RAPID COMMUNICATIONS

PHYSICAL REVIEW B VOLUME 48, NUMBER 15 15 OCTOBER 1993-I

Freezing of the quantum Hall liquid at $\nu = \frac{1}{7}$ and $\frac{1}{9}$

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(Received 7 June 1993)

We compare the free energy computed from the ground-state energy and low-lying excitations of the two-dimensional Wigner solid and the fractional quantum Hall liquid, at magnetic filling factors $\nu = \frac{1}{7}$ and $\frac{1}{9}$. We find that the Wigner solid melts into the fractional quantum Hall liquid at roughly the same temperature as that of some recent luminescence experiments, while it remains a solid at the lower temperatures characteristic of the transport experiments. We propose this melting as a consistent interpretation of both sets of experiments.

The phase boundary between the fractional quantum Hall liquid, or Laughlin liquid (LL), and the Wigner solid (WS) has been a subject of active research for some time now. Recent transport experiments[1–3] have shown clear evidence of an insulating state that first appears just above $\nu = 1/5$, disappears as ν approaches $1/5$, then reappears strongly at all smaller ν. This insulating phase has generally been interpreted to be a manifestation of a pinned Wigner solid, although recently an alternative theory[4] has been put forward where disorder plays a central role in driving the phase transitions.

The ground state in a two-dimensional electron system at filling factor $\nu = 1/7$ and $1/9$ remains unclear. Experimentally, there is a weak signature, a dip in the derivative of P_{xy} with respect to the magnetic field B, in the transport data at $\nu = 1/7$ (Refs. 2 and 5) at somewhat elevated temperatures in the best samples available today. But it disappears upon lowering the temperature to that typical of magnetotransport experiments, $\sim$ 20–100 mK. At $\nu = 1/9$, there has been no report of any transport anomaly. On the other hand, magneto-optical luminescence experiments,[6,7] done at somewhat higher temperatures of 400–600 mK, show features in the luminescence spectrum at $\nu = 1/7$ and $1/9$ that are similar to those at $\nu = 1/3$ and $1/5$. The authors have interpreted these features as due to the formation of the quantum Hall liquid at these filling factors. To explain the discrepancy with the transport studies, they suggest that the background resistivity, caused by magnetic-field-induced localization due to disorder, becomes so high at $\nu < 1/5$ that the quantum Hall states are unobservable in transport.

In this paper, we compare the free energies of the Wigner solid and the Laughlin liquid at $\nu = 1/7$ and $1/9$, and we find that at temperatures approximating those of the luminescence experiments the Wigner solid will melt into a Laughlin liquid, but at the temperatures of the typical transport experiments, the Wigner solid remains the stable state. Our calculation is necessarily only semiquantitative, as disorder effects at these low filling factors will have a significant effect on the ground-state energy of the solid and the roton gap of the liquid. Nonetheless, our results appear to provide a consistent interpretation for both sets of experiments.

Previous calculations of the ground-state energy of the liquid and solid[8,9] have used variational wave functions confined to the lowest Landau level. Then the only relevant dimensionless parameter is the magnetic filling factor ν, and it was found[9] that the melting transition takes place at $\nu_c = 1/6.5$. Physically, this high magnetic-field limit implies that as the magnetic field B goes to infinity the magnetic length ℓ, where $\ell^2 = \hbar c/eB$, goes to zero and the ion-disk radius $a = (\pi n)^{-1/2}$ goes to zero, but the filling factor $\nu = 2\ell^2/a^2$ remains constant. This works well at zero temperature, but at finite temperature, we find ourselves comparing thermal energies of the order $k_B T$ to Coulomb energies that go as $e^2/\ell \to \infty$, and we must forgo the assumption that no Landau-level mixing occurs and introduce another dimensionless parameter $r_s = a/a_B$, where $a_B = \hbar^2\epsilon/m^* e^2$ is the Bohr radius, ϵ is a dielectric constant, and m^* is an effective mass. For the electron-doped GaAs heterojunctions used in the experiments we use the values $\epsilon = 12.8$ and $m^* = 0.068$. Then a finite value B of the magnetic field requires a finite ion-disk radius a, and we can reasonably compare thermal energies $k_B T$ to Coulomb energies of the order e^2/a.

Price, Platzman, and He[10] have recently calculated the ground-state energy of the Laughlin liquid at $\nu = 1/7$ and $1/9$ as a function of r_s, using a variational wave function that included Landau-level mixing. Zhu and Louie[11] have also calculated the ground-state energy of the Wigner solid at these filling factors when $r_s = 2$ and 20, and they find that the ground-state energy of the solid is lower than that of the liquid for all r_s. Platzman and Price[12] have also calculated the free energy of both liquid and solid at $\nu = 1/3$ and $1/5$, and find a curious reentrant freezing behavior in certain ranges of r_s as the temperature is raised from zero. Here we will use the same method to find the free energies at $\nu = 1/7$ and $1/9$, and we will provide an estimated melting temperature as a function of r_s.

Because the temperatures of interest are so low, we need only know the lowest-lying modes ω_k of both the solid and the liquid to calculate the free energy. Then the free energy is

$$F = E + T \sum_k \ln(1 - e^{-\omega_k/T}), \tag{1}$$

where E is the ground-state energy and the ω_k are the lowest-lying excitations of either the liquid or the solid. For the Wigner solid these excitations are the lower-hybrid, essentially transverse magnetophonons $\omega_k^{\rm WS}$. We evaluated the free energy $F^{\rm WS}$ of the solid using the ground-state energy $E^{\rm WS}$ from Ref. 11, and the harmonic magnetophonons $\omega_k^{\rm WS}$ calculated in the same way as, e.g., Bonsall and Maradudin,[13] but with a strong magnetic field. The sum in (1) for the solid phase was evaluated by averaging over the Brillioun zone by the method of Cunningham,[14] so the result is exact in the harmonic approximation. It is useful, however, to examine the form of the transverse magnetophonons in the long-wavelength limit

$$\omega_k^{\rm WS} \approx 0.526 \left(\frac{\nu}{r_s} \frac{e^2/\epsilon}{2a_B} \right) (ka)^{3/2}. \tag{2}$$

Substituting into (1) we find

$$F^{\rm WS} \approx E^{\rm WS} - 0.701 \left(\frac{r_s}{\nu} \frac{2a_B}{e^2/\epsilon} \right)^{4/3} T^{7/3}, \tag{3}$$

and the free energy of the solid phase goes as $T^{7/3}$ at low temperatures.

On the liquid side, since we are interested primarily in the lowest-lying modes of the magnetoroton spectrum, we can approximate the magnetoroton mode by

$$\omega_k^{\rm LL} = \frac{(k - k_R)^2}{2m_R} + \Delta_R, \tag{4}$$

where m_R is an effective mass for the magnetorotons near the minimum. Here we have used the magnetoroton spectrum calculated in the lowest Landau level. Rappe, Zhu, and Louie[15] have very recently calculated the magnetoroton spectrum as a function of r_s for $\nu = 1/3$, and find that at $r_s = 20$, the magnetoroton spectrum has fallen only about 10% below the lowest Landau-level value. At $\nu = 1/7$ and $1/9$ the amount of Landau-level mixing found in Ref. 10 is more than an order of magnitude less than that found at $\nu = 1/3$, so using the lowest Landau-level spectrum is an excellent approximation. Assuming that only the modes in the vicinity of the minimum contribute, the free energy per particle of the liquid is

$$F^{\rm LL} = E^{\rm LL}(r_s) - (2\pi m_R)^{1/2} \frac{k_R \ell^2}{\nu} T^{3/2} e^{-\Delta_R/T}, \tag{5}$$

which goes exponentially at small $T \leq \Delta_R$.

Because the excitations of the liquid display a gap, at very low temperatures $T \ll \Delta_R$ even the lowest-lying modes at the minimum k_R remain unoccupied, while the low-lying modes of the solid, which have no gap, begin to fill immediately. The free energy of the solid then falls as a power of T, while the free energy of the liquid remains nearly constant. When the temperature begins to approach the roton gap energy, however, the free energy of the liquid begins to fall exponentially as the states become occupied. Because the density of states at the roton gap energy is very large, this exponential rise is very rapid and the liquid free energy quickly falls below that of the solid.

Figure 1 shows the difference in free energy $F^{\rm WS} - F^{\rm LL}$ as a function of T at $\nu = 1/7$ and $1/9$. The density $r_s = 2.3$ was chosen to match the density of one of the luminescence experiments.[6] The difference in ground-state energies favors the solid at zero temperature, and as the temperature begins to rise, the solid is favored slightly more since the magnetophonon modes are beginning to be occupied while the magnetoroton modes are not. Once the temperature becomes some substantial fraction of the magnetoroton gap, however, the exponential character of the free energy of the liquid begins to assert itself as the magnetoroton modes become available, and the liquid free energy rapidly drops below that of the solid. The slope at which the curves cross zero shows that the magnitude of the roton gap Δ_R has much more effect on the melting temperature than does any difference in ground-state energies $\Delta E = E^{\rm WS} - E^{\rm LL}$, unless ΔE is very small. Otherwise a change in ΔE, which shifts the curves up or down on the energy axis, changes the melting temperature by only a small amount.

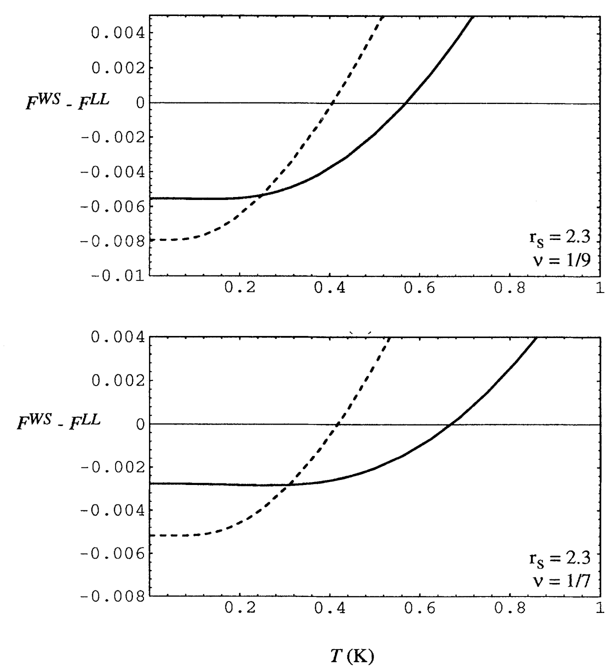

FIG. 1. The difference in free energy between the Wigner solid and the Laughlin liquid at $\nu = 1/7$ and $1/9$, with $r_s = 2.3$. The solid line is computed using the theoretical value for the gap Δ_R, assuming no disorder, and the dashed line includes an estimate of disorder-induced lowering of the Wigner solid ground-state energy and uses $\Delta_R = 0.4$ K at $\nu = 1/7$ and $\Delta_R = 0.25$ K at $\nu = 1/9$. The free energy is given in units of $(e^2/\epsilon)/2a_B$.

The phase boundaries for $\nu = 1/7$ and $1/9$ are shown in Fig. 2. The relationship of the melting temperature, shown as the solid lines, to the size of the magnetoroton gap, shown as the dash-dotted lines, is clearly visible. Melting, as a rule, will occur roughly at some constant fraction of the magnetoroton gap. The dashed lines in Fig. 2 show the predicted classical Kosterlitz-Thouless melting temperature. The melting temperatures we find are roughly comparable to the Kosterlitz-Thouless melting temperature, although there is no theoretical reason to expect them to be closely related.

Disorder in the sample will have varying effects on the ground-state and temperature-dependent parts of (5) and (3). Impurities will cause the background charge in the sample to become slightly nonuniform, and the Wigner solid will adjust by compressing somewhat in areas of high background charge and expanding somewhat in areas of low background charge. The Laughlin liquid, since it is incompressible, to lowest order cannot do this, and the difference in ground-state energies $\Delta E = E^{\mathrm{WS}} - E^{\mathrm{LL}}$ will change in favor of the solid at very low temperatures. In Ref. 10, the shift in energy due to impurities is estimated roughly as

$$E_{\mathrm{imp}} \approx \frac{1}{2} m v_t^2 \left(\frac{a}{\xi}\right)^2 = \frac{0.138}{r_s} \left(\frac{a}{\xi}\right)^2 , \tag{6}$$

where $v_t = (0.138 e^2/ma)^{1/2}$ is the transverse sound velocity in the absence of the magnetic field and ξ is the correlation length for the distorted WS. If we assume $\xi = 5a$, we find that the shift in ground-state energy is

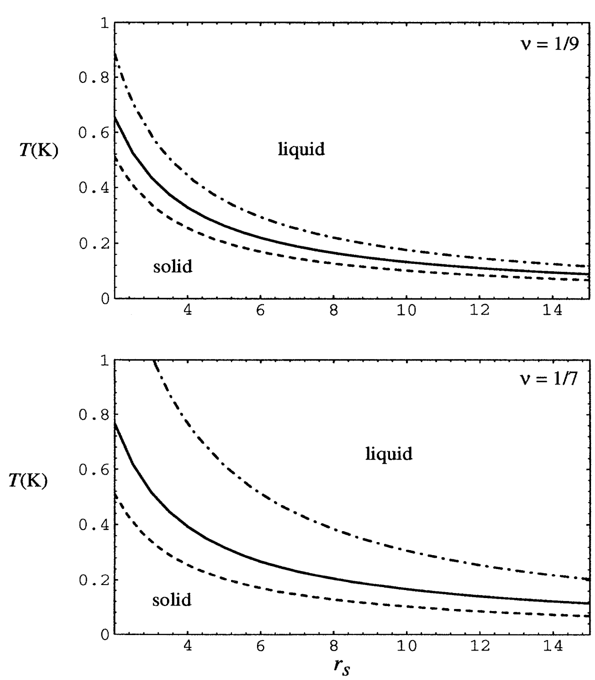

FIG. 2. The phase boundary between Wigner solid and Laughlin liquid (solid line) at $\nu = 1/7$ and $1/9$. The dashed line is the classical Kosterlitz-Thouless melting temperature and the dash-dotted line is the theoretical gap Δ_R. Disorder effects are not included in this diagram.

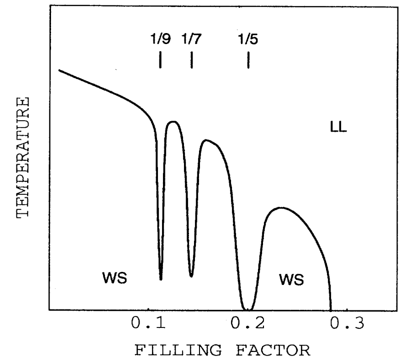

FIG. 3. A qualitative phase diagram at finite but constant r_s. The area marked "LL" is the liquid region, and the areas marked "WS" are the solid regions. Temperature is given in arbitrary units.

about $-0.0055/r_s$, very significant at $\nu = 1/7$ and only slightly less so at $\nu = 1/9$.

On the other hand, the magnetoroton gap of the liquid is significantly reduced by disorder.[16,17] At $\nu = 1/5$, the measured gap (presumably the quasielectron-quasihole gap) is 1.1 K, while the single-mode approximation of Ref. 18 which we have used gives $\Delta_R = 5.6$ K for the sample of Ref. 2. The luminescence measurements give gaps on the order of, but smaller than, 0.4 K at $\nu = 1/7$, and 0.25 K at $\nu = 1/9$ for a sample with $r_s = 2.3$. The presence of disorder will open a small gap in the magnetophonon spectrum, but this gap will have a negligible effect on the free energy of the solid, since it is centered at the origin of the Brillioun zone, where the density of states is small.

The result of moving the ground-state energy of the solid down by $-0.0055/r_s$ and using $\Delta_R = 0.4$ K at $\nu = 1/7$ and $\Delta_R = 0.25$ K at $\nu = 1/9$, reduced from the corresponding theoretical results, is shown in Fig. 1. In spite of the shift in ground-state energy favoring the solid at low temperatures, the exponential drop in the free energy of the liquid near the magnetoroton gap temperature moves the melting temperature down to about 400 mK for both $\nu = 1/7$ and $1/9$. Because the measured gaps are, strictly speaking, not the magnetoroton gap, but either the quasielectron-quasihole gap or the magnetoroton mode at small k, the actual gap Δ_R will be significantly lower than the measurements given above, and the melting temperature will be proportionately lower as well. Of course, we do not know the precise amount of disorder in the samples, but our calculation shows that the Wigner solid may melt at a temperature equivalent or slightly below those of the luminescence experiments, while remaining a solid at the lower temperatures of the transport experiments.

With this melting in mind, we would like to propose a modification, shown in Fig. 3, to the phase diagram given in Ref. 7. The phase boundary at $\nu = 1/7$ and $1/9$ no longer extends down to zero temperature, as the previous authors proposed. The ground state remains the Wigner solid at these filling factors, but as the temperature is raised the solid at $\nu = 1/7$ and $1/9$ quickly gives way to a fractional quantum Hall state. Our finite-temperature phase transition thus provides a possible explanation for both the relatively high-temperature magneto-optical results,[6,7] and the lower-temperature magneto-transport results.[2,5]

Work at Berkeley was supported by NSF Grant No. DMR91-20269 and by the Director, Office of Energy Research, Office of Basic Energy Sciences, Materials Sciences Division of the U.S. Department of Energy under Contract No. DE-AC03-76SF00098. Cray computer time for work at Berkeley was provided by the NSF at the San Diego Supercomputing Center and by the Department of Energy.

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