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Coupled fluxon modes in stacked Nb/AlO$_x$/Nb long Josephson junctions

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The current-voltage characteristics of N vertically stacked Nb-based long Josephson junctions ($N = 1, 2, 3$) are measured. For a single junction in the stack, we observe the splitting of the flux-flow mode into M modes, each of them being characterized by different magnetic field dependence and by different cavity-resonance (Fiske-mode) frequency. Our results suggest that a fluxon moving in one junction in the stack may experience $M = N$ different Swihart velocities due to the coupling to adjacent junction(s) through the common electrode(s) of the thickness smaller than the London penetration depth. In the twofold stacks, we find an indication for a coherent mode corresponding to coupled fluxon arrays moving in both junctions.

A system consisting of two Josephson tunnel junctions placed on top of one another was studied by Giaever,[1] who used it for demonstrating the existence of the ac Josephson effect. Linear electromagnetic wave properties of vertically stacked distributed Josephson tunnel junctions were investigated theoretically by Ngai.[2] More recently, the nonlinear magnetic flux dynamics in such systems became a subject of several theoretical studies.[3,4] Due to the interaction of fluxons moving in adjacent junctions in the stack, several interesting new physical effects have been predicted but not observed experimentally so far. The stacked arrangement of long Josephson junctions (of length $L \gg \lambda_J$ and of width $W < \lambda_J$, where λ_J is the Josephson penetration depth) also attracts interest because of potential improvements of the properties of fluxon devices in areas such as impedance matching, output power, and integration level. At the present stage of Nb thin film technology it is possible to fabricate high-quality stacked Josephson tunnel junctions and several groups have already reported experiments with small area stacked Nb-based junctions.[5–7]

In this paper we report on an experimental investigation of the fluxon dynamics in Nb/AlO$_x$/Nb long stacked Josephson junctions. We compared structures with similar dimensions and fabrication parameters but with different number of Josephson tunnel barriers $N = 1$, 2, and 3. A schematic view of the twofold stack is shown in the inset in Fig. 1. The details about the sample fabrication can be found elsewhere.[6] The thickness $d = 30\,\text{nm}$ of the common superconducting electrode between adjacent barriers in all the stacks was smaller than the London penetration depth $\lambda_L \approx 90$ nm. The characteristic features that we report here were found to be reproducible for four different sets of junctions having $L = 150$–400 μm and $W = 10$–40 μm. The typical value for λ_J in the single-barrier junctions was 25 μm.

In order to study the fluxon dynamic states, the current-voltage (IV) characteristics of the fabricated structures have been investigated as a function of the external magnetic field H applied in the plane of the tunnel barriers and perpendicular to the larger dimension L of the junctions. In the present layout we have no electric contacts to the common (internal) electrodes of the stacks. Thus, in the IV characteristics presented below, the stacked junctions are measured in series. The gap voltages V_Δ were found to be about 2.6 mV and varied for the different junctions in one stack within 1–2%. The difference between the critical current densities ($j_c \approx 200$ A/cm^2) in one stack was estimated from the maximum currents at the gap voltages and was found typically to be less than 15%. Below we discuss in detail the results obtained with the twofold stacks F223R and F229L, and the threefold stack F333L. Relevant parameters of the samples are listed in Table I.

For both twofold and threefold stacks in the zero magnetic field the switching from zero voltage state caused the switching of all the junctions to the sum gap voltage $N \times V_\Delta$ of the stack. However, the magnetic field dependence of the critical currents $I_c^A(H)$ and $I_c^B(H)$ corresponding to the different junctions A and B in one stack (in case of the twofold stack) was found to be almost linear in small fields but with significantly different slope dI_c/dH. The first critical field (extrapolated to zero current from the dI_c/dH slope) ratio H_{c1}^B/H_{c1}^A was found to be as high as 8 (for the stack F229L $H_{c1}^A \sim 1.0$ Oe). It has been already shown for small area stacks[8] ($L, W < \lambda_J$) that if the junctions in twofold stack are closely coupled ($d < 2\lambda_L$) and $j_c^A < j_c^B$, the magnetic field required to suppress I_c^B significantly increases and that for I_c^A slightly reduces. We observe here a similar effect with the stack of long closely coupled junctions. One may understand such behavior as a sort of shielding effect that the current flowing in the common electrode (due to the magnetic flux present in the junction A) produces on the junction B. The fluxon array which enters junction A tends to prevent fluxons from entering junction B. The switching of the whole stack from $V = 0$ to $V = N \times V_\Delta$ in zero field is most probably due to the inductive interaction between the junctions.

At $H \approx 0$ we did not observe any stable zero-field steps or low-order Fiske steps in the IV characteristics of the stacked samples. However, in the magnetic field range $H > H_{c1}^A$ we observed stable and reproducible flux-flow steps. Typical data for $H_{c1}^A < H < H_{c1}^B$ are shown in

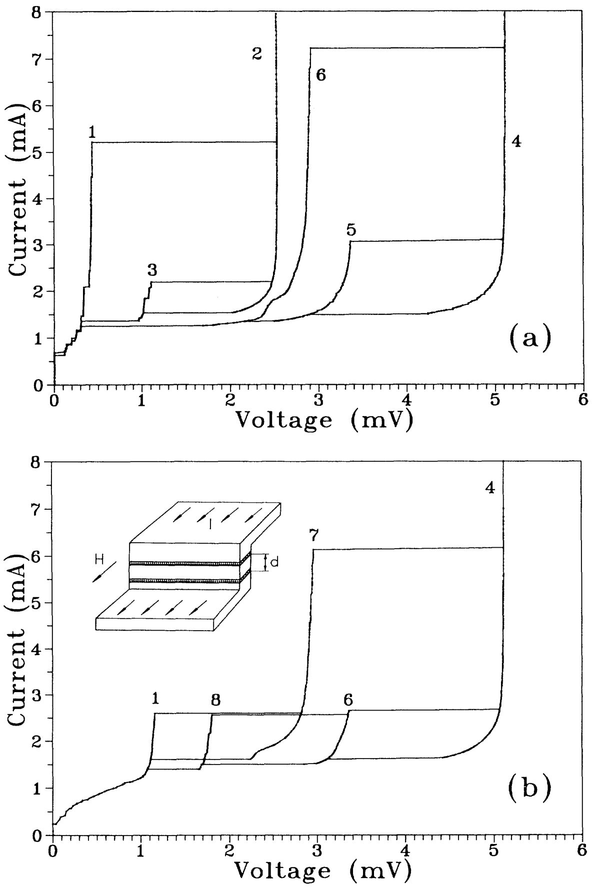

FIG. 1. *IV* characteristics of stack F229L consisting of two junctions measured in the external magnetic field: (a) $H = 6.8$ Oe; (b) $H = 17.0$ Oe. The horizontal arrows indicate the direction of switching while recording the curves. (1) V^A_{ff1}; (2) V^A_Δ; (3) V^A_{ff2}; (4) $V^A_\Delta + V^B_\Delta$; (5) $V^A_{ff2} + V^B_\Delta$; (6) $V^A_{ff1} + V^B_\Delta$; (7) $V^A_\Delta + V^B_{ff}$; (8) V^{AB}_{ff}. The inset shows a schematic view (not to scale) of the sample.

Fig. 1(a). We observe both gap voltage branches (curves 2 and 4) $V = V^A_\Delta$ and $V = V^A_\Delta + V^B_\Delta$. When the current I is increased from zero, the flux-flow step in A (curve 1) is traced up to $V = V_{ff1}$. At $I = 5.2$ mA the junction A switches to the gap $V = V^A_\Delta$. We observe here a single gap voltage because the other junction B is still in the stationary state ($V^B = 0$) since $H < H^B_{c1}$. Further increase of I leads to the switching of junction B at $I^B_c = 10$ mA. If the current instead is decreased with B not yet switched we observe another resonant step in A (curve 3) at $V \approx V_{ff2}$. This branch is traced by increasing the current again as seen in Fig. 1(a).

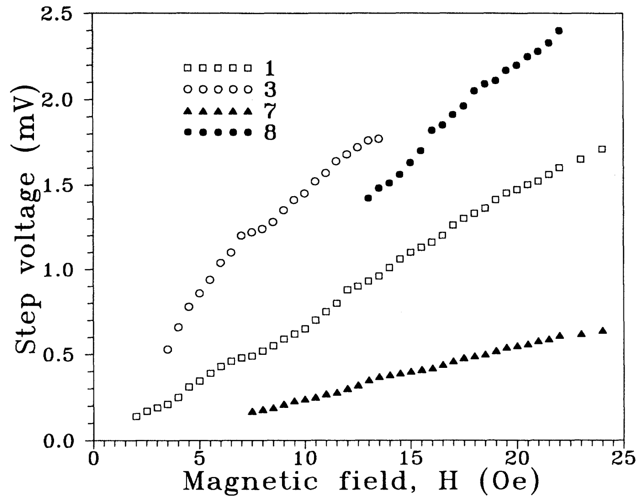

FIG. 2. Magnetic field dependence of the maximum flux-flow step voltages in stack F223R. Notations for the steps (1), (3), (7), and (8) are the same as in Fig. 1.

Magnetic field dependence of the maximum flux-flow step voltages is shown in Fig. 2. The maximum voltage V_{ff2} of step 3 was found to be linearly dependent on the magnetic field H, in a similar way as the flux-flow step 1 at V_{ff1}. However, the rate of the linear increase of the step voltage dV_{ff2}/dH is higher than dV_{ff1}/dH. As seen from Table I, for the single junction the flux-flow step rate was found to be close to that of dV_{ff2}/dH in the stack. We never observed two flux-flow steps in single barrier junctions.

The statement that at $V = V_{ff1}$ and $V = V_{ff2}$ we observe the flux-flow behavior in *only one junction* of the stack is supported by the observation of the analogous branches 6 and 5 at $V_\Delta < V < 2\,V_\Delta$ in Fig. 1(a). These are traced when the current is decreased from $V = 2\,V_\Delta$. Presumably, we observe the same branches 1 and 3 in junction A but added to the voltage $V^B \approx V_\Delta$. Because V^B is smoothly decreasing below V^B_Δ at small I, the replicas of the flux-flow branches of A at $V > V_\Delta$ are smoother than those near zero voltage. The high rate of the quasiparticle injection into the common electrode by junction B is supposed to increase the flux-flow losses in junction A. A knee in the curve 6 at $V \sim 2.4$ mV may tentatively be explained as the photon-assisted tunneling effect induced in junction B under the influence of the

TABLE I. Parameters of the samples.

Sample	No. of barriers	$L \times W$ (μm^2)	dV^A_{ff}/dH (mV/Oe)	dV^B_{ff}/dH (mV/Oe)	ΔV_{FS} (μV)
1	1	200 × 40	0.186	–	35
2	2	200 × 40	0.055, 0.196	0.029	18.5, 45
3	3	150 × 20	0.022, 0.031, 0.097	–	21, 33, 78
4	1	300 × 10	0.179	–	17.4
5	2	400 × 10	0.068, 0.162	0.025	13, 30

flux-flow radiation coming from junction A. This type of effect was observed in the experiment by Giaever.[1]

The picture obtained from fast sweep *IV* curves on a storage oscilloscope while slowly changing the magnetic field H from 3 to 7 Oe is shown in Fig. 3(a). As it is seen already from Fig. 1(a) and more clearly here, each flux-flow step consists of a set of cavity mode resonances (Fiske steps). For two different flux-flow steps of the junction A in the stack we observe *two different values of the Fiske step spacing.* For the *ff1* mode it is $\Delta V_{FS1} = 13.0 \pm 0.2\ \mu$V and for the *ff2* mode [seen on the right part of Fig. 3(a)] it is $\Delta V_{FS2} = 30.0 \pm 0.5\ \mu$V. Let us emphasize that these two separate Fiske step families appear in the different voltage ranges but are observed in *one junction* (A) of the double stack. Carrying out the same type of measurements with the threefold stack [shown in Fig. 3(b)] we observed three Fiske step families (at the voltages of about 0.25, 0.40, and 1 mV) with the spacings listed in Table I.

The observed splitting of the flux-flow regime and the Fiske resonances into two different modes for the double-junction stacks should be explained by the influence of junction B on the properties of the electromagnetic waves propagating in the closely coupled junction A. This problem was considered theoretically more than 20 years ago

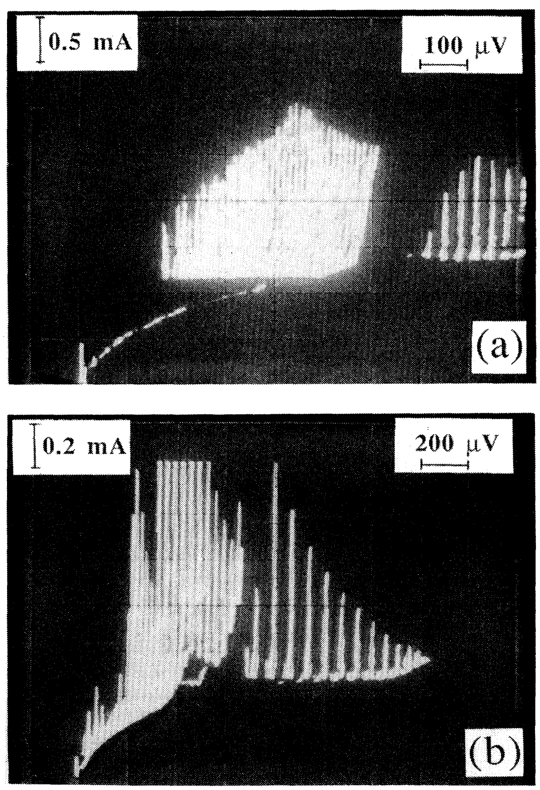

FIG. 3. Traces of the Fiske steps on the *IV* curve obtained by varying the external magnetic field. The images were recorded below the first gap voltage using a storage oscilloscope. (a) Two-junction stack F229L, horizontal scale 100 μV/div. (b) Three-junction stack F333L, horizontal scale 200 μV/div.

by Ngai[2] who predicted the existence of two distinct phase velocities $\bar{c}_-$ and $\bar{c}_+$ ($\bar{c}_- < \bar{c} < \bar{c}_+$) for the linear electromagnetic waves (surface plasmons) in the double-junction stack model related to Giaever's experiment.[1] To our knowledge, that effect has not been observed experimentally so far. In a similar model, but where solitons were considered instead of linear waves, Sakai, Bodin, and Pedersen[4] found two limiting fluxon velocities $\bar{c}_+$ and $\bar{c}_-$ (corresponding to coherent bunched and symmetric soliton modes, respectively) in the numerical simulations of a long double-junction stack. According to the simplest theoretical model[3] for two long junctions coupled inductively through their common superconducting electrode of thickness $d < 2\lambda_L$, the phase difference $\varphi(x,t)$ across each of them could be described by the equation

$$\varphi_{xx}^{A,B} - \varphi_{tt}^{A,B} = \sin\varphi^{A,B} + \alpha\varphi_t^{A,B} + \gamma - \delta\varphi_{xx}^{B,A}\ , \quad (1)$$

The spatial coordinate x is normalized to the single-junction Josephson penetration depth λ_J, the time t to the inverse plasma frequency ω_0^{-1}, α is the dissipation coefficient, and γ is the bias current. The coupling parameter δ in Eq. (1) is assumed to be small. Solving this equation for the small amplitude linear waves $\varphi = \varphi_0\exp[i(kx - \omega t)]$ with perturbative terms $\alpha = \gamma = 0$ yields the dispersion relation $\omega^2 = 1 + (1 \pm \delta)\,k^2$, where ω is normalized to ω_0. Thus, for $\delta \neq 0$ the plasma mode in each junction splits into two modes which are characterized by two different Swihart velocities

$$\bar{c}_{\pm} = \bar{c}\sqrt{1 \pm \delta}\ , \quad (2)$$

where $\bar{c}$ is the Swihart velocity for the single-barrier junction. For a junction of length L, the Fiske step voltage spacing $\Delta V_{FS} = \Phi_0\,\bar{c}/2L$ (Φ_0 is the magnetic flux quantum) provides a direct experimental measure of the Swihart velocity, which allows an experimental estimate of the coupling parameter δ. According to Eq. (2) both Fiske step spacings ΔV_{FS1} and ΔV_{FS2} in the double stack can be calculated from the single-junction ΔV_{FS} using the same value for δ. For the junction F113R, taking $\Delta V_{FS} = 35\ \mu$V with $\delta = 0.72$ we obtain $\Delta V_{FS1} = 18.5\ \mu$V and $\Delta V_{FS2} = 45.8\ \mu$V. These estimates are in good agreement with experimental data listed in Table I for the double stack F223R of the same dimensions. For the threefold stacks the strong coupling between the junctions leads to the appearance of three distinct Fiske step spacings in one junction as observed in Fig. 3(b). In Ref. 4 the coupling parameter was derived analytically from the ratio d/λ_L. Using that definition we obtain a coupling parameter δ to be about 0.6 to 0.7, which is close to the simple fit made here. This large value for δ means that the coupling between the neighboring junctions is very strong, so that the weak-coupling approximation (1) and (2) cannot be applied directly. The model (2) has been used before in order to describe the interaction between two long junctions which are placed closely *in plane.*[9] We would like to emphasize that *vertically stacked* junction system studied here is substantially different from the weakly coupled junction configuration in plane. The coupling between the stacked junctions is by 2 orders of magnitude higher, which leads to the new phenomenon which we observe.

The conventional expression for the flux-flow step asymptotic voltage in a single barrier junction is $V_{ff} = \Lambda\, H\, \bar{c}$, where magnetic penetration depth of the barrier Λ is typically close to $2\,\lambda_L$. The difference between $\bar{c}_+$ and $\bar{c}_-$ in the double-junction stacks should also lead to a difference in dV_{ff}/dH slopes for the higher and the lower modes. Since the corresponding penetration depths Λ_+ and Λ_- presumably differ from each other, the ratio between the slopes $r_{ff} = [dV_{ff2}/dH]/[dV_{ff1}/dH]$ and that between the Fiske step spacings $r_{FS} = \Delta\, V_{FS2}/\Delta\, V_{FS1} \approx 2.3$ is expected to be different. From the data in Table I we see that r_{ff} appears to be somewhat higher than r_{FS}.

Finally, we would like to discuss briefly the data presented in Fig. 1(b). This IV characteristics has been taken in high magnetic field $H > H^B_{c1}$ at which junction B also shows the flux-flow behavior at $V = V^A_\Delta + V^B_{ff}$. As shown in Fig. 2, the slope dV^B_{ff}/dH was less than a half of dV^A_{ff1}/dH while $\Delta\, V^B_{FS} \approx \Delta\, V_{FS1}$. We did not observe the higher flux-flow mode in junction B, probably because during the switching from $V = V^A_\Delta + V^B_\Delta$ junction A always switched first to $V = V^B_\Delta + V^A_{ff1}$ (at this large field the high mode *ff2* of A has to be already at $V \sim V^A_\Delta$ and it was not observed). When switching down from $V = V^B_\Delta + V^A_{ff1}$, we found another stable mode (called V^{AB}_{ff}) at a voltage close to but slightly higher than $V = V^B_{ff} + V^A_{ff1}$. Naturally, the explanation for this step would be that both junctions A and B are biased in the flux-flow state. In agreement with this assumption the measured slope dV^{AB}_{ff}/dH was close to $dV^A_{ff1}/dH + dV^B_{ff}/dH$ (see Fig. 2). Surprisingly, however, the measured Fiske step spacing $\Delta\, V^{AB}_{FS} = 48.5\,\mu$V for this mode is substantially larger than that of either junction A or B. The measurements of another double junction stack F223R revealed the similar feature with $\Delta\, V^{AB}_{FS} = 64\,\mu$V. This observation suggests that in this mode junctions A and B operate somewhat coherently (probably locked on the first and the second harmonic of the Josephson frequency). Because of a great interest to possible phase locking of stacked long Josephson junction oscillators the clarification of this point deserves additional study.

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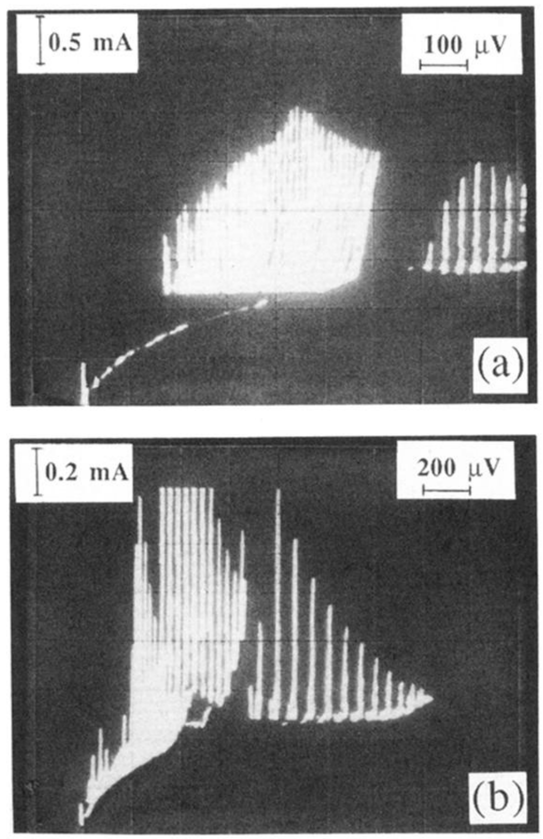

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