Screening of the electron-phonon interaction in quasi-one-dimensional semiconductor structures

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The screening of the electron-phonon interaction due to a quasi-one-dimensional (Q1D) electron gas is investigated. The contribution of the electron-phonon interaction to the ground-state energy of the Q1D *polaron gas* in different semiconductor quantum-well-wire structures is calculated within the Hartree-Fock and the random-phase approximation. The influences of the width of the quantum-well wire and the electron density on the ground-state energy of the *polaron gas* are studied. We found that the screening due to the electron gas reduces the effective electron-phonon coupling in a Q1D system appreciably and the contribution of the electron-phonon interaction to the ground-state energy of the polaron gas decreases with increasing electron density.

I. INTRODUCTION

In the last decade, there has been a lot of interest in the study of low-dimensional semiconductor systems. The reduction of spatial dimensions will influence the efficiency with which electrons can interact with phonons. The study of the polaron effects in two-dimensional (2D) semiconductor structures indicates that the electron-electron screening effect is important for the different polaron properties. Das Sarma studied the screening effect of the polaron gas in 2D systems using the Thomas-Fermi approximation[1] and the static RPA (random phase approximation).[2] The method of a Lee-Low-Pines unitary transformation was introduced by Lemmens, Devreese, and Brosens[3] to study the 3D polaron gas. Subsequently Wu, Peeters, and Devreese[4] extended this approach to investigate the 2D polaron gas within a dynamical screening scheme by taking into account the full frequency dielectric response. A detailed comparison was made with the results from different approximation schemes. Wendler[5] studied the influence of screening on the ground-state properties of the polaron gas in 2D systems within the full RPA. These works found that the properties of the polaron gas in 2D systems differ significantly from the one-polaron results.

Recent developments in semiconductor technology have made the quasi-one-dimensional (Q1D) semiconductor structures more and more important. Some studies on the electron-phonon interaction in Q1D systems have been done.[6–10] Very recently, Campos, Degani, and Hipólito[11] calculated the ground-state energy of the Q1D polaron gas in a rectangular quantum-well-wire structure. They show that the screening effects in such a Q1D structure are much more pronounced than those in the corresponding Q2D systems.

In the present paper, we report a systematic investigation of the ground-state energy of the Q1D polaron gas and the screening effect in different semiconductor quantum-well-wire structures. In Sec. II the unitary transformation method[3,4] is applied to study the electron-phonon interaction in the Q1D polaron gas including many-particle effects. The modification of the ground-state energy of the polaron gas and the effective electron-electron potential due to the electron-phonon interaction will be calculated. In Sec. III, our numerical results for different Q1D structures are presented within the RPA and the Hartree-Fock approximation (HFA) and three types of the most often used Q1D quantum-well-wire models are considered. For a comparison, we also calculate the one-polaron binding energy without screening but including the full set of intermediate states. Our calculation for rectangular quantum-well wires found that the numerical results in Ref. 11 overestimated the polaron binding energy considerably. Our conclusions are presented in Sec. IV.

II. FORMULATION AND CALCULATION

The Q1D system under consideration contains N electrons which are assumed to be free to move in the x direction and are confined in the yz plane. The phonons are assumed to be the 3D bulk LO-phonon modes. Such a system is described by the following Hamiltonian:

$$H=\sum_{j=1}^{N}\left(\frac{\mathbf{p}_j^2}{2m_b}+V_B(y_j,z_j)\right)+\sum_{i<j}v(\mathbf{r}_i-\mathbf{r}_j)+\sum_{\mathbf{q}}\hbar\omega_{\mathrm{LO}}(a_{\mathbf{q}}^{\dagger}a_{\mathbf{q}}+\tfrac{1}{2})+\sum_{j=1}^{N}\sum_{\mathbf{q}}(V_{\mathbf{q}}a_{\mathbf{q}}e^{i\mathbf{q}\cdot\mathbf{r}_j}+V_{\mathbf{q}}^{*}a_{\mathbf{q}}^{\dagger}e^{-i\mathbf{q}\cdot\mathbf{r}_j}), \tag{1}$$

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where $\boldsymbol{p}_j$ ($\boldsymbol{r}_j$) is the momentum (position) operator of the jth electron, m_b the electron band mass, $V_B(y,z)$ the confinement potential of the quantum-well wire, $a_{\boldsymbol{q}}^\dagger$ ($a_{\boldsymbol{q}}$) the creation (annihilation) operator of an optical-phonon with wave vector $\boldsymbol{q}$ and energy $\hbar\omega_{\rm LO}$, and

$$V_{\boldsymbol{q}} = -i\hbar\omega_{\rm LO}\left(\frac{\hbar}{2m_b\omega_{\rm LO}}\right)^{1/4}\left(\frac{4\pi\alpha}{Vq^2}\right)^{1/2}, \tag{2}$$

where α is the electron-phonon coupling constant. The term $v(\boldsymbol{r}_i-\boldsymbol{r}_j)$ in Eq. (1) represents the Coulomb interaction between two electrons, which is given by

$$v(\boldsymbol{r}_i-\boldsymbol{r}_j) = \frac{e^2}{\epsilon_\infty}\frac{1}{|\boldsymbol{r}_i-\boldsymbol{r}_j|}, \tag{3}$$

where ϵ_∞ is the optical dielectric constant. The one-electron wave function in such a system without electron-electron and electron-phonon interaction can be written as

$$\psi_{k_x,n,m}(x,y,z) = \frac{1}{\sqrt{L_x}}e^{ik_x x}\xi_n(y)\phi_m(z), \tag{4}$$

where $\xi_n(y)$ and $\phi_m(z)$ are the bound wave functions resulting from the quantization in the y and z direction, and k_x is the 1D wave vector along the quantum wire with length L_x.

To study the properties of the Q1D polaron gas we follow the Lee-Low-Pines unitary transformation approach which was introduced in Ref. 3 for a 3D polaron gas and later applied to study the Q2D polaron gas problem.[4] For simplicity we will restrict our problem to the extreme quantum limit where only the lowest subband ($n=0, m=0$) is occupied. This approximation is justified when the subband separation is much larger than the phonon energy for the thin wire at low temperature. Within this approximation the Fourier transform of the Coulomb potential in Eq. (3) is given by

$$v(q_x) = \frac{2e^2}{\epsilon_\infty}f(q_x), \tag{5}$$

where $f(q_x)$ is the form factor of the Q1D system. Then we employ a Lee-Low-Pines unitary transformation approximation. Here the unitary transformation U is taken to be

$$U = \exp\left(\sum_{j=1}^{N} s(x_j)\right) \tag{6}$$

with

$$s(x) = \sum_{\mathbf{q}}(\eta_{\mathbf{q}}a_{\mathbf{q}}e^{iq_x x} + \eta_{\mathbf{q}}^* a_{\mathbf{q}}^\dagger e^{-iq_x x}), \tag{7}$$

where $\eta_{\mathbf{q}}$ is to be determined variationally. Note that U in Eq. (6) depends only on the x component of the electron position vector. This unitary transformation is performed on the Hamiltonian H and the expectation value of the Hamiltonian is calculated over the states with zero real phonons and where the electron gas is in the ground state described by a wave function $\Psi_{\rm GS}$. The η_q is determined by minimizing the ground-state energy which leads to

$$\eta_{\boldsymbol{q}} = V_{\boldsymbol{q}}\frac{\langle\xi_0|e^{iq_y y}|\xi_0\rangle\langle\phi_0|e^{iq_z z}|\phi_0\rangle}{\hbar\omega_{\rm LO}S(q_x)+\hbar^2q_x^2/2m_b}S(q_x), \tag{8}$$

where $S(q_x)$ is the static structure factor of the Q1D electron system in the absence of the electron-phonon interaction.

After some algebra we obtain the ground-state energy of the Q1D polaron gas

$$E = \langle\Psi_{\rm GS}|H_e|\Psi_{\rm GS}\rangle + \frac{N}{2L_x}\sum_{q_x}v(q_x)[S(q_x)-1] + N\Delta E, \tag{9}$$

where ΔE is the contribution to the ground-state energy per particle of the polaron gas due to the electron-phonon interaction, which is given by

$$\begin{aligned}\Delta E &= -\sum_{\boldsymbol{q}}|V_{\boldsymbol{q}}|^2\frac{|\langle\xi_0|e^{iq_y y}|\xi_0\rangle\langle\phi_0|e^{iq_z z}|\phi_0\rangle|^2}{\hbar\omega_{\rm LO}S(q_x)+\hbar^2q_x^2/2m_b}S^2(q_x)\\ &= -\frac{2\alpha}{\pi}\hbar\omega_{\rm LO}k_{\rm LO}^{-1}\int_0^\infty dq_x\frac{S^2(q_x)}{S(q_x)+(q_x/k_{\rm LO})^2}f(q_x),\end{aligned} \tag{10}$$

where $k_{\rm LO}=(2m_b\omega_{\rm LO}/\hbar)^{1/2}$. Another interesting result which can be found in the transformed Hamiltonian is the effective electron-electron potential

$$v_{\rm eff}(\mathbf{r}-\mathbf{r}') = v(\mathbf{r}-\mathbf{r}') - 2\sum_{\mathbf{q}}(V_{-\mathbf{q}}^*\eta_{-\mathbf{q}}e^{\mathbf{q}_\perp\cdot\mathbf{r}'_\perp} + V_{\mathbf{q}}\eta_{\mathbf{q}}^*e^{\mathbf{q}_\perp\cdot\mathbf{r}'_\perp} - \hbar\omega_{\rm LO}\eta_{\mathbf{q}}^*\eta_{\mathbf{q}})e^{-iq_x(x-x')}, \tag{11}$$

where $\boldsymbol{q}_\perp$ ($\boldsymbol{r}_\perp$) is the component of $\boldsymbol{q}$ ($\boldsymbol{r}$) in the yz plane and the last term on the right-hand side corresponds to the modification of the electron-electron interaction due to the electron-phonon interaction. The effective electron-electron potential can be calculated as a function of the distance between two electrons along the quantum wire

$$V_{\rm eff}(x) = V_{ee}(x) - \frac{4\alpha}{\pi}\hbar\omega_{\rm LO}k_{\rm LO}^{-1}\int_0^\infty dq_x\cos(q_x x)\frac{S(q_x)+2q_x^2k_{\rm LO}^{-2}}{[S(q_x)+(q_x/k_{\rm LO})^2]^2}S(q_x)f(q_x), \tag{12}$$

where

$$V_{ee}(x) = \frac{2e^2}{\pi\epsilon_\infty}\int_0^\infty dq_x \cos(q_x x) f(q_x). \tag{13}$$

From Eqs. (10) and (12) we see that the static structure factor $S(q_x)$ is a key quantity which has to be calculated in order to determine the screening properties of the polaron gas system. $S(q_x)$ can be obtained from the dielectric function $\epsilon(q_x,\omega)$ through

$$S(q_x) = \frac{\hbar}{\pi n_e v(q_x)}\int_0^\infty d\omega \frac{-1}{\epsilon(q_x,\omega)}, \tag{14}$$

where n_e is the electron density and $v(q_x)$ the Fourier transform of the unperturbed Coulomb potential. In the present work we will calculate $S(q_x)$ within the RPA and the HFA.

A number of papers[12–15] have been published in which the dielectric response of a Q1D electron gas was investigated. As for the 2D and the 3D cases,[16] the RPA is justified for high electron densities. In fact, the RPA becomes exact in the limit $r_s \to 0$, i.e., the high density limit. For low electron densities, electron-electron correlation is more important and this is more so in Q1D systems. In typical samples of Q1D electron gases in GaAs one has $r_s \gg 1$ and the RPA is a fairly good approximation.

Within the RPA, $\epsilon(q_x,\omega) = 1 - v(q_x)\chi(q_x,\omega)$, where $v(q_x)$ is given by Eq. (5) and $\chi(q_x,\omega)$ is the polarization of the Q1D electron gas which is given by

$$\chi(q_x,\omega) = \chi_1(q_x,\omega) + i\chi_2(q_x,\omega) \tag{15}$$

with

$$\chi_1(q_x,\omega) = \frac{m_b}{\hbar^2\pi q_x}\ln\left|\frac{\omega^2-\omega_-^2}{\omega^2-\omega_+^2}\right|, \tag{16}$$

and

$$\chi_2(q_x,\omega) = \begin{cases} -m_b/\hbar^2 q_x, & \omega_+ > \omega > \omega_- \\ 0 & \text{otherwise}, \end{cases} \tag{17}$$

where $\omega_\pm = \hbar q_x^2/2m_b \pm \hbar k_F q_x/m_b$ and $k_F = \pi n_e/2$ is the Fermi wave vector. We found that the static structure factor is given by

$$S(q_x) = S_{eh}(q_x) + S_{\rm pl}(q_x) \tag{18a}$$

with the electron-hole contribution

$$S_{eh}(q_x) = \int_0^\infty dx \frac{q_x u^2(q_x)}{(1+x)^{3/2}(\omega'^2_+ + x\omega'^2_-)^{1/2}\{[1+u(q_x)\ln x]^2+\pi^2\}} \tag{18b}$$

and the plasmon contribution

$$S_{\rm pl}(q_x) = \frac{q_x u^2(q_x) e^{-u(q_x)}}{(\omega'^2_+ - \omega'^2_- e^{-u(q_x)})^{1/2}(1-e^{-u(q_x)})^{3/2}}, \tag{18c}$$

where $u(q_x) = 2\pi\hbar\omega_{\rm LO} q_x/(k_{\rm LO}^2 v(q_x))$, $\omega'_\pm = |q_x \pm 2k_F|$.

Within the HFA the static structure factor is calculated by

$$S(q_x) = \frac{1}{N}\int_0^\infty d\omega\, S(q_x,\omega) \tag{19}$$

with

$$S(q_x,\omega) = \sum_{\substack{k_x<k_F \\ |k_x\pm q_x|>k_F}} \delta(\omega - \omega_{k_x,q_x}), \tag{20}$$

where $\omega_{k_x,q_x} = \hbar q_x^2/2m_b \pm \hbar q_x k_x/m_b$. We found that the static structure factor has a simple form in this case,

$$S(q_x) = \begin{cases} q_x/2k_F, & q_x < 2k_F \\ 1, & q_x \geq 2k_F, \end{cases} \tag{21}$$

which does not depend on the form factor $f(q_x)$.

III. NUMERICAL RESULTS AND DISCUSSION

According to the derivation in the previous section the contribution of the electron-phonon interaction to the ground-state energy of the polaron gas depends on the electron density and the form factor, and the latter is determined by the confinement potentials of the Q1D semiconductor structures. In this section we will calculate the energy of the Q1D polaron gas in GaAs-based semiconductor quantum-well-wire structures with different confinement potentials. The relevant material parameters which are used in the calculations are $\hbar\omega_{\rm LO} = 36.25$ meV, $m_b = 0.067 m_e$, $\epsilon_\infty = 10.9$, and $\alpha = 0.068$.

Before we give the results for the screening effects on different polaron properties we calculate the binding energy of a one polaron in the Q1D system without screening ($n_e = 0$). Applying second-order perturbation theory the one-polaron binding energy due to the electron-phonon interaction is given by

$$\Delta E = -\sum_{n,m}\sum_{\mathbf{q}} \frac{|\langle n,m,k_x-q_x;\mathbf{q}|H_{ep}|0,0,k_x;0\rangle|^2}{\hbar\omega_{\rm LO} + E_{n,m} - E_{0,0} + \hbar^2 q_x^2/2m_b}$$

$$= -\frac{\alpha}{2\pi^2}\hbar\omega_{\rm LO} k_{\rm LO}^{-1}\sum_{n=0}^\infty\sum_{m=0}^\infty \int d^3q \frac{|\langle\xi_n|e^{iq_y y}|\xi_0\rangle\langle\phi_m|e^{iq_z z}|\phi_0\rangle|^2}{q^2[1+(E_{n,m}-E_{0,0})/\hbar\omega_{\rm LO} + (q_x/k_{\rm LO})^2]}, \tag{22}$$

where the ket $|n, m, k_x; \boldsymbol{q}\rangle$ describes a state composed of an electron in subband (n, m) with momentum $\hbar k_x$ in the x direction and a LO phonon with momentum $\hbar \boldsymbol{q}$. $E_{n,m}$ is the subband energy. Within the leading term approximation ($n = 0$, $m = 0$) the polaron binding energy becomes

$$\Delta E = -\frac{2\alpha}{\pi}\hbar\omega_{\rm LO}k_{\rm LO}^{-1}\int_0^\infty dq_x \frac{f(q_x)}{1+(q_x/k_{\rm LO})^2}, \qquad (23)$$

which can also be obtained from Eq. (10) by replacing $S(q_x)$ by 1 as it should be because $\lim_{n_e\to 0} S(q_x) = 1$. On the other hand, from Eq. (10) we know that $\Delta E \to 0$ when $n_e \to \infty$. In this case the electron-phonon interaction vanishes due to the strong electron-electron screening which is active at very high electron densities.

A. Two-dimensional parabolic confinement potential

In this case the confinement potential has the following form:

$$V_B(y,z) = \frac{1}{2}m_b(\Omega_y^2 y^2 + \Omega_z^2 z^2), \qquad (24)$$

where Ω_y and Ω_z are the confinement frequency of the parabolic potential well in the y and z direction, respectively. The corresponding form factor is given by

$$f(q_x) = \frac{1}{2}\int_0^\infty dx \frac{e^{-x}}{[(x+\hbar q_x^2/2m_b\Omega_y)(x+\hbar q_x^2/2m_b\Omega_z)]^{1/2}}. \qquad (25)$$

In the special case $\Omega_y = \Omega_z = \Omega$ the confinement potential becomes $V_B(y,z) = (1/2)m_b\Omega(y^2+z^2)$ and the form factor is given by

$$f(q_x) = -\frac{1}{2}\exp\left(\frac{\hbar q_x^2}{2m_b\Omega}\right)\mathrm{Ei}\left(-\frac{\hbar q_x^2}{2m_b\Omega}\right), \qquad (26)$$

where $\mathrm{Ei}(x)$ is the exponential integral function.

On the other hand, the one-polaron binding energy incorporating the full intermediate states can be obtained from Eq. (22),

$$\Delta E = -\frac{\alpha\hbar\omega_{\rm LO}}{\sqrt{\pi}}\int_0^\infty dx \frac{\ln[(\sqrt{x-\beta}+\sqrt{x})/\sqrt{\beta}]}{\sqrt{x-\beta}}e^{-x}, \qquad (27)$$

where $\beta = (\omega_{\rm LO}/\Omega)(1-e^{-(\Omega/\omega_{\rm LO})x})$. When $\Omega \to 0$, i.e., $\beta \to x$, Eq. (27) leads to the 3D polaron energy $\Delta E = -\alpha\hbar\omega_{\rm LO}$.

The contribution of the electron-phonon interaction to the ground-state energy in such a system is plotted as a function of $\omega_{\rm LO}/\Omega$ for different electron densities $n_e = 10^4$, 10^5, and 10^6 cm^{-1} in Fig. 1(a), and as a function of the electron density with $\Omega/\omega_{\rm LO}$=1, 10, and 100 in Fig. 1(b). The RPA and HFA results are given by the thick-solid and thick-dashed curves, respectively. The thin-solid curve in Fig. 1(a) represents the one-polaron binding energy within the leading term approximation and the thin dotted-dash curve gives the one-polaron result including the full intermediate states. The two thin horizontal dotted lines indicate the one-polaron 3D and 2D values of the binding energy, respectively. For $\Omega \to 0$ the result for a 3D system is reached and the dotted-dash

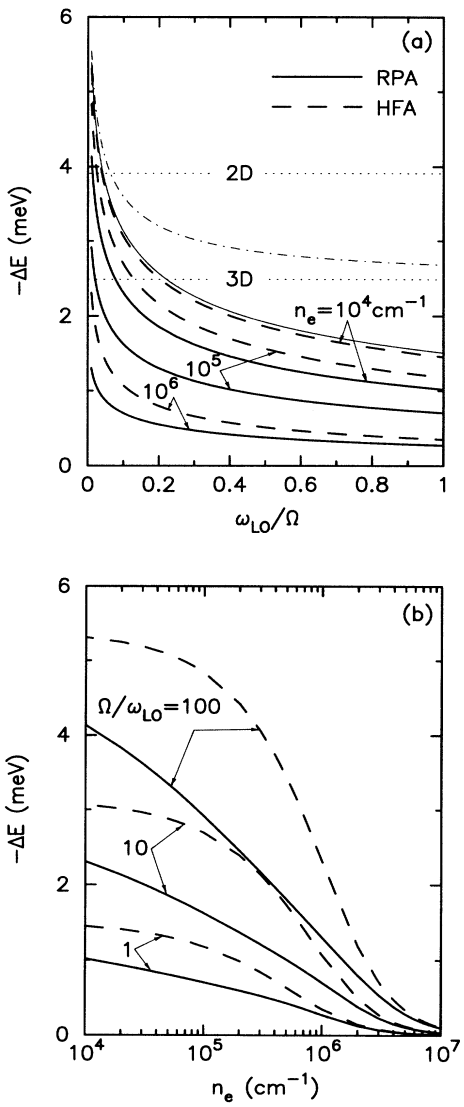

FIG. 1. The contribution of the electron-phonon interaction to the ground-state energy of the Q1D polaron gas as a function of (a) $\omega_{\rm LO}/\Omega$ with different electron densities $n_e = 10^4$, 10^5, and 10^6 cm^{-1}, and (b) the electron density with $\Omega/\omega_{\rm LO}$=1, 10, and 100. The thick solid and dashed curves indicate the RPA and HFA results, respectively. In (a) the thin solid curve gives the one-polaron binding energy within the leading term approximation and the thin dotted-dash curve represents the the one-polaron binding energy including full intermediate states. The thin horizontal dotted lines indicate the one-polaron binding energy in I2D and the 3D systems.

curve approaches the 3D polaron binding energy. Notice that when $n_e > 1.6 \times 10^6$ cm^{-1} the second subband becomes populated in the case of $\Omega/\omega_{\rm LO} = 1$, and the extreme quantum limit approximation is not valid. The corresponding results in Fig. 1(b) are only approximate in this density region.

B. Parabolic combined with triangular confinement potential

In this case, the confinement potential can be written as

$$V_B(y,z) = \frac{1}{2} m_b \Omega_y^2 y^2 + V(z), \tag{28a}$$

where we took a triangular potential in the z direction,

$$V(z) = \begin{cases} eFz, & z > 0 \\ \infty, & z < 0 \end{cases}, \tag{28b}$$

with F the effective electric field. In such a case, the wave function in the z direction is given by $\phi_0(z) = (b^3/2)^{1/2} z e^{-bz/2}$, where $b = 2(3em_bF/2\hbar^2)^{1/3}$, and the electron gas has an average width $W_z = 3/b$ in this direction. This confinement situation is realized in heterojunctions. The corresponding form factor is given by

$$f(q_x) = \int_0^\infty dx \frac{e^{-(\omega_{\rm LO}/\Omega_y)x^2}}{\sqrt{x^2 + q_x^2/k_{\rm LO}^2}} F(x, q_x) \tag{29a}$$

with

$$F(x, q_x) = \frac{8b^3 + 9b^2\mu + 3b\mu^2}{8(b+\mu)^3}, \tag{29b}$$

where $\mu = \sqrt{k_{\rm LO}^2 x^2 + q_x^2}$.

Now, let us consider the Q1D system which is composed of an ideal-2D (I2D) electron gas in the xy plane with a parabolic well potential in the y direction. This system can be reached by $b \to \infty$ ($W_z = 0$) for the confinement potential Eq. (28a) or $\Omega_z \to \infty$ for the confinement potential Eq. (24). In such a case, the form factor reduces to

$$f(q_x) = \frac{1}{2} \exp\left(\frac{\hbar q_x^2}{4m_b\Omega_y}\right) K_o\left(\frac{\hbar q_x^2}{4m_b\Omega_y}\right), \tag{30}$$

where $K_0(x)$ is the modified Bessel function of the second kind.

From Eq. (22) the one-polaron binding energy incorporating the full intermediate states in this system is given by

$$\Delta E = -\frac{2\alpha\hbar\omega_{\rm LO}}{\sqrt{\pi}} \int_0^\infty dx\, e^{-x^2} K\left(\sqrt{1 - \beta_y/x^2}\right), \tag{31}$$

where $\beta_y = (\omega_{\rm LO}/\Omega_y)(1 - e^{-(\Omega_y/\omega_{\rm LO})x^2})$ and $K(x)$ is the complete elliptic integral of the first kind. When $\Omega_y \to 0$, i.e., $\beta_y \to x^2$, Eq. (31) leads to the I2D result $\Delta E = -(\pi/2)\alpha\hbar\omega_{\rm LO}$.

The contribution of the electron-phonon interaction to the ground-state energy of the polaron gas is plotted as a function of $\omega_{\rm LO}/\Omega_y$ for $W_z = 0$ and with different electron densities $n_e = 10^4$, 10^5, and 10^6 cm^{-1} in Fig. 2(a). The RPA and HFA results are indicated by thick solid and thick dashed curves, respectively. The thin solid curve represents the one-polaron binding energy within the leading term approximation and the thin dotted-dash curve gives the one-polaron result including

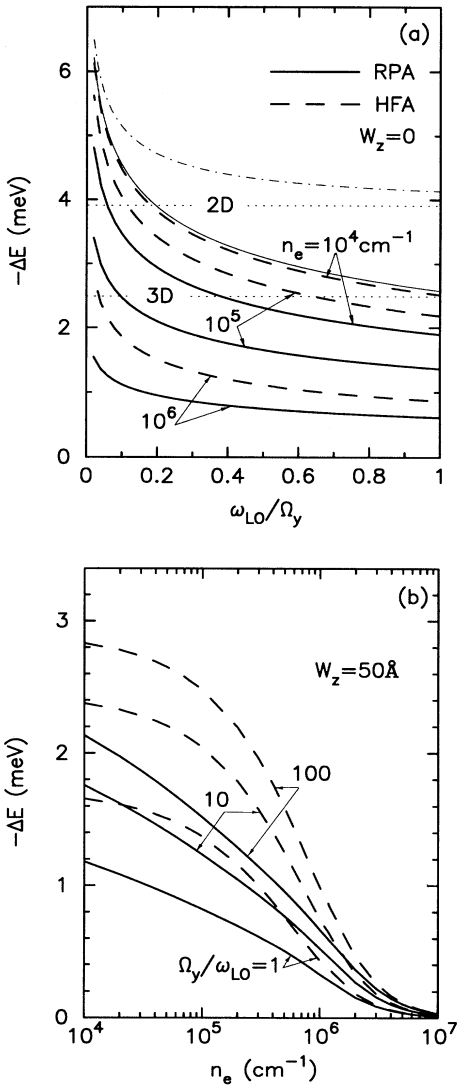

FIG. 2. The contribution of the electron-phonon interaction to the ground-state energy of the Q1D polaron gas as a function of (a) $\omega_{\rm LO}/\Omega_y$ with $W_z = 0$ and different electron densities $n_e = 10^4$, 10^5, and 10^6 cm^{-1}, and (b) the electron density with $W_z = 50$ Å and $\Omega_y/\omega_{\rm LO}$=1, 10, and 100. The thick solid and dashed curves indicate the RPA and HFA results, respectively. The thin solid curve gives the one-polaron binding energy within the leading term approximation and the thin dotted-dash curve represents the one-polaron binding energy including full intermediate states. The thin horizontal dotted lines indicate the I2D and the 3D value of the one-polaron binding energy.

full intermediate states. The two thin horizontal dotted lines indicate the 3D and I2D values of the one-polaron binding energy, respectively. For $\Omega_y \to 0$ an I2D system is reached and the dotted-dash curve approaches the I2D polaron value. In Fig. 2(b), the polaron energy due to the electron-phonon interaction is plotted as a function of the electron density for $W_z = 50$ Å and $\Omega_y/\omega_{\rm LO}$ =1, 10, and 100. When $n_e > 1.6 \times 10^6$ cm^{-1} for $\Omega/\omega_{\rm LO} = 1$ and $n_e > 5.0 \times 10^6$ cm^{-1} for $W_z = 50$ Å, the second subband will also be populated and the results are only very approximately valid.

It is seen that the electron-electron screening reduces the electron-phonon interaction appreciably with increasing electron density. When $n_e \to \infty$, $\Delta E \to 0$, while for $n_e \to 0$ the one-polaron binding energy is recovered within the leading term approximation. We find that the polaron correction energy within the RPA is smaller than that within the HFA. For large densities the RPA and HFA give nearly the same results.

The electron-phonon interaction leads to a modification of the Coulomb interaction between the electrons. We have calculated the effective electron-electron interaction potential in the Q1D system from Eqs. (12), (13), (18), and (29) within the RPA. In Fig. 3 the effective electron-electron potential is plotted as a function of the distance between the two electrons in the x direction for $W_z = 20$ Å, $\Omega_y/\omega_{\rm LO} = 1$ (thin curves), and 10 (thick curves) and with different electron densities $n_e = 10^5$, 10^6, and 10^7 cm^{-1}. The electron-electron Coulomb potentials without the electron-phonon interaction modification are given by the solid curves. The latter does not diverge for $x \to 0$ because we have averaged this potential over the confinement electron wave functions. We find that the modification due to the electron-phonon interaction is pronounced and it decreases with increasing average width of the polaron gas and with increasing electron density. When $n_e \to \infty$, $V_{\rm eff}(x) \to V_{ee}(x)$. For $\Omega_y/\omega_{\rm LO} = 1$ and $n_e = 10^7$ cm^{-1} we do not show the effective potential because in this case the second subband is populated, which was not included in our calculation.

C. Two-dimensional rectangular confinement potential

In this case, the confinement potential can be written as

$$V_B(y,z) = \begin{cases} 0, & |y| < L_y/2 \quad \text{and} \quad |z| < L_z/2 \\ \infty & \text{otherwise}, \end{cases} \tag{32}$$

where L_y and L_z are the well width in the y and z direction, respectively. The corresponding form factor is given by

$$f(q_x) = \int_0^\infty dq_y \frac{4\sin^2(q_y L_y/2) F(q_x, q_y)}{\{q_y L_y [1 - (q_y L_y/2\pi)^2]\}^2 \sqrt{q_x^2 + q_y^2}} \tag{33a}$$

with

$$F(q_x, q_y) = \frac{2}{\mu} + \frac{\mu}{4\pi^2 + \mu^2} - \frac{32\pi^4(1 - e^{-\mu})}{\mu^2(4\pi^2 + \mu^2)^2}, \tag{33b}$$

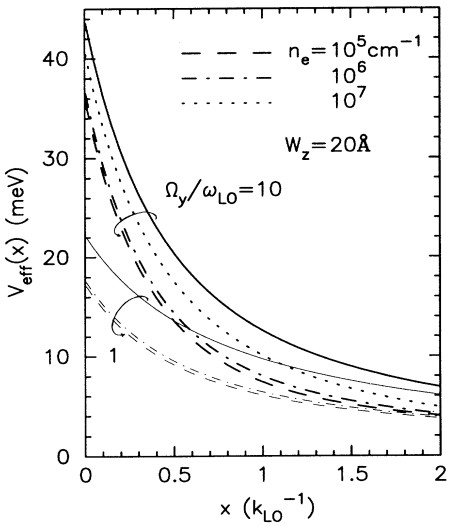

FIG. 3. The effective electron-electron interaction potential is plotted as a function of x (the distance between two electrons in the x direction) for different quantum wires with $W_z = 20$ Å, $\Omega_y/\omega_{\rm LO}$=1 (the thin curves), and 10 (the thick curves), and different electron densities $n_e = 10^5$, 10^6, and 10^7 cm^{-1}. The solid curves present the electron-electron interaction potential without electron-phonon modification.

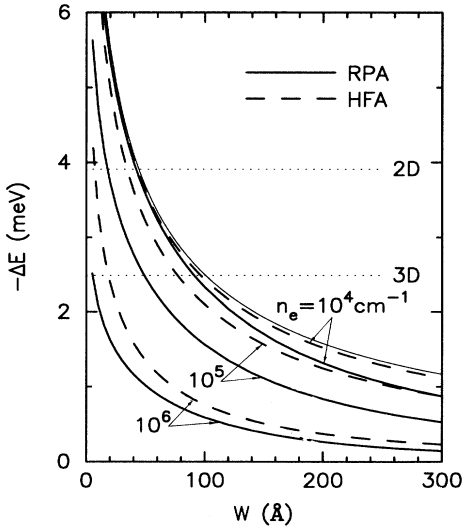

FIG. 4. The contribution of the electron-phonon interaction to the ground-state energy of the Q1D polaron gas as a function of the wire width W for different electron densities $n_e = 10^4$, 10^5, and 10^6 cm^{-1}. The thick solid and dashed curves indicate the RPA and HFA results, respectively. The thin solid curve gives the one-polaron binding energy within the leading term approximation. The thin horizontal dotted lines indicate the 2D and the 3D value of the one-polaron binding energy.

where $\mu = L_z\sqrt{q_x^2 + q_y^2}$.

We have calculated the polaron energy in the Q1D system with $L_y = L_z = W$. In Fig. 4 the contribution of the electron-phonon interaction to the ground-state energy of the polaron gas is plotted as a function of the width W for three different electron densities. In Ref. 11 Campos, Degani, and Hipólito gave the numerical results in such systems with W=50 Å and 100 Å. We find that their results are about a factor of 2 larger than as obtained from the present calculation. Notice that, different from the present calculation, their RPA result becomes larger than that within the HFA at high electron density.

IV. CONCLUSION

We have calculated the contribution of the electron-phonon interaction to the ground-state energy of the Q1D polaron gas within a dynamical screening scheme by taking into account the full frequency-dependent dielectric response. The screening effects due to electron-electron interaction are included into the electron-phonon interaction within the RPA and HFA. Our results show that the electron-electron screening reduces the electron-phonon interaction appreciably. For $n_e \to \infty$, the electron-phonon interaction correction energy goes to zero. For purposes of comparison we also calculated the one-polaron binding energy without screening within second-order perturbation theory. We found that, when the electron density $n_e \to 0$, the one-polaron binding energy within the leading term approximation is recovered both for the RPA and the HFA results. The electron-phonon coupling is enhanced by including the full intermediate states.

We also calculated the effective Coulomb interaction potential between electrons and obtained the corrections due to the electron-phonon interaction in a Q1D system. The calculation shows that the electron-phonon interaction correction to the electron-electron potential is pronounced and it decreases with increasing electron density.

The present model can be generalized to study the screening of the electron-phonon interaction of the Q1D polaron gas including higher subbands. In such a case one needs the dielectric function of an electron gas in a multi-subband system. This was done recently in Ref. 17 for a quasi-two-dimensional system and should be generalized to the multisubband quantum wire case. Then this result can be used to calculate the polaron effect which will be left for further study.

The present approach can also be used to calculate the effect of screening on the polaron binding energy for low density samples. Instead of using the RPA structure factor one should use the one which also incorporates electron-electron correlations and fluctuations, exchange, etc. For intermediate densities and, in particular, in the small density region, we know that the RPA overestimates screening. Nevertheless the present calculation, within the RPA, already shows that we are able to recover the correct zero density result for the polaron energy. Therefore we believe that the present calculation gives a reasonable interpolation for the polaron energy between the zero and high density limit.

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