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Coulomb blockade in the quantum-Hall-effect state

A. K. Evans, L. I. Glazman, and B. I. Shklovskii

Theoretical Physics Institute and School of Physics and Astronomy, University of Minnesota, 116 Church Street SE, Minneapolis, Minnesota 55455

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It is well known that at low temperature, a small two-dimensional (2D) electron gas shows peaks in the linear conductance at a series of sharply defined values of the external gate voltage. Recently published experimental studies have shown that under a large magnetic field, the value of the gate voltage required to give a peak oscillates as a function of the magnetic field. We explain these oscillations using a simplified model of the island. The model depends on the observation that in the area of each edge state in the island, the electric field is almost completely screened. This prompts us to treat each edge state as a conductor, and to use the Coulomb blockade approach to locate peaks in the conductance of the island. An unusual feature of the system is that the capacitances of the different regions can be controlled by the magnetic field. This, together with a Coulomb blockade within the dot, is what causes the oscillations. We compare our theory with the results of an existing experiment (P. L. McEuen, E. B. Forman, Jari Kinaret, U. Meirav, M. A. Kastner, N. S. Wingreen, and S. J. Wind [Phys. Rev. B **45**, 11 419 (1992)]) finding acceptable agreement. A similarity between this system and a single-electron pumping device is noted.

I. INTRODUCTION

Recently a number of experiments investigating electron conduction by tunneling onto a quantum dot have been performed.[1,2] An important feature observed in the experiments is a periodic peak structure in the linear conductance as a function of the gate voltage. This gate voltage is a parameter used to control the electrostatic potential induced in the dot. In principle, there are two alternative approaches to explaining the oscillations. If one neglects the Coulomb interaction between the electrons, then oscillations should be attributed to the successive depopulation of discrete spatially quantized levels. The alternative picture includes almost complete screening inside the dot of the external electric field. In this approach, conductance oscillations can be explained[3] in terms of the well-known "Coulomb blockade" theory.[2,4]

For a typical size of dot, with diameter d of 0.1–1.0 μm, the Coulomb blockade model is more realistic. This is because the charging energy significantly exceeds the spacing between spatially quantized levels if $d \gg a_b$, where a_b is the Bohr radius for electrons in GaAs: $a_b \sim 10$ nm. In this picture, the period ΔV_g in the gate voltage of the conductance oscillations is determined only by the capacitance C_g between the dot and the gate: $\Delta V_g = e/C_g$.

In this paper we are interested in how this result is modified if a quantizing magnetic field is applied. Charging of a dot in the quantum-Hall-effect state has been investigated experimentally.[5–7] In Ref. 5, a self-consistent numerical calculation was also performed, the results of which agree well with experiment.

One feature of the results of this calculation was that the dot's area was divided into concentric rings of alternating compressible and incompressible electron gas. In the incompressible regions, the number of filled Landau levels is an integer, while in the compressible regions, a level is partially filled. The calculations also show that in the compressible regions, the electric field is almost completely screened. In this respect the regions behave as if they were metallic. They are separated by insulating regions of incompressible electron gas.

Similar conclusions about the formation of insulating and conducting regions in strong magnetic fields have been reached in an analytical treatment of edge states which was based on the self-consistent approximation.[8] The width of these regions was also estimated analytically. A similar analytical treatment has been developed for edge states in a quantum wire and a quantum dot.[9] The self-consistent field varies slowly on the scale of the magnetic length $\lambda_B = (\hbar/eB)^{1/2}$, and this allows the mixing of Landau levels to be neglected. Tunneling between the levels is weak, and each electron belongs to a single level. This means that the number of electrons in each level is a well defined integer. The redistribution of charge between different conducting regions must occur in discrete units. In this paper, we study tunneling onto the dot and the distribution of discrete charge within the dot in the simplest case, where there are two conducting regions corresponding to the two different spin states of the lowest Landau level.

When an electron tunnels onto the dot, it must be placed in one of the two compressible areas. These are separated by an insulating region. This allows us (in the next section) to use the "Coulomb blockade" approach to treat electron movement between the two spin-split levels. The main modification to the single dot Coulomb blockade can be understood by considering a model including only three mutual capacitances: the capacitance C between the two conducting regions, and capacitances C_1,C_2 coupling these regions to the gate (see Fig. 1). An important feature of our model is that the capacitances C_1 and C_2 are not constant but depend monotonically

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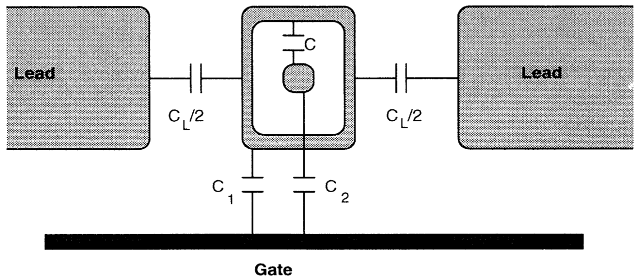

FIG. 1. Schematic diagram of the dot, showing conducting areas and mutual capacitances. For clarity, we do not show the capacitance between the inner conductor and the leads.

upon the magnetic field, varying in proportion to the areas of their respective conductors. The sum $C_g = C_1+C_2$ we take to be constant, because the area of the dot is almost independent of the magnetic field (see Sec. II).

In the experimental study,[5] it was found that the value of the gate voltage at which a conductance peak occurred did not remain constant, but oscillated as the magnetic field was varied. In our model, the capacitances C_1 and C_2 depend on the magnetic field. Oscillations in $V_g(B)$ arise from changes in the discrete distribution of charge in a circuit including the capacitances C_1, C_2, and C. The dependence of the capacitances on the magnetic field can be found from the self-consistent approximation.[8,9] This approximation is applicable if the system of conducting and insulating regions is macroscopic, i.e., all its characteristic sizes exceed significantly the interelectron distance. Under this condition, corrections to our simple approach caused by the correlation effects and by the finite spatial extent of Landau wave functions are small.

In assuming that the energy of all the electrons in the system can be written in terms of the capacitances of the conducting regions, we are ignoring the presence of the fully filled part of the lower level. This is partially justified by the fact that electrons in fully filled states are "fixed" in these states (at low temperature) and cannot be compressed. They do not take part in the redistribution of charge which occurs when we change from the continuous charge distribution (which corresponds to zero magnetic field) to the discrete division of charge between the two levels. In this paper, we are interested in the modification to the Coulomb blockade caused by this redistribution, and so our conclusions are not affected by electrons in the fully filled region of the level. The Zeeman splitting energy difference between the two levels also does not affect the expansion of the energy in terms of capacitances: this is because the splitting only adds to the energy a term linear in the charge on one of the levels. (For the same reason, the mutual capacitance of two conductors is not affected by a difference in the work functions of their materials.)

When $B = 0$, there is no insulating strip and the distribution of charge density across the dot is continuous. If the capacitances to the leads $C_L/2$ (see Fig. 1) are neglected, the electrostatic energy is[2,4]

$$U_0(N) = \frac{(Ne)^2}{2C_g} - NeV_g. \tag{1}$$

In a large magnetic field, the constraint that the charge must be transferred in discrete steps causes some redistribution of the charge density, and a consequent increase in the energy, making the energy larger than the minimum corresponding to a continuous distribution (see Fig. 2). In the limit $C \gg C_1 + C_2$, this difference in energy can be written as

$$\Delta U = \frac{[e\delta N(B,N)]^2}{2C}, \tag{2}$$

where $e\delta N(B,N)$ is the additional charge placed on one of the Landau levels by the constraint of discreteness. The exact value of δN depends on the capacitances C_1 and C_2, and also on N, which is controlled by the gate voltage. However, it is clear that the constraint can always be satisfied by $|\delta N| < 1/2$.

At low temperature and low bias voltage, a peak in the conductance occurs when the state of the system with N electrons on the dot is degenerate with the state having $N+1$ electrons:[2,4]

$$U(N+1) - U(N) = E_F, \tag{3}$$

where E_F is the Fermi energy for electrons in the leads. With $U = U_0 + \Delta U$, using Eqs. (1) and (2) we find

$$V_g = -\frac{E_F}{e} + \frac{(2N+1)e}{2C_g} + \frac{e}{2C}\{[\delta N(B,N+1)]^2 - [\delta N(B,N)]^2\}. \tag{4}$$

This formula allows us to estimate the amplitude of oscillations of $V_g(B)$. Since $|\delta N| < 1/2$, the maximum deviation of V_g from the value corresponding to the undivided dot is $\pm e/8C$, and so the maximum possible peak-to-peak amplitude is $e/4C$.

It is possible to estimate the number of oscillations of $V_g(B)$ in the interval of magnetic field between the point

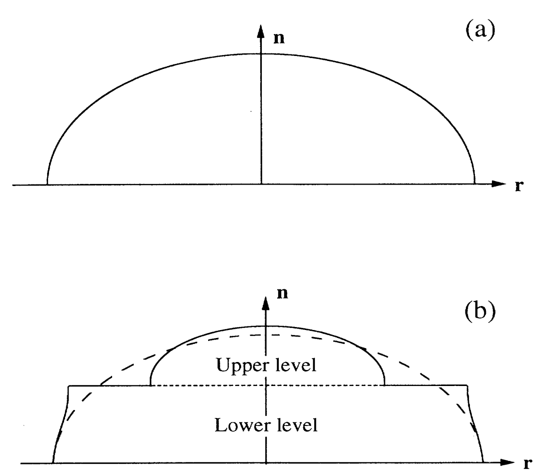

FIG. 2. Sketch of electron density distribution across the dot, (a) in zero magnetic field, and (b) in a strong magnetic field, when only two levels are occupied.

where the inner conductor has its maximum extent and the point where it is completely removed. As the magnetic field is increased and the inner conductor becomes smaller, electrons are transferred, one by one, from it to the other conductor. With the transfer of each charge, there is a buildup and release of the charge $e\delta N$ on the capacitance C. If the number of electrons initially on the inner conductor (that is, on the upper level) is N_0, there will be N_0 oscillations in this interval. Since initially the upper level occupies approximately the same area as the (fully filled) lower level, N_0 will be about half the total number of electrons in the dot: $N_0 \sim N/2$. In Ref. 5, it is estimated that $N \sim 40$, so there will be around 20 oscillations.

These oscillations are studied quantitatively in the next section. There we predict that the maximum amplitude of oscillations will occur when the areas of the two conducting regions are approximately equal.

In the same section we present a "phase diagram" for the system, which lays out the way in which the ground state depends upon the two parameters of gate voltage and magnetic field. A similarity is noted between the diagram for our system and one for a reversible single electron pump which has been constructed by Pothier *et al.*[10]

In Sec. III, we make numerical estimates of the mutual capacitances, basing them on the self-consistent analytical theory of Ref. 8. We use these estimates to compare our theory with the results of experiment.

II. CONDUCTANCE PEAK STRUCTURE DERIVED FROM THE ELECTROSTATIC MODEL

A circuit equivalent to the device of Fig. 1 is shown in Fig. 3. In terms of the polarization charges p, q, r, and s, the electrostatic energy is

$$U = \frac{p^2}{2C_1} + \frac{q^2}{2C_2} + \frac{r^2}{2C} + \frac{s^2}{2C_L} - V_g(p+q). \tag{5}$$

The charge discreteness requirement for the lower level gives

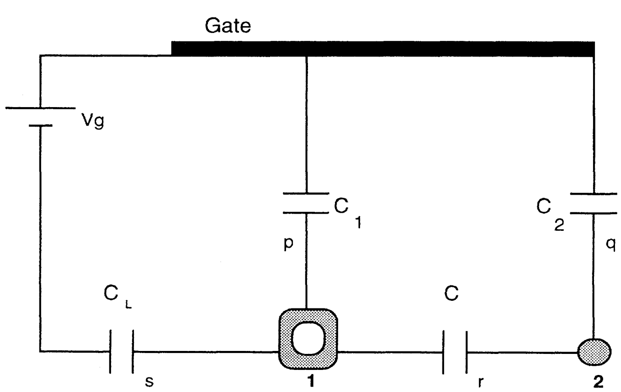

FIG. 3. Equivalent circuit for the dot in a quantizing magnetic field. Capacitances of the four tunnel junctions and polarization charges on these capacitors are shown; 1 is the outer conducting area, 2 is the inner conducting area.

$$en_1 = p + s - r. \tag{6}$$

For the upper level,

$$en_2 = q + r. \tag{7}$$

Here n_1 and n_2 are integers which represent the numbers of electrons on the two levels. There are also two constraints which require the voltages across the capacitors between leads and gate to sum to V_g. These are

$$V_g = \frac{p}{C_1} - \frac{s}{C_L} \tag{8}$$

and

$$V_g = \frac{q}{C_2} - \frac{r}{C} - \frac{s}{C_L}. \tag{9}$$

Using these four equations, we can eliminate the four polarization charges to find the energy of the system as a function of n_1 and n_2. For most of the calculations in this paper, we will neglect the lead-dot capacitance C_L. This is because including C_L yields results which differ very little from the results we give, while making the equations much more complex. We will estimate corrections due to a nonzero value of C_L at the end of this section. The assumption $C_L = 0$ implies that the charge $s = 0$. Applying the constraints then gives

$$U = \frac{N^2e^2}{2C_g} - eNV_g + \frac{e^2(C_1n_2 + n_1C_2)^2}{2C_g(C_1C_2 + CC_g)}, \tag{10}$$

where $C_g = C_1 + C_2$ and $N = n_1 + n_2$.

The case of zero magnetic field, where we can consider the whole dot as a single conducting region, corresponds to the limit $C \to \infty$. Equation (10) then reduces to (1). According to (3), peaks in the conductance occur at a series of discrete values of V_g, given by

$$V_g = -\frac{E_F}{e} + \frac{(2N+1)e}{2C_g}, \tag{11}$$

and separated by intervals of $\Delta V_g = e/C_g$.

Estimating C_1 and C_2 in the presence of a magnetic field, we assume that the distance between the gate and the two-dimensional electron gas forming the dot is much smaller than the size of the dot. The capacitances C_1 and C_2 are then proportional to the areas of their respective conductors. It was shown in Ref. 8 that the width of the depleted region between the dot and the confining gate is determined mostly by the confining potential and is independent of the magnetic field. It was also shown that the width of the incompressible area separating two conducting regions is small in comparison to the width of the conducting regions. Therefore the sum of the capacitances $C_g = C_1 + C_2$ can be regarded as independent of B and proportional to the area of the dot, even though both C_1 and C_2 depend on B.

The first two terms in (10) are independent of the way charge is distributed between the two conductors: that is, they depend only on N. The third term therefore determines the charge distribution inside the dot. The equilibrium value of n_1 can be found by minimizing this third term with respect to variations in n_1, holding N

fixed. The result is

$$n_1' = \frac{C_1 N}{C_g}. \tag{12}$$

Since charge is quantized, n_1 will increase in steps:

$$n_1 = \mathrm{Int}\left(n_1' + \frac{1}{2}\right) = \mathrm{Int}\left(\frac{C_1 N}{C_g} + \frac{1}{2}\right). \tag{13}$$

Here Int denotes an integer part of a number.

We now use the condition (3) for a peak in the low-temperature conductance. An electron moving onto the island may go to the inner or to the outer conductor. In the first case n_1 is constant, and in the second n_2 is constant. First consider addition to the outer conductor. Equations (3) and (10) with n_2 held constant give

$$V_g = -\frac{E_F}{e} + \frac{(2N+1)e}{2C_g} + \frac{eC_2^2}{2C_g(C_gC + C_1C_2)} + \frac{eC_2(n_2' - n_2)}{C_gC + C_1C_2}. \tag{14}$$

As can be seen from Eqs. (12) and (13), $(n_2' - n_2)$ oscillates around zero as C_1 changes, with peak-to-peak amplitude 1. According to (14), this causes oscillations in V_g, with amplitude $eC_2/(C_gC + C_1C_2)$. The dependence on C_2 of the last two terms in Eq. (14) is shown in Fig. 4(a). Because of the symmetry between the inner and outer conductors in this approximation, a similar calculation for addition to the inner conductor gives a diagram almost identical to Fig. 4(a). This second diagram is a reflection of the first about the vertical line through the center, $C_1 = C_2 = C_g/2$. Note that the lower bound for the oscillations in Fig. 4 is the same for both cases. We can plot both sets of lines on a single diagram: this is Fig. 4(b).

For each value of the capacitance C_1, only the lower of the two lines on this diagram has physical significance. To see this, consider what happens as we start at a value of V_g well below the lines and increase V_g slowly. At first, N has some well defined value, say N_1, which minimizes the energy. When the lower of the two lines on the diagram is reached, there is a degeneracy between the state of the system in which $N = N_1$ and the state in which $N = N_1 + 1$. This is what causes a peak in the conductance. Above the line, the $N = N_1 + 1$ state is the state of lowest energy. When the second line is reached, there is no change, because the line represents a degeneracy between two states which are not the equilibrium states of the system: the $N = N_1$ state and a second $N = N_1 + 1$ state obtained by adding an electron to a different conductor from the first.

We can therefore remove the upper of the two degeneracy lines at each point, to obtain a plot of the value of V_g at which a conductance peak occurs, against the capacitance C_2. This is Fig. 4(c). On all these diagrams, the horizontal axis corresponds to the value of V_g given by the first two terms of (14), or equivalently, the value given by Eq. (11).

As the magnetic field increases and capacitance C_2 is increased at the expense of C_1, the position of the conductance peak oscillates. This is the effect we discussed qualitatively in the Introduction.

The zigzag line in Fig. 4(c) divides the (B, V_g) plane into two regions. In the region below the line, the equilibrium state of the system has total charge N, and in the region above, the total charge is $N+1$. On the line, there is a degeneracy between two states having different values of N. This is the cause of the peak in the conductance.

The line in Fig. 4(c) is not the only locus of degeneracy between equilibrium states of the system: there are also lines of degeneracy between states with the same N, caused by redistribution of discrete charge within the dot. These lines do not give rise to peaks in the dc conductance through the dot. Figure 5 is a diagram showing all the degeneracy lines in part of the plane. This "phase diagram" of the system bears a striking similarity to the stability diagram of a reversible single-electron pump which has been constructed by Pothier *et al.*[10] The

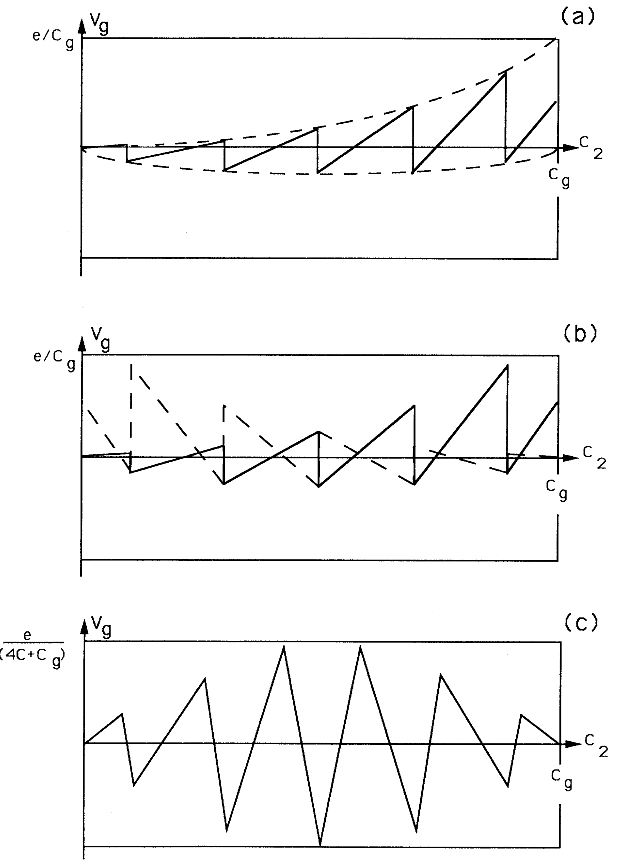

FIG. 4. Construction of the form of oscillations in V_g vs C_2. Conductance C_2 is a monotonous function of magnetic field B, see text. (a) Gate voltage $V_g(C_2)$ determined by the degeneracy condition for bringing an electron into the outer conductor, $U(n_1, n_2) = U(n_1 + 1, n_2)$, see Eq. (14). (b) The result of combining (a) with $V_g(C_2)$ corresponding to the degeneracy condition for bringing an electron into the inner conductor (dashed line), $U(n_1, n_2) = U(n_1, n_2 + 1)$. (c) The "physical" part of $V_g(C_2)$ corresponding to the observable conductance peaks. The monotonous part of $V_g(C_2)$ given by the third term in (14) is omitted.

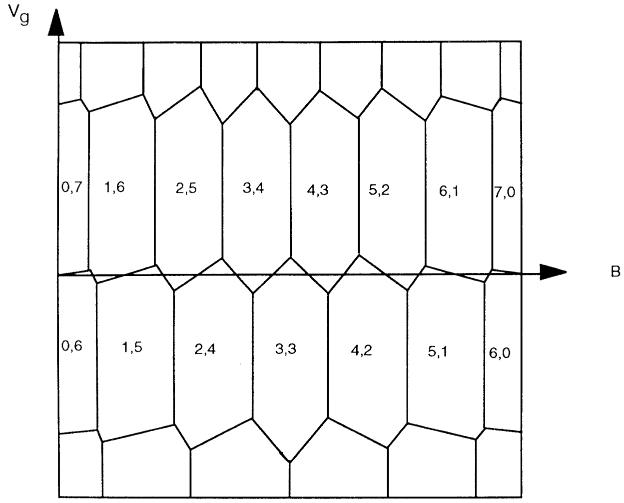

FIG. 5. "Phase diagram" for the system. Each hexagon represents the area in the (B, V_g) plane in which a certain state of the system is the ground state. States are labeled by the numbers of electrons on the two conducting rings: n_2,n_1. The total charge $N = n_1 + n_2$ is the same for all hexagons in the same horizontal row, and differs by 1 between adjacent rows.

resemblance is not a coincidence. The circuit used to pump electrons is very similar to our model of the island, with two "grains," each having a discrete charge. The two grains are coupled to each other and to leads by tunnel junctions, as are our two conducting areas. The parameters V_g and B are replaced by two independent gate voltages for the two grains. However in the pump, each grain is coupled to only one of the leads, while each conductor in our dot is coupled to both leads. Unfortunately the Coulomb island of the present paper could not be used to pump electrons in the same style, because of this difference in the way grains and leads are coupled.

The peak-to-peak amplitude of the oscillations varies as B is changed. Oscillations in $V_g(B)$ are suppressed at small C_1 and C_2, and most pronounced when $C_1 \sim C_2$. Because the number of oscillations N_0 is large (probably around 20—see the Introduction), it is reasonable to determine the B dependence of peak-to-peak amplitude. To do this we must determine the points of intersection of the two sets of lines in Fig. 4(b). The result, for large N, is

$$\delta V_g = \frac{eC_1C_2}{C_g(C_gC + C_1C_2)}. \tag{15}$$

If C_1 and C_2 vary so that $C_g = C_1 + C_2$ is constant, then the maximum amplitude of oscillations will be $e/(4C + C_g)$ and will occur when $C_1 = C_2$; that is, when the areas of the two conducting regions in our model are equal.

The construction we have used, which gives a phase diagram of hexagonal cells, can be used for any value of the ratio C/C_g. In the limit $C/C_g \to 0$, the vertical sides of the hexagons shrink to zero length and cells in the phase diagram become four-sided. This corresponds to a system of two separate quantum dots which do not interact electrostatically. In the opposite limit, $C/C_g \to \infty$, the system becomes equivalent to a single quantum dot, as we mentioned in the Introduction.

If the capacitance C_L between the leads and the outer conductor is included in the model, the equations become much more cumbersome, but exactly the procedure followed above can be used to analyze the problem. One important effect of including C_L is that the expression for the energy (10) is no longer symmetric with respect to the exchange of the two conductors. As before, there are two expressions for V_g, analogous to (14); but now the equation for tunneling onto the inner conductor is not simply the result of exchanging C_1,n_1 with C_2,n_2 in the equation for tunneling onto the outer conductor. Hence Fig. 4(c) becomes asymmetric: in fact the amplitude is decreased for smaller values of C_2 (or equivalently, of B). The expression for the amplitude is

$$\delta V_g = \frac{eC_1C_2}{C_g(C_gC + C_1C_2 + C_LC_2 + CC_L) - 4C_1C_2C_L}. \tag{16}$$

Since C_L is smaller than the other capacitances in the system (see the next section), it is reasonable to expand this expression to first order in C_L. The result is

$$\delta V_g = \frac{eC_1C_2}{C_g(C_gC + C_1C_2)} + C_L\frac{e[4C_1C_2 - C_g(C_2 + C)]}{C_g^2(C_gC + C_1C_2)^2}. \tag{17}$$

Since $4C_1C_2 < C_gC$ (See Sec. III), the effect of $C_L \neq 0$ is to decrease the amplitude. This effect is most pronounced at smaller C_2: that is, at smaller values of B. Another change to the phase diagram is that the lines which are vertical in Fig. 5 become inclined.

We will now show that (10) is equivalent to the electrostatic energy of Eqs. (1) and (2) in the Introduction. To do this, we introduce the difference between the discrete charge en_1 on one of the Landau levels and the corresponding continuous charge en_1' [see (13)]: $e\delta N = e(n_1 - n_1') = e(n_2' - n_2)$. From (10), we find

$$U = \frac{(Ne)^2}{2C_g} - NeV_g + (\delta N)^2\frac{e^2C_g}{2(C_gC + C_1C_2)}. \tag{18}$$

This formula agrees with Eqs. (1) and (2) when $C \gg C_g$. When $\delta N = 0$, it reduces to (1). Therefore (18) can be viewed as the first term of a power series expansion in δN, a small parameter which expresses the deviation of the system from the minimum energy corresponding to a continuous charge distribution.

III. ESTIMATES OF CAPACITANCES AND COMPARISON WITH EXPERIMENT

To compare the results of the previous sections to experimental data available in Ref. 5, we need to know the values of the capacitances C_1, C_2, C, C_L. Finding accurate estimates of these values is difficult for several reasons. First, the correspondence between our model and the real device is approximate: for example, the capacitance C varies somewhat with the magnetic field, as we mention later in this section. Also we have neglected any change in the sizes of the conducting areas which may occur as electrons are transferred between them. Second, although the positions of the gates in the device are precisely known, the shape and size of the 2D electron gas confined by the gates are not. Third, finding each capacitance, even when the dimensions are known, requires the solution of a nontrivial 3D electrostatics problem.

With these difficulties in mind, we will settle for very approximate estimates of the capacitances, which will serve only to show that the results of our calculations are of the same order of magnitude as the results of experiment.

All the conductors are embedded in $Al_xGa_{1-x}As$ or GaAs. Each of these semiconductors has a dielectric constant ϵ of approximately 12. We neglect the difference.

It is easy to find an estimate of C_g, the total gate capacitance. The Coulomb island can be regarded as a rectangle of area 350 $\times$700 nm,[2] which is 100 nm away from a broad plane which forms the gate. Since the size of the island is much larger than its distance from the gate, we can treat C_g as a parallel-plate capacitor, and find $C_g \sim 190\epsilon$ nm.

Next we estimate C, the mutual capacitance of the two conducting rings. Using the fact that the incompressible strip is narrow compared to the typical radius of the conducting areas, we can determine the charge distribution in the vicinity of this strip from an auxiliary problem. We substitute two straight conducting strips, separated by an insulating strip, for the two compressible regions. This makes the problem two-dimensional. We assume that the width a of the insulating strip is much smaller than the length of the system. If the electrodes were semi-infinite half-planes, the charge distribution induced on them by a potential difference V would be[11]

$$\sigma(x) = \frac{V}{2\pi^2[x^2 - (a/2)^2]^{1/2}}, \tag{19}$$

where x is the distance from the middle of the insulating strip. The total charge on the strips in the interval $a/2 < | x | < x_1$ depends logarithmically on x_1 if $x_1 \gg a$. Because of this weak logarithmic dependence, we can estimate C by cutting off the charge distribution in our auxiliary system at a length equal to the separation d between gate and dot. In this approximation, for $d \gg a$, the capacitance is

$$C = \frac{\epsilon l}{2\pi^2} \ln\left(\frac{4d}{a}\right). \tag{20}$$

We estimate a with the help of a result from Ref. 8. There, an equation is derived for a in terms of b, the width of the neighboring compressible strip, and a_b, the Bohr radius in the semiconductor. A modification is required to change the energy separation between levels from the Landau level separation $\hbar\omega_c$ to the Zeeman splitting energy. This gives an additional factor $(m/m_{\text{eff}})^{1/2} = 0.26$ for GaAs, where m and m_{eff} are the true mass and the effective mass of an electron in the semiconductor. An exchange interaction between electrons may modify the Zeeman splitting energy, forcing us to introduce an effective g factor g_{eff}.[12,13] The result is

$$a = \left(\frac{g_{\text{eff}} b a_b m}{\pi m_{\text{eff}}}\right)^{1/2}. \tag{21}$$

In experiment,[13] effective g factors have been observed in the range $2 - 15$. For the island, $a_b \sim 10$ nm, and we estimate $b \sim 100$ nm, so $a \sim 7 - 19$ nm. With the length $l \sim 1500$ nm, (20) gives $C \sim 230 - 310\epsilon$ nm.

Estimating the value of C_L, the capacitance between leads, and the outer conductor is difficult. However, the distance from dot to leads is at least ten times larger than the distance between the two conductors forming the capacitance C. We can see from Eq. (20) that the capacitance between two conducting planes depends logarithmically on this separation. The length of the capacitor [l in (20)] is also smaller for C_L than for C. We therefore expect that the value of C_L will be less than half that of C.

To compare the results of our model with the experimental results of Ref. 5 we calculate the ratio between the maximum amplitude of oscillations of a single peak, δV_g, and ΔV_g , the separation in V_g of successive peaks. Measurements in Ref. 5 give the values $\delta V_g = 0.1$ mV and $\Delta V_g = 1.2$ mV. An experimental value for the ratio is therefore 0.08. We will assume that this is the maximum value. It corresponds to the tenth oscillation after the start of the depopulation of the upper level, and in Ref. 5 the number of electrons on the dot is estimated to be around 40. In any case, our result for the amplitude varies little except at the extreme values of C_1 and C.

The separation between successive peaks in the conductance is independent of the magnetic field in our calculations: $\Delta V_g = e/C_g$. The maximum amplitude of oscillations is $\delta V_g = e/(4C + C_g)$. With the capacitances estimated in the previous section, the ratio[14] is

$$\frac{\delta V_g}{\Delta V_g} = \frac{C_g}{4C + C_g}. \tag{22}$$

This yields a value for the ratio of between 0.13 and 0.17. Including the maximum value of C_L estimated earlier, and using the expression (16) for the amplitude of oscillations δV_g, gives the slightly smaller values of 0.10 to 0.13 for the ratio. Since the effect is relatively small, the inaccuracies in our calculation of the value of C_L are not very important.

Our result is surprisingly close to the experimental value. The estimates we made of the capacitances are so rough that this agreement must be partly due to chance. However, this result does show that our theory predicts an amplitude which is of the correct order of magnitude. The unreliable way we take the filled portion of the lower Landau level into account (see the Introduction) probably also contributes to the error, and there is the possibility that C may vary with the magnetic field. This may also be the reason for the increase in $V_g(B)$ as B increases, which occurs in the experimental results, but is not explained by our calculations.

IV. CONCLUSION

We have constructed a simple model of charging effects in a quantum dot in a large magnetic field. The model can be solved analytically, and provides an intuitive understanding of the way charge distributes itself within the dot. Our work is complementary to the numerical approach of Ref. 5: it accounts simply for most important features of the case where there is only one occupied Landau level, while the computational approach reproduces experimental results more exactly and is easily applied to the dot in smaller magnetic fields, when the number of occupied levels is greater.

The validity of our model depends on observations about edge states made in the computational study[5] and also in an analytical treatment of the subject.[8,9] In particular, we assume that the electric field in areas where a Landau level is partially filled is screened almost completely, so that we can treat such areas as conductors. Also we assume that there is a large number of electrons on each level, so that the self-consistent approximation applies. The validity of the Coulomb blockade approach depends upon the assumption that electron transitions between two levels are inhibited sufficiently for the number of electrons in each level to be a well-defined integer. This is ensured for our two spin-split levels by the conservation of the electron spin quantum number.

The equations we use can be partly justified by considering a power series expansion of the energy about the minimum corresponding to a continuous charge distribution, which leads to an equation of the form (18), but does not give the coefficients in the series. We have used the mutual capacitances of the various conducting areas to supply the coefficients. This is more difficult to justify because it does not take into account electrons which are in the fully filled part of the lower level, and not in any of the compressible states which form our conductors. However, electrons in the fully filled states are prevented from moving in response to an electric field. This means that they do not take part in the redistribution of the charge density on the dot which occurs as charge hops in discrete steps from conductor to conductor. Since it is the energy change in this redistribution which causes the effects we have studied, the fully filled states will at most add an uninteresting constant to the interaction energy, and can be ignored.

The main feature which our model explains is the oscillations in $V_g(B)$, the gate voltage required to give a peak in the dc conductance, as a function of the magnetic field. These oscillations are a result of an "internal Coulomb blockade" between the two conducting regions in the dot, whose areas are dependent on the magnetic field. We predict that the amplitude of oscillations will be greatest when the two conductors have equal areas, and give an equation (15) for the amplitude as a function of the mutual capacitances of the different conducting areas in the dot. By estimating these capacitances from the geometry of the system, we are able to compare our results with the experimental results of Ref. 5, and find reasonable agreement.

With $C_L = 0$, as the conductance peaks oscillate, a maximum of $V_g(B)$ for one peak occurs at the same value of B as a minimum for the next. This is why the vertical lines in Fig. 5 are vertical. This agrees with the results of the numerical calculation in Ref. 5, where the Coulomb interaction between dot and leads was not taken into account. As we mention in Sec. II, when C_L is introduced, the vertical lines in Fig. 5 become inclined, and the correspondence is removed. A mismatch of this type was observed in the experimental results of Ref. 5, and we attribute this to the effect of the dot-lead interaction.

The theory given here applies only to the case where a single, spin-split Landau level is occupied, but it might be extended to more complex states of the system. There are also cases where nominally distinct Landau levels may merge because of thermally activated electron transport, reducing the effective number of separate levels.

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[14] Actual values for ΔV_g and δV_g given by our equations (for $C_L = 0$) are reasonably close to the experimental numbers. $\Delta V_g = 0.66$ mV and $\delta V_g = 0.09 - 0.12$ mV, compared to 1.2 mV, 0.1 mV.