Comments

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Coupling of electrons to interface phonons in semiconductor quantum wells

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In a recent paper [B. K. Ridley and M. Babiker, Phys. Rev. B **43**, 9096 (1991)] the authors have argued that when retardation is taken into account the interface phonons of semiconductor superlattices obtained from the usual dielectric continuum approach are transverse polariton modes and that they therefore do not couple to electrons via the strong Fröhlich interaction. Here we show that the interface polaritons obtained from this dielectric continuum approach, in general, have both transverse and longitudinal components and that the dispersion and character of the modes are affected by retardation only for small wave vectors ($k \lesssim \omega_{\text{phonon}}/c$). For wave vectors of interest in electron-phonon scattering, the interaction is given to a good approximation by the usual Fröhlich interaction neglecting retardation.

The effects of confinement on the optical phonons and the electron-phonon interaction in polar semiconductor quantum wells and superlattices have been of considerable interest recently. In such systems theoretical and experimental studies indicate that the optical modes consist of interface modes and confined bulk modes.[1] Here we address an issue related to the interface modes. Theoretical work has invoked a strong interaction between electrons and the interface modes via the Fröhlich interaction,[2] which arises from the scalar potential generated by the optical lattice vibrations. This interaction has been used to account for experimental results such as recent work involving nonequilibrium phonon populations.[3] These interface modes are analogous to the Fuchs-Kliewer surface modes of a free dielectric slab,[4] and they have been studied extensively for quantum wells within a dielectric continuum approach.[5–7] A continuum approach for phonons in these systems has the advantage of providing analytic results for the electron-phonon coupling. In recent work based on lattice dynamics[8–10] and in other work using a more extensive continuum approach[11] it has been shown that these interface modes are described to a good approximation by the simple dielectric continuum approach.

In a recent paper[12] Ridley and Babiker note that when retardation (the finite velocity of light c) is included in the usual dielectric continuum approach the interface modes are polaritons. They argue that these polaritons are transverse modes and therefore do not have a scalar potential ϕ. Thus they do not couple to electrons via the strong Fröhlich interaction ($H_{\text{int}} = -e\phi$), but they may couple via the weaker $\mathbf{p} \cdot \mathbf{A}$ interaction, where $\mathbf{A}$ is the vector potential.

In the dielectric continuum model the electric field satisfies $\nabla \cdot \mathbf{E} = 0$ almost everywhere, but Ridley and Babiker incorrectly take this as evidence that $\mathbf{E}$ is purely transverse and thus that it possesses no scalar potential ϕ. In order that $\mathbf{E}$ be purely transverse, its divergence must vanish everywhere.[13] In fact $\nabla \cdot \mathbf{E}$ diverges at the interfaces of the quantum well owing to the presence of oscillating bound surface charge densities there. Thus $\mathbf{E}$ is not purely transverse.

In the present Comment we give the interface polariton modes of a quantum well including retardation using the dielectric continuum approach. We show that the modes in general have both longitudinal and transverse components and that they have a scalar potential associated with them. We point out that in electron-phonon scattering the interaction is given well by the usual Fröhlich result calculated neglecting retardation.

In the bulk it is well known that the optical phonons separate into transverse optical modes and longitudinal optical modes. In general longitudinal modes have vanishing curl everywhere, and transverse modes have vanishing divergence everywhere. Light couples only to the transverse modes forming bulk phonon-polaritons.[14] The coupling is strongest in the region where the light line ($\omega = ck/\sqrt{\varepsilon_\infty}$) crosses the bulk unretarded TO phonon frequency ω_{TO}. We denote the region where this occurs by $k \sim k_c \equiv \sqrt{\varepsilon_\infty}\omega_{\text{TO}}/c$ ($\sim 10^{-5}$ Å^{-1} for most solids). In the presence of a surface or interface, on the other hand, the modes of the system have mixed transverse and longitudinal character when retardation is included. A similar situation for plasmons and phonons of a surface has been

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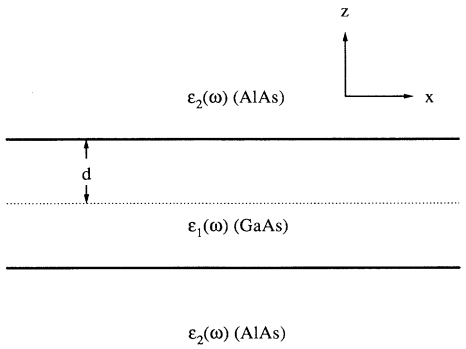

FIG. 1. Geometry of the quantum well system used here.

studied previously.[15]

Here we consider the system shown in Fig. 1 which consists of a quantum well of GaAs between two semi-infinite layers of AlAs. Similar results are obtained both for quantum wells of other materials and for superlattices. An electromagnetic wave in this system must satisfy Maxwell's equations for a system without free charges and currents:

$$\nabla \cdot \mathbf{D} = 0, \tag{1}$$

$$c\nabla \times \mathbf{B} = \partial \mathbf{D}/\partial t, \tag{2}$$

$$c\nabla \times \mathbf{E} = -\partial \mathbf{B}/\partial t, \tag{3}$$

$$\nabla \cdot \mathbf{B} = 0, \tag{4}$$

where $\mathbf{D} = \varepsilon \mathbf{E}$. In the usual dielectric continuum approach the dielectric functions are taken to be isotropic and homogeneous and to have the form

$$\varepsilon_i(\omega) = \varepsilon_{\infty,i} \frac{\omega^2 - \omega_{\mathrm{LO},i}^2}{\omega^2 - \omega_{\mathrm{TO},i}^2}, \tag{5}$$

where i=1,2 for $|z| < d$, $|z| > d$. The quantities $\omega_{\mathrm{LO},i}$ and $\omega_{\mathrm{TO},i}$ correspond to bulk LO and TO phonon frequencies in the absence of retardation.

In discussing the interface polaritons we follow a procedure like that used for surface modes in Ref. 15. From translational invariance the electric and magnetic fields are of the form $\{\mathbf{E}(\mathbf{r},t), \mathbf{B}(\mathbf{r},t)\} = \{\mathbf{E}(z), \mathbf{B}(z)\} \exp[i(\bar{k} \cdot \bar{r} - \omega t)]$, where $\bar{k}$ and $\bar{r}$ are the wave vector and position components parallel to the quantum well. Here we take $\bar{k} \parallel \hat{x}$. Interface polaritons have frequencies different from the bulk phonons of either system, so $\varepsilon_i(\omega) \neq 0$. For $|z| \neq d$ the wave equation is obtained by combining Eqs. (1), (2), and (3), which yields

$$\left(\frac{\partial^2}{\partial z^2} - \alpha_i^2 \right) \mathbf{E} = 0, \tag{6}$$

where $\alpha_i^2 = k^2 - \varepsilon_i \omega^2/c^2$.

Modes for which α_1^2 and α_2^2 are both positive are interface polaritons. There are four such modes. The symmetric modes have electric fields given by $E_y = 0$,

$$E_x(z) = \begin{cases} e^{-\alpha_2|z|} \cosh \alpha_1 d & \text{if } |z| > d \\ e^{-\alpha_2 d} \cosh \alpha_1 z & \text{if } |z| < d \end{cases} \tag{7}$$

and

$$E_z(z) = ik \times \begin{cases} \alpha_2^{-1} e^{-\alpha_2 z} \cosh \alpha_1 d & \text{if } z > d \\ -\alpha_1^{-1} e^{-\alpha_2 d} \sinh \alpha_1 z & \text{if } |z| < d \\ -\alpha_2^{-1} e^{\alpha_2 z} \cosh \alpha_1 d & \text{if } z < -d, \end{cases} \tag{8}$$

and the antisymmetric modes have fields $E_y = 0$,

$$E_x(z) = \begin{cases} e^{-\alpha_2 z} \sinh \alpha_1 d & \text{if } z > d \\ e^{-\alpha_2 d} \sinh \alpha_1 z & \text{if } |z| < d \\ -e^{\alpha_2 z} \sinh \alpha_1 d & \text{if } z < -d \end{cases} \tag{9}$$

and

$$E_z(z) = ik \times \begin{cases} \alpha_2^{-1} e^{-\alpha_2|z|} \sinh \alpha_1 d & \text{if } |z| > d \\ -\alpha_1^{-1} e^{-\alpha_2 d} \cosh \alpha_1 z & \text{if } |z| < d. \end{cases} \tag{10}$$

From the continuity of E_x and D_z at $|z| = d$ we obtain the interface polariton dispersion relations:

$$-\frac{\varepsilon_2(\omega)}{\varepsilon_1(\omega)} = \frac{\alpha_2(k,\omega)}{\alpha_1(k,\omega)} \times \begin{cases} \tanh d\alpha_1(k,\omega) & \text{for the symmetric mode} \\ \coth d\alpha_1(k,\omega) & \text{for the antisymmetric mode.} \end{cases} \tag{11}$$

For the system studied here there are two roots for each of these two transcendental equations. Equations (11) have solutions only for $\varepsilon_2/\varepsilon_1 < 0$. These are the reststrahlen regions of the two bulk materials, which do not overlap for AlAs and GaAs. The interface polariton modes of a quantum well system of half-thickness $d = 1$ μm (Ref. 16) are shown by the dotted lines in Fig. 2. There is a symmetric and an antisymmetric interface polariton associated mainly with each of the two materials. Figure 2 also shows the dispersion relations for the unretarded bulk phonons, the unretarded interface phonons, and the bulk polaritons. We see that for $k \gg k_c$ the interface mode dispersions with and without retardation are very nearly the same.

Next we consider the transverse and longitudinal components of the interface polariton fields and their coupling to electrons. The total electric field is given in terms of the scalar and vector potentials by $\mathbf{E} = -\nabla\phi - i\omega\mathbf{A}/c$. In the Coulomb gauge ($\nabla \cdot \mathbf{A} = 0$) this separates the field uniquely into its longitudinal ($-\nabla\phi$) and transverse ($-i\omega\mathbf{A}/c$) parts.[13] For $|z| \neq d$ the scalar potential satisfies $\nabla^2\phi = 0$, and for the symmetric mode is given by

$$\phi(x,z) = iC_s e^{ikx} \times \begin{cases} e^{-k|z|} \cosh kd & \text{for } |z| \geq d \\ e^{-kd} \cosh kz & \text{for } |z| \leq d, \end{cases} \tag{12}$$

where C_s is a constant. The potential generated by the antisymmetric mode is

$$\phi(x,z) = iC_a e^{ikx} \times \begin{cases} e^{-kz} \sinh kd & \text{for } z \geq d \\ e^{-kd} \sinh kz & \text{for } |z| \leq d \\ -e^{kz} \sinh kd & \text{for } z \leq -d, \end{cases} \tag{13}$$

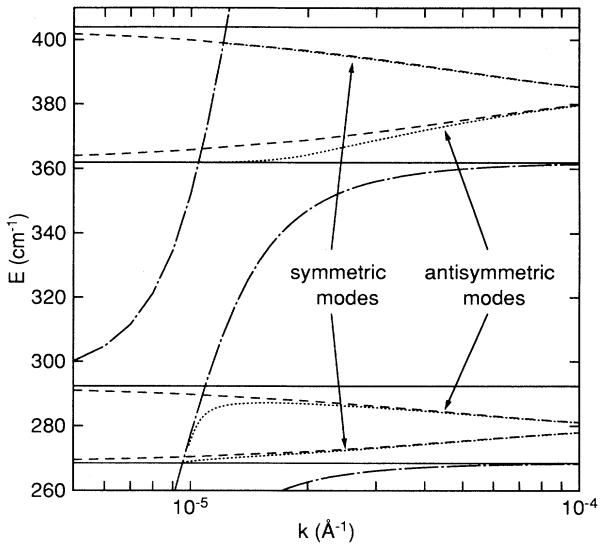

FIG. 2. Dispersion relations of interface polaritons are given by the dotted lines. The interface phonon dispersion relations in the absence of retardation are given by the dashed lines. The upper set of interface modes are associated mainly with AlAs, and the lower set mainly with GaAs. Solid horizontal lines give the unretarded bulk phonon frequencies, and dot-dashed lines give the bulk polariton dispersion curves. The well half-thickness is $d = 1$ μm. The parameters used in these calculations are $\omega_{\mathrm{LO},1} = 292$ cm^{-1}, $\omega_{\mathrm{TO},1} = 268$ cm^{-1}, $\varepsilon_{\infty,1} = 10.89$, $\omega_{\mathrm{LO},2} = 404$ cm^{-1}, $\omega_{\mathrm{TO},2} = 362$ cm^{-1}, and $\varepsilon_{\infty,2} = 8.16$.

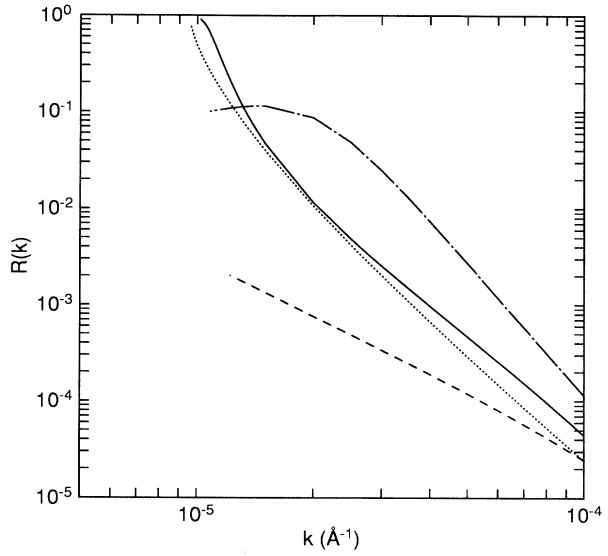

FIG. 3. The ratio $R(k) \equiv \int_{-\infty}^{\infty} dz|\mathbf{T}(z)|^2 / \int_{-\infty}^{\infty} dz'|\mathbf{E}(z')|^2$, which gives the ratio of the integrated value of the square of the transverse component of the electric field to that of the total electric field for the four polaritons from Fig. 2 as a function of wave vector parallel to the interface. The dotted, solid, dashed, and dot-dashed lines represent $R(k)$ for the symmetric GaAs-like, antisymmetric GaAs-like, symmetric AlAs-like, and antisymmetric AlAs-like modes, respectively.

where C_a is another constant. The constants are determined by the condition that $\nabla \cdot \mathbf{A}$ vanishes everywhere, including $|z| = d$:

$$C_s = e^{-\alpha_2 d}\left(\frac{\cosh \alpha_1 d}{\alpha_2} + \frac{\sinh \alpha_1 d}{\alpha_1}\right), \tag{14}$$

$$C_a = e^{-\alpha_2 d}\left(\frac{\cosh \alpha_1 d}{\alpha_1} + \frac{\sinh \alpha_1 d}{\alpha_2}\right). \tag{15}$$

The transverse part of $\mathbf{E}$ is obtained from $\mathbf{T} \equiv \mathbf{E} + \nabla\phi = -i\omega\mathbf{A}/c$. The quantity $R(k) \equiv \int_{-\infty}^{\infty} dz|\mathbf{T}(z)|^2 / \int_{-\infty}^{\infty} dz'|\mathbf{E}(z')|^2$ satisfies $0 \leq R(k) \leq 1$ and gives a measure of the degree of transverse character in each mode. In Fig. 3 we show $R(k)$ for a well of half-thickness $d = 1$ μm for each of the four interface polariton modes from Fig. 2. Here it is seen that for $k \gg k_c$ the modes have very small transverse components and are mostly longitudinal.

In the preceding we have argued that the interface polariton modes of a quantum well in the continuum approximation have a longitudinal as well as a transverse part and thus that they have a scalar potential ϕ associated with them. The basis of this argument involves the requirement that the longitudinal part of $\mathbf{E}$ have vanishing curl everywhere and that the transverse part of $\mathbf{E}$ have vanishing divergence everywhere.[13] For a quantum well $\nabla \cdot \mathbf{E}$ vanishes everywhere except at the interfaces, where it diverges because of the oscillating bound charge densities there. The electric field is not purely transverse, as claimed by Ridley and Babiker, because $\nabla \cdot \mathbf{E}$ does not vanish everywhere. The longitudinal part of $\mathbf{E}$ and the corresponding scalar potential ϕ arise from the bound charges at the interfaces and are obtained from Eq. (6) and the usual electromagnetic boundary conditions. A simple system, which is conceptually similar, is a point charge q at the origin, for which $\nabla \cdot \mathbf{E}$ vanishes everywhere except at the origin, where it diverges; in this case the electric field is given by $\mathbf{E} = -\nabla\phi$, where $\phi = q/r$, and $\mathbf{E}$ is thus purely longitudinal.

From the results shown in Figs. 2 and 3 we see that retardation affects the character of the modes in the region $k \lesssim k_c$, which is of interest in some light scattering situations. For electron-phonon scattering, however, this is a small part of the region of k space of interest. In the region $k \gg k_c$ the electric field can be obtained to a good approximation by the gradient of a scalar potential, and the interaction with electrons is given by the Fröhlich interaction neglecting retardation as has been done previously.[2,5–7] An integral over k space involving the phonons is made in calculating, for example, electron intrasubband and intersubband transition rates due to electron-phonon scattering.[2] We have made calculations of these rates[2] and find that the integrand is smooth for $k \lesssim 1/d$ and falls off for $k \gtrsim 1/d$. Corrections due to retardation come from the region $k \lesssim k_c$. For typical well widths ($d \sim 100$ Å) this correction is at most $O(10^{-4})$ of the total transition rate and can be neglected.

In this Comment we have shown that the interface polaritons in a semiconductor quantum well structure obtained from the dielectric continuum approach have both longitudinal and transverse character. For the large wave vectors of interest in electron-phonon scattering their transverse component is small and the electron-phonon interaction is given well by the Fröhlich interaction neglecting retardation. Similar results can also be obtained for analogous systems such as quantum wires and quantum dots.

Note added in proof. It has come to our attention that B. f. Zhu has made in a Comment [B. f. Zhu, Phys. Rev. B **46**, 13 619 (1992)] on the paper by Ridley and Babiker some of the point made here, including the Fuchs-Kliewer interface modes have a Frölich coupling to electrons.

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