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Scaling in spin-degenerate Landau levels in the integer quantum Hall effect

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In a low-mobility $\mathrm{In}_x\mathrm{Ga}_{1-x}\mathrm{As/InP}$ heterostructure, we find that the quantized Hall resistance plateau, $\rho_{xy}=h/ie^2$ with $i=3$, starts developing only when T is below 50 mK. For higher T, the plateau and the associated minimum in the diagonal resistivity ρ_{xx} can be made to appear by tilting the sample with respect to the B field. We have studied the T dependence of the ρ_{xx} minimum and deduced the thermal-activation energy (Δ). We find that Δ is 67 times smaller than the Landau-level broadening deduced from the electron mobility. This result implies that there is no gap in the density of states and that, for $T>50$ mK, the Landau level is spin degenerate. In this higher T range, we observe a $T^{-0.21}$ power law for the maximum value of $d\rho_{xy}/dB$ between the $i=2$ and 4 plateaus, and $T^{-0.42}$ for the minimum value of $d^2\rho_{xx}/dB^2$. These exponents are a factor 2 smaller than those for spin-polarized levels.

The experimental discovery of the integer quantum Hall effect[1] (IQHE) has posed fundamental problems on the scaling theory of localization in disordered electronic films.[2] Important progress[3-6] has been made in recent years, following the theoretical breakthrough[7] on the subject of Anderson delocalization in strong magnetic fields by Pruisken and co-workers. In a two-dimensional electron gas (2DEG) at low temperature (T) and high magnetic fields (B), the appearance of the quantum Hall steps in the Hall resistance ρ_{xy} ($=h/ie^2$, where h is the Planck constant, e the electron charge, and i an integer) is now understood in terms of a continuous metal-insulator transition with critical singularities in the resistances ρ_{xx} and ρ_{xy} as T approaches zero. More specifically, we demonstrated[4] that both the maximum in $d\rho_{xy}/dB$, $(d\rho_{xy}/dB)^{\max}$, and the inverse of the half-width in ρ_{xx}, $(\Delta B)^{-1}$, between two adjacent quantum Hall plateaus at low T followed the power law $T^{-\kappa}$ ($\kappa=0.42$), independent of the Landau-level index. Moreover, the minimum value of $d^2\rho_{xx}/dB^2$ $[(d^2\rho_{xx}/dB^2)^{\min}]$ exhibited $T^{-0.84}$. These results verify the theoretical prediction[8] that both resistances, ρ_{xx} and ρ_{xy}, depend on the single scaling variable $T^{-\kappa}(B-B^*)$ only. Here, $\kappa=p/2\nu$, and p is the temperature exponent of the inelastic scattering time;[9] ν is the critical exponent for the zero T localization length ξ. B^* denotes the critical B where an infinitely sharp Hall step develops as T approaches zero. In this paper, we study the T-dependent transport between plateaus in spin-degenerate Landau levels in low-mobility $\mathrm{In}_x\mathrm{Ga}_{1-x}\mathrm{As/InP}$ heterostructures. Two results stand out. First, our data indicate that the formation of IQHE does not need a gap in the single-particle density of states. Second, we find that the exponent of the power law is 0.21, half the value for spin-nondegenerate levels.

The sample used in this work is an $\mathrm{In}_x\mathrm{Ga}_{1-x}\mathrm{As/InP}$ heterostructure grown by vapor phase epitaxy. It has electron density $n_{2\mathrm{D}}=2.0\times10^{11}$ cm^{-2}, and mobility $\mu=16\,000$ cm^2/V s at 4.2 K. Conventional Hall bars of the size of several millimeters were used to make the transport measurement in a dilution refrigerator for 25 mK $<T<$ 800 mK. The ρ_{xx} and ρ_{xy} were measured simultaneously by using two lock-in amplifiers with an excitation current of 1 to 0.5 nA at an oscillating frequency of 13.8 Hz. We have used an *in situ* rotating sample holder to tilt the sample with respect to B. The tilt angle θ is the angle between B and the surface normal. Since the cyclotron motion depends only on the B component normal to the sample surface, a Hall voltage measured at $B=B_1$ and $\theta=0$ will be obtained at $B=B_1/\cos\theta$ for $\theta\neq0$.

Figure 1 shows the ρ_{xx} and ρ_{xy} as a function of B at three different T's for $\theta=0$. Above $T\sim50$ mk, there is no indication of the $i=3$ plateau. However, by rotating the sample with respect to the applied B to increase the

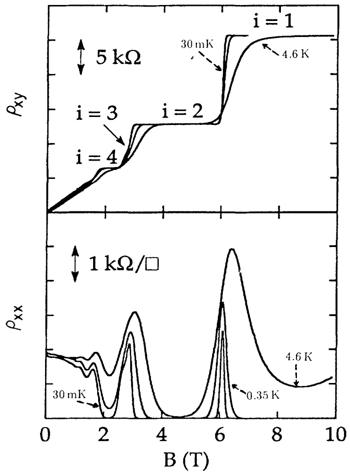

FIG. 1. Magnetotransport coefficients ρ_{xy} and ρ_{xx} vs B at $T=4.6$, 0.35, and 0.03 K in a low-mobility $\mathrm{In}_x\mathrm{Ga}_{1-x}\mathrm{As/InP}$ heterostructure for $\theta=0$.

Zeemann energy, the plateau becomes apparent. Figure 2(a) shows ρ_{xy} and ρ_{xx} as a function of the perpendicular component of B for four different θ's achieved by using our *in situ* rotating sample holder at 50 mK. It is clear that the $i=3$ plateau becomes wider and wider with increasing θ. We have measured the T dependence of the ρ_{xx} at the $i=3$ minimum for several θ's, and the data show thermally activated behavior over at least a decade in ρ_{xx} within our T range. From the Arrhenius plot of the data taken at each θ, we extract an activation energy Δ. Figure 2(b) shows Δ as a function of the total B. All the data points fall on a straight line passing through the origin (for $\theta=0$, Δ cannot be measured due to the limited T range). The Δ is a measure of energy between two regions of extended states if the thermal energy is smaller than their separation. We noticed that the thermal energy at 800 mK (our highest T in this experiment) is 0.69 and 0.002 meV at 25 mK. Since the measured $\Delta=0.06$ meV at $\theta=64°$, this picture of thermal activation into the extended states across an energy gap is reasonable in our T range except at the high T end.

The lower bound for the broadening of the Landau level (Γ), estimated from the measured mobility using the self-consistent Born approximation result, is 3.9 meV.[10] Our measured $\Delta=0.06$ meV at $\theta=64°$ is much smaller than Γ. This result indicates that the density of states (DOS) for the $N=1\uparrow$ and $1\downarrow$ levels overlap and has no gap. The same statement holds for $\theta=0$, at which the extrapolated $\Delta=0.023$ meV. These results indicate that the spin polarization is hardly visible in the DOS for all values of θ involved. From the semiclassical picture of transport in a B field, the behavior of the conductivity follows qualitatively the DOS. Therefore, ρ_{xx} should show a peak at filling factor 3 at all T. On the other hand, our data in Fig. 1 indicate that the $i=3$ IQHE will appear if T is much lower than 50 mK. This is the first experimental evidence that the plateau in ρ_{xy} and the vanishing of ρ_{xx} will appear despite the fact that the DOS at the Fermi energy is a maximum. Our result and earlier experimental studies[11–13] showing nonzero DOS in between two Landau levels indicate that the IQHE does not depend on the existence of a gap in the DOS.[14]

In our sample, there is no $i=3$ IQHE above 50 mK and the system is therefore spin degenerate. As shown in Fig. 1, the ρ_{xx} peak becomes narrower in the transition region between the $i=2$ and 4 plateaus, and ρ_{xy} becomes steeper as T decreases. These behaviors are qualitatively similar to those between the $i=1$ and 2 plateaus. In Fig. 3(a) we have plotted $(d\rho_{xy}/dB)^{\max}$ vs T for the $N=0\downarrow$

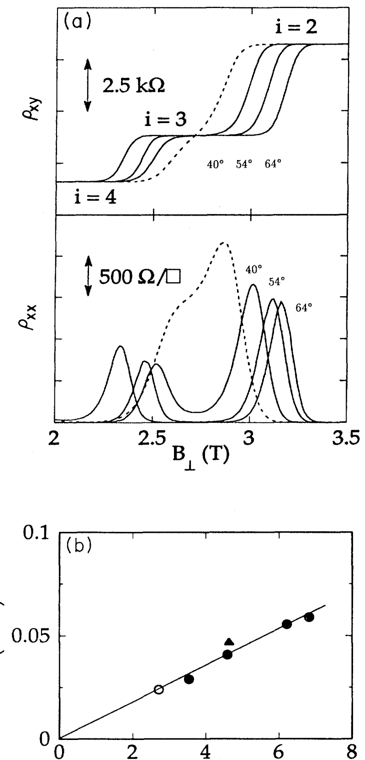

FIG. 2. (a) ρ_{xy} and ρ_{xx} vs $B_\perp$ at $T=50$ mK for tilt angle $\theta=0°$ (the dashed line), 40°, 54°, and 64°. (b) The activation energy Δ as a function of total B. The solid circles are Δ obtained from each θ. The open circle is the extrapolated value for $\theta=0$. The solid triangle is obtained from another $\mathrm{In}_x\mathrm{Ga}_{1-x}\mathrm{As/InP}$ sample with $n_{2D}=3.4\times10^{11}$ cm^{-2}, $\mu=33\,000$ cm^2/V s.

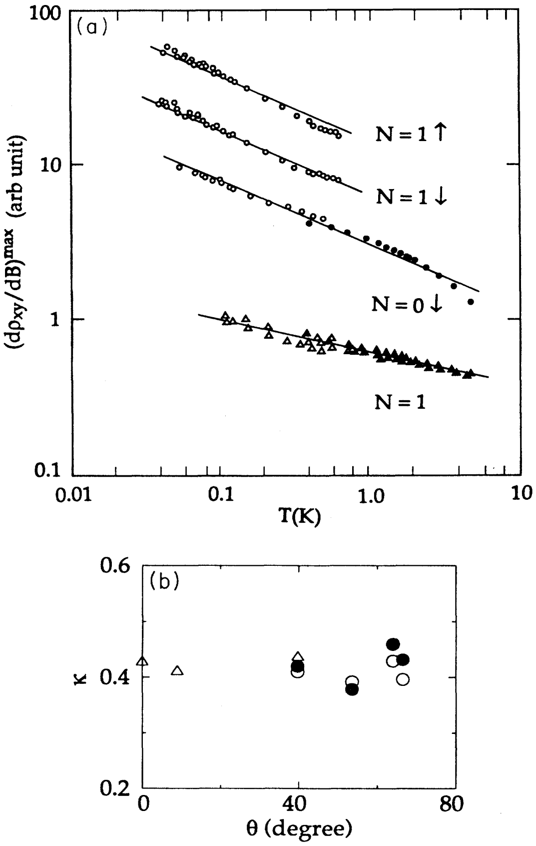

FIG. 3. (a) $(d\rho_{xy}/dB)^{\max}$ vs T for $N=1$, $N=0\downarrow$ Landau levels at $\theta=0$, for $N=1\uparrow$ and $1\downarrow$ at $\theta=64°$. The straight lines are drawn with slopes of 0.42 (for $N=0\downarrow$, $N=1\uparrow$, and $N=1\downarrow$) and 0.21 ($N=1$). (b) All the slopes in the $(d\rho_{xy}/dB)^{\max}$ vs T data for spin-resolved levels. The triangles are for $N=0\downarrow$, solid circles $N=1\uparrow$, and open circles $N=\downarrow$.

and the spin-degenerate $N=1$ Landau levels. The $(d\rho_{xy}/dB)^{\max}$ for $N=0\downarrow$ behaves like $T^{-0.42}$, whereas the power law $T^{-0.21}$ is observed for the $N=1$ Landau level. The $T^{-0.42}$ power law for the spin-polarized $N=0\downarrow$ level is the same as that reported in Ref. 4 and is a manifestation of transition of electronic states at the Fermi level from localized to delocalized states.[8] On the other hand, the $T^{-0.21}$ power law for the spin-degenerate $N=1$ level is a surprising result and has also been found in higher spin-degenerate Landau levels of the sample used in Ref. 4.

Two additional measurements have been carried out to investigate the result of the $T^{-0.21}$ power law. First, we perform the $(d\rho_{xy}/dB)^{\max}(T)$ measurement for the spin split $N=1\downarrow$ and $1\uparrow$ when $\theta\neq 0$. The data for $\theta=64°$ are shown in the upper two curves of Fig. 3(a). They follow the same $T^{-0.42}$ power law as that for $N=0\downarrow$ level. The exponents of the power law for all θ's and Landau levels are plotted in Fig. 3(b). They all fall within a 10% range of 0.42. Our previous study of GaAs samples[15] has shown that when the correlation length of the background potential fluctuations is large compared with the inelastic scattering length l_{in}, then the exponent deduced form $(d\rho_{xy}/dB)^{\max}(T)$ in a limited T range is not 0.42 and is due to a crossover effect. In this sample, since the localization to delocalization transition is observed for the spin split $N=1\downarrow$ and $1\uparrow$ levels, the potential fluctuations in this sample must be such that the scaling regime is reached within the T range in our experiment. By rotating the sample, the potential fluctuations are expected to remain unchanged. Therefore, the $T^{-0.21}$ power law cannot be a crossover effect due to long-range potential fluctuations as observed in some GaAs samples.[15] Second, we find that $(d^2\rho_{xx}/dB^2)^{\min}$ behaves as $T^{-0.42}$ when $T>50$ mK. The exponent in this case is two times that for $(d\rho_{xy}/dB)^{\max}$. This result supports our interpretation that the observed $T^{-0.21}$ power is due to genuine scaling in the spin-degenerate Landau level.

On the other hand, our experimentally measured exponent κ is a ratio of two microscopic exponents,[8] namely $\kappa=p/2\nu$, where p is the T exponent for the inelastic scattering length, and ν is the exponent for the divergent electron localization length. Since only the ratio (κ) of the otherwise unknown exponents p and ν has been measured, our experiments do not decide which of the two processes (inelastic scattering vs localization) is actually affected by spin degeneracy. We should emphasize that it is not clear from the experiment that the new exponent is the result of a new universality class. It is also possible that the dominant inelastic scattering process is changed in spin-degenerate levels. In any case, measurements of inelastic scattering length in high B are needed.

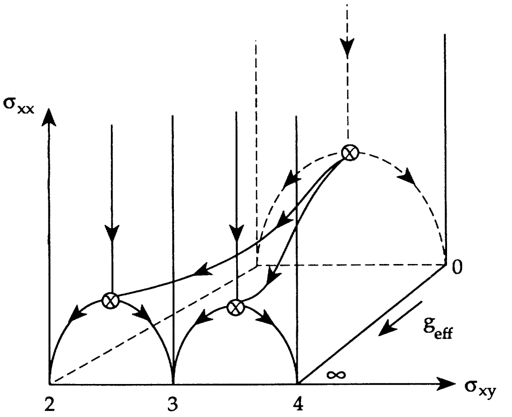

FIG. 4. Three-dimensional σ_{xx} vs σ_{xy} scaling diagram with the effective g factor connecting the complete spin-degenerate case at $g_{\text{eff}}=0$ to the spin-polarized one at $g_{\text{eff}}=\infty$.

It is instructive to construct a three-dimensional scaling diagram which is completely consistent with the overall behavior of our data as a function of B, T, and θ (Fig. 4). Besides the conductances σ_{xx} and σ_{xy}, it contains an effective g factor as a third axis. It is helpful to think of the third variables as $g_{\text{eff}}\sim(g/\cos\theta)T^{-\kappa}$, which is a measure for the "effective" spin polarization as seen in the transport data. It is understood that only the projection of the flow lines onto the $(\sigma_{xx},\sigma_{xy})$ plane is experimentally observable. According to Fig. 4, however, we are really dealing with a crossover problem between the unstable fixed point in the spin-degenerate case at $g_{\text{eff}}=0$ and the spin polarization fixed points which are observed when g_{eff} becomes sufficiently large. This crossover is controlled by the Zeemann energy in our experiment.

In summary, we have studied T-dependent transport between two adjacent quantum Hall plateaus in the case of spin-degenerate Landau levels. Our results indicate that the formation of the IQHE does not need a gap in the single-particle density of states. Furthermore, we find that the transport in spin-degenerate levels is different from that in spin-polarized levels. The spin interactions may change either the inelastic scattering process or result in a different universality class.[16]

Note added in proof: The dependence of σ_{xx} on frequency (f) between $f=0.2$ to 14 GHz has been studied recently by L. W. Engel *et al.*[17] They found $(\Delta B)^{-1}\sim f^{-\gamma}$ with $\gamma=0.41$ and 0.2 for spin-polarized and spin-degenerate levels, respectively. This result supports the existence of a different universality class for spin-degenerate levels.

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