}}. \tag{9}$$

This formula was obtained by McLaughlin and Scott by a perturbation theory,[13] although the qualitative features of the $v(\beta,\eta)$ dependences at $\eta \ll 1$ virtually follow from a dimensional analysis of Eq. (1).

The field distribution $H(x,y,t)$ in the J fluxon can be obtained from Eq. (4), where one can replace the slowly varying phase gradient $\partial\varphi/\partial u$ by its value at $u=y$. Then performing the integration (see, e.g., Ref. 15) and using Eq. (6), one finds

$$H(x,y,t)=\frac{\phi_0}{2\pi\lambda L}\frac{\exp(-|x|/\lambda)}{\cosh(y-vt)/L}\ , \tag{10}$$

where $L=\lambda_J\sqrt{1-v^2/c_s^2}$. Current lines in the J vortex are shown in Fig. 1(c).

III. *AJ* VORTEX

Now we consider the dynamics of the AJ vortex at $j_c > j_l$. In this case one has to analyze the integral equation (3), which, in the coordinate frame moving with the speed v, takes the form

$$\nu^2\varphi''-\nu\eta\varphi'=\frac{\varepsilon}{\pi}\int_{-\infty}^{\infty}K_0(|\zeta-u|\varepsilon)\varphi''(u)du-\sin\varphi+\beta\ . \tag{11}$$

Here $\nu=v/c_s$ is the dimensionless fluxon velocity, $\varepsilon=\lambda_J/\lambda=(j_l/j_c)^{1/2}$, and j_l is given by Eq. (2). In the nonlocal regime $j_c \gg j_l$, $\varepsilon \ll 1$, the phase $\varphi(y)$ changes over the length l much smaller than λ; therefore, the function $\varphi''(u)$ in Eq. (11) decays much faster than the kernel $K_0(\zeta)$. This allows one to replace $K_0(u)$ by its expansion at small argument $K_0(u)=\ln(2/u)-C$, where $C=0.577$ is the Euler constant.[15] Then the nonlinear integral equation (11) has the exact static solution $\varphi(y)=\pi+2\tan^{-1}(y/l)$,[16] which describes the Abrikosov vortex with highly anisotropic Josephson core of length l.[7] For the moving AJ vortex, the solution of Eq. (11) is sought for in the similar form

$$\varphi(y,t)=\theta+\pi+2\tan^{-1}\left[\frac{y-vt}{L}\right]\ , \tag{12}$$

with the constants θ, v, and L to be found from Eq. (11). By substituting Eq. (12) into Eq. (11), one can prove that the ansatz (12) indeed gives an exact solution of Eq. (11) in the overdamped regime $\eta \gg 1$ for which the term $\nu^2\varphi''$ in Eq. (11) is negligible (see Appendix A). The values θ, L and v are given by

$$\theta=\sin^{-1}\beta\ , \tag{13}$$

$$L=(1-\beta^2)^{-1/2}l\ , \tag{14}$$

$$v=-\frac{\beta v_0}{\sqrt{1-\beta^2}}\ , \tag{15}$$

where $v_0=c_s\varepsilon/\eta=Rc^2/8\pi\lambda^2$. As follows from Eqs. (14) and (15), the length $L(\beta)$ of the vortex core and the velocity $v(\beta)$ monotonically increase with β and diverge as j approaches $j_c(\beta\to 1)$. Therefore the increase of v results in the expansion of the AJ vortex at $\eta \gg 1$, unlike the Lorentz contraction of the J vortex at $\eta \ll 1$. However, at $\beta \to 1$ formulas (14) and (15) should be modified, since at large $L\sim\lambda$ the scale over which the kernel $K_0(u)$ changes turns out to be comparable to that of $\varphi''(u)$ and the term $\nu^2\varphi''$ in Eq. (11) becomes essential even if $\eta \gg 1$.

When taking account of the term $\nu^2\varphi''$, formula (12) is no longer the exact solution of Eq. (11). However, Eq. (12) can be used as a basis for a self-consistent approximate scheme based on exact conservation laws, which can be obtained as follows. First, we multiply Eq. (11) by φ' and integrate over ζ, taking into account the boundary conditions $\varphi'(\pm\infty)=0$ and $\varphi(\infty)-\varphi(-\infty)=2\pi$. Then the integrals of the first, third, and fourth terms in Eq. (11) vanish, and we arrive at the following formula which reflects the balance of the friction and Lorentz forces:

$$\nu\eta\int_{-\infty}^{\infty}\varphi'^2 d\zeta=-2\pi\beta\ . \tag{16}$$

To obtain the second integral relation, we multiply Eq. (11) by φ'' and integrate over ζ. This gives

$$\begin{aligned}\nu^2\int_{-\infty}^{\infty}\varphi''^2 d\zeta=&\frac{\varepsilon}{\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\varphi''(\zeta_1)\varphi''(\zeta_2)\\&\times K_0(|\zeta_1-\zeta_2|\varepsilon)d\zeta_1 d\zeta_2\\&-\int_{-\infty}^{\infty}\varphi''\sin\varphi d\zeta\ .\end{aligned} \tag{17}$$

Formulas (16) and (17) enable one to calculate the length of the core L and the velocity v of the moving vortex self-consistently by substituting Eq. (12) into Eqs. (16) and (17). Then integrations in Eqs. (16) and (17) result in the following equations for the variational parameters L and v: $2\pi\nu\eta/s=-2\pi\beta$ and $\pi\nu^2/s^3=\pi\varepsilon/s^2-\pi\cos\theta/s$, where $s=L/\lambda_J$. Hence

$$L=\frac{l}{\beta^2/\eta^2+\sqrt{1-\beta^2}}\ , \tag{18}$$

$$v=-\frac{\beta v_0}{\beta^2/\eta^2+\sqrt{1-\beta^2}}\ . \tag{19}$$

At $\eta^2(1-\beta^2)^{1/2} \gg \beta^2$, formulas (18) and (19) reduce to Eqs. (14) and (15). However, at $\beta\approx 1$ the term $\nu^2\varphi''$, which comes from a nonzero capacitance of the contact, eliminates the singularities in $L(\beta)$ and $v(\beta)$. The maximum values $L(1)=l\eta^2$ and $v(1)=\eta\varepsilon c_s$ increase with η; therefore, this approach is valid if $L(1)\ll\lambda$, i.e., $\eta<\varepsilon^{-1}$. Note that qualitative dependences of $L(\beta)$ and $v(\beta)$ are insensitive to details of $\varphi(y)$ and virtually follow from a dimensional analysis of Eqs. (16) and (17). Indeed, if the phase kink $\varphi(y)$ has a characteristic width L, one can estimate the derivatives in Eqs. (16) and (17) as $d\varphi/dy\sim 2\pi/L$ and $d^2\varphi/dy^2\sim 2\pi/L^2$, and obtain Eqs. (18) and (19) with an accuracy to numerical coefficients.

Substituting Eq. (12) into Eq. (4) and making use of the inequality $L\ll\lambda$, one can calculate the field $H(x,y,t)$ in the moving AJ fluxon by analogy to the static case.[7] The result can be presented in the form

$$H(x,y,t)=\frac{\phi_0}{2\pi\lambda^2}K_0\left[\frac{\sqrt{(L+|x|)^2+(y-vt)^2}}{\lambda}\right]\ . \tag{20}$$

At $L=0$, Eq. (20) reduces to the well-known solution of the London equation for an A vortex with a pointlike core for which $H(x,y)$ has a logarithmic singularity at $x=y=0$.[9] By contrast, the field

$$H(0)=(\phi_0/2\pi\lambda^2)[\ln(2\lambda/L)-C]$$

in the center of an AJ vortex remains finite, due to the finite size L of the phase core. The field distribution (20) allows a clear geometrical interpretation;[7] namely, the current lines in the half-plane $x>0$ coincide with those of a fictitious A vortex placed in the point $x=-L$, $y=0$ [likewise, in order to obtain $H(x,y)$ at $x<0$, one should put the A fluxon in the point $x=L$, $y=0$]. As j_c approaches j_d, the spacing $2L$ between these fictitious A vortices becomes of order ξ; thereby, the AJ vortex turns into an A vortex with a normal core. On the other hand, an increase of the vortex velocity v leads to an increase of $L(v)$, which may cause a dynamic crossover between AJ and J fluxons at large v for which $L(v)>\lambda$. It should be emphasized that at large distances $x^2+y^2 \gg L^2$ from the core the field $H(x,y)$ in AJ vortex coincides with that for A fluxon; therefore, the magnetic interaction of AJ vortices remains the same as that for A fluxons.

IV. NONLINEAR RESISTIVITY

A. Voltage distribution

Now we calculate the current-voltage characteristic due to the viscous motion of vortices along the crystalline defects, neglecting for simplicity the pinning effects. If vortices move with a constant velocity [$\varphi(y,t)=\varphi(y-vt)$], the Josephson voltage $V(y,t)$ across the contact is given by

$$V(y,t)=-\frac{\hbar v}{2e}\frac{\partial\varphi}{\partial y} . \tag{21}$$

This voltage is applied to the Josephson contact, unlike the measured macroscopic voltage $U(y,t)$ across a sample with the defect. Here U is defined as a line integral of $\mathbf{E}(x,y)$ along a straight line AB perpendicular to the defect, the distance AB being much larger than λ (Fig. 2). To obtain a relationship between the local $V(y,t)$ and the macroscopic $U(y,t)$, we use the integral form of the Maxwell equation,

$$c\oint \mathbf{E}\,dl=-\int\frac{\partial H}{\partial t}dx\,dy , \tag{22}$$

where the line integral is taken along the contour $ABCD$ shown in Fig. 2. This integral just gives the local voltage across the sample, $U(y,t)$, since the fields $E(x,y)$ and $H(x,y)$ exponentially decay over the length λ; therefore, only the part AB of the contour contributes to the integral. For the steady-state flux motion ($\partial H/\partial t=-v\partial H/\partial y$), formula (22) takes the form

$$U(y,t)=-\frac{v}{c}\int_{-\infty}^{\infty}H(x,y-vt)dx , \tag{23}$$

where the field distribution $H(x,y)$ for an arbitrary relationship between λ and λ_J is given by Eq. (4). Substituting Eq. (4) into Eq. (23) and performing the integration (see, e.g., Ref. 15), one obtains

$$U(y,t)=-\frac{\phi_0 v}{4\pi\lambda c}\int_{-\infty}^{\infty}e^{-|y-x|/\lambda}\frac{\partial\varphi}{\partial u}(u-vt)du . \tag{24}$$

Hence it follows that, unlike $V(y)$, the relationship between the macroscopic voltage $U(y)$ and the phase gradient $\partial\varphi/\partial y$ is nonlocal. If, however, the phase $\varphi(y)$ varies slowly over spatial scales $\sim\lambda$, then one can put $\exp(-|y|/\lambda)=2\lambda\delta(y)$, which reduces Eq. (24) to Eq. (21). The latter corresponds to the weak-coupling regime ($\lambda_J \gg \lambda$) for which the macroscopic voltage distribution $U(y)$ caused by moving J vortices coincides with the local Josephson voltage $V(y)$ across the contact. By contrast, in the nonlocal strong-coupling regime ($\lambda_J \ll \lambda$, $j_c \gg j_l$), the voltage distributions $U(y)$ and $V(y)$ caused by moving AJ vortices become different due to the different spatial scales of $H(y)$ and $\varphi(y)$.

As an illustration, we consider $U(y)$ for a periodic chain of AJ vortices with the period a much larger than the core length L. In this case the derivative $\partial\varphi/\partial u$ in Eq. (24) can be written in the form

$$\frac{\partial\varphi}{\partial u}=2\pi\sum_{n=-\infty}^{\infty}\delta(u-na-vt) . \tag{25}$$

Substituting Eq. (25) into Eq. (24), one obtains, after summation,

$$U(y)=-\frac{\phi_0 v}{2\lambda c}\frac{\cosh(y-a/2-vt)/\lambda}{\sinh a/2\lambda},\quad 0<y<a , \tag{26}$$

where the function $U(y)$ is periodic with period a. In this case the macroscopic voltage distribution $U(y)$ is much smoother than the local $V(y)$ across the contact. For a single AJ vortex, formula (26) yields

$$U(y,t)=-(\phi_0 v/2\lambda c)\exp(-|y-vt|/\lambda) ,$$

which also follows from Eq. (24) with $\partial\varphi/\partial u=2\pi\delta(u-vt)$.

In conclusion of this section, we derive a formula for the mean voltage U averaged over the space variations of $\varphi(y)$ along the contact of length L_0. Integrating Eq. (24) over y, one obtains

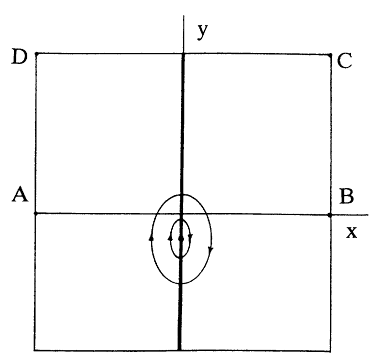

FIG. 2. Voltage distributions $V(y)$ and $U(y)$ around a fluxon moving along the planar defect parallel to the y axis. Here the Josephson voltage $V(y)$ is the local potential difference $\Delta\Phi(y)$ across the contact, whereas the macroscopic voltage $U(y)=\Phi_A-\Phi_B$ is the potential difference between two points A and B which are far away from the fluxon.

$$U=-\frac{\phi_0 v}{2\pi c L_0}\int_0^{L_0}\frac{\partial\varphi}{\partial u}du=-\frac{\phi_0 v}{ca}\ . \tag{27}$$

As follows from Eq. (21), the averaging of the local $V(y)$ gives the same result. Therefore, despite the difference of the spatial distributions of $V(y)$ and $U(y)$, their mean values prove to be equal.

B. Current-voltage characteristic

Now we use Eq. (27) to calculate the mean voltage V caused by the moving vortex chain, where the spacing $a(H)$ between fluxons depends upon the external magnetic field H. We consider here the case $a \gg L$ for which the vortex cores do not overlap; therefore, the velocity $v(j)$ is determined by the above formulas for a single vortex. In particular, for a J vortex, one has $a=\phi_0/2\lambda B$, where $B(H)$ is the magnetic induction. Then Eq. (27) reduces to $V(j)=-2\lambda B v(j)/c$, where $v(j)$ at $\eta \ll 1$ is given by Eq. (9). Hence

$$V(j)=\frac{R_J j}{\sqrt{1+j^2/j_0^2}}\ , \tag{28}$$

$$j_0=\frac{4}{\pi R}\left[\frac{\hbar j_c}{2eC}\right]^{1/2}\ . \tag{29}$$

At $j \ll j_0$ the V-I curve is linear, the specific resistance R_J per unit area of the contact being

$$R_J=\frac{RB(H)}{4}\left[\frac{\pi\lambda e}{\hbar j_c}\right]^{1/2}\ . \tag{30}$$

Formulas (28)–(30) are valid in the field region $H-H_{c1}<H_{c1}$, where $a \gg \lambda_J$. Here $H_{c1}=\phi_0/\pi^2\lambda\lambda_J$ is the lower critical field for J vortices, and the dependence of B on H was calculated by Owen and Scalapino.[17] Above the bulk lower critical field $H>H_{c1b}$, there appear intragrain A fluxons, which gives rise to a dependence of the effective Josephson penetration depth λ_J upon H.[18]

The V-I characteristic for AJ vortices at $\eta \gg 1$ can be obtained from Eqs. (15) and (27), whence

$$V=\frac{R_{AJ}j}{\sqrt{1-j^2/j_c^2}}\ . \tag{31}$$

At $j \ll j_c$ the V-I curve is linear, with the specific resistance R_{AJ} given by

$$R_{AJ}=\frac{c\phi_0}{8\pi a\lambda^2 j_c}R=\frac{2\pi\lambda}{a}\frac{j_l}{j_c}R\ . \tag{32}$$

We consider here the dependence $a(H)$ for two characteristic field regions $H_{c1}<H<H_{c1b}$, where H_{c1b} and H_{c1} are the corresponding lower critical fields for A and AJ vortices. The values H_{c1b} and H_{c1} are given by (see Refs. 9 and 7, respectively),

$$H_{c1b}=\frac{\phi_0}{4\pi\lambda^2}\left[\ln\frac{\lambda}{\xi}+\gamma_1\right]\ , \tag{33}$$

$$H_{c1}=\frac{\phi_0}{4\pi\lambda^2}\left[\ln\frac{\lambda}{l}+\gamma_2\right]\ , \tag{34}$$

with $\gamma_1=0.497$ (Ref. 19) and $\gamma_2=1-C=0.423$.[20] Note that H_{c1} at defects is smaller than the bulk H_{c1b} due to the gain in the core energy of the AJ vortex as compared to the normal core of the A fluxon ($l \gg \xi$). Therefore the magnetic flux first penetrates planar defects, where the equilibrium vortex density is higher than in the bulk. For instance, at $H_{c1}<H<H_{c1b}$ the intergrain A vortices are absent, and the magnetic flux penetrates a superconductor in the form of AJ vortex chains stretched out along planar defects. The equilibrium spacing $a(H)$ between AJ vortices in the chain is determined by the minimum of the thermodynamic potential, which yields (see Appendix B)

$$a(H)=\lambda\ln\frac{\phi_0\gamma_3}{\lambda^2(H-H_{c1})}\ , \tag{35}$$

with $\gamma_3 \sim 1$. This dependence is similar to that for A vortices in the vicinity of H_{c1b}.[9]

At $H>H_{c1b}$ the situation is complicated by the presence of intragrain A fluxons. However, at $H \gg H_{c1b}$ the

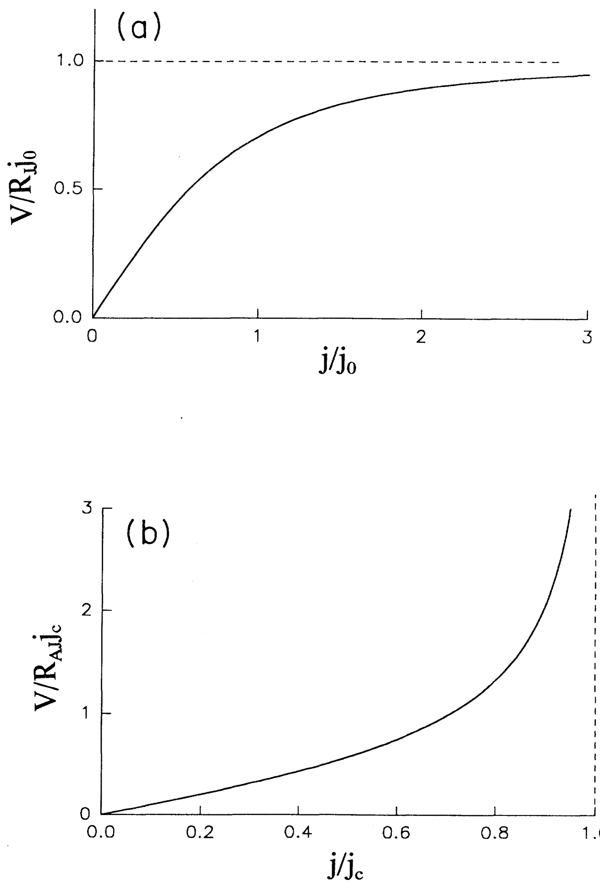

FIG. 3. Voltage-current characteristics of (a) a J vortex at $\eta \ll 1$ and (b) an AJ vortex at $\eta \gg 1$ described by Eqs. (28) and (31), respectively.

equilibrium density of vortices is mostly determined by their magnetic repulsion, regardless of details of the core structure.[9] In this case the difference in the core energies of A and AJ fluxons becomes negligible as compared to the characteristic energy of vortex interaction which is the same for both A and AJ vortices. Therefore the equilibrium densities of A and AJ vortices coincide over a wide field region $H_{c1} \ll H \ll H_{c1}(j_c/j_l)^2$, where the phase cores of AJ fluxons do not overlap ($a \gg \lambda_j^2/\lambda$). Hence it follows that $a(H)=(\phi_0/H)^{1/2}$, and formula (32) for the flux-flow resistance R_{AJ} takes the form

$$R_{AJ}=\frac{cR}{8\pi\lambda^2 j_c}\sqrt{\phi_0 H} \ . \tag{36}$$

This square-root field dependence of $R_{AJ}(H)$ differs from the linear dependence of the flux-flow resistivity upon H given by the Bardeen-Stephen model, due to the one-dimensional character of the vortex motion along planar defects.[21]

For both types of vortices, the flux-flow resistances R_J and R_{AJ} are proportional to R, although their field dependences are different. The V-I curves become nonlinear at high j due to the dependence of the core length L upon v, the character of the nonlinearity of $V(j)$ being qualitatively different for $\eta \gg 1$ and $\eta \ll 1$ (Fig. 3). For instance, as a result of the Lorentz contraction, the length of the J vortex at $\eta \ll 1$ decreases as v increases [see Eq. (6)]. By contrast, the core size $L(v)$ of the AJ vortex at $\eta \gg 1$ increases as v increases, the singularity in Eq. (31) at $j=j_c$ being eliminated by the effect of the nonzero capacitance of the contact [see Eqs. (18) and (19)]. A qualitatively similar $v(j)$ dependence for the J vortex at $\eta \gg 1$ has been obtained in Ref. 11 by means of computer simulations. Note that the above results give an asymptotically exact description of two opposite regimes, namely, $j_c \ll j_l$, $\eta \ll 1$ (J vortex in contact with high-resistivity R in the case of weak Josephson coupling) and $j_c \gg j_l$, $\eta \gg 1$ (AJ vortex in contact with low-resistivity R in the case of strong Josephson coupling).

V. DISCUSSION

The viscous motion of vortices along planar crystalline defects can be essentially nonlinear, which can contribute to the nonlinearity of I-V characteristics of superconductors in the flux-flow regime. As follows from the above results, any planar defect with $j_c < j_d$ results in the increase of l; thereby, the defect can turn into a channel for the easier vortex motion. Note that the flux motion along the percolating network can considerably affect temperature and field dependences of the flux-creep rate in high-T_c oxides.[4,22] A similar dissipative network for the flux motion has been obtained by Jensen *et al.*[23] by means of computer simulations of A vortices in a random pinning potential, although this effect seems to be the most pronounced for low-j_c planar defects which give rise to weakly pinned J vortices with $l \sim \lambda_J$. Recently, the preferential flux penetration along crystalline defects (twins) has been directly observed by Duran *et al.*[24] by means of a magneto-optical technique. Apart from the weaker flux pinning, this effect could be also due to the local reduction of the lower critical field H_{c1}, which results in a higher vortex density at planar defects due to the gain in the core energy. This may pertain to the elevated vortex density along the twin planes observed in decoration experiments on $YBa_2Cu_3O_{7-x}$ single crystals.[25,26]

The structure of a single vortex and the character of nonlinearity of the I-V curve in the flux-flow regime can be quite sensitive to the relationship between j_c and j_l, as well as to the value of the dimensionless damping constant $\eta=1/RC\omega_J$. There are two characteristic limiting cases $j_c \ll j_l$, $\eta \ll 1$ and $j_c \gg j_l$, $\eta \gg 1$ for which exact formulas (28) and (31) for $V(j)$ can be obtained. The first case could be considered as a model of high-angle grain boundaries in high-T_c superconductors which have low j_c and high contact resistance R due to local oxygen deficiency, long-range strain fields, etc.[1] Vortices localized at such defects seem to be J vortices described by Eq. (1) within the framework of local Josephson electrodynamics. As follows from Eq. (28), the V-I characteristic for J vortices at $\eta \ll 1$ has an upward curvature, with $V(j)$ approaching $R_J j_0$ at $j \gg j_0$ [Fig. 3(a)].

The opposite limiting case $j_c \gg j_l$, $\eta \gg 1$ may correspond to coherent planar defects (twins, low-angle grain boundaries, etc.) with high j_c and low contact resistance R. Such defects play the role of "hidden" weak links which do not affect the magnetic structure of A fluxons, but cause strong deformation of its core, which turns into a highly anisotropic phase kink of length l along the defect plane and of width ξ in the perpendicular direction. Such an AJ vortex is described by a nonlocal Josephson electrodynamics which results in the V-J characteristics (31) with downward curvature and a singularity at $j=j_c$ [Fig. 3(b)]. Besides the high-T_c oxides, a similar situation may occur in low-T_c superconductors as well, for example, in optimized high-j_c NbTi alloys, where the strong pinning is due to a dense network of thin $[d \approx (0.2-2)\xi]$ α-Ti ribbons.[27] Such ribbons may give rise to AJ vortices which can more easily move along the network than in the transversal direction for which the ribbons act as very strong pinning centers. For a periodic stack of s-i-s-i $\cdots$ or s-n-s-n $\cdots$ layers, this can be described within the framework of the intrinsic pinning model[28] for which the linear dynamics of vortices along the layers was considered by Clem and Coffey.[29]

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APPENDIX A: EXACT SOLUTION OF EQ. (11)

In the case $j_l \ll j \ll j_c$ and $\eta \gg 1$, Eq. (11) takes the form

$$v\eta\varphi'=\frac{\varepsilon}{\pi}\int_{-\infty}^{\infty}\ln(|\zeta-u|)\varphi''(u)du+\sin\varphi-\beta \ . \tag{A1}$$

Substituting $\varphi''(u)=-4su/(s^2+u^2)^2$ with $s=L/\lambda_J$ into

Eq. (A1), the integral term can be transformed by means of the identity (see, e.g., Ref. 15, p. 556)

$$4s\int_{-\infty}^{\infty}\ln(|\zeta-u|)\frac{u\,du}{(u^2+s^2)^2}=-2\pi\frac{\zeta}{\zeta^2+s^2}\ . \quad (A2)$$

Then Eq. (A1) becomes

$$\frac{2\nu\eta s}{\zeta^2+s^2}=\frac{2\zeta\varepsilon}{\zeta^2+s^2}-\frac{2s\zeta\cos\theta}{\zeta^2+s^2}-\frac{(s^2-\zeta^2)\sin\theta}{\zeta^2+s^2}-\beta\ . \quad (A3)$$

This equation can be written as

$$[c_1\zeta^2+c_2\zeta+c_3]/(s^2+\zeta^2)=0\ ,$$

with $c_{1,2,3}$ depending on θ, s, and ν. By equating the coefficients $c_{1,2,3}$ to zero, one obtains

$$\sin\theta=\beta\ , \quad (A4)$$

$$s\cos\theta=\varepsilon\ , \quad (A5)$$

$$-\nu\eta=s\sin\theta\ , \quad (A6)$$

which reduces to Eqs. (13)–(15).

APPENDIX B: EQUILIBRIUM DENSITY OF *AJ* VORTICES

We consider here two limiting cases $H-H_{c1}\ll H_{c1}$ and $H\gg H_{c1}$. In the first case, there is only a periodic chain of *AJ* vortices in the contact, and the bulk *A* fluxons are absent. If the period of the chain, $a(H)$, is much larger than l, the core structure of *AJ* vortices does not affect their equilibrium density, which is mostly determined by the vortex magnetic interaction. The latter is the same as that for *A* fluxons, which enables one to write the thermodynamic potential G of the vortex chain per unit length in the standard form (see, e.g., Ref. 9)

$$G=\frac{\phi_0}{4\pi a}\left[H_{c1}-H+\frac{\phi_0}{2\pi\lambda^2}\sum_{n=1}^{\infty}K_0\left[\frac{na}{\lambda}\right]\right]\ . \quad (B1)$$

At $H-H_{c1}\ll H_{c1}$, the distance $a(H)$ exceeds λ; therefore, only the vortex interaction with the nearest neighbors is essential. In this case one can retain only the term with $n=1$ in Eq. (B1) and use the asymptotic formula $K_0(z)=(\pi/2z)^{1/2}\exp(-z)$.[15] Then Eq. (B1) becomes

$$G=\frac{\phi_0}{2\pi a}\left[H_{c1}-H+\frac{\phi_0}{2\lambda^2}\left[\frac{\lambda}{2\pi a}\right]^{1/2}\exp\left[-\frac{a}{\lambda}\right]\right]\ . \quad (B2)$$

The value a is determined by the minimum of G, which yields

$$H-H_{c1}=\frac{\phi_0}{2\lambda^2}\left[\frac{a}{2\pi\lambda}\right]^{1/2}\left[1+\frac{3\lambda}{2a}\right]\exp\left[-\frac{a}{\lambda}\right]\ . \quad (B3)$$

Equation (B3) determines the equilibrium period $a(H)$ of the *AJ* vortex chain at $H-H_{c1}\ll H_{c1}$. With a logarithmic accuracy, a solution of Eq. (B3) is given by Eq. (35), where γ_3 is a number of order of unity.

At $H>H_{c1b}$ the intragrain *A* fluxons should be taken into account. However, at $H\gg H_{c1b}$ the density of *AJ* vortices coincides with that of *A* vortices, since both of them are determined by the same magnetic vortex interaction, which is insensitive to details of the core structure.

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