Conductivity-peak broadening in the quantum Hall regime

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We argue that hopping conductivity dominates on both sides of σ_{xx} peaks in low-mobility samples and use a theory of hopping of interacting electrons to estimate a width $\Delta\nu$ of the peaks. Explicit expressions for $\Delta\nu$ as a function of the temperature T, current J, and frequency ω are found. It is shown that $\Delta\nu$ grows with T as $(T/T_1)^\kappa$, where κ is the inverse-localization-length exponent. The current J is shown to affect the peak width like the effective temperature $T_{\rm eff}(J) \propto J^{1/2}$ if $T_{\rm eff}(J) \gg T$. The broadening of the Ohmic ac-conductivity peaks with frequency ω is found to be determined by the effective temperature $T_{\rm eff}(\omega) \sim \hbar\omega/k_B$.

I. INTRODUCTION

The integer quantum Hall effect in a disordered two-dimensional electron gas manifests itself the more clearly the lower the temperature T. The steps connecting adjacent plateaus in the dependence of the Hall conductance σ_{xy} on the filling factor ν narrow with decreasing T and so do the peaks in the longitudinal conductance σ_{xx}. In a number of experiments[1–6] a remarkable result has been obtained: the width $\Delta\nu$ of the peaks shrinks as $T \to 0$ according to a power law $\Delta\nu \propto T^\kappa$. The exponent $\kappa \simeq 0.4$ was found in Refs. 1 and 2 to be universal; neither the Landau level index nor the electron mobility are relevant at low temperatures. The measurements have been performed down to temperatures as low as a few tens of millikelvins, thus giving a definite indication that extended electron states exist at only one energy within the broadened Landau level. Other states should be localized. Although the question as to the nature of the localization still remains unresolved, various computer simulations[7–11] strongly support this concept yielding the power-law divergence of the localization length $\xi(E) \propto |E|^{-\gamma}$, $\gamma \simeq 2.3$, as the electron energy E approaches the Landau level center ($E = 0$). Recently, the same value of γ has been directly measured by studying how $\Delta\nu$ scales with the sample size in the low-T limit.[4]

The conventional explanation of the scaling dependence $\Delta\nu \propto T^\kappa$ is as follows.[7,12] It is assumed that at a finite temperature there exists a phase-coherence length L_ϕ the shorter the higher T. One believes that if $L_\phi \ll \xi(E_F)$, E_F being the Fermi energy, the localization is destroyed and the electron system exhibits metallic behavior. Similarly to the theory of weak localization, L_ϕ is expressed in terms of the diffusion coefficient $\mathcal{D}$ and the phase-breaking time τ_ϕ: $L_\phi \sim (\mathcal{D}\tau_\phi)^{1/2}$. The time τ_ϕ is set to be proportional to T^{-p} with the exponent p which depends on the inelastic-scattering mechanism. These arguments lead to the conclusion that the width of the conducting energy band vanishes with decreasing T as $T^{p/2\gamma}$, so that $\kappa = p/2\gamma$ (to describe the experimental data in this way, one has to admit that $p \simeq 2$).

Although such an approach looks very attractive, introducing the phase-breaking time to account for the temperature-induced delocalization at $\sigma_{xx} \sim e^2/h$ is not obvious. There is no generally accepted theory for τ_ϕ in the quantum Hall regime. Here we suggest an explanation of the scaling behavior $\Delta\nu(T)$ in terms of the strong localization (approaching a peak from the region where $\sigma_{xx} \ll e^2/h$). We start with the notion that the only possible mechanism of transport in the strongly localized electron system is hopping. Consequently, the temperature-induced conductivity far from a peak should be exponentially small. As E_F approaches the level $E = 0$, the exponential factor must grow rapidly due to the divergence of the localization length. Our basic idea is to define the width of a peak by determining the position of the Fermi level at which the exponential factor of the hopping conductivity becomes of the order of unity.

In the present paper, we realize this program for three experimentally interesting cases differing in the origin of broadening: this may be temperature, current, or frequency. In Sec. II, the width due to the temperature broadening is shown to have a form

$$\Delta\nu = \left(\frac{T}{T_1}\right)^{1/\gamma}, \tag{1}$$

with the exponent $\kappa = 1/\gamma$, which is in accordance with the experimental data mentioned above. The characteristic temperature T_1 which results from our consideration is compared with that which is observed experimentally. Good agreement is achieved for single Landau levels which do not overlap with each other. As for anomalously small experimental values of T_1 in the case of two close spin-split σ_{xx} peaks, we relate them to the anomalous behavior of unsplit peaks that has been observed recently.[13,14] A width of steps in the Hall conductivity is discussed. Section III is devoted to evaluation of the width of peaks at very low temperatures as a function of current J. The following power-law dependence is obtained:

$$\Delta\nu = \left(\frac{J}{J_1}\right)^{1/2\gamma}, \quad (2)$$

which is in a reasonable agreement with what is observed. In Sec. IV we consider the Ohmic high-frequency conductivity $\sigma_{xx}(\omega)$. In the low-temperature limit $\hbar\omega \gg k_BT$, the conductivity is found to have a very simple form

$$\sigma_{xx}(\omega) = \tfrac{1}{6}\varepsilon\xi\omega, \quad (3)$$

where ε is the lattice dielectric constant. Thus, unlike the dc case, the hopping ac conductivity decreases as E_F moves away from a peak according to a power law. Equation (3) enables us to estimate the width of a peak of $\sigma_{xx}(\omega)$ as

$$\Delta\nu = \left(\frac{\omega}{\omega_1}\right)^{1/\gamma}. \quad (4)$$

Both Eqs. (3) and (4) agree with recent experimental results.[15] At the end of Sec. IV, we turn to describing a magnetic field dependence of the dc photoconductivity excited by a microwave radiation. A width of peaks of the photoconductivity is shown to coincide with that of $\sigma_{xx}(\omega)$ peaks. All the aforesaid results are obtained on the assumption that the dielectric constant ε does not diverge as the Fermi level approaches the Landau level center. In Sec. V, another scenario is considered in which ε for the length scale of the order of ξ grows with ξ. It results in new dependences of σ_{xx} on T, J, and ω. We are not able to choose between these two scenarios on pure theoretical grounds but we argue that the experimental data now existing do not support the scenario of diverging ε. Section VI contains concluding remarks.

A brief version of this paper has been published earlier.[16]

II. TEMPERATURE BROADENING OF THE CONDUCTIVITY PEAKS

We start with an expression for the temperature-induced hopping conductivity σ_{xx} far away from a peak where the conductivity is exponentially small as compared to e^2/h. It is known that variable-range hopping near the Fermi level[17] is dominant in the low-temperature limit. Therefore, to calculate the temperature dependence of the conductivity in this regime, one should know how the density of states behaves at $E \to E_F$. According to Refs. 18 and 19, interaction between localized electrons creates the Coulomb gap near the Fermi level, so that the single-electron density of state $g(E)$ vanishes at $E = E_F$. A form of the Coulomb gap in the two-dimensional case is given by

$$g(E) = \frac{2}{\pi}\frac{\varepsilon^2}{e^4}|E - E_F|. \quad (5)$$

As a consequence, the temperature dependence of σ_{xx} is[18,19]

$$\sigma_{xx} = \sigma_0 e^{-(T_0/T)^{1/2}}, \quad (6)$$

where

$$T_0(\nu) = C\frac{1}{k_B}\frac{e^2}{\varepsilon\xi(\nu)}, \quad (7)$$

$\xi(\nu)$ denotes the localization radius of the states on the Fermi level for a given ν, k_B is the Boltzmann constant, and $C \simeq 6$ in two dimensions.[20] This temperature dependence was observed in the middle of the Hall plateaus.[21,22] Note that Ono[23] also derived Eq. (6) (with a different expression for T_0) assuming a finite density of states at the Fermi level but using unperturbed wave functions of isolated impurities $\psi(\rho) \propto e^{-\rho^2/4\lambda^2}$, where λ is the magnetic length (see also Refs. 24 and 25). It is known,[26] however, that tails of wave functions are actually of a simple exponential form $e^{-\rho/\xi}$ due to multiple scattering of a tunneling electron. Of course, this form of the tails is consistent with a number of numerical calculations.[7–11] That is why the exponential factor of σ_{xx} has the same form as without magnetic field.

As mentioned above, the length $\xi(\nu)$ diverges as ν approaches a half integer ν_0:

$$\xi(\nu) = \xi_0|\nu - \nu_0|^{-\gamma}, \quad \gamma \simeq 2.3. \quad (8)$$

Correspondingly, the value of T_0 tends to zero as $\nu \to \nu_0$. Hence, at a given temperature, there should exist a characteristic value of ν at which the exponential factor in Eq. (6) becomes of the order of unity. It is natural to assume that it is the difference between this value and ν_0 that determines the half-width of a resistivity ρ_{xx} peak $\Delta\nu$. In this case, solving equation $T_0(\nu) \sim T$ with the use of the relations (7) and (8) immediately yields a power-law dependence of $\Delta\nu$ on T:

$$\Delta\nu = \left(\frac{T}{T_1}\right)^{\kappa}, \quad (9)$$

with $\kappa = 1/\gamma$ and

$$T_1 = A\frac{1}{k_B}\frac{e^2}{\varepsilon\xi_0}, \quad (10)$$

where A is a numerical coefficient. For $\gamma \simeq 2.3$ we arrive at the experimental value $\kappa \simeq 0.4$. As for the characteristic temperature T_1, to our knowledge, it is the first time an explicit expression for T_1 is given. Note that T_1 is of the order of T_0 in the middle of an adjacent plateau. Equations (9) and (10) might be interpreted in terms of the conventional theory[7,12] if $L_\phi \sim e^2/\varepsilon T$.

To compare with what is experimentally observed, we should define the elementary length ξ_0 depending on the properties of a random potential. Provided the potential fluctuations are short range, so that their correlation radius is less than or of the order of the magnetic length λ, one may expect that ξ_0 for the lowest Landau levels is $\sim \lambda$ (strictly speaking, ξ_0 should depend on the strength of disorder but only logarithmically[26]). One believes that fluctuations of this kind are realized in $In_xGa_{1-x}As/InP$ heterostructures, the experiment on which Ref. 1 most clearly confirms the universality of the exponent κ. Extracting $\Delta\nu$ from the data for ρ_{xx} presented in Ref. 1 we obtain $T_1 \simeq 600$ K for $\nu = \frac{3}{2}$ (the $N = 0\downarrow$ Landau level). Substituting then λ for ξ_0 in

Eq. (10) we find $A \simeq 4$, which looks reasonable. [This value of T_1 is also in reasonable agreement with values $T_0 \sim 500$ K observed in low-mobility $GaAs/Ga_{1-x}Al_xAs$ (Ref. 21) for $\nu = 2$.] It should be noted, however, that when the peaks corresponding to the spin-split $N = 1$ level are treated in the same way, $T_1 \sim 30$ K is obtained. This temperature is much smaller than what one could expect according to Eq. (10) with $\xi_0 \sim \lambda$ and $A \simeq 4$. It is worth comparing $T_1 \sim 30$ K with the measured value of T_0 for hopping at $\nu = 3$ which was found to be 7.8 K for similar samples.[22] This value is of the same order of magnitude as T_1 and also much less than what would be expected from Eq. (7). The fact that both the characteristic temperatures are so small for the spin-split $N = 1$ level indicates that the length ξ_0 for this level is much larger than λ. Anomalously small temperatures T_0 were observed also for all minima with large N [$T_0 \simeq 11$ K for $\nu = 6$ in $In_xGa_{1-x}As/InP$,[22] and $\simeq 10$, 5.9, and 2.7 K for $\nu = 8$, 10, and 12, respectively, in low-mobility $GaAs/Ga_{1-x}Al_xAs$,[21] note that the value of $T_0 \simeq 41$ K for $\nu = 3$ indicated in Table 1 of Ref. 21 seems to be a misprint: extracting T_0 from the data for $\ln(\sigma T)$ vs $T^{-1/2}$ presented in Fig. 2 of the same paper, one would obtain $T_0 \simeq 7$ K]. This also means extremely large values of ξ in the middle of corresponding plateaus. Using Eq. (6) with $C \simeq 6$ we get $\xi \simeq 30\lambda$ for $\nu = 8$ and $\xi \simeq 100\lambda$ for $\nu = 12$ in $GaAs/Ga_{1-x}Al_xAs$.[21] We will return to the problem of large ξ below.

Now let us look more closely at the starting point of our theory: the conductivity σ_{xx} on both sides of the peak was claimed to be due to variable-range hopping (6). The question is, can activation to the extended states existing in the middle of the Landau level compete with the variable-range hopping? We argue that it cannot. To make sure of this, we first consider a single Landau level of the width Γ which is much smaller than the energy distances to the adjacent levels. For example, such is the $N = 0 \downarrow$ level in the experiments on $In_xGa_{1-x}As/InP$ (Ref. 22) and $GaAs/Ga_{1-x}Al_xAs$ (Refs. 21 and 27) samples. Let us compare the contributions to the conductivity from activation and variable-range hopping provided the Fermi level is separated from the center of the Landau level by its width Γ. Note that we consider low-mobility samples that do not display the fractional quantum Hall effect. Therefore the characteristic Coulomb energy $\sim 0.1e^2/\varepsilon\lambda$ is supposed to be small as compared with Γ. In that case, the contribution from activation is given by $\ln\sigma_{xx} \sim -\Gamma/T$ while that from hopping by $\ln\sigma_{xx} \sim -[(e^2/\varepsilon\lambda)/T]^{1/2}$. It is clear that hopping dominates not only at $T \to 0$, which is usual, but even at T of the order of the Coulomb energy (at which temperature $\Delta\nu \sim 1$). Now it is easy to understand that if the Fermi level is closer to the Landau band center the conditions are still more favorable to hopping because $\xi(\nu)$ grows rapidly with decreasing $|\nu - \nu_0|$. It can then be shown in the same way that hopping with an energy transfer larger than $[TT_0(\nu)]^{1/2}$, the typical transfer according to Eq. (6), is also of no importance. Hence, variable-range hopping near the Fermi level is dominant everywhere inside the peak of the density of states (and outside the σ_{xx} peak). In other words, we conclude that the width $\Gamma[T/(e^2/\varepsilon\lambda)]^{\kappa}$ of the energy band corresponding to $|\nu - \nu_o| \lesssim \Delta\nu$ is always much greater than T.

Let us turn to the question about the conductivity when the Fermi level lies in the gap between the Landau levels. In wide gaps, some approximately constant "background" in the density of states is observed.[27,28] However, it is an experimental fact that the fraction of the total density of states corresponding to the gap is small.[27,28] Therefore the Fermi level may lie in the gap only if ν is very close to an integer. Since the density of states is small, the average distance between the gap states is much larger than their localization radius. It follows that hopping near the Fermi level cannot compete at high enough T with hopping associated with activation to the states in the peak of the density of states. Such an activation-type conduction has been studied in Refs. 27 and 28. As T goes down, the concentration of electrons activated to the bottom of the peak of the density of states decreases rapidly and hopping near the Fermi level becomes dominating. For $\nu = 2$ it happens at $T \lesssim 1$ K both for $In_xGa_{1-x}As/InP$ (Ref. 22) and $GaAs/Ga_{1-x}Al_xAs$.[21,27] Thus the activation-type conductivity does exist in the wide energy gaps but only at high temperatures and in a narrow range of ν around an integer, for which reason it does not influence our estimate of $\Delta\nu$.

For the Landau levels with large N, the spin-split levels, or for the levels corresponding to mixed states in double quantum wells, the σ_{xx} peaks are observed in the magnetic fields that may not be sufficient to form gaps in the density of states. So the Landau levels overlap while the σ_{xx} peaks may not. In this case, variable-range hopping near the Fermi level should be the only mechanism of conduction at any ν and T. This is confirmed by the fact that the conductivity at $\nu = 3, 4, 6, 8, 10, 12$ obeys the law (6) even for the highest temperatures.[21,22] However, as pointed out above, rather small values of T_0 as well as T_1 are observed for these narrow gaps, e.g., for the gap which separates the centers of the $N = 1\uparrow$ and $N = 1\downarrow$ levels. Our approach enables us to relate the anomaly in the values of T_0 and T_1 with another striking phenomenon reported in Ref. 13: if the only one σ_{xx} peak corresponds to the $N = 1$ level, i.e., its spin splitting is not resolved, the width of the peak follows $T^{\kappa/2}$ instead of T^{κ} as for each of the $\uparrow$ and $\downarrow$ peaks taken separately. The same phenomenon was observed also for the unsplit $N = 2$ level.[14] According to direct measurements,[14] the localization length exponent in the latter case is much greater than 2.3. Indeed, in our picture, the only thing that may account for the change of the exponent in the dependence $\Delta\nu(T)$ is a stronger divergence of the localization length as compared with Eq. (8). For example, the value of ξ for the $N = 1$ level should behave as

$$\xi(\nu) \sim \xi_0'|\nu - 3|^{-2\gamma} \tag{11}$$

[if two spin levels overlap strongly, the values of ν corresponding to the extended states are close to an integer in contrast to a half integer in Eq. (8)]. The length ξ_0' must be proportional to the constant of spin-orbit interaction, so that Eq. (11) may be valid only for $\xi(\nu) \gtrsim \lambda|\nu-3|^{-\gamma}$. By analogy with the derivation of Eqs. (9) and (10), the assumption (11) yields the width of the unsplit level

$$\Delta\nu = (T/T_1')^{1/2\gamma}, \quad k_B T_1' \sim e^2/\varepsilon\xi_0'. \tag{12}$$

We think that, even if a σ_{xx} peak is not spin split, there exist two different energy levels corresponding to delocalized states. The levels are separated by a Zeeman energy E_g. The reason why two peaks may not be observable is that the hopping conductivity between these two levels is not small. As the Fermi level approaches closely any of the levels, $\xi(\nu)$ must diverge with the usual exponent γ. Therefore, when $|\nu-3|$ becomes $\sim E_g/\Gamma \ll 1$, one should expect a crossover from the dependence (11) to that which is similar to (8) but with much larger "elementary length" $\sim \xi_0'(\Gamma/E_g)^\gamma$ resulting from matching in the crossover point. The divergence of $\xi(\nu)$ should take place at $\nu = 3\pm\delta\nu$, where $\delta\nu \sim E_g/\Gamma$. Thus our conjecture is that the localization length behaves as follows:

$$\xi \sim \xi_0'\left[\frac{\Gamma^2}{|E^2-\frac{1}{4}E_g^2|}\right]^\gamma, \quad E_g \ll \Gamma, \tag{13}$$

where the energy E is reckoned from the middle of the gap. At $E=0$ we get $\xi \sim \xi_0'(\Gamma/E_g)^{2\gamma} \gg \lambda$. Consequently, if the two σ_{xx} peaks are resolved and the hopping conductivity in the middle between them is observed, the value of T_0 should be strongly reduced in comparison with that for large gaps in the density of states:

$$k_B T_0 \sim \frac{e^2}{\varepsilon\xi_0'}\left(\frac{E_g}{\Gamma}\right)^{2\gamma}. \tag{14}$$

This equation gives also the characteristic temperature at which the two peaks merge. We would like to draw attention to the extremely strong dependence of T_0 on the ratio E_g/Γ. Indeed, a very high sensitivity of properties of σ_{xx} peaks to the electron mobility[14] as well as to the angle between the normal to the plane in which electrons move and the direction of the magnetic field[29] has been observed for the $N=1$ level. To conclude this discussion, we have to say that we have no clear idea about the mechanism of doubling of the localization-length exponent due to spin-orbit interaction. What we do here with respect to close spin levels is a phenomenological description of the experimental data from the point of view of the approach based on the concept of hopping.

As mentioned above, the localization lengths for minima with large ν (obtained from the experimental data for T_0) are also extremely large. In our opinion, this effect can be related only to overlap of neighboring Landau levels broadened by a disorder. Numerical calculations[30] show that the value of ξ between two lowest lying Landau levels (spin has not been taken into account) indeed very strongly increases with overlap. For example, according to Ref. 30, $\xi \sim 10\lambda$ even when a half-width of the $N=0$ level Γ is only $\frac{2}{3}$ of the cyclotron energy $\hbar\omega_c$. Another argument for the growth of ξ follows from a conventional understanding of the limit $\omega_c \to 0$. It is an accepted notion that the localization length at the Fermi level ξ_F is exponentially large at $E_F\tau \gg 1$ in the limit of small magnetic field:[31] $\ln\xi_F \sim E_F\tau/\hbar$, where τ is the free path time at zero field. According to Ref. 32, a total number $m+1$ of times delocalized states cross the Fermi level as the magnetic field changes from ∞ to 0 is equal to $E_F\tau/\hbar$ (spin is neglected here). This number is finite because no delocalized states should remain under the Fermi level at zero field.[31] The function $\xi_F(\nu)$ exhibits local minima at $\nu=\nu_i$ when E_F is in the middle between two consequent energy levels corresponding to delocalized states. Let us consider a sequence of m lengths $\xi_i \equiv \xi_F(\nu_i)$ in these minima. It is natural to think that $\xi_F(\nu_i)$ grows with increasing ν_i because eventually it should become exponentially large. Each of the lengths ξ_i being substituted for $\xi(\nu)$ in Eq. (7) gives a characteristic temperature $T_i \sim e^2/\varepsilon\xi_i$. According to our approach, as T lowers with respect to T_i, the ith minimum in the dependence $\sigma_{xx}(\nu)$ becomes much deeper than e^2/h, i.e., a new peak of σ_{xx} appears. Thus the sequence $\{\xi_i\}$ determines evolution of the phase diagram disorder magnetic field with changing T (in the integer quantum Hall regime in low-mobility samples). Two phase diagrams have been suggested recently.[33,34] In one of them,[33] all boundaries between phases with different quantized σ_{xy} exist at an arbitrary small magnetic field. In the other one,[34] the boundaries disappear with decreasing magnetic field. It is suggested in Ref. 34 that the diagram[34] turns into the diagram[33] at $T \ll T_c$, where the characteristic temperature T_c is exponentially small, $\ln T_c \sim -E_F\tau/\hbar$. In our picture, only those phases survive at finite temperature for which T_i is larger than T. Thus there is a wide temperature range $T_c \lesssim T_{i=m} \lesssim T \lesssim e^2/\varepsilon\lambda$ in which the number of phases increases with lowering T.

We concentrated above on σ_{xx} and did not consider the Hall conductivity σ_{xy}. The width of steps in the dependence of σ_{xy} on ν is observed to be of the same order as that of σ_{xx} peaks. This fact seems quite natural from the point of view of the present approach. Following the aforesaid procedure, one should evaluate a correction $\Delta\sigma_{xy}$ to a quantized value of σ_{xy} in the range of ν where σ_{xx} is exponentially small. Naturally, the correction in this range should be exponentially small as well. The width of a step in σ_{xy} can then be found by equating the exponent of $\Delta\sigma_{xy}$ to unity. We are not going to discuss here a very interesting problem of a ratio $\Delta\sigma_{xy}/\sigma_{xx}$ (or the problem of a ratio σ_{xy}/σ_{xx} in a weak magnetic field which is similar in many respects) when both of these values are exponentially small (some conflicting results can be found in Refs. 35 and 24 and references therein). We would like only to emphasize that the exponential factors of $\Delta\sigma_{xy}$ and σ_{xx} in any model of variable-range hopping[35] become of the order of unity simultaneously, which means similar scaling behavior of $\Delta\nu$ for σ_{xy} and σ_{xx}.

III. CURRENT INDUCED BROADENING OF THE CONDUCTIVITY PEAKS

The suggested approach permits us to elucidate yet another interesting phenomenon observed at very low temperatures. It was found in Refs. 36 and 4 that the width $\Delta\nu$ of the σ_{xx} peaks grows with increasing current J, i.e., with the increase of the Hall electric field $\mathcal{E}_H$. Let us show that the dependence $\Delta\nu(\mathcal{E}_H)$ can be understood in terms of the theory of hopping in a strong electric field.[37,38] This theory is based on the fact that there exists a quasi-Fermi-level inclined by the electric field $\mathcal{E}$. Zero-temperature hopping with phonon emission then becomes possible and, even though there are no absorption processes, the local Fermi distribution with an effective temperature $\sim e\mathcal{E}\xi$ is formed.[37,38] On this account, the exponent of the current-voltage characteristics at $T = 0$ may be obtained from that of the Ohmic conductivity by replacing $T \to e\mathcal{E}\xi/2$. In the quantum Hall regime, if the Ohmic transport obeys the law (6), the zero-temperature conductivity should behave with increasing electric field as

$$\sigma_{xx} = \sigma_0 e^{-(\mathcal{E}_{H0}/\mathcal{E}_H)^{1/2}}, \quad \mathcal{E}_{H0} = \frac{2k_B T_0}{e\xi}. \tag{15}$$

Similarly to the case of Ohmic conductivity the width of the σ_{xx} peak is found from the equation $\mathcal{E}_{H0}(\xi) \sim \mathcal{E}_H$. Solving this equation for ξ we get $\xi \sim (e/\varepsilon\mathcal{E}_H)^{1/2}$, which yields

$$\Delta\nu = \left(\frac{\mathcal{E}_H}{\mathcal{E}_{H1}}\right)^{\mu} = \left(\frac{J}{J_1}\right)^{\mu}, \tag{16}$$

where

$$\mathcal{E}_{H1} = B\frac{e}{\varepsilon\xi_0^2}, \quad J_1 \sim \sigma_{xy}(\nu_0)\mathcal{E}_{H1}, \tag{17}$$

$\mu = 1/2\gamma = \kappa/2$ and B is a numerical coefficient. Comparing Eq. (16) with Eqs. (9) and (10) one can notice that the field $\mathcal{E}_H$ leads to the same broadening of the peak as if there was the temperature

$$T_{\text{eff}} = \frac{1}{k_B}\left(\frac{A^2}{B}\frac{e^3}{\varepsilon}\right)^{1/2}\mathcal{E}_H^{1/2}. \tag{18}$$

This relation is remarkably universal: it contains only one parameter of the sample, its dielectric constant ε. The sensitivity of $\Delta\nu$ to $\mathcal{E}_H$ may be viewed as due to heating in the critical region of the metal-insulator transition. In this connection note the unusual square-root dependence of T_{eff} on $\mathcal{E}_H$. The increase in $\Delta\nu$ with $\mathcal{E}_H$ was clearly observed in Refs. 36 and 4, however, no treatment in terms of power dependences was presented. Our analysis of the lowest-temperature data of both the experiments shows that they can indeed be described by introducing $T_{\text{eff}} \propto \mathcal{E}_H^{1/2}$.

Another effect we wish to mention in this section is a saturation of $\Delta\nu$ with decreasing J in small samples. It is experimentally established[4,14] that a σ_{xx} peak stops narrowing as T lowers down to a characteristic temperature T_2 which depends on the sample size L. To evaluate T_2, we follow Refs. 4 and 14 and equate L and the localization length at $\Delta\nu = (T_2/T_1)^{\kappa}$. As a result, T_2 turns out to be $\sim e^2/\varepsilon L$, and the corresponding width $\Delta\nu \sim (\xi_0/L)^{\kappa}$. It has been shown above that $\xi(\nu)$ may be governed by the Hall electric field, too. Therefore one should expect the saturation with decreasing $\mathcal{E}_H$ at the same value of $\Delta\nu$ if $T \ll T_2$. We find that the characteristic Hall field in which this occurs is $(\mathcal{E}_H)_2 \sim e/\varepsilon L^2$.

To conclude this section, we would like to suggest a method of measuring ξ which is based on observation of the non-Ohmic phenomena far away from a peak of σ_{xx}. Suppose both the current dependence of σ_{xx} at low enough temperature and its temperature dependence under the Ohmic conditions are measured at the same ν. If they agree with Eqs. (6) and (15), the value of $\xi(\nu)$ may be obtained as a ratio $2k_B T_0(\nu)/e\mathcal{E}_{H0}(\nu)$.

IV. BROADENING OF THE ac-CONDUCTIVITY PEAKS WITH FREQUENCY

In this section we address ourselves to evaluation of the Ohmic zero-temperature conductivity in the quantum Hall regime at a finite frequency ω. Low-temperature experiments[39,40] have demonstrated a violation of a strict quantization of $\sigma_{xy}(\omega)$ in the microwave range, $\omega/2\pi \sim$ 30–60 GHz, and its complete destruction at $\omega/2\pi \gtrsim 150$ GHz. Recently, broadening of narrow σ_{xx} peaks has been observed at a few tens of millikelvins as the frequency changed in the range $\sim$ 0.2–15 GHz.[15] Similarly to the phenomenon of temperature-induced broadening, the width $\Delta\nu$ of the peaks has been found to exhibit a power-law behavior $\Delta\nu \propto \omega^{\kappa'}$ with the exponent $\kappa' \simeq 0.4$. It has been noticed that the value $\Delta\nu$ at a given ω is of the same order as if the measurements were performed at a temperature $\sim \hbar\omega/k_B$. Such a correspondence with respect to widths of plateaus of σ_{xy} has been reported also in Ref. 40.

In a spirit of the approach presented in this paper, it is natural to think that the broadening with increasing frequency is related to the ac-hopping conductivity. In contrast to the dc case, the hopping conductivity at a finite frequency is determined by sparsely distributed pairs of localized states, typical separation between two sites of a pair being much shorter than that between pairs. There exist two different mechanisms of absorption of quanta $\hbar\omega$ depending on a ratio $\hbar\omega/k_B T$. At $\hbar\omega \ll k_B T$, a dissipation is associated with relaxation losses whereas in the high-frequency limit, $\hbar\omega \gg k_B T$, it is due to resonant phononless transitions of electrons from one site of a pair to another. Both these mechanisms are strongly affected by electron-electron interaction if the Coulomb energy on a length scale equal to a typical arm of a pair is larger than $\hbar\omega$. As shown in Ref. 41, the conductivity is enhanced by the interaction, so that $\sigma(\omega) \propto \omega$ as $\omega \to 0$ rather than ω^2 as it follows from a one-electron consideration.[17] However, explicit expressions presented in Ref. 41 refer only to three dimensions. Relaxation losses of two-dimensional electrons at $H = 0$ were considered by Efros.[42] The following formula was obtained:

$$\sigma(\omega) = \tfrac{1}{12}\varepsilon\xi\omega, \quad \hbar\omega \ll k_B T. \tag{19}$$

This expression is valid also for the conductivity $\sigma_{xx}(\omega)$ in the quantum Hall regime if $\sigma_{xx}(\omega) \gg \sigma_{xx}(\omega = 0)$, where the dc conductivity is given by Eq. (6). The last inequality and the condition $\hbar\omega \ll k_B T$ are met simultaneously only in the region of plateau far enough from the conductivity peak. Presented at the end of this section is a calculation of $\sigma_{xx}(\omega)$ in the opposite limit $\hbar\omega \gg k_B T$. It yields

$$\sigma_{xx}(\omega) = \tfrac{1}{6}\varepsilon\xi\omega, \quad \hbar\omega \gg k_B T. \tag{20}$$

This expression differs from Eq. (19) only by a factor of 2. It is interesting to note also that taking electron-electron interaction into account led to disappearance of the electron charge in both of them. Another observation is that, in both cases, $\sigma_{xx}(\omega)$ diverges together with ξ as $E_F \to 0$. Therefore Eq. (20) for the zero-temperature conductivity enables us to estimate the width of a peak in $\sigma_{xx}(\omega)$ by making use of the condition that the conductivity near the peak is of the order of e^2/h. With the help of Eq. (8) this gives

$$\Delta\nu = \left(\frac{T_{\text{eff}}(\omega)}{T_1}\right)^{1/\gamma}, \quad T_{\text{eff}}(\omega) \gg T, \tag{21}$$

where $k_B T_{\text{eff}} = D\hbar\omega$, $D \sim 1$ being a constant. The characteristic frequency ω_1 [Eq. (4)] is equal then to $(A/D)(e^2/\hbar\varepsilon\xi_0)$. Thus, when broadening of a peak is considered, the value $\hbar\omega/k_B$ indeed plays the role of an effective temperature as it was observed in Ref. 15. Consequently, the width of a conductivity peak at a finite frequency exceeds the width of the peak in dc measurements only if $\hbar\omega$ is greater than $k_B T$. However, away from a peak, the regime of relaxation losses may be observable as well. We would like to stress that both Eqs. (19) and (20) then make it possible to measure directly the value of ξ as a function of ν, that is, to realize a kind of "localization-length spectroscopy" (this possibility for $H = 0$ was first noted in Ref. 42). Yet another way of using high-frequency measurements away from peaks is to treat $\sigma_{xx}(\omega)$ together with the temperature dependence of σ_{xx} at the same ν. At $\hbar\omega \gg k_B T$, for example, combining Eqs. (20) and (7) we get a measurable quantity

$$I = k_B T_0 \frac{\sigma_{xx}(\omega)}{\omega} = \frac{C}{6} e^2, \tag{22}$$

which should not depend on ν. An experimental observation of this invariance would verify our approach as based on the Coulomb gap theory.

Now let us turn to derivation of Eq. (20). The ac conductivity in an electric field $\mathcal{E}_0 \cos\omega t$ is equal to $2Q(\omega)/\mathcal{E}_0{}^2$, $Q(\omega)$ being the energy dissipated per unit time and unit area. A general expression for $Q(\omega)$ at $\hbar\omega$ much smaller than the energy spacing on a scale of the localization length has a form[41]

$$Q(\omega) = \int d^2\rho \int d\Omega\, F(\Omega, \rho) q(\omega, \Omega, \rho), \tag{23}$$

where

$$q(\omega, \Omega, \rho) = \frac{2\pi}{\hbar}\hbar\omega \left(\frac{1}{2} e\mathcal{E}_0 \rho \frac{I(\rho)}{\hbar\omega}\right)^2 \times \delta\{\hbar\omega - [(\hbar\Omega)^2 + 4I^2(\rho)]^{1/2}\} \tag{24}$$

is the contribution to Q from a pair with a distance between sites equal to ρ and a difference between energies of localized states which form the pair (leaving a quantum overlapping of the states out of account) equal to $\hbar\Omega$, and $F(\Omega, \rho)$ is the probability density to find a pair with given Ω and ρ within unit area. In Eq. (24), $I(\rho)$ stands for the overlap integral of two states of the pair and the expression in parentheses is the matrix element of transition within the pair. Due to the Coulomb gap effects, the distribution function F does not depend on Ω in the range $\hbar\Omega \ll e^2/\varepsilon\rho$ and is equal to[42]

$$F(\Omega, \rho) = \frac{2\hbar}{3\pi^2}\left(\frac{\varepsilon^2}{e^4}\right)^2 \left(\frac{e^2}{\varepsilon\rho}\right)^3. \tag{25}$$

Representing the overlap integral in the form $I(\rho) \sim (e^2/\varepsilon\xi)\exp(-\rho/\xi)$ and performing integration with a logarithmic accuracy with respect to a large parameter $e^2/\hbar\omega\varepsilon\xi$ we obtain Eq. (20). The main contribution to the integral (23) is given by pairs with $|\rho - \rho_\omega| \lesssim \xi$, ρ_ω being equal to $\xi \ln(e^2/\hbar\omega\varepsilon\xi)$. However, in contrast to the three-dimensional case,[41] no logarithmic factor arises in the final formula (2), so that the dependence $\sigma_{xx}(\omega)$ is pure linear as $\omega \to 0$.

The rest of this section deals with the dc photoconductivity σ_{ph} under microwave illumination with a frequency $\omega \gg k_B T/\hbar$. The last inequality means that absorption is described by Eq. (20). A theory of the hopping photoconductivity due to a long-wave excitation in a three-dimensional system of localized electrons was developed in Ref. 43. It is based on the fact that, having absorbed a quantum $\hbar\omega$ within a pair of close states, an electron has a much greater probability to recombine with a "geminate" hole rather than with any other. Thus typically an electron experiences a nonradiative transition directly back to the initial state and so does not contribute to the photoconductivity. A basic idea of Ref. 43 is that there exist very rare pairs each situated at the beginnings of two chains of localized states ("electron and hole wires") so as to allow geminate electrons and holes to leave each other with much greater probability than in typical pairs. It is these rare pairs that give the main contribution to the photoconductivity (calculation of their statistical weight is the main subject of Ref. 43). Extending this approach to the case of the quantum Hall effect, we obtain with logarithmic accuracy

$$\sigma_{\text{ph}} \propto \exp\left[-\frac{1}{2\ln 2}\left(\ln\frac{e^2}{\varepsilon\rho_\omega\hbar\omega}\right)^2\right]. \tag{26}$$

This is just the same expression as found in Ref. 43: when $\hbar\omega$ is much less than a width of the Coulomb gap, the leading term in the exponent of σ_{ph} does not depend on dimensionality [such a universality is inherent in the Coulomb gap theory and manifests itself also in a similar form of the temperature dependence (6) for two and three dimensions[18,19]]. The value of σ_{ph} as a function of ν at a given ω exhibits peaks due to changing $\rho_\omega \propto \xi$. Equating the exponent in Eq. (26) to unity, we find that the width $\Delta\nu$ of the peaks is of the order of that for the ac conductivity at the same frequency ω. As for minima of σ_{ph}, they are much deeper than those of $\sigma_{xx}(\omega)$.

V. DOES THE DIELECTRIC CONSTANT GROW NEAR THE PEAKS OF σ_{XX} ?

Until now we assumed the dielectric constant ε to be independent of the Fermi level position. This requires a special study because the localization length ξ diverges as E_F approaches the Landau level center and the insulator-metal-insulator transition takes place at $E_F = 0$. In the three-dimensional case, in the vicinity of the metal-insulator transition, the dielectric function $\varepsilon(k)$ is believed to exhibit a strong wave-vector dependence on scales $k^{-1} \lesssim \xi$.[44] As a result, the value of $\varepsilon(k)$ at $\xi \lesssim k^{-1}$ is supposed to be, in the insulator phase, as large as $(\xi/\xi_0)^{\alpha_3}$. The exponent α_3 may lie in the range $0 \le \alpha_3 \le 2$. Analogously, one may well think that the dielectric function $\varepsilon(k)$ in the quantum Hall regime, defined according to the expression $V(k) = 2\pi e^2/k\varepsilon(k)$ for a two-dimensional Fourier component of the interaction energy between two electrons, behaves like $\varepsilon(k\lambda)^{-\alpha_2}$ at $\lambda \lesssim k^{-1} \lesssim \xi$, $0 \le \alpha_2 \le 1$, and so is of the order of $\varepsilon(\xi/\lambda)^{\alpha_2}$ at $k \sim \xi^{-1}$, ε being the lattice dielectric constant. However, unlike the three-dimensional case, $\varepsilon(k)$ of two-dimensional electrons should decay with decreasing k at $k^{-1} \gg \xi$. Explicitly, it should change with k as $\eta\varepsilon k\xi(\xi/\lambda)^{\alpha_2}$ for $\xi \ll k^{-1} \ll \xi(\xi/\lambda)^{\alpha_2}$, where η is a numerical coefficient. This means that, similarly to the case of a thin film with a large dielectric constant,[45] the interaction energy in the range $\xi \ll \rho \ll \xi(\xi/\lambda)^{\alpha_2}$ decays according to a logarithmic law rather than proportionally to ρ^{-1}:

$$V(\rho) = \frac{e^2}{\eta\varepsilon\xi}\left(\frac{\lambda}{\xi}\right)^{\alpha_2} \ln\left[\frac{\xi}{\rho}\left(\frac{\xi}{\lambda}\right)^{\alpha_2}\right]. \tag{27}$$

At $\rho \gg \xi(\xi/\lambda)^{\alpha_2}$, when electric field lines completely leave the plane, $V(\rho)$ crosses over to $e^2/\varepsilon\rho$. The logarithmic interaction leads to the "hard" Coulomb gap and the Arrhenius-like temperature dependence of the hopping conductivity:[46]

$$\sigma_{xx} = \sigma_0 e^{-\frac{\tilde{T}_0}{T}\mathcal{L}}, \tag{28}$$

where

$$\tilde{T}_0 = \frac{e^2}{\eta\varepsilon\xi}\left(\frac{\lambda}{\xi}\right)^{\alpha_2}, \quad \mathcal{L} = \ln\left[\frac{T}{\tilde{T}_0}\left(\frac{\xi}{\lambda}\right)^{\alpha_2}\right]. \tag{29}$$

In the same way as before, the width $\Delta\nu$ of a peak can then be found from the equation $\tilde{T}_0(\nu)\mathcal{L} \sim T$. It yields

$$\Delta\nu = (T/\tilde{T}_1)^{\frac{1}{\gamma(1+\alpha_2)}}, \tag{30}$$

where $\tilde{T}_1 \sim (e^2\mathcal{L}/\varepsilon\lambda)$. To get the experimental value of the exponent κ in that case, one should assume that $\gamma \simeq 2.3/(1+\alpha_2)$, which is possible for the localization-length exponent in the many-electron problem but contradicts the results of numerical computation for the one-electron exponent γ. Equations (28) and (29) are valid in a range of temperature $(\lambda/\xi)^{\alpha_2}\tilde{T}_0 \ll T \ll \tilde{T}_0$. At $T \sim \tilde{T}_0(\lambda/\xi)^{\alpha_2}$ a crossover from Eq. (28) to Eq. (6) takes place.

We cannot answer the question of whether the value of α_2 is equal to zero or is finite; both scenarios are possible from the theoretical point of view. Numerical calculations in the critical region $\xi/\lambda \to \infty$ are necessary to solve this problem definitely. Nevertheless, in our opinion, there exists strong experimental evidence that $\alpha_2 \lesssim 0.2$, which probably means that actually $\alpha_2 = 0$. First, according to the experimental data[22] for T_0 at $\nu = 3$ in $In_xGa_{1-x}As/InP$ samples, the localization length ξ in the middle of a minimum separating two σ_{xx} peaks, $N = 1\uparrow$ and $N = 1\downarrow$, is very large. Using Eq. (7) with $C \simeq 6$ and the value of $T_0 \simeq 7.8$ K (Ref. 22) we estimate the ratio ξ/λ at ~ 80. It follows that, if the scenario with large $\varepsilon(k \sim \xi^{-1})$ would be realized, there should be observed a large range of temperature within which the dependence (28) is valid. However, at $\nu = 3$, as well as in other minima where the value T_0 is very small, Eq. (6) describes well the whole range of T corresponding to hopping conductivity.[21,22] The second experimental indication that $\alpha_2 \ll 1$ is related to a saturation of $\Delta\nu$ with decreasing T observed in small samples.[4,14] The temperature of saturation T_2 (see discussion at the end of Sec. III) can be found by equating the sample size L and the localization length at $\Delta\nu$ given by Eq. (30), which yields $T_2 \sim (e^2/\varepsilon L)(\lambda/L)^{\alpha_2}$. We would like to emphasize that a ratio L/λ at which the measurements[4,14] were performed is in the range $10^3 - 10^4$. Therefore the value of T_2 at large L would be strongly reduced as compared to $e^2/\varepsilon L$ and would decay with L faster than $1/L$. However, the experimental value of $T_2 \simeq 3e^2/\varepsilon L$, which means that α_2 cannot be greater than 0.2. Besides, T_2 decreases with increasing L even slower than $1/L$, which is completely inconsistent with the large $\varepsilon(k)$ scenario. There is yet another piece of evidence against this scenario. It is related to the current dependence of $\Delta\nu$. If $\varepsilon(k \sim \xi^{-1})$ diverges with ξ, dependence $T_{\text{eff}} \propto (\mathcal{E}_H)^\delta$ with $\delta = \frac{1+\alpha_2}{2+\alpha_2}$ would take place instead of Eq. (16). At $\alpha_2 = 0$ the exponent $\delta = 1/2$, whereas at $\alpha_2 = 1$ it is equal to 2/3. According to our analysis of the experimental data,[4,14] the value of δ is close to 1/2, which means that α_2 is small.

In spite of the lack of evidence in support of the large $\varepsilon(k)$ scenario, we think that another verification of nondivergency of $\varepsilon(k)$ would be useful. We suggest studying a temperature dependence of the variable-range hopping conductivity near peaks to discriminate between Eqs. (6) and (28). Moreover, such measurements could verify our approach as using the Coulomb gap concept. Provided the temperature behavior is described by Eq. (6), one could try to fit the dependence T_0 on $|\nu - \nu_0|$ by the power law $T_0 \propto |\nu - \nu_0|^\beta$. Our prediction is that the exponent β is equal to κ^{-1} where κ determines the temperature dependence of the width of σ_{xx} peaks.

VI. CONCLUSIONS

Before concluding remarks we would like to point out two difficult problems which arise within our approach to estimation of $\Delta\nu$. The first of them is related to the prefactor σ_0 in Eq. (6). As a matter of fact, our theory can work literally only if

$$\sigma_0 = \frac{e^2}{h} f\left(\frac{T}{T_0}\right), \tag{31}$$

where $f(x)$ is a dimensionless function (strictly speaking, it might depend on N). Experimental data[21,22] for a number of minima of σ_{xx} do not contradict this law. Plotted as $\ln(\sigma_{xx}T)$ vs $T^{-1/2}$ they show straight lines, which enables one to use the function f of a form $f = f_\infty(T_0/T)$ for fitting the data. We found that the values of f_∞ for different minima lie within a range 0.2–0.6, the values for minima with small T_0 being close to 0.5. However, Eq. (31) offers a theoretical problem: it contains only the ratio of k_BT to the Coulomb energy on the length scale of the order of ξ besides to the universal constants. It is clear that such a prefactor cannot be derived within the framework of a conventional theory of phonon-assistant hopping (in the limit of weak electron-phonon coupling). Probably, to get Eq. (31), one should use as a starting point an idea of hopping due to only electron-electron scattering at finite temperatures suggested in Ref. 47.

The second problem refers to the fractional quantum Hall regime. Recently, a remarkable observation of the scaling behavior $\Delta\nu \propto T^\kappa$ in this regime has been reported.[48,3] According to Ref. 48, the exponent $\kappa \simeq 0.4$ is the same as for the integer quantum Hall effect. Together with an experimental observation of the variable-range-hopping law (6) for fractional gaps,[49] this gives an indication that our results may be applicable to the fractional regime, too. If that is the case, the characteristic temperatures T_1 and T_0 contain the fractional charges and so should be much smaller than those for the integer effect. The experimental values of T_1 and T_0 are actually very small.[48,49] Note that one could use the invariant I [Eq. (22)] to compare effective charges of hopping excitations for different plateaus. The theoretical problem is to understand the nature of localization in the fractional quantum Hall regime (Ref. 34 and references therein) and, correspondingly, to determine which excitations hop.

In conclusion, this paper emphasizes an important role which the hopping conductivity plays in the quantum Hall effect. A new approach to calculating the width $\Delta\nu$ of σ_{xx} peaks is formulated. It is based on considering the hopping conductivity far away from a peak where the hopping length typically exceeds the localization radius and finding the distance in ν from the middle of the peak at which these two lengths become of the same order of magnitude. Dependences of $\Delta\nu$ on temperature, current, and frequency are found. They seem to be in good agreement with existing experimental data. It is shown that the hopping conductivity away from peaks measured as a function of T, J, and ω can give direct information about the most important value in the quantum Hall effect—the localization length ξ as a function of ν. We argue that the length ξ in the middle of a plateau directly determines the temperature at which the plateau disappears and two peaks surrounding it merge. The point is that the peaks may be observed as distinct only if the hopping conductivity between them is small enough.

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