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Flux-line pinning by competing disorders

Terence Hwa and David R. Nelson
Department of Physics, Harvard University, Cambridge, Massachusetts 02138

Valerii M. Vinokur
Materials Science Division, Argonne National Laboratory, Argonne, Illinois 60439
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Localization of flux lines by columnar defects is investigated in the presence of point disorder. A $(1+1)$-dimensional model is analyzed in some detail. When point disorder is weak, the phase transition is found to be of the Bose-glass type. Some aspects of the Bose-glass phase itself may be marginally unstable to weak point disorder, but only beyond an astronomically large crossover scale. Correlated disorder is always irrelevant when point disorder is very strong. The Bose-glass transition is also studied in a small transverse magnetic field. Critical exponents for the commensurate-incommensurate transition that occurs in this case are derived using scaling arguments.

I. INTRODUCTION

The importance of flux-line pinning in preserving the superconductivity of a type-II superconductor in a magnetic field has long been recognized.[1] For the cuprate superconductors, the flux lines form an entangled line liquid at high temperature.[2] The line liquid cannot be pinned effectively and has a linear resistivity.[3,4] At lower temperature, the flux lines are preempted from forming an Abrikosov lattice be weak defects in the underlying crystal such as oxygen vacancies.[5] It has been suggested that these weak pointlike defects may then *collectively* pin the flux-line network in a possible "vortex-glass" phase.[6–8] Unfortunately, evidence supporting the existence of such a phase in a bulk sample is still inconclusive at the moment. The phase transitions seen experimentally by Koch *et al.*[9] and by Gammel, Schneemeyer, and Bishop[10] may in fact be due to correlated pinning by twin boundaries and/or screw dislocations. The recent observation[11] that a *first-order* transition replaces the putative vortex-glass singularities of Ref. 10 in twin-free samples supports this hypothesis. Intriguing evidence for a vortex-glass behavior in numerical simulations[12] is thus far restricted to models with *zero* external magnetic field and an *infinite* London penetration depth. The behavior for directed vortex lines with finite range interactions could be different.

More recently, enhanced pinning has been reported in $YBa_2Cu_3O_{7-\delta}$ crystals with random *columnar* defects produced by energetic heavy ion radiation.[13] The theory of flux-line pinning by such columnar defects has been considered by Nelson and Vinokur.[14] At low temperature, the flux lines are strongly localized to the columnar defects, forming a "Bose glass"[15] with zero dc resistivity. A similar theory would apply to pinning dominated by mosaics of twin boundaries or forests of screw dislocations.[14]

In this paper, we analyze competition between the columnar defects which promote flux-line localization and point defects which promote flux-line wandering. We focus on a $(1+1)$-dimensional model where the flux lines are confined to a plane, since this is the only case where the existence of a vortex-glass phase has been explicitly demonstrated.[6,16–18] We find weak point disorder to be irrelevant at the line liquid-Bose-glass transition originally discussed by Giamarchi and Schulz.[19] The stability of the Bose-glass phase itself is marginal. Some aspects of the Bose-glass ground state may be modified by rare regions of weak point disorder beyond an astronomically long crossover length. However, important dynamic properties dominated by rare regions of columnar defects continue to be of the Bose-glass type. In the presence of strong point disorder, columnar defects are irrelevant. In this case, both the glass transition and the low-temperature phase are dominated by point disorder. Qualitatively similar conclusions may apply to competition between the Bose-glass and the putative vortex-glass phase in $(2+1)$ dimensions.[20] We also study the effect of point disorder on a system with a small transverse magnetic field, which arises when the applied field and the columnar pins are slightly misaligned. We find the Bose-glass transition induced by tilting the magnetic field in this fashion changes from the standard commensurate-incommensurate (CIC) transition[21] to the random CIC transition[22] in $(1+1)$ dimensions. Analogous critical behavior in $(2+1)$-dimensional systems with negligible point disorder is obtained using scaling arguments. This theory predicts how the linear resistivity arises at the tilt-induced transition out of the Bose-glass phase. Our conclusions for $(1+1)$-dimensional systems could be tested directly by experiments on two-dimensional Josephson junctions with an in-plane magnetic field[23] and with line pins inserted by microfabrication.

II. COMPETING DISORDERS

We start with a review of existing models. Following Ref. 2, we describe the configurations of the flux-line net-

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work by the free energy

$$\mathcal{F}=\int dz \sum_n \left\{ \frac{\tilde{\epsilon}_1}{2}\left[\frac{d\mathbf{r}_n}{dz}\right]^2 + \sum_{m\neq n}\frac{v_0}{2}\delta^{d_\perp}[\mathbf{r}_m(z)-\mathbf{r}_n(z)] + V_0[\mathbf{r}_n(z),z]+V_1[\mathbf{r}_n(z)]\right\}, \quad (2.1)$$

where $\mathbf{r}_n(z)$ is a $d_\perp$-dimensional vector denoting the transverse coordinates of the nth flux line directed along the z direction, $\tilde{\epsilon}_1$ is the elastic energy cost for transverse wandering, and v_0 is the strength of the line-line repulsion. Point disorder is described by the random potential $V_0(\mathbf{r},z)$ and the columnar pins by $V_1(\mathbf{r})$. We assume Gaussian distributions with means $\overline{V}_0$, $\overline{V}_1$ and variances

$$\overline{V_0(\mathbf{r},z)V_0(\mathbf{0},0)}=\Delta_0\delta^{d_\perp}(\mathbf{r})\delta(z) ,$$
$$\overline{V_1(\mathbf{r})V_1(\mathbf{0})}=\Delta_1\delta^{d_\perp}(\mathbf{r}) .$$

Obviously, $d_\perp=2$ describes flux lines in a bulk sample. However, a good starting point for the investigation of competitions between point and columnar defects is in $d_\perp=1$. The $(1+1)$-dimensional model describes flux lines confined to a plane[6,16–18] as shown in Fig. 1. It is also a model of vortex lines in a rough two-dimensional Josephson junction (see the Appendix and Ref. 23).

In $(1+1)$ dimensions, $\mathbf{r}_n(z)$ becomes a scalar function, allowing a continuum description. We write Eq. (2.1) as

$$\mathcal{F}=\int dx\, dz \left[\frac{v_0}{2}(\partial_x A)^2+\frac{\tilde{\epsilon}_1 a}{2}(\partial_z A)^2 +(V_0+V_1)\partial_x A+V_0'\partial_z A\right], \quad (2.2)$$

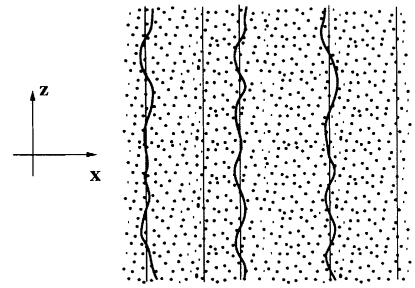

FIG. 1. An illustration of flux lines confined in $(1+1)$ dimensions, with both point disorder (dots) and columnar pins (straight lines).

with

$$A(x,z)=\sum_n \theta[x-r_n(z)] , \quad (2.3)$$

$\theta(\xi)$ being the Heaviside step function. Here, a is the mean spacing between lines and we have also included the random potential $V_0'(x,z)$ which couples to the local tilt of the flux lines. Although this term is not present in the original model, it is generated by the point disorder upon renormalization. We take $V_0'(x,z)$ to be a Gaussian random variable again, with $\overline{V_0'}=0$ and $\overline{V_0'(x,z)V_0'(0,0)}=\Delta_0'\delta(x)\delta(z)$.

The field $A(x,z)$ can be thought of as the y component of the magnetic vector potential. The discrete nature of A, as expressed in Eq. (2.3), is a consequence of flux quantization and is crucial in the formation of glassy phases. Its effect is manifested in the appearance of a periodic potential in the coarse-grained free energy. Upon introducing a displacementlike field $u(x,z)$, applying the coarse-graining procedure outlined in the Appendix, and then averaging over disorder using the replica trick, we obtain the following replicated free energy,

$$\frac{\mathcal{F}_n}{T}=\int dx\, dz \sum_{j,\alpha,\beta}\left\{\frac{1}{2}[K_j\delta_{\alpha,\beta}-\Delta_j]\partial_j u_\alpha\cdot\partial_j u_\beta - g_0\cos[2\pi(u_\alpha(x,z)-u_\beta(x,z))] - g_1\int dz'\cos[2\pi(u_\alpha(x,z)-u_\beta(x,z+z'))]\right\} . \quad (2.4)$$

Here, $\alpha,\beta\in\{1,\ldots,n\}$ are replica indices, and $j\in\{x,z\}$. The parameters $K_x=v_0/T$ and $K_z=\tilde{\epsilon}_1 a/T$ are the bulk and tilt modulus, respectively.[4] They describe the linear elasticity of the flux-line array. The bare coupling constants describing the nonlinear (cosine) interactions are $g_0=\Delta_0/(aT)^2$ and $g_1=\Delta_1/(aT)^2$. We have also set $\Delta_x=\Delta_0/T^2$ and $\Delta_z=\Delta_0'/T^2$ to simplify notation.

We performed a renormalization-group (RG) analysis of the model (2.4). Upon rescaling by a factor e^l, we find the following recursion relations for small g_0 and g_1 in the limit $n\to 0$,

$$\frac{dK_x}{dl}=0 , \quad (2.5)$$

$$\frac{dK_z}{dl}=C_1 g_1+O(g_1^2) , \quad (2.6)$$

$$\frac{d\Delta_{x,z}}{dl}=C_2 g_0^2+O(g_0 g_1, g_1^2) , \quad (2.7)$$

$$\frac{dg_0}{dl}=[2-K^{-1}]g_0-C_3 g_0^2+O(g_0 g_1, g_1^2) , \quad (2.8)$$

$$\frac{dg_1}{dl}=\left[3-\left[1+\frac{D}{2}\right]K^{-1}\right]g_1+O(g_1^2) . \quad (2.9)$$

Here the C's are positive constants, and the bulk modulus K_x is not renormalized to any order due to a statistical invariance under tilt.[24] We also introduced the dimensionless parameters

$$D \equiv \frac{\Delta_x}{K_x} + \frac{\Delta_z}{K_z} \quad \text{and} \quad K^{-1} \equiv \frac{2\pi}{\sqrt{K_x K_z}} \sim T \ ,$$

to characterize the strength of the point disorder and the rigidity of the line network, respectively. Equations (2.5)–(2.9) combine previous results on Bose-glass and vortex-glass transitions. if $\Delta_{x,z} = g_0 = 0$, then we recover[25] at temperature $K_{\rm BG}^{-1} = 3$, the Bose-glass transition first studied by Gianmarchi and Schulz.[19] The tilt modulus K_z remains finite at the transition but diverges upon entering the Bose-glass phase.[14,15] This signals the localization of flux lines to the columnar defects, since K_z is proportional to the renormalized tilt modulus $\tilde{\epsilon}_1$. If $g_1 = 0$, i.e., if correlated disorder is absent, we recover at $K_{\rm VG}^{-1} = 2$, the vortex-glass transition considered by Fisher.[6] Now, both the bulk and tilt modulus remain finite below the transition, leading to a line of fixed points describing a weakly pinned vortex-glass phase.[17,18]

The most important feature of the recursion relations of the combined problem is the eigenvalue of g_1 in Eq. (2.9). We see that point disorder reduces the Bose-glass transition temperature $K_{\rm BG}^{-1}$ from 3 to $3/(1+D/2)$. For weak point disorder (i.e., $D \ll 1$), the point disorder coupling g_0 is irrelevant at $K_{\rm BG}^{-1}$. Thus the glass transition continues to be of the Bose-glass type. However for strong point disorder ($D > 1$), where random pinning energies becomes comparable to the elastic energy, we have $K_{\rm VG}^{-1} > K_{\rm BG}^{-1}$, and the glass transition is changed to the vortex-glass type.

We next investigate the low-temperature glass phase itself. In the weak point disorder regime, the stability of the Bose-glass phase is determined by the RG flow of D. The Bose glass is stable if D decreases, but is unstable if D increases to $O(1)$. Since K_z diverges in the Bose-glass phase, while K_x remains finite, the flow of D is determined by the flow of Δ_x, i.e., the renormalized strength of point disorder. Unfortunately, the RF flow is only known close to the glass transition (see Fig. 2). The flow at low temperatures is not directly accessible via the perturbation approach discussed above.

We can, however, study the effect of point disorder on the low-energy excitations of the Bose-glass ground state. In cases where the columnar pins outnumber the flux lines, the ground state (in the absence of point disorder) is simply the state in which each flux line is localized to a columnar pin, "filling" the pins in decreasing order of pinning strength. (We assume the line-line repulsion is short range but strong enough to forbid double occupancy of any columnar pin.) As indicated in Ref. 14, the dominant excitation about the ground state is the generation of a "superkink" (see Fig. 3) by the most weakly bound flux line from a columnar pin (A) right below the chemical potential (μ) to a pin (B) slightly above μ. The energy $E_{\rm sk}$ of a superkink of width W and length L is easily estimated using the variable range hopping approach.[26] It is given by

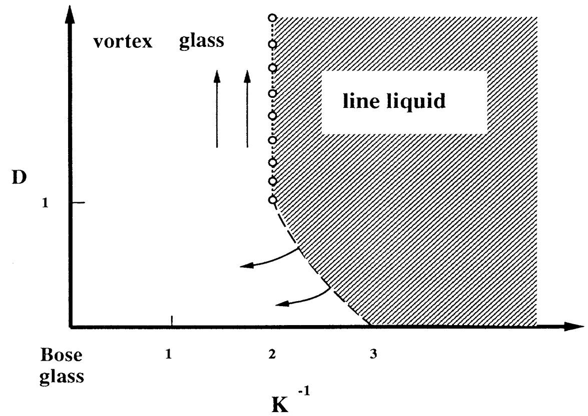

FIG. 2. A renormalization-group flow diagram near the glass transition in (1+1) dimensions. The dashed line is the critical temperature of the Bose-glass transition, $K_{\rm BG}^{-1} = 3/(1+D/2)$. The line with open circles is the critical temperature of the vortex-glass transition, $K_{\rm VG}^{-1} = 2$. A line liquid, stable to both types of disorder appears in the shaded region. In the region $D > 1$, the parameters flow to the vortex-glass phase, while along the line $D = 0$, they flow to the Bose-glass phase. The flow trajectories in the region $1 > D > 0$ and $K^{-1} < 2$ is not accessible by the RG analysis described in the text.

$$E_{\rm sk} = 2E_{\rm k}\frac{W}{d} + \epsilon(W)L \ , \tag{2.10}$$

where $E_{\rm k}$ is the energy of a single kink and $\epsilon(W)$ is the difference in pinning energy between columns A and B. Let the density of states at the chemical potential be $n_0(\mu)$. Then the smallest $\epsilon(W)$ one can find in a typical

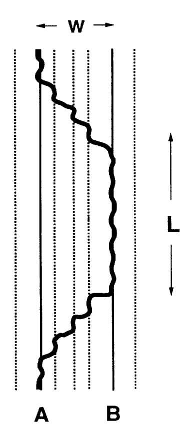

FIG. 3. An illustration of a superkink excitation. Column A is a pin right below the chemical potential and column B is the strongest unoccupied pin within a distance W of column A. Columns with weaker pinning are shown as dashed lines. The average interpin distance $d \ll W$. The length of the superkink is L. The difference in pinning energies between columns A and B is $\epsilon(W)$.

region of linear size W is of the order

$$\epsilon(W)\sim\frac{1}{n_0(\mu)W^{d_\perp}}$$

in $d_\perp$ transverse dimensions. By optimizing $E_{\rm sk}$, we find the best conformation of the superkink to be $W\sim L^{1/(d_\perp+1)}$, with a *diverging* energy scale

$$E^*_{\rm sk}(L)\sim E_{\rm k}\left[\frac{L}{E_k n_0(\mu)}\right]^{1/d_\perp+1} .$$

Thus it is energetically *unfavorable* to create large superkinks at low temperatures. (This simple consideration neglects interaction among flux lines. Incorporating a short-range interaction, one finds[14] that the qualitative results remain unchanged. In particular, the renormalized density of states $n(\mu)$ is found to be reduced but still finite.)

In the presence of point disorder, the most weakly bound flux line is affected by unavoidable variations in the pinning strengths of both columns A and B as it attempts to make a superkink. The typical fluctuation due to point disorder is of the order $\delta E(L)\sim(\Delta_0 L)^{1/2}$ for superkinks of length L centered at various points along the z direction. This fluctuation becomes bigger than the energy scale $E^*_{\rm sk}(L)$ above a length

$$L_c\sim\Delta_0^{-(d_\perp+1)/d_\perp-1)}n(\mu)^{-2/(d_\perp-1)} .$$

Thus, in $d_\perp>1$, large superkinks (of the type illustrated in Fig. 3) spontaneously proliferate along the z direction. This signals the instability of the Bose-glass ground state.

Clearly, $d_\perp=1$ is the marginal dimension. Here, one might naively expect a reduced but still diverging energy scale for small Δ_0. But fluctuations coming from rare regions of both columnar and point disorders can change results considerably. To illustrate what can happen, let us consider the probability of the most weakly bound flux line to form superkinks spontaneously in a typical $(1+1)$-dimensional sample.[27] Since a superkink of length l can only form if $|\delta E(l)|>E^*_{\rm sk}(l)\sim l^{1/2}$, we see that it occurs very rarely (with a probability $p\sim\exp[-c/\Delta_0]$) at any arbitrary point along the z direction.[28] However, the probability of having a superkink at successively larger length scales, $(2l,4l,8l$, etc.) is also given by p. Then the accumulated probability of having some superkink of any size from l to $L=2^n l$ is given by $1-(1-p)^n$. The accumulated probability becomes significant at a scale $p\ln L_c\sim1$, giving the crossover scale $L_c\sim\exp\{\exp[c/\Delta_0]\}$ beyond which point disorder must be relevant. Thus if we divide the flux line into segments of length L_c, then each segment will typically contain a superkink of size up to L_c. This suggests the instability of a *typical* Bose-glass ground state even in $(1+1)$ dimensions.

There are of course also rare samples (or rare regions of a big sample) which are void of "good" columnar pins. For example, if the next best pin within a distance W of a weakly bounded flux line is higher in energy by $\epsilon(W)\sim1$, then the minimum energy required to form a superkink of length L becomes $E^*_{\rm sk}\sim W\sim L$, which is certainty *not* sensitive to a small amount of point disorder. The probability of encountering such a region is $\sim\exp[-n(\mu)W]$, since the smallest energy difference in a typical region is $1/[n(\mu)W]$. Thus the occurrence of such a region is very rare for large W and there would normally be no observable effects. However, it is important to observe that in $(1+1)$ dimensions, such rare regions are the "bottle necks", which control transport properties (e.g., the resistivity) of a large system.[29] It then follows that the important *dynamic* properties of the flux lines remain to be of the Bose-glass type.

The above considerations indicate that the behavior of the flux lines in the marginal dimension $d_\perp=1$ is quite complicated. Both columnar and point disorders may be important in the thermodynamic limit. It all depends on which observable one looks at. But in any case, the crossover (if at all) away from the simple Bose-glass phase would occur at an astronomically large scale for small Δ_0. Thus the Bose-glass phase exists $(1+1)$ dimensions for all practical purposes in this limit. In higher dimensions, the energy scale $E_{\rm sk}$ is always short circuited by the point disorder. However the crossover length, of $O[\Delta_0^{-3}n(\mu)^{-1}]$ in $(2+1)$ dimensions, can still be quite large, especially when the range of the flux-line interaction (of the order of the London penetration length) is long and thereby the density of states $n(\mu)$ is low.[14,30] We do not know the nature of the phase beyond the crossover length. It could be the vortex glass (if it exists in higher dimensions) or a mixed anisotropic phase involving both columnar and point disorder. On the other hand, simple power counting using the boson Hamiltonian of Ref. 15 indicates that weak point disorder is again *not* relevant at the Bose-glass transition in $(2+1)$ dimensions.[31] It is therefore plausible that the above conclusions in $(1+1)$ dimensions are at least qualitatively correct for bulk $(2+1)$-dimensional samples.

III. RESPONSE TO A TRANSVERSE FIELD

In the remaining part of this paper, we will assume that a Bose-glass phase exists over the relevant length scales in the presence of weak point disorder, and study the effect of a small transverse magnetic field $\mathbf{H}_\perp$ on the glass transition. The transverse field represents misalignment between the applied field and the average columnar pin direction, which is straightforward to control experimentally. As discussed in Ref. 14, application of such a field provides a convenient way to determine whether point or correlated disorder dominates a particular experiment. We first discuss the problem in $(1+1)$ dimensions, where the transverse field can be incorporated into Eq. (2.4) as

$$\mathcal{F}_n(H_\perp)=\mathcal{F}_n-\frac{\phi_0}{4\pi}H_\perp\sum_\alpha\int dx\,dz(\partial_z u_\alpha) , \qquad (3.1)$$

where ϕ_0 is the magnetic flux quantum. The field $H_\perp$ attempts to tilt the flux lines in the x direction. In the Bose-glass ground state, there is an energy barrier preventing the flux lines from tilting. The barrier arises because the energy gained from tilting by an angle θ is only of $O(H_\perp \theta L)$, L being the linear dimension of the system, while the energy cost is of $O(E_k \theta L/a)$, since $\theta L/a$ kicks must be created. The presence of point disorder reduces the barrier, but is not enough to eliminate it (unless $L > L_c$). Thus there is a critical field $H_\perp^{c1} \sim E_k$ below which the Bose glass acts as a "transverse Meissner phase" where the transverse magnetic susceptibility is zero. At higher temperature, thermal fluctuations renormalize E_k and lower the tilt barrier. In particular, $H_\perp^{c1}$ must vanish as we approach the Bose-glass transition temperature T_{BG} from below, since the kink energy vanishes there (see Fig. 4). The shape of $H_\perp^{c1}(T)$ in the vicinity of T_{BG} can be straightforwardly determined from scaling. According to Eq. (3.1), $H_\perp$ has the length dimension $[x^{-1}]$. Using the known behavior of the correlation length at the Bose-glass transition,[19] we find $H_\perp^{c1}(T) \sim \exp[-c/(T_{BG}-T)]$. Thus the transverse critical field has a sharp upward cusp, similar to the one found for commensurate-incommensurate (CIC) transitions of atoms on a periodic substrate.[32] In (2+1) dimensions, the transverse Meissner phase is still expected to exist, since the energy barrier against tilting remains unchanged. However, the shape of the cusp near T_{BG} is modified to a power law.[14]

For $H > H_\perp^{c1}(T)$, the transverse susceptibility is finite,[33] since kinks proliferate throughout the system. It is energetically favorable for the kinks to line up in order to insure single occupancy of the columnar pins. Thus the kinks form "chains" of average spacing w that run across the sample in the $H_\perp$ direction (see Fig. 5). To reduce the free energy, the chains themselves can wander in the z direction by kink motion along the columnar pins. However, the transverse excursion is limited by encounters with the neighboring chains, since the chain-chain interaction is repulsive. This reduction in entropy discourages kink proliferation.

In (1+1) dimensions and without point disorder, the physics describing the kink chains right above $H_\perp^{c1}(T)$ is

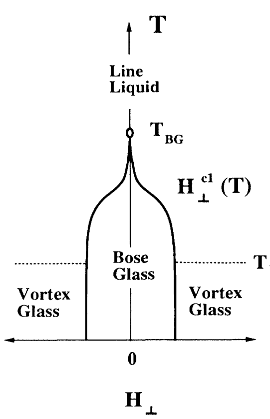

FIG. 4. Phase diagram in the $(H_\perp, T)$ plane, showing the Bose-glass, vortex-glass, and line liquid phases for weak point disorder. The phase transition across the solid line $H_\perp^{c1}(T)$ is of the random CIC type. But critical temperature at the tip of the cusp is described by the Bose-glass transition.

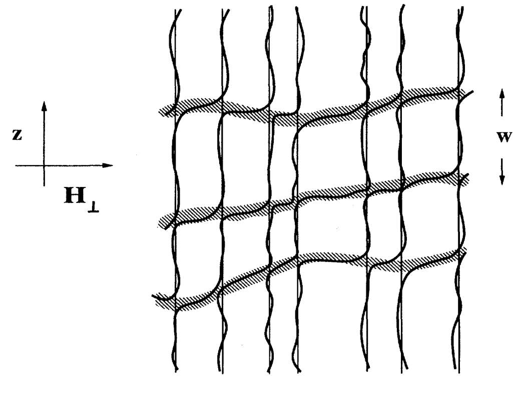

FIG. 5. For $H_\perp > H_\perp^{c1}$, each flux line (solid line) "hops" from one occupied columnar pin to another via kinks. The kinks link up to form chains in order to keep each columnar pin singly occupied. (Note that only columnar pins with binding energies below the chemical potential are relevant here. Other pins that do not participate in the process are now shown in the figure.) The chains (shaded regions) run across the sample in the $\perp$ direction. The average interchain distance is w.

just that of the usual CIC transition.[21] Balancing the magnetic energy gain $(\sim\delta)$ with the confinement entropy cost $(\sim w^{-2})$, where $\delta = H_\perp - H_\perp^{c1}(T)$ is the distance from $H_\perp^{c1}(T)$ and w is the average chain separation distance (see Fig. 5), one obtains the preferred chain density $n_{\text{chain}} = w^{-1} \sim \delta^{1/2}$. This transition is *not* sensitive to the quenched fluctuations in the strength of columnar pins, which only provide identical overall energy fluctuations to each individual chain. In the presence of point disorder, a directed chain can reduce its free energy more substantially by wandering further in the transverse direction to take advantage of fluctuations in random energy. A single chain of length l wanders a distance $l^{2/3}$ to lower its free energy by $l^{1/3}$.[22] Confining a chain to a distance w therefore leads to a loss of random energy of $O(w^{-1})$ per length per chain, in addition to the entropy cost. This increases in the free energy of confinement then leads to a reduced chain density, $n_{\text{chain}} \sim \delta$. This expression was first derived by Kardar using the Bethe ansatz.[22] For $H_\perp > H_\perp^{c1}(T)$, there will ultimately be a vortex-glass transition at low temperatures when point disorder is present (see Fig. 4).

The above analysis can be extended straightforwardly to (2+1) dimensions. Let $\mathbf{H}_\perp$ be in the x direction. In the absence of point disorder and at low temperatures, we expect the kinks to organize into "sheets" (as in a smectic-A liquid crystal), equally spaced and stacked on top of each other in the z direction. This is a consequence of translational symmetry (in z) and interkink repulsion. Within a sheet, we can again think of the kinks as chains, directed by $\mathbf{H}_\perp \| \hat{\mathbf{x}}$ through a forest of columnar defects. The configuration of the directed chains within a sheet is similar to that of the (1+1) dimensional flux lines with point disorder. Let the intersheet spacing be w_z, and interchain spacing within a sheet by w_y, then, the free energy (per chain) is

$$f=-\delta+O(w_y^{-2}+w_z^{-2})+O(w_y^{-1}) . \quad (3.2)$$

Here, the first term is the energy gained by a chain when it follows the tilt field, the second term is the entropy cost of confinement, and the third term is the loss in random energy as explained above. From Eq. (3.2), we find the optimal separations distances to be $w_y\sim\delta^{-1}$ and $w_z\sim\delta^{-1/2}$, leading to a preferred density, $n_{\rm chain}=(w_y w_z)^{-1}\sim\delta^{3/2}$.

The critical behavior close to $H_\perp^{c1}(T)$ can be measured through the behavior of the linear resistivity ρ near the glass transition. In the Bose-glass phase, the *I-V* characteristics are nonlinear,[14] with zero linear resistivity. However, the glass turns into a line liquid with finite linear resistivity as one crosses $H_\perp^{c1}(T)$ for $T>T_{\rm VG}$ (see Fig. 4). The rise in ρ results from the appearance of free-moving chains of kinks. Therefore, $\rho\sim n_{\rm chain}\sim\delta^{\nu}$, with $\nu=\frac{1}{2}$ (or $\frac{3}{2}$) for columnar pins only in (1+1) [or (2+1)] dimensions. Including weak point disorder, we have $\nu=1$ in (1+1) dimensions. The singular behavior of the tilt modulus is given by $K_a\sim n_{\rm chain}^{-1}\sim\delta^{-\nu}$ near the CIC transition.

On the other hand, the resistivity at the tip of the phase boundary is controlled by the *critical dynamics* of the Bose-glass transition. For a (1+1)-dimensional system (Fig. 1), a current J applied perpendicular to the plane of the flux lines generates a Lorentz force which pushes the lines in the x direction.[34] The resistivity is simply the response of the system to J, which can be incorporated into the free energy of Eq. (A4) as

$$\mathcal{F}\to\mathcal{F}[u,J]=\mathcal{F}[u]-J\int dx\, dz\, u(x,z) .$$

For small J, the dynamics of this system is given by the variation of the effective free energy as

$$\frac{\partial u}{\partial t}=-\frac{\Gamma}{T}\frac{\delta}{\delta u}\mathcal{F}[u,J]+\eta(x,z,t) ,$$

where the parameter Γ characterizes the "mobility" of the flux-line network and is proportional to the resistivity, and η is the thermal noise, characterized by its second moment,

$$\langle\eta(x,z,t)\eta(x',z',t')\rangle=2\Gamma\delta(x-x')\delta(z-z')\delta(t-t') .$$

Extending the RG calculation of Sec. II to dynamics as was done for a related problem in Ref. 35, we find the following recursion relation for Γ,

$$\frac{d\Gamma}{dl}=-\Gamma[a_0 g_0+a_1 g_1] , \quad (3.3)$$

where a's are positive constants in the vicinity of the glass transition. Since the coupling constant g_1 (or g_0) become nonzero in the Bose-glass (or the vortex-glass) phase, we see explicitly here that $\Gamma(l\to\infty)\to 0$, i.e., zero resistivity in the glass phases. The behavior of Γ at the glass transition is more interesting; it depends crucially on the flows $g_0(l)$ and $g_1(l)$, which can be obtained by solving Eqs. (2.5) through (2.9). For large l, we find $g_1(l)\sim l^{-2}$ at the Bose-glass transition and $g_0(l)\sim l^{-1}$ at the vortex-glass transition. Using these results in Eq. (3.3), we find $\Gamma_{\rm BG}(l)\to$const and $\Gamma_{\rm VG}(l)\sim l^{-a_0}$. Thus, the faster decrease in $g_1(l)$ leads to a *discontinuous* drop in the resistivity to zero at the Bose-glass transition.[36] But at the vortex-glass transition, $g_0(l)$ decreases slowly enough to give a *continuous* vanishing of the resistivity, of the form $\rho(T)\sim(T-T_{\rm VG})^{1.58}$, as one approaches the transition from above.[35] Although $\rho(T)$ is discontinuous at $T_{\rm BG}$ when columnar defects dominate in (1+1) dimensions, it is expected also to have a power-law form in (2+1) dimensions.[14]

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APPENDIX: DERIVATION OF THE CONTINUUM FREE ENERGY

In this appendix, we derive an effective free energy describing the behavior of the flux-line array at distance scales much larger than the interline spacing a. The effective free energy was introduced in Ref. 6 based on symmetry grounds. Here, we derive it explicitly from the free energy of the directed lines, i.e., from Eq. (2.1) and Eq. (2.2). Unfortunately, the derivation does *not* extend immediately to flux lines in (2+1) dimensions.[20]

We note that although the free energy (2.2) has a simple quadratic form, the complicated dependence on $r_n(z)$ is hidden in Eq. (2.3). However, simplifications occur upon coarse graining the system. To this end, we first rewrite Eq. (2.3) using the Poisson summation formula,

$$A(x,z)=\int d\sigma\,\theta[x-r(\sigma,z)]\sum_{m=-\infty}^{\infty}e^{2\pi i m\sigma} , \quad (A1)$$

where the last factor fixes the integration variable σ to be nonzero only at integer values. The step function in the integrand sets the upper limit of integration at σ^* such that $r(\sigma^*,z)=x$. This implicitly defines σ^* as a function of x and z, i.e.,

$$\sigma^*=\phi(x,z) .$$

In terms of the field ϕ, Eq. (A1) simply becomes

$$A(x,z)=\phi(x,z)+\pi^{-1}\sin[2\pi\phi(x,z)]+\cdots . \quad (A2)$$

We can identify $\phi(x,z)$ as a *phase* field, since the transformation $\phi\to\phi+$integer merely shifts the line labels, leaving the problem invariant. The invariance is a result of the equivalence of the flux lines as required by flux quantization. Substituting Eq. (A2) into the free energy (2.2) and keeping only the relevant terms, we obtain the following effective free energy,

$$\mathcal{F}=\int dx\,dz\left[\frac{v_0}{2}(\partial_x\phi)^2+\frac{\tilde{\varepsilon}_1 a}{2}(\partial_z\phi)^2+(V_0+V_1)\partial_x\phi\{1+2\cos[2\pi\phi(x,z)]\}+V_0'\partial_z\phi\right]. \tag{A3}$$

It is interesting to point out that Eq. (A3) is precisely the energy describing a two-dimensional Josephson junction,[23] with $\phi(x,z)$ being the gauge-invariant phase difference across the junction, and the random potential V's describing the variations in local critical currents due to nonuniformities in the junction thickness. Here, the periodicity in ϕ appears directly as a consequence of magnetic-flux quantization.

From Eq. (A3), we see that the average profile of ϕ is $-x/a$, where $a=\overline{V_0+V_1}/v_0$ is the average line separation. (The average chemical potential V_0 is proportional to the applied magnetic field H_z.) To study the fluctuations, it is convenient to introduce a displacementlike field, via $\phi(x,z)=-x/a-u(x,z)$. Using the field u in Eq. (A3), and shifting away $V_1(x)$ from the quadratic part with the transformation $u(x,z)\rightarrow u(x,z)+v_0^{-1}\int^x dx' V_1(x')$, we obtain the effective free energy

$$\mathcal{F}=\int dx\,dz\left[\frac{v_0}{2}(\partial_x u)^2+\frac{\tilde{\varepsilon}_1 a}{2}(\partial_z u)^2-V_0\partial_x u-V_0'\partial_z u-\frac{2}{a}V_0(x,z)\cos[2\pi(x/a+u(x,z))]\right]. \tag{A4}$$

Replicating the above and performing disorder average finally yields Eq. (2.4).

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