Selective manipulation of the emission spectrum of an electron in a biased double-well heterostructure driven by a free-electron laser

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We study the emission spectrum of an electron in a biased double-well heterostructure driven by a cw laser field. It is shown that, in general, the emission spectrum consists of triplets at all harmonics of the laser frequency. We analytically and numerically show that it is possible, by making the proper choice of parameters, to selectively eliminate each line in the spectrum.

Recently, interesting effects have been found in the interaction of electrons in symmetric double-well heterostructures with laser fields in the IR and microwave regions. Among these are suppression of tunneling,[1–6,8] the induction of a static dipole moment,[2,5–8] and low-frequency generation.[2,5–7] These effects have been studied in isolated systems as well as in systems which include interaction with a dissipative bath.[9,10] In a series of papers it has been shown, using a two-level approximation, that the emission spectrum of the electron resulting from such interactions consists of odd harmonics of the laser frequency ω as well as split even harmonic doublets.[2,6,8] The components of the doublets interfere destructively and disappear when they merge,[6] unless ω is not much higher than the splitting between the two levels Δ.[6] The effects listed above require that the light-matter interaction energy $(E\mu)$ be larger than the splitting between levels and also be of the order of the photon energy. Quantum-well heterostructures are most suitable for observation of such effects due to their large dipole moments ($10-10^3$ larger than that of molecular systems) and adjustable small splitting between levels.[11] The intensities required can be obtained by using a free-electron laser which produces intense 20-μs square pulses that are tunable in the region of $6-170\ \mathrm{cm}^{-1}$.

Although these effects were obtained for both optical and tunneling initial conditions,[5–8] in general they are stronger for tunneling initial conditions. By tunneling initial condition we mean that the electron is initially localized in one of the wells (prepared in a coherent state), and by optical initial condition we mean that the electron is initially in an eigenstate, and therefore has the probability of being in both wells. However, though possible,[12] the tunneling initial condition is hard to achieve. This problem is overcome when the double-well structure is biased (see Fig. 1) because the electron in an eigenstate is naturally localized in one of the wells. The system loses the inversion symmetry giving rise to various effects. For example, the laser field can, by a proper choice of parameters, delocalize the initially localized electron.[13]

The purpose of this paper is to study the emission spectrum resulting from the interaction of an electron in a biased system with a laser field, and manipulate the spectrum by choice of parameters such as laser frequency, amplitude, and bias.

We consider the system described by the following Hamiltonian:

$$H=\Delta\sigma_x+V(t)\sigma_z \ , \tag{1}$$

where Δ is the splitting parameter, $V(t)$ is a driving force, hereafter we consider the units where $\hbar$ is equal to 1. For superlattices,

$$V(t)=[E_0\cos(\omega t)+E_s]\mu_{12} \ , \tag{2}$$

where E_0 is the amplitude of the cw field, E_s is a constant field used for breaking the symmetry, σ_x and σ_z are the Pauli matrices, and μ_{12} is the transition dipole between the two levels. The Hamiltonian (1) may be easily reduced to the form which is convenient for the description of optical properties by making use of the following unitary transformation:

$$U=\exp(i\pi\sigma_y/4) \ . \tag{3}$$

The Hamiltonian (1) then takes the form

$$H=UHU^{-1}=-V(t)\sigma_x+\Delta\sigma_z \ . \tag{1'}$$

The time-dependent dipole moment may be defined as follows:

$$\mu(t)=\langle\psi|\sigma_x(t)|\psi\rangle \ , \tag{4}$$

and in the representation (1)

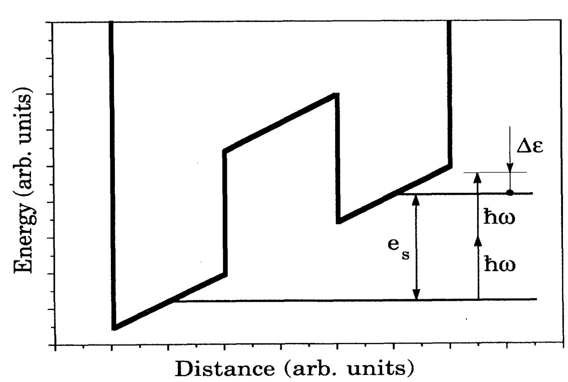

FIG. 1. The biased double-well heterostructure.

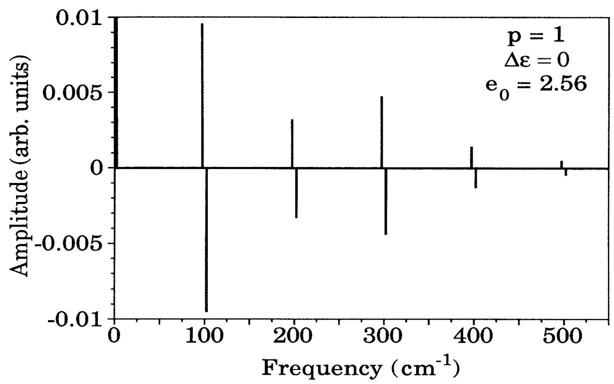

FIG. 2. Numerically calculated emission spectrum with $\Delta=5$ cm^{-1}, $\hbar\omega=100$ cm^{-1}, $e_s=1.0$, $e_0=2.56$.

$$\mu(t)=[a_1^*\langle 1|+a_2^*\langle 2|]|\sigma_z(t)|[a_1|1\rangle+a_2|2\rangle] \ . \quad (4')$$

The dynamics of $\mu(\tau)$ is governed by the integro-differential equation

$$d\mu/d\tau=-(\Delta/\hbar\omega)^2\int_0^\tau d\tau_1\mu(\tau_1)\times\cos[e_0\sin(\tau)-e_0\sin(\tau_1)+e_s(\tau-\tau_1)] \ , \quad (5)$$

where

$$\mu(0)=\mu_{12} \ , \quad (6)$$

$$\tau\equiv\omega t \ , \quad (7)$$

$$e_0\equiv 2E_0\mu_{12}/\hbar\omega \ , \quad (8)$$

$$e_s\equiv 2E_s\mu_{12}/\hbar\omega \ . \quad (9)$$

In order to solve Eq. (5) with the initial condition (6), we expand the kernel in a Fourier series with Bessel function Fourier coefficients[14]

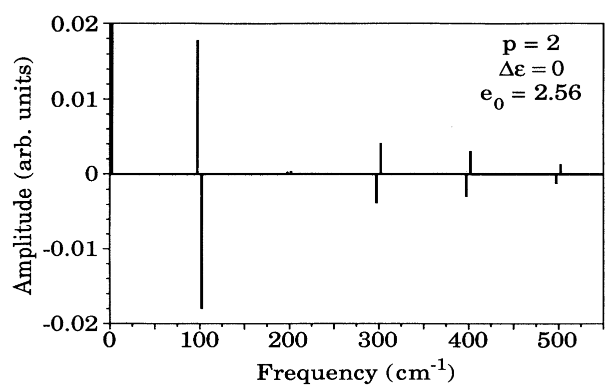

FIG. 3. Numerically calculated emission spectrum with $\Delta=5$ cm^{-1}, $\hbar\omega=100$ cm^{-1}, $e_s=2.0$, $e_0=2.56$.

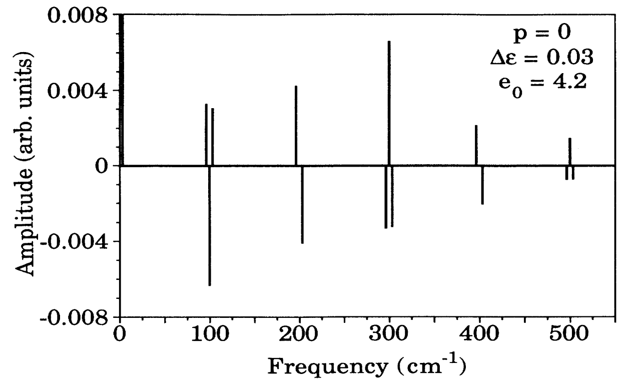

FIG. 4. Numerically calculated emission spectrum with $\Delta=5$ cm^{-1}, $\hbar\omega=100$ cm^{-1}, $e_s=0.03$, $e_0=4.2$.

$$\begin{aligned} d\mu/d\tau=&-(\Delta/\hbar\omega)^2\mathrm{Re}\int_0^\tau d\tau_1\mu(\tau_1)J_{-p}^2(e_0)\times\exp[i\Delta\epsilon(\tau-\tau_1)] \\ &-(\Delta/\hbar\omega)^2\mathrm{Re}\sum_{n,m=-\infty}^{\infty}{}' J_m(e_0)J_n(e_0)\times\int_0^\tau d\tau_1\mu(\tau_1)\times\exp[i(e_s+n)\tau-i(e_s+m)\tau_1] \ , \end{aligned} \quad (10)$$

where the prime in the double sum in the second term is used to exclude the term with $m=n=p$, and p in the first part of Eq. (10) is the nearest number of photons required to match the bias

$$e_s\equiv p+\Delta\varepsilon \ , \quad (11)$$

where $\Delta\varepsilon$ is the mismatch. Figure 1 illustrates the biased wells with mismatch corresponding to $p=2$. For cases with small mismatch parameter,

$$\Delta\varepsilon\ll 1 \quad (12)$$

and

$$\Delta/\hbar\omega\ll 1 \ , \quad (13)$$

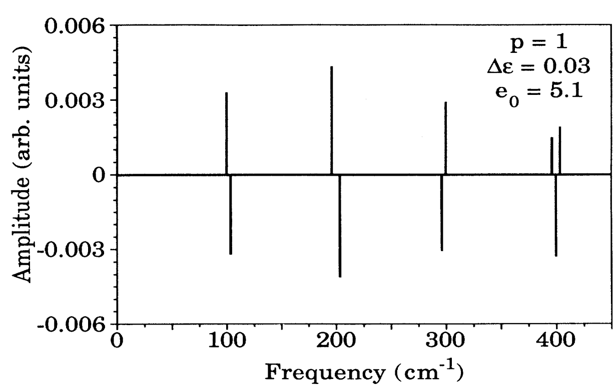

FIG. 5. Numerically calculated emission spectrum with $\Delta=5$ cm^{-1}, $\hbar\omega=100$ cm^{-1}, $e_s=1.03$, $e_0=5.1$.

the following low-frequency solution has been found in Ref. 13:

$$\mu_0(\tau)/\mu_{12}=(\Delta\varepsilon)^2/\varpi_p^2+\Delta_p^2/\varpi_p^2\cos\{\tau\varpi_p\} \ . \tag{14}$$

Here,

$$\Delta_p\equiv(\Delta/\hbar\omega)J_p(e_0) \tag{15}$$

and

$$\varpi_p\equiv[\Delta_p^2+(\Delta\varepsilon)^2]^{1/2} \ . \tag{16}$$

Expression (14) is the exact solution of Eq. (10) when the second term in it is neglected due to the fact that integration yields terms multiplied by frequency denominators. Since the first term contains the smallest frequency $\Delta\varepsilon$ [see Eq. (12)], it dominates the dynamics.

In order to solve for the high-frequency part of the spectrum, we present the dipole in the following form:

$$\mu(\tau)=\mu_0(\tau)-(\Delta/\hbar\omega)^2\mathrm{Re}\sum_{n,m=-\infty}^{\infty}{}' J_m(e_0)J_n(e_0)\int_0^\tau d\tau_1\int_0^{\tau_1}d\tau_2\mu(\tau_2)\exp[i(e_s+n)\tau_1-i(e_s+m)\tau_2] \ . \tag{17}$$

Keeping only the terms with $m=-p$ and performing first iteration with respect to $\mu_0(\tau)$, one obtains the terms in the spectrum not included in (14),

$$\mu(\tau)=\mu_0(\tau)-(\Delta/\hbar\omega)^2J_p(e_0)/\varpi_p^2\sum_{k=1}^{\infty}(-1)^k\{(A_{p,k}-B_{p,k})\cos(k\tau)+B_{p,k}\cos[(k-\varpi_p)\tau]-A_{p,k}\cos[(k+\varpi_p)\tau]\}/2k \ , \tag{18}$$

where

$$A_{p,k}\equiv(\varpi_p-\Delta\varepsilon)J_{p+k}(e_0)+(\varpi_p+\Delta\varepsilon)J_{p-k}(e_0) \ , \tag{19}$$

$$B_{p,k}\equiv(\varpi_p+\Delta\varepsilon)J_{p+k}(e_0)+(\varpi_p-\Delta\varepsilon)J_{p-k}(e_0) \ . \tag{20}$$

As is evident from Eqs. (18)–(20), in general, the spectrum consists of triplets, $k\omega$ and $\omega(k\pm\varpi_p)$, centered at all harmonics of the laser frequency. This is different from the unbiased case where the spectrum consists of doublets at even harmonics with vanishing amplitudes at odd ones.[6,8] In addition to the external parameters e_0 and $\Delta/\hbar\omega$ which control the dynamics of unbiased systems, here we also have the number of photons required for the resonance p and mismatching $\Delta\varepsilon$. By changing the values of these parameters we can control the emission spectrum. In what follows we test the analytical solution (18)–(20) by performing a numerical integration of Eq. (10) and Fourier transforming the time-dependent dipole moment to obtain the spectrum.

For example, in Fig. 2 we use exact "resonance," $\Delta\varepsilon=0$, to eliminate the central peaks and obtain only $\omega(k\pm\Delta_p)$ where Δ_p is defined by Eq. (15). Here, k includes both even and odd harmonics. When, in addition to $\Delta\varepsilon=0$, e_0 is taken to be a zero of the J_p Bessel function, the two satellites merge and destructively interfere yielding vanishing spectrum except of a zero-frequency term [see Eq. (14)]. By making the proper choice of e_0 while still keeping $\Delta\varepsilon=0$, it is possible to eliminate any one of the doublets shown in Fig. 2. In Fig. 3 we show an example with $p=2$ and $J_0(e_0)+J_4(e_0)=0$, the amplitude of the second-harmonic doublet is negligibly small as predicted by Eqs. (18)–(20). The analysis above is valid also for weak bias, $p=0$ (but $\Delta\varepsilon\neq0$). In such cases the spectrum consists of triplets at odd harmonics and doublets at even ones due to the amplitude of the central peaks being proportional to $J_k(e_0)-J_{-k}(e_0)=J_k(e_0)[1-(-1)^k]$. Figure 4 shows an example with $p=0$ and $\Delta\varepsilon=0.03$, and the spectrum obeys the analysis. It is also possible to eliminate any one component of a triplet by making one of the amplitudes $A_{p,k}$, $B_{p,k}$, or $B_{p,k}-A_{p,k}$ equal to zero. An example of this effect is shown in Fig. 5 where the left-hand line of the first-harmonic triplet, the central line of the second, and the right-hand line of the third one are missing.

In conclusion, we have studied the emission spectrum of an electron in a biased double-well heterostructure driven by a cw laser field. We have shown that, in general, the emission spectrum consists of a static component, low frequency ϖ_p (in ω units), and triplets at frequencies k and $k\pm\varpi_p$ for $k=1,2,3,\ldots$. The ampli-

TABLE I. Possibilities of selective manipulation.

Elimination of	p	$\Delta\varepsilon$	e_0
all k	$\neq0$	0	arbitrary
one k	arbitrary	arbitrary	$J_{p-k}(e_0)=J_{p+k}(e_0)$
$(k-\varpi_p)$	arbitrary	arbitrary	$(\varpi_p-\Delta\varepsilon)J_{p-k}(e_0)+(\varpi_p+\Delta\varepsilon)J_{p+k}(e_0)=0$
$(k+\varpi_p)$	arbitrary	arbitrary	$(\varpi_p+\Delta\varepsilon)J_{p-k}(e_0)+(\varpi_p-\Delta\varepsilon)J_{p+k}(e_0)=0$
whole k triplet	$\neq0$	0	$J_{p-k}(e_0)=-J_{p+k}(e_0)$
everything but static	arbitrary	arbitrary	$J_p(e_0)=0$
static	arbitrary	0	$J_p(e_0)\neq0$
all $2k$	0	$\neq0$	arbitrary

tudes of all lines and $\bar{\omega}_p$ depend on the bias e_s, the laser intensity e_0, and the mismatch $\Delta\varepsilon$ as defined in Eqs. (8), (9), and (11). We have analytically and numerically showed that it is possible, by making the proper choice of the above parameters, to selectively eliminate each line in the spectrum, a whole triplet, all of the exact harmonics, the static component, and the whole spectrum but the static component. The possibilities of selective manipulation are summarized in Table I.

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