Experimental realization of mode locking during intrinsic quasiperiodicity in *p*-type germanium

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During low-temperature impact-ionization breakdown in extrinsic germanium, quasiperiodic current oscillations arise spontaneously. If the two competing frequencies (strictly speaking, the corresponding oscillatory components) can be localized in two spatially separated regions of the sample, mode locking can hardly be observed. However, if the two frequencies can be detected all over the sample, the frequency ratio undergoes an exemplary mode-locking sequence under variation of a transverse magnetic field. We find that the mode lockings are in accordance with the Farey-tree configuration, originally predicted to hold for periodically driven systems.

I. INTRODUCTION

The theoretical predictions made for the complex dynamical behavior of a nonlinear dissipative system with two competing frequencies were the subject of many experimental investigations in hydrodynamics,[1] chemical reaction kinetics,[2] or semiconductor breakdown,[3] for example. They started from a discrete mapping, the so-called "sine circle map,"

$$\Theta_{n+1}=\Theta_n+\Omega-\frac{K}{2\pi}\sin(2\pi\Theta_n) \pmod 1 , \tag{1}$$

where Θ_n denotes the angular variable, Ω the frequency ratio, and K the coupling constant. The mean number of rotations per iteration is given by the winding number $W=\lim_{n\to\infty}(\Theta_n-\Theta_0)/n$. Equation (1) describes the angular dynamics in the Poincaré section through a two-torus on which the continuous dynamics of a quasiperiodic motion takes place. Moreover, it provides several universal conjectures for the dynamics of two coupled oscillators.[4] Taking into account that such theory assumes a system with only one intrinsic frequency, which is modulated periodically by an external driving force, most of the experiments undertaken were realized under the following conditions: A periodic modulation of some control parameter was used to generate quasiperiodicity in a hitherto periodically oscillating system. Advantages are the direct accessibility of both model parameters, namely, the coupling strength and the frequency ratio, via two corresponding experimental parameters, namely, the amplitude and frequency of the external modulation. Predictions such as the mode-locking phenomenon (i.e., synchronization of the two frequencies leading to a rational winding number), the hornlike structure of the locked regions in the K-Ω phase diagram (i.e., Arnol'd tongues), as well as their hierarchical ordering according to the Farey tree, even the fractal dimension of the devil's staircase for $K=1$, could be confirmed experimentally with astonishingly high accuracy.

However, in contrast to these investigations, experiments focusing on the mode-locking phenomenon in undriven (i.e., not periodically driven) systems with two intrinsic, self-generated oscillation frequencies are scarce so far.[5–7] This paper reports on experimental investigations of self-generated quasiperiodicity and mode locking that develop during impurity-impact-ionization-induced avalanche breakdown in *p*-type germanium at low temperatures. The outline of the paper is as follows. In Sec. II, we give a brief description of the experimental setup and the underlying physics. In Sec. III, we present results of different mode-locked oscillatory behavior obtained from the analysis of quasiperiodic states in two parameter regimes. Finally, in Sec. IV, we discuss the results and deduce an explanation for the distinct properties of both states. Moreover, a comparison of our results with predictions of the theory for externally driven systems is performed.

II. EXPERIMENT

Our system consists of a homogeneously doped *p*-type germanium single crystal with an acceptor concentration of about 10^{14} cm^{-3}. The sample of dimension $(8.0\times2.0\times0.2)$ mm^3 was furnished with ohmic contacts, evaporating aluminum and alloying it afterwards with germanium by heat treatment. The electric circuit used in the following measurements is shown schematically in Fig. 1. The outer contacts of the sample were connected to a dc voltage source V_0 via a variable load resistor R_L, allowing to adjust the current flow I_S. A dc magnetic field B perpendicular to the current and the broad sample surface could also be applied using a superconducting coil. A copper metal shield surrounding the sample avoided disturbances by external irradiation. The two

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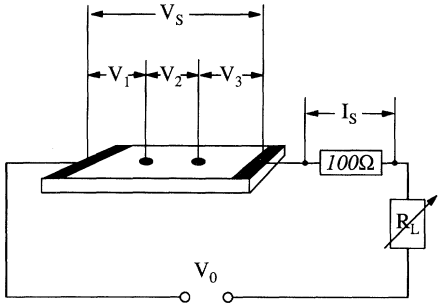

FIG. 1. Schematical illustration of the experimental setup. The dark areas on the surface of the semiconductor sample indicate the ohmic contacts.

inner contacts (of about 0.2 mm diameter) allowed a rough localization of potentially existing oscillating centers. Finally, the current I_S and the voltage V_S were determined via the voltage drops upon a constant 100-Ω load resistor and the sample, respectively.

At liquid-helium temperatures, the semiconductor sample becomes nearly insulating. Because of the reduction of the thermal energy, almost all charge carriers are frozen out at the impurities, having a binding energy of about 10 meV. Upon application of an electric field of a few V/cm, an avalanchelike multiplication of the mobile charge carriers takes place, leading to impurity-impact-ionization breakdown. A nonequilibrium phase transition from a weakly conducting ($T\Omega$ resistance range) to a strongly conducting state (100 Ω resistance range), accompanied by spatiotemporal structure formation, takes place. Current filaments as spatial structures and spontaneous current (and/or voltage) oscillations as temporal structures can be related to the presence of negative differential conductance in the S-shaped current-voltage characteristic.[8] A model based on a semiconductor-physical ansatz which is able to explain such phase-transition phenomena was put forward by Schöll.[9]

III. RESULTS

In dependence of the external parameters (i.e., magnetic field B, sample current I_S, and temperature T), our semiconductor system reveals many different modes of self-generated oscillatory behavior, thus representing an interesting study object of nonlinear dynamics. Under variation of the appropriate parameter, the well-known routes from regular to chaotic motion, like intermittency,[10] period doubling,[11] and quasiperiodicity,[12] can be observed. In this section, we present two quasiperiodic regimes with different features.

In the first regime, the quasiperiodic signal is composed of two incommensurate frequencies, the oscillatory components of which are generated in spatially separated parts of the sample. This can be verified by looking at the power spectra of the voltage drops $V_1(t)$, $V_3(t)$, and $V_S(t)$. Superposition of the spectra obtained from the partial voltages $V_1(t)$ and $V_3(t)$ corresponds to that of the global voltage $V_S(t)$.[13]

The successive emergence of both frequencies under variation of a control parameter (i.e., either decreasing the sample current or increasing the transverse magnetic field) can be described as follows. Starting from a stable fixed point, the system first bifurcates to spontaneous limit cycle oscillations (discernible in part 1 of the sample). Before the periodic oscillation becomes stable, a stochastical switching between both states can be observed. Upon further varying the control parameter, the second frequency arises from a similar transition to another limit cycle oscillation that takes place in a different part (i.e., part 3) of the sample. The resulting current signal then undergoes an intermittentlike transition of stochastically switching between periodicity with a single intrinsic frequency and quasiperiodicity with two incommensurate intrinsic frequencies. Finally, the quasiperiodic signal becomes stable.

Protruding characteristics of the present quasiperiodic oscillatory state are its robustness and insensibility to disturbances over a relatively large parameter regime and the nearly ideal time trace shown in Fig. 2. In what follows, we refer to the temporal voltage profile $V_S(t)$. Superimposed to the steady dc voltage of a few volts, the oscillatory ac signal typically displays a relative amplitude of about 10^{-3} and frequencies in the range between 0.1 and 10 kHz. The corresponding phase portrait in Fig. 3 shows off the toroidal structure of the underlying attractor, as is expected for a quasiperiodic dynamics.

In order to come across the mode-locking phenomenon, the sample current I_S was varied by changing the bias voltage V_0 accordingly, while all other parameters were kept constant. (Note that the transverse magnetic field B turned out to be another appropriate control parameter leading to equivalent results.) Hereto, we recorded the time traces for different parameter values and calculated the corresponding power spectra from which the two fundamental frequencies f_1 and f_2 were extracted. Their control parameter dependence is plotted in Fig. 4. Determining the actual ratio of the frequencies and rescaling it to the unit interval[14] leads to the according development of the winding number shown in Fig. 5. Presence of a weak coupling can be deduced from the fact that the two frequencies are locked onto a 1/1 ratio (corresponding to the 0/1 plateau in the course of the winding number) over a finite interval of the control parame-

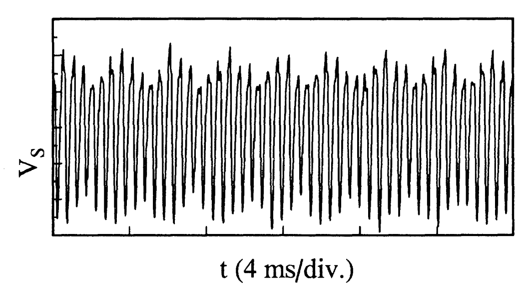

FIG. 2. Temporal structure of the self-generated quasiperiodic voltage signal obtained at the constant parameters time-averaged voltage $\bar{V}_S=2.178$ V, current $I_S=2.527$ mA, transverse magnetic field $B=11.5$ G, and temperature $T=2.1$ K.

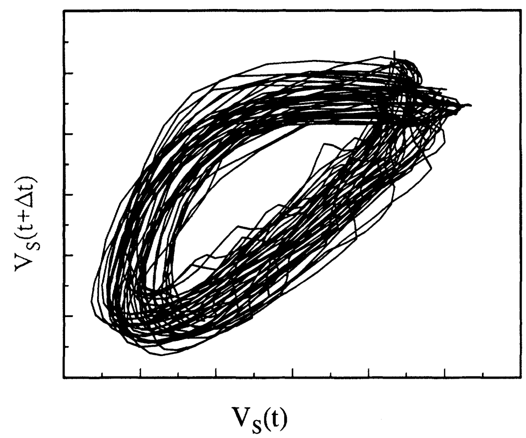

FIG. 3. Phase portrait of the quasiperiodic voltage signal of Fig. 2 constructed via the time-delay method ($\Delta t = 0.48$ ms).

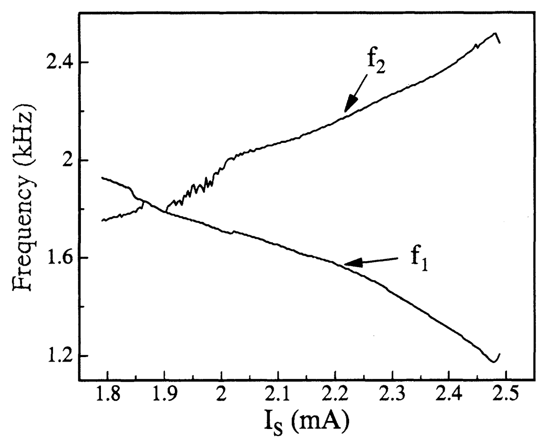

FIG. 4. Development of the two fundamental intrinsic frequencies under variation of the sample current obtained at the constant parameters $\overline{V}_S = 2.15$ V, $B = 10.0$ G, and $T = 1.8$ K.

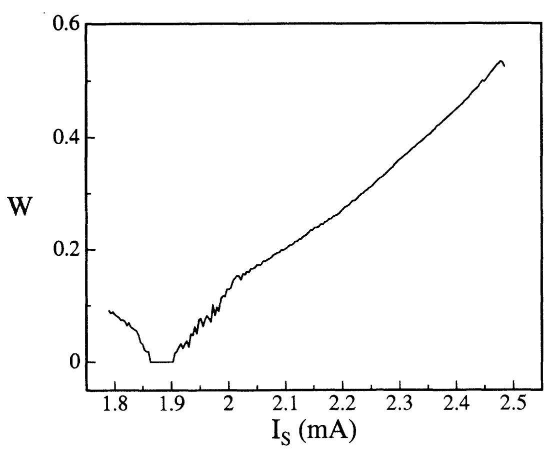

FIG. 5. Winding number as a function of the sample current obtained from the data of Fig. 4.

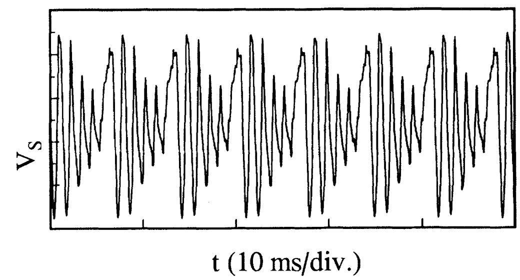

FIG. 6. Temporal structure of the self-generated mode-locked voltage signal obtained at the constant parameters $\overline{V}_S = 2.21$ V, $I_S = 2.53$ mA, $B = -0.4$ G, and $T = 2.01$ K.

ter, as is expected for the case of two coupled nonlinear oscillators. We emphasize that any attempts to increase the coupling strength via changing other parameters (magnetic field and/or temperature) were not successful. It turned out that strongest coupling could be accomplished just by applying the parameter set of Figs. 4 and 5. At a higher temperature ($T = 2.1$ K), even the 1/1 frequency locking step disappeared.

Next, let us turn to the second parameter regime of self-generated quasiperiodicity. Therefore, we reverse the polarity of the bias voltage V_0, tune the time-averaged sample voltage and current to the values $\overline{V}_S = 2.21$ V and $I_S = 2.53$ mA, respectively, and further apply a weak transverse magnetic field. In contrast to the first regime, there is only a very small parameter range where the present quasiperiodic state persists. The individual power spectra obtained from different parts of the sample reveal no significant difference from each other. Thus, unlike in the former case, oscillating centers cannot be detected.

An exemplary time series of the voltage signal in question is shown in Fig. 6. Since both frequencies are locked onto a rational ratio (1/5), the spectrum reveals peaks only at a multiple of the smaller fundamental[15] frequency f_1, while the second one f_2 must be assigned to the fifth peak (harmonic) of the power spectrum plotted in Fig. 7.

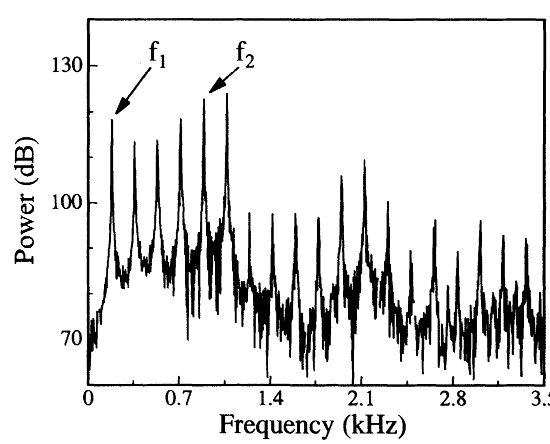

FIG. 7. Power spectrum of the mode-locked voltage signal of Fig. 6.

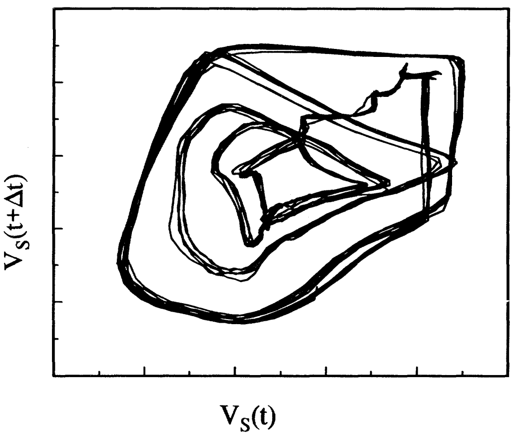

FIG. 8. Phase portrait of the mode-locked voltage signal of Fig. 6 constructed via the time-delay method ($\Delta t = 0.24$ ms).

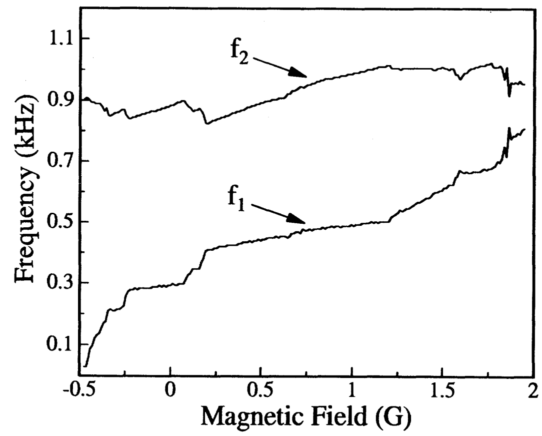

FIG. 10. Development of the two fundamental intrinsic frequencies under variation of the transverse magnetic field obtained at the constant parameters $\bar{V}_S = 2.21$ V, $I_S = 2.53$ mA, and $T = 2.01$ K.

The closed mode-locking structure of the underlying attractor becomes evident in the phase portrait of Fig. 8. A minute change of the transverse magnetic field leads to a quasiperiodic state with two incommensurate frequencies, as shown in Fig. 9. The formerly closed trajectory (cf. Fig. 8) traces out an invariant two-torus in phase space. Note that, because of the extreme sensitivity of the semiconductor to smallest changes of the sample current and a somewhat stronger robustness respective to the transverse magnetic field, the latter one turned out to represent the more appropriate control parameter for the subsequent investigations.

Following the scheme used in the first regime of quasiperiodicity, the development of the two intrinsic (i.e., self-generated) frequencies under variation of a control parameter (here, the transverse magnetic field) was detected. As we can gather from Fig. 10, both frequencies change their magnitude in a synchronous way over the whole parameter range examined. Plotting the ratio of the two frequencies versus the applied magnetic field (again rescaled to the interval [0,1]) now yields a staircaselike pattern, characterized by distinguished plateaus at each rational value of the winding number, according to the theoretical predictions for the devil's staircase (Fig. 11). The hierarchical ordering of the widths of the plateaus can be followed up to the 6/11 locking that belongs to the sixth generation in the Farey tree, as can be recognized in Fig. 12 (a closeup of Fig. 11). The pronounced scattering in the winding number at higher magnetic fields (around the 2/3 step) can be related to chaotic breakouts that take place due to overlapping Arnol'd tongues. Additional measurements performed at lower temperatures (with all parameters kept constant) clearly unveil a switching process between the two mode-locked states 1/2 and 1/3. This finding confirms our assumption that Arnol'd tongues do overlap in the present semiconductor system (in contrast to observations made for other experiments[2,7]).

Finally, an estimation of the relating fractal dimension

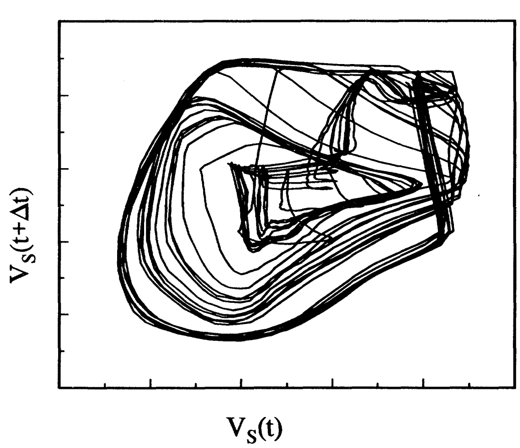

FIG. 9. Phase portrait of the quasiperiodic voltage signal obtained at the constant parameters $\bar{V}_S = 2.21$ V, $I_S = 2.53$ mA, $B = -0.38$ G, and $T = 2.01$ K ($\Delta t = 0.24$ ms).

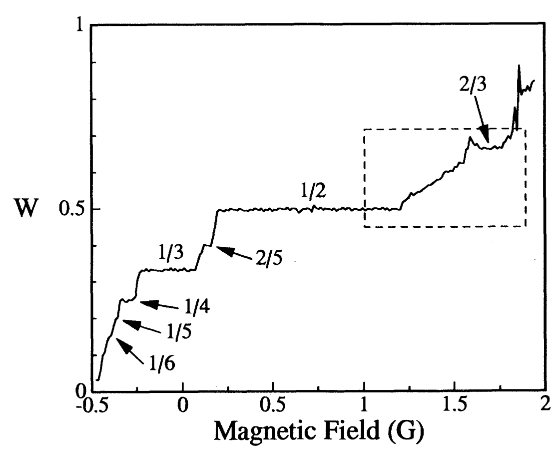

FIG. 11. Winding number as a function of the transverse magnetic field obtained from the data of Fig. 10. The dashed rectangular area is magnified in Fig. 12.

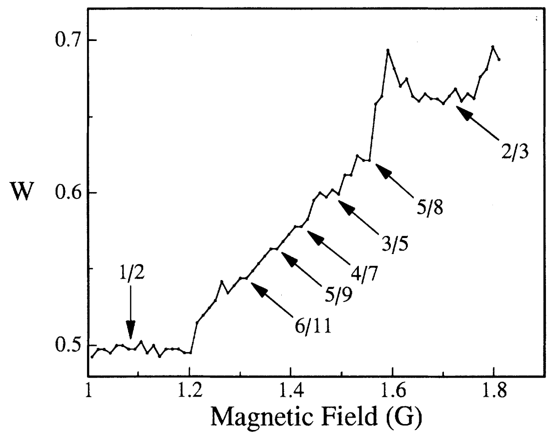

FIG. 12. Blowup of Fig. 11.

(all values of about 0.5) indicates that the coupling strength already exceeds the one corresponding to the critical line in theory. Systematic investigations on the parameter dependence of the coupling strength led to the following correlations: It increases with decreasing sample current, increasing transverse magnetic field, and decreasing temperature. Finding a parameter range of nearly critical coupling strength turned out to be practically impossible. Before reaching the aim, the present quasiperiodic state immediately becomes unstable if one leaves the close vicinity of the above parameter values.

IV. DISCUSSION AND CONCLUSION

In contrast to most publications dealing with quasiperiodicity, we have presented the behavior of two oscillatory components (respectively, their frequencies) that are intrinsic to our semiconductor system. The first quasiperiodic state, although having some nearly ideal characteristics, did not fulfill our expectations. The coupling strength between both frequencies could not be increased in a sufficient way, in order for the frequencies to undergo a distinct mode-locking sequence. One reason for this lack was found to be the fact that the frequencies are generated by two spatially separated oscillation centers. Their existence was detected by concerning the partial voltage drops (V_1 and V_3) of the sample, each revealing only one of the two intrinsic frequencies by which the quasiperiodic signal of the total voltage drop (V_S) is composed.

However, in the second case of quasiperiodic behavior, where the partial voltage drops did not show any significant difference from each other (i.e., the two frequencies could not be localized), the prevailing coupling strength was strong enough to cause the two intrinsic frequencies to lock onto different rational ratios over a finite parameter range, as predicted by theory. An estimation of the fractal dimension of the obtained staircase as well as the presence of chaotic breakouts at higher magnetic fields give rise to the assumption that the coupling strength already exceeds the critical value where the Arnol'd tongues begin to overlap. Unfortunately, the coupling strength could not be decreased down to criticality [corresponding to $K=1$ in the sine circle map, Eq. (1)]. There, the quasiperiodic state becomes unstable.

Nevertheless, we point out that, in the first instance, any direct comparison with the well-known phase and parameter space of externally (periodically) driven model systems seems to be inadequate for the present undriven experimental system. The critical coupling strength, in theory corresponding to a critical line at $K=1$ in the K-Ω plane, in practice turns out to be a quite complex and irregular function of all relevant parameters, thus leading to a (more or less) complicated surface in the high-dimensional parameter space. A measurement along this critical surface would require the simultaneous variation of at least two control parameters under the assumption that the course of the surface is already known.

In summary, one can say that some predictions made by theory for externally driven systems were also realized in our nondriven experiment. The tendency for both coupled frequencies to lock onto rational ratios, even the hierarchical ordering of the widths of the mode-locked steps in the winding number according to the Farey tree, could be clearly detected in the present intrinsic system (up to the limits of our resolution). There remain some other interesting investigations, such as the transition from self-generated quasiperiodicity to chaotic motion along the golden mean [while keeping the winding number at a constant value of $(\sqrt{5}-1)/2$]. Hereto, the coupling strength needs to be adjustable over an adequate range.

ACKNOWLEDGMENTS

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[14]For convenience, we map the interval [1,2] of the winding number measured onto the interval [0,1], in accordance with the modulo-1 symmetry of the Arnol'd tongues.

[15]Actually, the determination of the fundamental frequencies in an intrinsic oscillatory system is ambiguous or purely arbitrary, to some extent. As Haucke and Ecke (Ref. 5) have demonstrated, the concrete choice does not affect the quantities relevant for comparison with theory.