Correlation effects on the tunneling of electrons from the surface of liquid helium

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We report calculations of the effect of interelectron interactions on the tunneling rates of electrons from the surface of liquid helium in which the short-range correlations are treated in a harmonic lattice model of the electron system. We consider two *quasistatic* approximations of this lattice model, corresponding to the extreme cases in which the remaining electrons have no chance to readjust to the motion of the escaping electron (*frozen*) or follow the motion adiabatically (adiabatic). By comparing our results to recent experimental data we find little difference between these two extreme situations, and conclude that the quasistatic approximation is sufficient to describe the physics of the existing experiments. We comment on the experimental conditions under which *dynamical* correlation effects may become observable.

I. INTRODUCTION

The system of electrons on the surface of liquid helium is a remarkable many-body system in which the electron surface density, n_s, and hence the relative strength of electronic correlations can be varied over a wide range. Under typical experimental conditions, $T \approx 0.1$–1 K and $n_s < 2 \times 10^9$ cm^{-2}, the Fermi energy, $E_F = \pi\hbar^2 n_s/m \lesssim 10$ mK, is much smaller than the temperature and thus this system behaves as a classical two-dimensional (2D) electron gas (m is the electron mass). In this regime the ratio between Coulomb and thermal energies defines the plasma parameter, $\Gamma = e^2(\pi n_s)^{1/2}/T$ (e is the electron charge), which can be continuously varied from the weakly correlated ($\Gamma \ll 1$) to the strongly correlated ($\Gamma \gg 1$) regimes. Of particular interest is the behavior close to $\Gamma = 128$, where the system is known to crystallize into a triangular lattice.[1]

The study of tunneling of electrons from the surface of liquid helium[2–4] provides another (albeit indirect) probe of many-particle correlations and, more importantly, may ultimately lead to novel effects and new device applications. The problem is also of some theoretical interest since tunneling from the two-dimensional electron gas is a complicated dynamical process which involves, in principle, the rearrangement of all the degrees of freedom of the many-body system.

The first attempt to study the escape rate of a SSE in the tunneling regime was made by Goodkind and coworkers.[2,3] These experiments were done in an external *pressing* field $0 < E_\perp < E_\perp^0 = 2\pi e n_s$, with 0.5 K $\lesssim T \lesssim 1$ K, and found that at the lowest temperature the escape rate of a surface electron (SE) becomes virtually independent of temperature. However, the observed rates were much faster than the theoretical estimates of tunneling rates, and it is now believed that these experimental results, recently reconfirmed by experiments by Andrei *et al.*,[4] were not related to tunneling.[4–6] An attempt to understand the physical origin of the anomalous behavior was made by Azbel and Platzman[7] who considered the many-body effects on the evaporation of a SSE and concluded that a temperature independent ("pseudotunneling") regime was indeed possible.

The first conclusive observations of tunneling of SE's were recently made by Andrei *et al.*[4] These authors used very low temperatures $T \gtrsim 30$ mK and a *pulling* external field to create the experimental conditions under which the tunneling rate becomes accessible. Indeed, in a certain range of densities (see below) their data are consistent with tunneling through an effective many-particle potential proposed earlier.[8,12]

The purpose of this paper is twofold: First of all we would like to develop a *quantitative* description of the available experimental data and decide to what extent many-particle correlations are important in understanding tunneling from this system. The upshot of our discussion will be that although correlation effects completely dominate the experimental rates, all the effects observed so far can be understood analytically on the basis of a simple *quasistatic* treatment of correlations. The second point of the paper is to set the limits of applicability of the quasistatic approximation and to define the experimental conditions under which the more interesting *dynamical* effects become observable. Our calculations are based on the effective potential obtained in Ref. 9 in the harmonic lattice model of the electron solid. This approach is appropriate for the description of the present experiments[4] which are carried out in the crystalline phase. It can also be applied to the case of a strongly correlated electron *liquid* because the effective potential is mainly determined by *short-range* correlations; remarkably, already for $\Gamma > 50$, these are quantitatively described by the pair correlation function of the harmonic triangular lattice (for more details see Ref. 10).

In the next section we will follow this approach and obtain the dependence of the electronic correlation potential, $U_{ee}(z)$ and the tunneling rates, on density and external field $E_\perp$. In Sec. III we will analyze our results and comment on the relation to previous work. Finally, in Sec. IV we will present the comparison with the available experimental data, and in the last section we suggest experimental conditions under which dynamical effects on

the tunneling rates should become observable and briefly summarize our conclusions.

II. THE EFFECTIVE POTENTIAL AND THE TUNNELING RATE

A. Approach

In the system under consideration the motion of an electron in the direction perpendicular to the surface (hereafter taken as the z direction) is determined by a potential energy, $U(z)$, consisting of three contributions: The first is the attractive potential to the image charge below the surface; the second is the potential $eE_\perp z$ due to the external electric field; and finally, a third contribution, $U_{ee}(z)$, from the interaction of the escaping electron with the other electrons on the surface:

$$U(z) = -\frac{\lambda e^2}{z} + eE_\perp z + U_{ee}(z - \langle z\rangle_1),\ \lambda = \frac{\epsilon - 1}{4(\epsilon + 1)}. \tag{1}$$

Here $\epsilon = 1.0572$ is the dielectric constant of liquid helium and $\langle z\rangle_1$ is the average distance of the electrons from the helium surface. The function $U(z)$ is shown in Fig. 1 for different values of the pressing field. The case $E_\perp = E_\perp^0 = 2\pi e n_s$ corresponds to the equilibrium case for which the lifetime of surface electrons is virtually infinite. Such a situation is reached experimentally by charging the helium surface by a continuously acting electron source. The surface ceases to charge when the total electric field above the two-dimensional electron system, $E_\perp^> = E_\perp - 2\pi e n_s$, becomes zero. For $E_\perp < E_\perp^0$, the ground state of the SE is metastable (Fig. 1, curve 2) and in a finite time the surface of the helium discharges to a new equilibrium value of the concentration $n_s = E_\perp/2\pi e$.

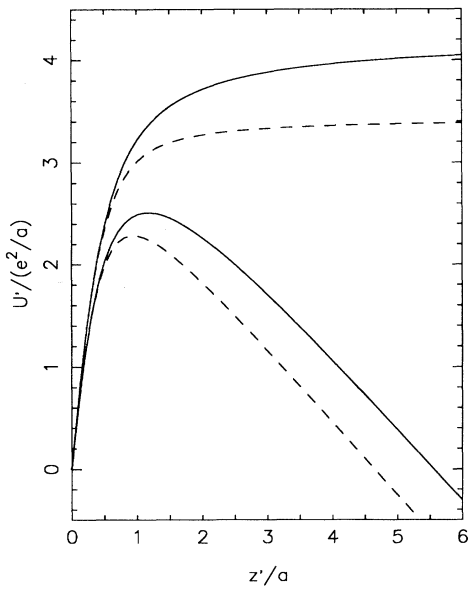

FIG. 1. Frozen (dashed lines) and adiabatic (solid lines) potentials without image charge contribution ($U' = U + \lambda e^2/z$) as a function of the distance z' from the electron crystal: (1) equilibrium pressing field $E_\perp = E_\perp^0 = 2\pi e n_s$; (2) the case $E_\perp < E_\perp^0$ ($E_\perp = 0.9E_\perp^0$).

Before presenting our calculations we would like to make some remarks motivating a number of the approximations involved. A rigorous treatment of the problem of the lifetime of a surface electron (SE) requires the consideration of the *dynamical* response of the electron gas to the escaping of the electron. Since solving the problem in general is extremely difficult, especially in the tunneling regime, we will limit our consideration to the two extreme quasistatic limits: the so-called frozen and adiabatic approximations.[9,6] In the frozen approximation one imagines that the escape is very fast and the remaining surface electrons do not have a chance to rearrange themselves during the tunneling process. In the crystal this simply means that electrons on the helium surface remain localized in the sites of the undeformed triangular lattice. The opposite *adiabatic* limit corresponds to a situation in which the positions of the surface electrons are determined by the equilibrium configuration in a 2D electron system with the escaping electron located at a given height z above the surface. Obviously, this leads to a partial "screening" (i.e., a shrinking) of the hole left behind by the tunneling electron and thus to a reduction of the correlation contribution to the effective potential. The actual value of the escape rate should lie in between the values calculated in the frozen and adiabatic approximations.

In the static approximation the dependence of the tunneling rate, W, on concentration and external field is determined by the the potential $U(z)$ with $U_{ee}(z)$ dependent on coordinate z, but not on time. In the WKB approximation the formula for the tunneling rate $W = \tau^{-1}$ of a SE from the ground state level can be easily derived by matching the hydrogenic wave function $\psi(z) = 2\gamma^{3/2}\exp(-\gamma z)$ ($\gamma = a_B^{-1} = m\Lambda_0 e^2/\hbar^2$) with a WKB wave function at $z \gtrsim z_1 \approx 2a_B$ (see, for example, Ref. 11). The resulting formula has the form

$$W = \frac{2\Delta_1}{\hbar}\exp\left[-2\left(\frac{1}{\hbar}\int_{z_1}^{z_2}\sqrt{2m[U(z) - \Delta_1]} + 1\right)\right], \tag{2}$$

where the limits of integrations satisfy the equation $U(z_{1,2}) - \Delta_1 = 0$ and Δ_1 is the energy of the ground state surface level [for the discussion of the formula (2) see Sec. IV]. Under typical experimental conditions, the dependence of Δ_1 on the external field $E_\perp$ is fairly weak and can be calculated by perturbation theory, $\Delta_1 = \Delta_1^{(0)} + eE_\perp\langle z\rangle_1$, with $\Delta_1^{(0)} = -7.6$ K.

B. Correlation contribution to the tunneling potential

We now turn to the discussion of the interaction potential $U_{ee}(z')$ (here $z' = z - \langle z\rangle_1$) which was already calculated in Ref. 9. Here we will only outline the general approach and present our results. $U_{ee}(z')$ can be expressed in terms of the interaction potential of the escaping electron in the presence of the surface electron gas

and a uniform neutralizing background, $U_c(z')$, $U_{ee}(z') = U_c(z') - eE_\perp^0 z'$, where the contribution $U_c(z')$ incorporates all effects due to correlations.

In principle, the value of $U_c(z')$ can be determined from the work necessary to move an electron from the charged layer to a height z' above the surface [for simplicity, we will omit the prime on z in the calculation of $U_c(z')$]:

$$U_c(z) = \int_0^{z'} e[E_\perp^0 + E_{ee}(z')]dz. \tag{3}$$

Here $E_{ee}(z)$ is the z projection of the field due to the remaining surface electrons. If the electron density is microscopically uniform then for $z > 0$ we have $E_{ee}(z) = -2\pi e n_s = -E_\perp^0$ and, therefore, $U_c(z) = 0$. In a crystallinelike state, the electronic field goes to zero near the surface ($z \ll n_s^{-1/2}$) and tends to the limiting value $-E_\perp^0$ for large values of $z > n_s^{-1/2}$.

Rather than obtaining $U_c(z)$ directly from the formula (3) it is more convenient to eliminate the integration over z and reduce the calculation to a purely two-dimensional problem. This can be done by reinterpreting the expression for $U_c(z)$ in terms of the energy of a defect in a planar Wigner crystal with a compensating background; this problem can then be solved by using methods developed for treating three-dimensional problems with long-range interactions. More precisely, an electron at a height z above an empty site, $\boldsymbol{l}$, should be interpreted as a substitutional impurity in a two-dimensional Wigner crystal, where z is a *parameter* that determines the interaction of the impurity with the particles of the crystal. As $z \to \infty$ the substitutional impurity corresponds to a vacancy, while for $z = 0$ it becomes a particle of the host two-dimensional crystal.

The impurity energy, $U_c(z)$, is equal to the difference $\Phi(z) - \Phi(0)$, where $\Phi(z)$ is the energy of a closed system (electrons plus background) under the condition that one particle is located a distance z above an empty site $\boldsymbol{l_0}$. $\Phi(z)$ consists of three contributions: the interaction energy Φ_{ee} of the surface electrons with one another, the interaction energy of the electrons with the positive background, Φ_{ep}, and the potential energy of the background, Φ_{pp}:

$$\Phi_{ee} = \frac{e^2}{2} \sum_{\boldsymbol{l} \neq \boldsymbol{l}' \neq \boldsymbol{l}_0} |\mathbf{R}_{\boldsymbol{l}\boldsymbol{l}'} + \mathbf{u}_{\boldsymbol{l}} - \mathbf{u}_{\boldsymbol{l}'}|^{-1} + e^2 \sum_{\boldsymbol{l} \neq \boldsymbol{l}_0} [(\mathbf{R}_{\boldsymbol{l}\boldsymbol{l}_0} + \mathbf{u}_{\boldsymbol{l}})^2 + z^2]^{-1/2}, \tag{4}$$

$$\Phi_{ep} = -n_s e^2 (N-1) \int_{(S)} \frac{d^2 r}{r} - n_s e^2 \int_{(S)} \frac{d^2 r}{(r^2 + z^2]^{1/2}}. \tag{5}$$

Here $\mathbf{R}_{\boldsymbol{l}\boldsymbol{l}'}$ is the vector joining sites $\boldsymbol{l}$ and $\boldsymbol{l}'$, $\mathbf{u}_{\boldsymbol{l}}$ is the displacement of an electron from site $\boldsymbol{l}$, N is the number of sites, and integration in (5) is carried out over the full two-dimensional surface, S.

In the harmonic approximation the potential energy of the system can be written as

$$\Phi = \Phi_0 + \frac{1}{2} \sum A_{\boldsymbol{l}\boldsymbol{l}'}^{ij} u_{\boldsymbol{l}}^i u_{\boldsymbol{l}'}^j + e^2 \sum_{\boldsymbol{l} \neq \boldsymbol{l}_0} \mathbf{R}_{\boldsymbol{l}\boldsymbol{l}_0} \mathbf{u}_{\boldsymbol{l}} \left(\frac{1}{R_{\boldsymbol{l}\boldsymbol{l}_0}^3} - \frac{1}{(\mathbf{R}_{\boldsymbol{l}\boldsymbol{l}_0}^2 + z^2)^{3/2}} \right), \tag{6}$$

where $A_{\boldsymbol{l}\boldsymbol{l}'}^{ij}$ is the matrix of the force constants of a two-dimensional Wigner crystal, and $\mathbf{u}_{\boldsymbol{l}}$ is assumed to be zero for $\boldsymbol{l} = \boldsymbol{l_0}$. Up to an additive z-independent contribution Φ_0 has the form

$$\Phi_0(z) = e^2 \lim_{S \to \infty} \left(\sum_{\boldsymbol{l} \neq 0} \frac{1}{(R_{\boldsymbol{l}}^2 + z^2)^{1/2}} - n_s \int_{(S)} \frac{d^2 r}{(r^2 + z^2)^{1/2}} \right). \tag{7}$$

1. *Frozen potential*

First, we will discuss the result for the energy of the impurity in an undeformed crystal, $U_{c,0}(z) = U_c(z; \mathbf{u}_{\boldsymbol{l}} = 0)$, which determines the tunneling rates in the frozen approximation. Clearly, $U_{c,0}(z) = \Phi_0(z) - \Phi_0(0)$, where, from (7),

$$\Phi_0(z) = e^2 \lim_{k \to 0} \left(\chi(\mathbf{k}, 0, z) - \frac{1}{z} - \frac{2\pi n_s}{k} e^{-kz} \right), \tag{8}$$

with

$$\chi(\mathbf{k}, \mathbf{r}, z) = \sum_{\boldsymbol{l}} \frac{e^{-i\mathbf{k}\cdot\mathbf{R}_{\boldsymbol{l}}}}{[(\mathbf{R}_{\boldsymbol{l}} + \mathbf{r})^2 + z^2]^{1/2}}. \tag{9}$$

The explicit form of $\chi(\mathbf{k}, \mathbf{r}, z)$ can be easily calculated by the Ewald summation method,[12]

$$\chi(\mathbf{k}, \mathbf{r}, z) = (\beta_0/\pi)^{1/2} \frac{1}{a} \left[\sum_{\mathbf{g}} e^{i(\mathbf{k}+\mathbf{g})\cdot\mathbf{r}} \psi_{-1/2} \left(\beta_0 \frac{z^2}{a^2}, \frac{|\mathbf{k}+\mathbf{g}|^2 a^2}{4\beta_0} \right) + \sum_{\boldsymbol{l}} e^{-i\mathbf{k}\mathbf{R}_{\boldsymbol{l}}} \phi_{-1/2} \left(\beta_0 \frac{(\mathbf{R}_{\boldsymbol{l}} + \mathbf{r})^2 + z^2}{a^2} \right) \right]. \tag{10}$$

Here a is the lattice constant, $\beta_0 = 2\pi/\sqrt{3}$, and $\mathbf{g}$ denotes vectors of the two-dimensional reciprocal lattice. The functions $\psi_\nu(x, y)$ and $\phi_\nu(y)$ are defined by the relations

$$\psi_\nu(x, y) = \int_1^\infty t^\nu exp(-x/t - yt)dt, \quad \phi_\nu(y) = \psi_\nu(0, y), \tag{11}$$

and are expressed fairly simply in terms of the error functions.[12] These functions decrease exponentially for large values of the argument y; we will thus confine ourselves to the first two terms in the sums over l and $\mathbf{g}$ in (10), corresponding to the contributions with $g = 0, 2\beta_0/a$, and $l = 0, 1$.

In this approximation the expression for $U_c(z)$ can be

written down analytically for arbitrary values of z:

$$U_{c,0}(z) = \frac{e^2}{a}[4.21 - \mathrm{erf}(\beta_0^{1/2}\tilde{z})/\tilde{z} + 2\beta_0\tilde{z}\ \mathrm{erf}(\beta_0^{1/2}\tilde{z}) - (\beta_0/\pi)^{1/2}(2e^{-\beta_0\tilde{z}^2} - 6\{\psi_{-1/2}[\beta_0\tilde{z}^2,\beta_0] + \phi_{-1/2}[\beta_0(\tilde{z}^2+1)]\})], \quad (12)$$

where $\tilde{z} = z/a$. In turn, this leads to the following asymptotic form of $U_c(z)$ for large and small values of the argument: (a) for $z \geq a$,

$$U_{c,0}(z) \simeq 4.21e^2/a - e^2/z, \quad (13)$$

$$U_{c,0}(z) \simeq eE_\perp^0 z - 5.5z^2\frac{e^2}{a^3}. \quad (14)$$

We note that, as expected, our value of $U_{c,0}(\infty) = 4.21e^2/a$ is consistent with cohesive energy (per particle) of the Wigner crystal, $E_W = -U_{c,0}(\infty)/2 \approx 2.1e^2/a$.[13]

2. *Adiabatic potential*

Next we will consider the crystal-deformation correction, $\delta U_d(z)$, to the defect energy in the adiabatic approximation:

$$\delta U_d(z) = U_c(z) - U_{c,0}(z) = \Phi(z) - \Phi_0(z), \quad (15)$$

where the displacements u_l are determined from the conditions, $\partial\Phi/\partial u_l^i = 0$, calculated for a fixed position of the impurity. These conditions translate into

$$\sum_{l'} A_{ll'}^{ij} u_{l'}^j = F_{l-l_0}^i(z) - F_{l-l_0}^i(0), \quad (16)$$

with

$$\mathbf{F}_l(h) = e^2\frac{\mathbf{R}_l}{(R_l^2+h^2)^{1/2}}. \quad (17)$$

According to (16), the field of displacements in the two-dimensional Wigner crystal induced by the substitutional impurity under consideration is equivalent to the deformation caused by a vertical dipole, whose negative charge is located at a height $h = z$ and positive charge at $h = 0$. The force with which such a dipole acts on a particle of the crystal at the site $\boldsymbol{l} = \boldsymbol{l}_0$ is denoted by $\boldsymbol{\mathcal{F}}_l(z) = \mathbf{F}_l(z) - \mathbf{F}_l(0)$. By applying the Fourier transformation to the equilibrium equation (16) we obtain

$$A^{ij}(\mathbf{k})u_{\mathbf{k}}^j = \mathcal{F}^i(z,\mathbf{k})\exp(-i\mathbf{k}\cdot\mathbf{R}_{l_0}), \quad (18)$$

where $A_{ii}(k)$, $u_{\mathbf{k}}^i$, and $\mathcal{F}(z,k)$ are the Fourier transforms $A_{ll'}^{ij}$, u_l^i, and $\mathcal{F}_l^i(z)$, respectively. We find the solution of Eq. (18) with the aid of the static Green's function $G(\mathbf{k}) = A^{-1}(\mathbf{k})$,

$$u^i(\mathbf{k}) = G^{ij}(\mathbf{k})\mathcal{F}^i(z,\mathbf{k})\exp(-i\mathbf{k}\cdot\mathbf{R}_{l_0}), \quad (19)$$

leading, through integration over the Brillouin zone (BZ), to the following expression for the displacement field, $\mathbf{u}_l$:

$$u_l^i = \frac{1}{n_s}\int_{\mathrm{BZ}} G^{ij}(\mathbf{k})\mathcal{F}^i(z,\mathbf{k})\exp(-i\mathbf{k}\cdot\mathbf{R}_{ll_0}). \quad (20)$$

By substituting (7) and (20) into (15) we obtain, after several transformations, the desired expression for the deformation correction δU_d to the defect energy,

$$\delta U_d = \frac{1}{2n_s}\int_{\mathrm{BZ}} G^{ij}(\mathbf{k})\mathcal{F}^i(z,\mathbf{k})\mathcal{F}^j(z,\mathbf{k})\frac{d^2k}{(2\pi)^2}. \quad (21)$$

When calculating the functions $\mathcal{F}^i(z,\mathbf{k})$ and $A_{ij}(\mathbf{k})$, we must again make use of the Ewald method since the lattice sums involved converge slowly. Both $\mathcal{F}^i(z,\mathbf{k})$ and the matrix elements, $A_{ij}(k)$, can be expressed in terms of $\chi(\mathbf{k},\mathbf{r},z)$ in (10) as follows:

$$\mathcal{F}^i(z,\mathbf{k}) = -e^2\lim_{r\to 0}\frac{\partial}{\partial r_i}[\chi(\mathbf{k},\mathbf{r},z) - \chi(\mathbf{k},\mathbf{r},0)], \quad (22)$$

$$A^{ij}(\mathbf{k}) = -e^2\lim_{r\to 0}\frac{\partial^2}{\partial r_i\partial r_j}[\chi(\mathbf{k},\mathbf{r},0) - \chi(0,\mathbf{r},0)]. \quad (23)$$

Finally, Eqs. (10) and (21)–(23) enable us to numerically determine the deformation correction $\delta U_d(z)$ to the correlation potential which will be used below to estimate the tunneling rates in the adiabatic limit.

III. THEORETICAL RESULTS

A. Correlation contribution to the potential

Although $\delta U_d(z)$ is, in general, accessible only numerically, in the limiting cases $z \ll a$ and $z \gg a$ we can obtain simple analytical forms

$$\delta U_d \approx -0.73e^2/a + (3/4)e^2/z,\ z \gg a,$$
$$\delta U_d \sim -e^2/a(z/a)^4,\ z \ll a. \quad (24)$$

The deformation correction to the vacancy energy, $\delta U_d(\infty) = -0.73e^2/a$, and the resulting total vacancy energy, $U_v = U_{c,0}(\infty) + \delta U_d(\infty) = 3.48e^2/a$, should be compared with the corresponding quantities, $\delta U_d(\infty) = -0.78e^2/a$ and $U_v = 3.43e^2/a$, obtained in the Monte Carlo calculations of the energy of a vacancy in a two-dimensional Wigner crystal by Fisher *et al.*[14] The small difference between these results and ours (of the order of 1.5%) is clearly associated with the omission in (6) of the higher terms of the expansion of the energy in the displacements u_l. The error is expected to be greater in the liquid phase.

From (24) it is clear that for $z \geq a$ there is a substantial deformation contribution to $U_c(z)$ caused by the escape of an individual electron—this corresponds to the compression of the correlation hole left behind by the escaping electron. More explicitly, when (13) and (24) are taken into account, the asymptotic form of $U_c(z)$ for $z \geq a$ is

$$U_c(z) = 3.48\frac{e^2}{a} - \frac{e^2}{4z}. \quad (25)$$

In the range of small z ($z \ll a$) the deformation correction, $\delta U_d(z) \sim (z/a)^4$, is negligible. In this limit the asymptotic form of $U_c(z)$ coincides with that of $U_{c,0}(z)$ and the total electron energy $U(z)$ in an arbitrary press-

ing field is given by

$$U_c(z) = eE_\perp z - cz^2e^2/a^3, \tag{26}$$

where $c = 5.5$ for the lattice model considered here. We expect that this value should serve as a lower bound in the real electron system in which case coefficient c is determined by the form of the pair correlation function ($c = \infty$ in an ideal gas). Spectroscopy experiments of surface electrons[15] show that in the range $9 < \Gamma < 44$ the coefficient c differs little from its value in the lattice model and even for $\Gamma \sim 10$ it is bigger by no more than 20%. We also note that the correlation hole model proposed in Ref. 8 leads to $c_{\rm Iye} = 5.0$, a value lower than that obtained for the crystal. The Iye potential thus considerably overestimates the correlation effects in the liquid phase.

Our numerical results for the function $U_c(z)$ are shown in Fig. 1. Using the function $U_c(z)$ we can now calculate the tunneling rate using the WKB expression for tunneling (2).

B. Tunneling rates

The first calculation of tunneling rates of SE's was presented by Yücel and Andrei[5] who solved the Schrödinger equation numerically using the Iye potential[8] [corresponding to Eq. (26) with $c = 5$]. These authors used very low temperatures ($T \geq 30$ mK) and *pulling* external field sufficient to produce measurable rates. Under these conditions only the small z behavior of the potential is important (the outer turning point is much smaller than a, $z_2 \ll a$) and the form in (26) is indeed appropriate. However, as already mentioned above, the Iye potential underestimates the rates. In addition, the analysis of Ref. 5 does not take into account that the SE's are located a finite distance from the helium surface, $\langle z \rangle_1 = 1.5a_B$ [see (1)]. This omission leads to an overestimate of the rates, and partially compensates the result of the inaccuracy of the Iye potential. Finally, the explicit numerical procedure used in Ref. 5 is difficult to apply in the case of a more complicated form of an effective correlation potential which should be used in the general case.

The use of the WKB formula (2) requires some discussion. First of all, in our case the use of the WKB approximation is justified since, under the conditions of present experiments, the electric field which tends to destroy the quasibound state is always much smaller than the "atomic" field, $E_\perp \ll \Lambda_0 e/a_B^2$. In fact, we checked the validity of the WKB formula (2), by comparing it with the numerical results of Ref. 5 for the case of the Iye potential: We found that the accuracy of the formula (2) is about 10% in the appropriate range of fields and densities. In addition, explicit calculations using (2) require particular care due to the characteristic *exponential* dependence of the escape rate W on various parameters.

First of all, the Stark correction to the energy of quasibound states should be taken into account even in the case of very small external fields. This leads to a linear change in the binding energy, $\Delta_1 = \Delta_1^{(0)} + eE_\perp \langle z \rangle_1$ with $\langle z \rangle_1 = 1.5a_B$; since the upper limit on the integral in (2) is much larger than the Bohr radius, $z_2 \gg a_B$, this in turn produces a nontrivial correction to the exponent of expression (2). This issue becomes especially clear in the single particle limit, $n_s \to 0$, in which case the asymptotic form of W at small $E_\perp$ reads (in dimensionless units)

$$W = 4\left(\frac{2}{eE_\perp}\right)^2 \exp\left(-\frac{2}{3}\frac{|2\Delta_1|^{3/2}}{|eE_\perp|}\right). \tag{27}$$

From this expression it is easy to see that in the limit $E_\perp \to 0$ the linear Stark effect leads to a numerical factor, $\exp(3) \sim 20$, in the rate W. We also note that the exponential factor in (27) is the same as in the 3D Stark effect[11] as well as in the 1D symmetric [$\psi_1(-z) = \psi_1(z)$] hydrogen atom[16] [recall that in the case of SE's the wave function $\psi_1(z)$ is not symmetric $\psi_1(z) = 0$ for $z < 0$]. However, the prefactor is different in all those cases and, in particular, there is no factor of 4 in the formula for W in the case of the symmetric 1D hydrogen atom in the Ref. 16. This is due to the difference in boundary conditions between our case and that considered in Ref. 16. In addition, the correction to Δ_1 is not important in the latter case since, in the symmetric 1D hydrogen atom, the Stark effect is quadratic in the strength of the electric field.

Another correction to Δ_1 which leads to observable effects in the rate is the change in the binding energy due to the finite value of the barrier at $z = 0$ ($V_0 = 1$ eV) and the finite width of vapor-liquid helium interface.[17] The latter can be included by replacing the usual image charge potential[17] by the phenomenological potential for the polarization attraction, $\Lambda_0 e^2/(z + \beta)$ ($\beta = 1.01$ Å). The corresponding correction to the binding energy due to both of the above mentioned effects was given in Ref. 17:

$$\Delta_1 = \Delta_1^0 + \frac{2\hbar^2}{m}\left[\beta - \left(\frac{\hbar^2}{2mV_0}\right)^{1/2}\right]. \tag{28}$$

C. Accuracy of quasistatic approximations

Before discussing the comparison with experiment we would like to make some general comments concerning our use of the quasistatic approximation. First of all, we notice that in the case of pulling external field $\delta > 1$ and only the small z behavior is essential ($z_m < z_2 \ll a$). As discussed above, in this limit the deformation correction to $U_c(z)$ is negligible and thus the frozen approximation is appropriate in both thermal activation and tunneling regimes.

In the opposite limit, $\delta \lesssim 1$, the relevant value of z becomes comparable with the lattice constant, a, and the frozen and adiabatic potentials can different considerably—in this regime *dynamical* effects are likely to become important. Let us estimate some of the relevant time scales in the problem: For the thermally activated escape of a SE the characteristic time (or inverse "attempt frequency") is given by $\tau_{\rm tr} \sim a/v_{\rm th}$, where $v_{\rm th} \sim \sqrt{T/m}$ is the thermal velocity. On the other hand, the time scale for the dynamical response of the electron crystal is determined by the Debye frequency of the Wigner crystal, $\tau_{ee}^{-1} \sim \omega_D \sim (e^2/ma^3)^{1/2}$. Since in our

case $\Gamma \gg 1$, $\tau_{ee}/\tau_{\rm tr} \sim 1/\Gamma^{1/2} \ll 1$, the adiabatic potential is appropriate for calculating thermal activation rates.

The tunneling case is more complicated. As discussed, for example, by Landauer and Buttiker,[18] the characteristic time associated with the tunneling process is the absolute value of the imaginary time which the particle "spends" under the barrier, $\tau_{\rm tr}$. In the case of intermediate pressing fields ($\delta \sim 0.5$) the second turning point z_2 is of the order of lattice constant $z_2 \sim a$ and the time $\tau_{\rm tr}$ can be estimated as $\tau_{\rm tr} \sim a/v_{\rm bar}$, where $v_{\rm bar}$ is the absolute value of the velocity under the barrier, $v_{\rm bar} = \sqrt{(U-\Delta_1)/m} \sim (e^2/ma)^{1/2}$. Thus, we have $\tau_{\rm tr} \sim (e^2/ma^3)^{-1/2} \sim \tau_{ee}$ and hence dynamical effects should be important when the external field is in the region of intermediate pressing fields ($\delta \lesssim 0.8$). As already mentioned above, in this case an upper bound on the dynamical effect can be estimated from the difference between our results in the "adiabatic" and "frozen" approximations.

IV. COMPARISON WITH EXPERIMENT

The observation of tunneling of a SE was recently reported by Andrei *et al.*[4] At sufficiently low temperature ($T < 0.2$ K) the experimental escape rate becomes independent of temperature and its density and field dependence are consistent with the tunneling mechanism. The parameters characterizing the typical experimental arrangement are the potential difference between the electrodes, V, and the distances from the helium surface to the lower and upper electrodes, h and d, respectively. In terms of these, the external electric field, $E_\perp$, and parameter δ can be written as

$$E_\perp = 4\pi e n_s \frac{d}{d+h/\epsilon} - \frac{V}{d+h/\epsilon}, \tag{29}$$

$$\delta = \frac{d-h/\epsilon}{d+h/\epsilon} + \frac{V}{2\pi e n_s (d+h/\epsilon)}. \tag{30}$$

There are a number of uncertainties in determining the various parameters in (29) and (30): As noted in Ref. 4, a small deviation of the helium level from the middle position between the electrodes (more precisely from $h = \epsilon d$) leads to an additional electric field, which noticeably affects the rates. In addition there is some uncertainty in the external electric field due to accumulation of electrons on the electrodes when the SE's are created. This means that the resulting electric field can be slightly different from the field calculated from formula (29). In our calculations we use $h = \epsilon d$ (no image charges on electrodes), and the fields $E_\perp = 32.9$, 28.8, 18.5, 14.4 V/cm for the experimental voltages $V = 7.55$, 6.5, 4.5, 3.5 V, respectively [the first two values of $E_\perp$ are higher than from the formula (29) by about 7%, while the last two are the same as those obtained from (29)].

The escape rates of Ref. 4 are reproduced in Fig. 2 (T=0.4 K), as a function of density and potential difference $V > 0$ between the top and bottom plates. The only data attributed to tunneling lie to the left of the sharp jumps at $n = n_c$ (marked by vertical arrows) and above the flat portions at $W \sim 3 \times 10^{-4}$ sec^{-1}. Andrei *et al.* argue that the data at $n > n_c$ were affected by the resonance ionization of helium atoms (in the film covering the experimental cell) induced by the escaping electrons. The saturation of the rates at $W \approx 3 \times 10^4$ sec^{-1} is virtually independent of the pulling field or electron density and its origin is not understood in detail (see a more detailed discussion of this issue in Ref. 4).

Our results together with the experimental data[4] are shown in Fig. 2. In the region of the data relevant to tunneling (defined above) the agreement between theory and experiment is good. As one can see from the Fig. 2 the results of the calculations of W in the frozen (solid lines) and adiabatic (dashed lines) approximations practically coincide under the conditions of the experiment.[4] As already explained above, in the presence of pulling fields, $\delta > 1$, the values of z in the integral (2) are small, $z < z_2 \ll a$, and hence $\delta U_d \propto (z/a)^4 \ll c(z/a)^2$.

As was already pointed out, dynamical effects on tunneling rates become important in the presence of pressing fields, $\delta < 1$. However, for the densities used in Ref. 4 the rates in this case would be too small to be observable with existing experimental techniques. In Fig. 3 we show our calculations of tunneling rates as functions of the parameter δ for three different densities. The rates calculated in the adiabatic approximation are significantly faster than that in the frozen approximation, indicating that in order

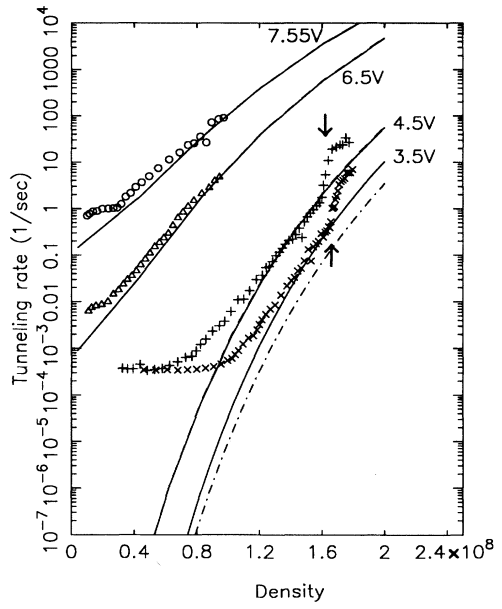

FIG. 2. Experimental data from Ref. 4 (symbols) and our calculations of tunneling rates $W(n_s)$ in the frozen (solid lines) and adiabatic (dashed lines) approximations, for a series of top plate voltages (the bottom plate is grounded $V = V_T^b$). Calculations for the two top lines were made for the pulling fields which are slightly higher than obtained from the formula (29) (see text). Vertical arrows indicate the critical density n_c above which the data were affected by the resonance ionization of helium atoms by escaping electrons. The dot-dashed line represents our calculations for the Iye potential at $V_t = 3.5$ V.

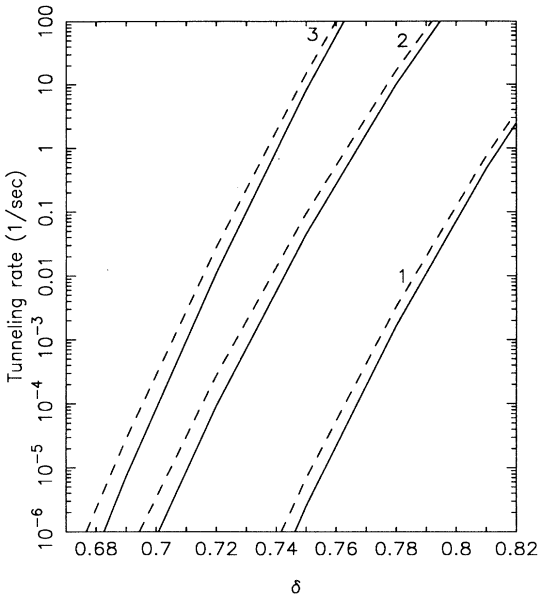

FIG. 3. Theoretical dependence of tunneling rates on the parameter δ in the frozen (solid lines) and adiabatic (dashed lines) approximations, for a series of densities. Curves 1, 2, and 3 correspond to the densities $n_s = 5 \times 10^9$, 1.6×10^9, 9×10^8 cm^{-2}, respectively.

to observe dynamical effects on tunneling rates one requires densities higher than $n_s \gtrsim 9 \times 10^8$ cm^{-2}. Since the possible electron densities on the surface of bulk helium are limited by the stability condition, $n_s < 2 \times 10^9$ cm^{-2}, the most promising candidate for these studies would be electrons on helium *films*, in which case densities of an order of 10^{11}cm^{-2} can be achieved.

V. CONCLUSIONS

Making use of the harmonic lattice model we have calculated the tunneling rate from the system of electrons on liquid helium in the two extreme ("frozen" and "adiabatic") quasistatic limits. In the case of pulling or weak pressing external fields both these approximations lead to practically identical results since in these limits only the behavior at small z is important, where the deformation correction to the effective potential is negligible. Such conditions are realized in the experiments of Andrei *et al.*[4] and our quasistatic calculations are in a good agreement with their data.

Our calculations show that dynamical effects on tunneling rates may be important in the limit of intermediate pressing fields in which case frozen and adiabatic approximations lead to significantly different results. We find also that in order to make the tunneling rates observable in this case densities higher then $n_s \gtrsim 9 \times 10^8$ cm^{-2} must be used. The most promising candidate for these studies would be electrons on helium films, for which densities of an order of 10^{11}cm^{-2} could be achieved.

Finally, we would like to note that despite the use of a lattice model our results should also be valid for the electron liquid with $\Gamma \gtrsim 50$; this is because the effective potential is mainly determined by the short-range correlations which are virtually identical in the crystal and strongly correlated liquid. This is consistent with the experimental results of Andrei *et al.* which show no influence of a liquid-crystal transition on the escape rate.

Note added: We recently became aware of work by Saville and co-workers[19] who have measured tunneling rates with qualitatively similar density dependence as in Ref. 4. They also analyze their data by comparing with unpublished calculations of adiabatic and frozen potentials.[20] In the adiabatic case, calculated by a numerical Monte Carlo method, their results lead to faster rates than ours, implying a considerably larger deformation contribution to the potential. Although the reason for this discrepancy is not understood in detail, we note that in a small system, such as considered numerically, a broken sixfold symmetry due to finite size effects can easily lead to a substantial asymmetry in the displacement field around the impurity. In turn, this would result in a corresponding increase in the magnitude of the deformation contribution. A careful analysis of the finite size dependence of the numerically evaluated potential would be useful in clarifying this issue.

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