PHYSICAL REVIEW B VOLUME 48, NUMBER 14 1 OCTOBER 1993-II

Eliashberg equations and superconductivity in a layered two-dimensional metal

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(Received 10 December 1992)

The Eliashberg equations for superconductivity in a layered two-dimensional (2D) metal, taking into account the interaction between an electron and an arbitrary boson excitation, are obtained. The theory is applied to the case of a plasmon-mediated electron-electron interaction and it is shown that the interelectronic Coulomb repulsion parameter μ is connected with both the bare Coulomb interaction and the single-particle excitation. An evaluation of this parameter shows that while remaining always positive, it decays with the decrease of the interlayer distance. It is also shown that the electron-plasmon interaction in a layered 2D system leads to a logarithmic divergence of the renormalization factor $Z(\omega)$ as $\omega \rightarrow 0$ and $T \rightarrow 0$. This gives rise to a logarithmic dependence of $\lambda_{\rm pl}$, the electron-plasmon interaction constant, on temperature for $T \geq T_c$ and a logarithmic dependence on the gap parameter Δ for $T \ll T_c$. It is found that at small interlayer distance, i.e., for strong intelayer plasmon exchange, the cutoff frequency for plasmon exchange reaches its maximum value at the Fermi energy ε_F, which leads to the important conclusion that T_c is proportional to $\varepsilon_F e^{-1/\lambda_{\rm eff}}$.

I. INTRODUCTION

High-temperature superconductivity has been discovered in a number of families of compounds. Despite strenuous efforts directed in understanding the various experimental investigations, the microscopic mechanism for high-temperature superconductivity in these compounds remains obscure. It is well known from experimental facts that all of these compounds have a layered structure characterized by copper-oxygen layers in which the charge carriers are concentrated. It is also known that high-T_c superconductors exhibit a whole collection of unusual properties in both the superconducting and normal states, such as an anomalously weak isotope effect,[1,2] an increasing of T_c in these compounds with number of layers per unit cell,[3,4] linear temperature dependence of the resistivity in a wide temperature region in the normal state,[5] logarithmic dependence of the effective mass[6] on temperature, etc.

This collection of the unusual properties of high-T_c compounds led to the various theoretical models to explain the high T_c's observed. Among the several potential nonphonon mechanisms proposed to explain the dramatic increase in the superconductivity transition temperature is the plasmon mechanism.[7–9] Unfortunately, all of the known plasmon models suffer from a series of drawbacks. In particular, the simple average over the Fermi surface of the unscreened Coulomb interaction in a pure two-dimensional (2D) case,

$$\langle V(k) \rangle_{\rm FS} = \int_0^{2p_F} \frac{V(k)dk}{\sqrt{4p_F^2-k^2}} ,$$

leads to divergence at the lower limit. This divergence does not allow evaluation of the electron-plasmon interaction constant. It is not evident how to calculate the interelectron Coulomb repulsion parameter and how to determine the cutoff frequency in a layered two-dimensional system. These uncertainties are obstacles to the development of the theory of the high-T_c superconductors.

In this study we attempt to remove the drawbacks enumerated above. In a related recent paper[10] we have shown that in the framework of standard Fermi-liquid approach the electron-plasmon interaction in a layered 2D Fermi liquid can lead to results which are similar to the phenomenology of the normal state of a marginal Fermi-liquid model.[11] In this paper we first obtain the Eliashberg equations for a layered 2D system where the interaction between an electron and an arbitrary boson excitation is taken into account. On the basis of these equations, we show that the interelectron Coulomb repulsion parameter μ is connected with both the bare Coulomb interaction and the single-particle excitation. Our calculation shows that this parameter decays with

0163-1829/93/48(14)/10504(10)/$06.00

the decrease of interlayer distance. Next, we show that the attractive electron-electron interaction due to the plasmon exchange has a logarithmic divergence on frequency, i.e., $V(\omega)\sim\ln\omega$, which gives rise to a logarithmic divergence of the renormalization factor $Z(\omega)$. This in its turn leads to the logarithmic temperature dependence of the electron-plasmon interaction constant $\lambda_{\rm pl}$ for $T\geq T_c$ and the logarithmic dependence of the parameter $\lambda_{\rm pl}(\Delta)$ on Δ, the gap, for $T\ll T_c$. This result is similar to that obtained by using the phenomenology of the marginal Fermi-liquid theory.[12] In addition, in this paper, we have calculated the superconducting transition temperature due to the plasmon-mediated electron-electron interaction and have shown that for small interlayer distances, i.e., for strong interlayer plasmon exchange, the cutoff frequency for plasmon exchange reaches its maximum value $\approx\varepsilon_F$, which leads to the important result that T_c is proportional to ε_F.

II. ELIASHBERG EQUATIONS FOR A LAYERED 2D SYSTEM

It is known that in the superconducting state the normal and anomalous self-energy components associated with the electron-boson interaction are determined by the system of integral equations[13]

$$\begin{aligned}\Sigma_n(\omega_n,\mathbf{p})&=-T\sum_{\mathbf{k},\omega_{n'}}G(\omega_{n'},\mathbf{p}-\mathbf{k})D(\omega_n-\omega_{n'},\mathbf{k})\ ,\\ \Sigma_s(\omega_n,\mathbf{p})&=T\sum_{\mathbf{k},\omega_{n'}}F(\omega_{n'},\mathbf{p}-\mathbf{k})D(\omega_n-\omega_{n'},\mathbf{k})\ ,\end{aligned}\tag{1}$$

$$\begin{aligned}G(\omega_n,\mathbf{p})&=\frac{\omega_n+\xi_{\mathbf{p}}+\Sigma_n(-\omega_n,\mathbf{p})}{\Omega(\omega_n,\mathbf{p})}\ ,\\ F(\omega_n,\mathbf{p})&=\frac{\Sigma_s(\omega_n,\mathbf{p})}{\Omega(\omega_n,\mathbf{p})}\ ,\end{aligned}\tag{2}$$

and

$$\begin{aligned}\Omega(\omega_n,\mathbf{p})=&[\omega_n-\xi_{\mathbf{p}}-\Sigma_n(\omega_n,\mathbf{p})]\\ &\times[\omega_n+\xi_{\mathbf{p}}+\Sigma_n(-\omega_n,\mathbf{p})]-[\Sigma_s(\omega_n,\mathbf{p})]^2\ ,\end{aligned}\tag{3}$$

where G and F are the normal and anomalous electron Green's functions; Σ_n and Σ_s are the normal and anomalous self-energies, respectively; D is the boson Green's function; ω_n's are the discrete imaginary "frequencies"; and $\xi_{\mathbf{p}}$ $(=\varepsilon_{\mathbf{p}}-\varepsilon_F)$ is the energy measured relative to the Fermi energy with $\varepsilon_{\mathbf{p}}=\mathbf{p}^2/2m^*$ as the 2D single-particle energy. Using the spectral properties of the Green's functions $G(\omega_n,\mathbf{p})$, $F(\omega_n,\mathbf{p})$ and the self-energies $\Sigma_n(\omega_n,\mathbf{p})$ and $\Sigma_s(\omega_n,\mathbf{p})$, we can go from (1)–(3) to the integral equations

$$\begin{aligned}\Sigma_n(\mathbf{p},\omega)&=-2\sum_{\mathbf{k}}\int_{-\infty}^{\infty}\frac{d\omega'}{2\pi}\int_{-\infty}^{\infty}\frac{d\omega_1}{2\pi}\frac{\mathrm{Im}G(\mathbf{p}-\mathbf{k},\omega_1)\,\mathrm{Im}D(\mathbf{k},\omega')}{\omega'+\omega_1-\omega-i\delta}\left[\tanh\frac{\omega_1}{2T}+\coth\frac{\omega'}{2T}\right]\ ,\\ \Sigma_s(\mathbf{p},\omega)&=2\sum_{\mathbf{k}}\int_{-\infty}^{\infty}\frac{d\omega'}{2\pi}\int_{-\infty}^{\infty}\frac{d\omega_1}{2\pi}\frac{\mathrm{Im}F(\mathbf{p}-\mathbf{k},\omega_1)\,\mathrm{Im}D(\mathbf{k},\omega')}{\omega'+\omega_1-\omega-i\delta}\left[\tanh\frac{\omega_1}{2T}+\coth\frac{\omega'}{2T}\right]\ .\end{aligned}\tag{4}$$

These expressions apply to a system of any dimensions. Following Eliashberg,[13] we know that in the isotropic case the self-energies Σ_n and Σ_s are practically independent of p for $p\sim p_F$. So we can neglect the p dependence in Σ_n and Σ_s and consider that these functions depend only on ω. Let us now introduce two new functions, the renormalization parameter $Z(\omega)$ and the gap parameter $\Delta(\omega)$, to replace Σ_n and Σ_s:

$$\Sigma_n=\omega[1-Z(\omega)],\quad \Sigma_s=Z(\omega)\Delta(\omega)\ .\tag{5}$$

First, we consider the second equation in (5). Using (2)–(4), we can represent the expression for the gap, for a layered 2D system with cylindrical topology of the Fermi surface, in the form

$$\begin{aligned}Z(\omega)\Delta(\omega)=&\frac{4c}{(2\pi)^5}\int_{-\pi/c}^{\pi/c}dk_z\int\int_{-\infty}^{\infty}d\omega'\,d\omega_1\int k_{\parallel}dk_{\parallel}\int_0^{\pi}d\phi\,\mathrm{Im}\frac{Z(\omega_1)\Delta(\omega_1)\mathrm{sgn}\omega_1}{Z^2(\omega_1)\left[\omega_1^2-\Delta^2(\omega_1)\right]-\xi^2_{\mathbf{p}_{\parallel}-\mathbf{k}_{\parallel}}}\\ &\times\frac{\mathrm{Im}D(\mathbf{k},\omega')}{\omega'+\omega_1-\omega-i\delta}\left[\tanh\frac{\omega_1}{2T}+\coth\frac{\omega'}{2T}\right]\ ,\end{aligned}\tag{6}$$

where $\mathbf{k}_{\parallel}(k_x,k_y)$ and k_z are the momentum components in the plane and normal to the plane, respectively, and c is the interlayer distance. The angular integration over ϕ is determined by the residues of the poles of the denominator in (6) and can be carried out analytically. For a 2D electronic spectrum, the result is

$$\int_0^{\pi}d\phi\,\mathrm{Im}\frac{Z(\omega)\Delta(\omega)\mathrm{sgn}\omega}{Z^2(\omega)\left[\omega^2-\Delta^2(\omega)\right]-\xi^2_{\mathbf{p}_{\parallel}-\mathbf{k}_{\parallel}}}=-\frac{\pi\Delta(\omega)\mathrm{sgn}\omega}{2v_Fk_{\parallel}\sqrt{\omega^2-\Delta^2(\omega)}}[F_-(\omega)+F_+(\omega)]\ ,\tag{7}$$

where

$$F_-(\omega)=\left\{1-\frac{k_\parallel^2}{4p_F^2}\left[1-\frac{2m^*Y(\omega)}{k_\parallel^2}\right]^2\right\}^{-1/2},\quad 1>\frac{k_\parallel}{2p_F}\left|1-\frac{2m^*Y(\omega)}{k_\parallel^2}\right|,$$
$$F_+(\omega)=\left\{1-\frac{k_\parallel^2}{4p_F^2}\left[1+\frac{2m^*Y(\omega)}{k_\parallel^2}\right]^2\right\}^{-1/2},\quad 1>\frac{k_\parallel}{2p_F}\left|1+\frac{2m^*Y(\omega)}{k_\parallel^2}\right|, \tag{8}$$

with

$$Y(\omega)=Z(\omega)\sqrt{\omega^2-\Delta^2(\omega)}\,\mathrm{sgn}\omega\ .$$

It will be seen later that the frequency dependence of $F_\pm(\omega)$ leads to the convergence of integrals at the lower limit. Substitution of (7) and (8) in (6) and some straightforward manipulation give, for a layered metal,

$$Z(\omega)\Delta(\omega)=-\frac{c}{(2\pi)^4v_F}\int_{-\pi/c}^{\pi/c}dk_z\left\{\left[\int_{-\varepsilon_F}^{0}d\omega_1\int_{k_-(\omega_1)}^{k_+(\omega_1)}dk_\parallel+\int_0^\infty d\omega_1\int_{-k_-(\omega_1)}^{k_+(\omega_1)}dk_\parallel\right]F_-(\omega_1)\right.$$
$$\left.+\left[\int_{-\infty}^{0}d\omega_1\int_{-k_-(-\omega_1)}^{k_+(-\omega_1)}dk_\parallel+\int_0^{\varepsilon_F}d\omega_1\int_{k_-(-\omega_1)}^{k_+(-\omega_1)}dk_\parallel\right]F_+(\omega_1)\right\}$$
$$\times\frac{\Delta(\omega_1)\mathrm{sgn}\omega_1}{\sqrt{\omega_1^2-\Delta^2(\omega_1)}}\int_{-\infty}^{\infty}d\omega'\frac{\mathrm{Im}D(\mathbf{k},\omega')}{\omega'+\omega_1-\omega-i\delta}\left[\tanh\frac{\omega_1}{2T}+\coth\frac{\omega'}{2T}\right], \tag{9}$$

where

$$k_\pm=p_F[1\pm\sqrt{1+Y(\omega)/\varepsilon_F}\,]\ .$$

Since all quantities in (9) other than the Green's function $D(\mathbf{k},\omega)$ are independent of k_z, we can introduce an average over k_z on the boson Green's function as

$$\bar{D}(k_\parallel,\omega)=\frac{c}{2\pi}\int_{-\pi/c}^{\pi/c}D(k_\parallel,k_z,\omega)dk_z\ . \tag{10}$$

Using (10) in (9) and the condition $Y(\omega)<\varepsilon_F$, we finally can represent (9) in the form

$$Z(\omega)\Delta(\omega)=\int_0^\infty d\omega'\,\mathrm{Re}\left\{\frac{\Delta(\omega')}{\sqrt{\omega'^2-\Delta^2(\omega')}}\right\}$$
$$\times\int_0^\infty d\Omega\,S(\Omega,\omega')\left\{\left[\tanh\frac{\omega'}{2T}+\coth\frac{\Omega}{2T}\right]\left[\frac{1}{\omega'+\omega+\Omega+i\delta}+\frac{1}{\omega'-\omega+\Omega-i\delta}\right]\right.$$
$$\left.-\left[\tanh\frac{\omega'}{2T}-\coth\frac{\Omega}{2T}\right]\left[\frac{1}{\omega'-\omega-\Omega-i\delta}+\frac{1}{\omega'+\omega-\Omega+i\delta}\right]\right\}, \tag{11}$$

where

$$S(\Omega,\omega')=-\frac{1}{(2\pi)^3v_F}\int_{Y(\omega')/v_F}^{2p_F}[F_-(\omega')+F_+(\omega')]\,\mathrm{Im}\bar{D}(k,\Omega)dk\ . \tag{12}$$

Expression (12) represents the kernel of the electron-boson interaction in a layered 2D system. In (11) and (12) we made the change of variables $\omega_1=\omega'$, $\omega'=\Omega$, and $k_\parallel=k$. The equation for $\omega[1-Z(\omega)]$ given by the first equation of (5) can be obtained by using the same consideration as in the derivation of (11). The results can be written as

$$\omega[1-Z(\omega)]=\int_0^\infty d\omega'\,\mathrm{Re}\left\{\frac{\omega'}{\sqrt{\omega'^2-\Delta^2(\omega')}}\right\}$$
$$\times\int_0^\infty d\Omega\,S(\Omega,\omega')\left\{\left[\tanh\frac{\omega'}{2T}+\coth\frac{\Omega}{2T}\right]\left[\frac{1}{\omega'+\omega+\Omega+i\delta}-\frac{1}{\omega'-\omega+\Omega-i\delta}\right]\right.$$
$$\left.+\left[\tanh\frac{\omega'}{2T}-\coth\frac{\Omega}{2T}\right]\left[\frac{1}{\omega'-\omega-\Omega-i\delta}-\frac{1}{\omega'+\omega-\Omega+i\delta}\right]\right\}. \tag{13}$$

The system of equations (11)–(13) represents the Eliashberg equations for a layered 2D system and, but for the kernel of the electron-boson interaction (12), has the same form as the 3D case. Here we should make a comment regarding the upper limit of integration over ω' in (11) and (13). According to (11) and (13), all excitations will give contribution to the normal Σ_n and anomalous Σ_s self-energies, even those excitations the energy scale of which exceeds the Fermi energy. However, it follows from (9), which is an exact expression, that with the increase of frequency ω, the lower limit $k_-(\omega)$ of the $k_\parallel$ integration approaches $k_+(\omega)$, the upper limit. This happens when ω is comparable with the Fermi energy ε_F. So for the upper limit of integration over ω' in the Eliashberg equations (11) and (13), we can use a cutoff frequency ω_∞ which is comparable with Fermi energy ε_F. In the framework of the isotropic 3D Eliashberg theory, the cutoff energy parameter is not well defined. This fact leads to uncertainties in the calculation of the superconducting transition temperature. Note that the kernel of the electron-boson interaction (12) for a layered 2D system is different from the well-known expression for the kernel of the electron-phonon (electron-boson) interaction in the framework of the 3D Eliashberg theory[13] in that (12) depends on two frequencies. This feature has a principal significance for both the superconducting state and quasiparticle damping[10] in a layered 2D system.

III. INTERELECTRON COULOMB REPULSION PARAMETER

The Coulomb repulsion parameter μ was first introduced in the Eliashberg equation by Morel and Anderson.[14] Since then, this parameter has been calculated a number of times. Recently, some articles have appeared where it has been shown that this parameter could change sign,[15] leading to the possibility of an increase of the superconducting transition temperature. We show here that Coulomb repulsion parameter is connected with the bare Coulomb interaction as well as the single-particle excitation which is responsible for screening in the electron gas. Thus the Coulomb repulsion parameter, in principal, cannot change sign. Let us consider the second equation of (1) where there is a boson Green's function. We can represent boson Green's function in Eq. (1) in the form

$$D=\sum_i D_i=D_{\rm ph}+D_{\rm pl}+D_{\rm sp}+V(k)\ , \tag{14}$$

where $D_{\rm ph}$ is the phonon Green's function, $D_{\rm pl}$ is the plasmon Green's function, $D_{\rm sp}$ is the Green's function which is connected with single-particle excitation, and $V(k)$ is the unscreened (bare) Coulomb interaction, which for a layered 2D system can be written as

$$V(\mathbf{k})=V_0(k_\parallel)f(\mathbf{k})\ , \tag{15}$$

where

$$f(\mathbf{k})=\frac{\sinh ck_\parallel}{\cosh ck_\parallel-\cos ck_z} \tag{16}$$

and $V_0(k_\parallel)=2\pi e^2/k_\parallel\epsilon_i$ is the bare Coulomb interaction in a pure 2D case with ϵ_i as the appropriate lattice dielectric constant. It is easy to show that the contribution of the bare Coulomb interaction to the Σ_s, the anomalous self-energy, can be represented as

$$\Sigma_s^0(\mathbf{p})=\sum_{\mathbf{k}}\int_{-\infty}^{\infty}\frac{d\omega'}{2\pi}\,{\rm Im}F(\omega',\mathbf{p}-\mathbf{k})V(\mathbf{k})\tanh\frac{\omega'}{2T}\ . \tag{17}$$

Using the same transformation as in the derivation of (11), we can represent the contribution of the bare Coulomb interaction to the gap parameter as

$$Z_c^0\Delta_c^0=-\int_0^\infty d\omega'\,{\rm Re}\left\{\frac{\Delta(\omega')}{\sqrt{\omega'^2-\Delta^2(\omega')}}\right\}\mu_0(\omega')\tanh\frac{\omega'}{2T}\ , \tag{18}$$

where

$$\mu_0(\omega')=\frac{1}{(2\pi)^2v_F}\int_{Y(\omega')/v_F}^{2p_F}\bar{V}(\mathbf{k})[F_-(\omega')+F_+(\omega')]dk\ . \tag{19}$$

Here $\mu_0(\omega)$ is the bare Coulomb repulsion parameter. In (19), $\bar{V}(\mathbf{k})$ signifies the same average as (10). Using (15) and (16) in (10), it is easy to show that $\bar{V}(\mathbf{k})=V_0(k_\parallel)$. Here we should note that the frequency dependence of (19) causes $\mu_0(\omega')$ to have a finite value.

In order to evaluate the total Coulomb repulsion parameter as well as the attractive electron-plasmon interaction parameter in a layered 2D system, we next introduce the Green's functions for plasmon and single-particle excitations:

$$D(\mathbf{k},\omega)=V(\mathbf{k})\left[\frac{1}{\epsilon(\mathbf{k},\omega)}-1\right]\ . \tag{20}$$

Here $V(\mathbf{k})$ is the bare Coulomb interaction in a layered system defined by (15) and (16) and $\epsilon(\mathbf{k},\omega)$ is the dielectric function which can be represented in random-phase approximation (RPA) as

$$\epsilon(\mathbf{k},\omega)=1+V(\mathbf{k})\Pi(\mathbf{k}_\parallel,\omega)\ , \tag{21}$$

where $\Pi(\mathbf{k}_\parallel,\omega)$ is the polarization propagator for the pure 2D system. The various properties of the Green's function $D(\mathbf{k},\omega)$ have been discussed in some detail in a previous publication,[10] and here we quote several relevant results that will be utilized in this paper. The real part of the average Green's function appearing in (12) in the region of plasmon excitation has been shown to be[10]

$$\begin{aligned}{\rm Re}\bar{D}_{\rm pl}(k_\parallel,\omega)=-V_0(k_\parallel)+&\frac{V_0(k_\parallel)\omega^2}{\sqrt{(\omega^2-\omega_+^2)(\omega^2-\omega_-^2)}}\\&\times[\Theta(\omega-\omega_+)-\Theta(\omega_--\omega)]\ ,\end{aligned} \tag{22}$$

where $\Theta(x)$ is the step function, $\omega_+=\omega_p(k_\parallel)/(\tanh ck_\parallel/2)^{1/2}$ is the pure optical plasmon frequency, and $\omega_-=\omega_p(k_\parallel)(\tanh ck_\parallel/2)^{1/2}$ is the proper acoustic plasmon frequency.[16] Here $\omega_p(k_\parallel)$ is the frequency for a pure 2D plasmon mode with the approximate dispersion relation[10] $\omega_p{}^2(k_\parallel)\approx e^2 p_F v_F k_\parallel/\epsilon_i$. We consider this approximation only for the reason of simplifying further calculations. The imaginary part of the plasmon Green's function is expressed as[10]

$$\mathrm{Im}\bar{D}_{\mathrm{pl}}(k_\parallel,\omega)=-\frac{V_0(k_\parallel)\omega^2}{\sqrt{(\omega_+^2-\omega^2)(\omega^2-\omega_-^2)}}\ ,\qquad \omega_-\le\omega\le\omega_+\ . \tag{23}$$

The region of the plasmon excitation for the layered system is shown in Fig. 1 where the optical (ω_+) and acoustic (ω_-) limits are shown by solid curves (1) and (2), respectively. The dashed line $[\omega_*=\omega_p(k_\parallel)(\tanh ck_\parallel/2)^{1/2}]$ in Fig. 1 represents the curve which distinguishes between the optical and acoustic natures of the plasmon band. Here we should also note that, as will be shown below, the existence of the plasmon band is the principal reason for the high temperature of superconductivity in a layered 2D system.

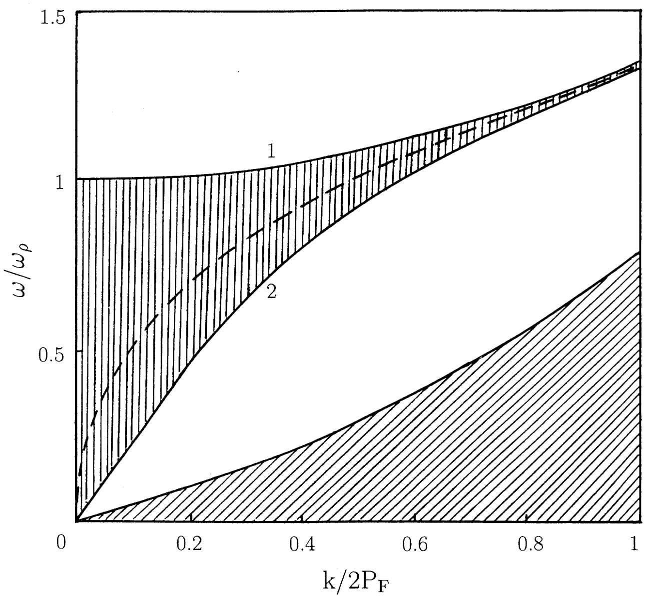

FIG. 1. Representation of the pseudo-optical plasmon band contributing to the effective electron-electron interaction (vertically hatched region) in a layered 2D electron gas. The upper solid curve (1) is the pure optical limit, and the lower solid curve (2) represents the pure acoustic limit. The region of the low-frequency single-particle excitations is shown by slanted hatching. The dashed curve represents the frequency above (below) which the plasmon has an optical (acoustic) nature.

Now we consider the Green's function for the single-particle excitation for a layered 2D system. The imaginary part of this Green's function has been shown to be[10]

$$\mathrm{Im}\bar{D}_{\mathrm{sp}}(k_\parallel,\omega)\cong -V_0^2(k_\parallel)\,\mathrm{Im}\Pi(k_\parallel,\omega)\sinh^2 ck_\parallel\frac{B}{(B^2-1)^{3/2}}\ , \tag{24}$$

where

$$B=\cosh ck_\parallel[1+V_0(k_\parallel)\Pi(k_\parallel,0)\tanh ck_\parallel]\ . \tag{25}$$

The real part of the Green's function for the single-particle excitation has the form

$$\mathrm{Re}\bar{D}_{\mathrm{sp}}(k_\parallel)=-V_0(k_\parallel)+V_{\mathrm{scr}}(k_\parallel)\ , \tag{26}$$

where $V_{\mathrm{scr}}(k_\parallel)$ is the screened Coulomb repulsion for a layered 2D metal:

$$V_{\mathrm{scr}}(k_\parallel)=\frac{V_0(k_\parallel)}{\sqrt{1+V_0^2(k_\parallel)\Pi^2(k_\parallel,0)+2V_0(k_\parallel)\Pi(k_\parallel,0)\coth ck_\parallel}}\ . \tag{27}$$

Now we turn to the calculation of the contribution of the single-particle excitations to the anomalous self-energy Σ_s or $Z(\omega)\Delta(\omega)$ for a layered 2D system. Since the single-particle contribution to the anomalous self-energy has a weak frequency dependence, we set $\omega=0$ in (11). This enables us to represent the single-particle contribution to the equation for gap as

$$Z_{\mathrm{sp}}\Delta_{\mathrm{sp}}=4\int_0^\infty d\omega'\,\mathrm{Re}\left\{\frac{\Delta(\omega')}{\sqrt{\omega'^2-\Delta^2(\omega')}}\right\}\tanh\frac{\omega'}{2T}\int_0^\infty\frac{\Omega}{\Omega^2-\omega'^2}S_{\mathrm{sp}}(\Omega,\omega')d\Omega\ . \tag{28}$$

Using the fact that $S_{\mathrm{sp}}\sim -\mathrm{Im}D_{\mathrm{sp}}$ and using the Kramers-Kronig relation for single-particle excitation Green's function, we finally can represent total Coulomb part of the equation for the gap in the form

$$Z_c\Delta_c=Z_c^0\Delta_c^0+Z_{\mathrm{sp}}\Delta_{\mathrm{sp}}=-\int_0^\infty d\omega'\,\mathrm{Re}\left\{\frac{\Delta(\omega')}{\sqrt{\omega'^2-\Delta^2(\omega')}}\right\}\mu(\omega')\tanh\frac{\omega'}{2T}\ , \tag{29}$$

where

$$\mu(\omega)=\frac{1}{(2\pi)^2 v_F}\int_{Y(\omega)/v_F}^{2p_F}V_{\mathrm{scr}}(k)[F_-(\omega,k)+F_+(\omega,k)]dk\ . \tag{30}$$

For example, using (30), we can calculate μ for two interesting cases: a pure 2D case and a layered 2D system. Substituting (27) in (30) and assuming $\omega=0$, we obtain, for a pure 2D case ($2p_F c\gg 1$),

$$\mu=\begin{cases}\dfrac{\alpha}{\pi\sqrt{1-\alpha^2}}\ln\dfrac{\sqrt{1+\alpha}+\sqrt{1-\alpha}}{\sqrt{1+\alpha}-\sqrt{1-\alpha}}, & \alpha<1,\\ \dfrac{2\alpha}{\pi\sqrt{\alpha^2-1}}\tan^{-1}\sqrt{(\alpha-1)/(\alpha+1)}, & \alpha>1\,,\end{cases} \tag{31}$$

where $\alpha=e^2/\epsilon_i v_F$ is the electron-electron interaction constant.

It follows from this expression that, in the high-density limit ($\alpha \ll 1$),

$$\mu \cong \frac{\alpha}{\pi}\ln\frac{2}{\alpha}$$

and that it assumes the maximum value of $\mu=\frac{1}{2}$ at the low-density limit ($\alpha \gg 1$).

In the case of a layered 2D system, we have ($2p_F c \ll 1$)

$$\mu=\frac{\alpha}{\pi\sqrt{1+\alpha^2(1+1/2\gamma)}}K[1/\sqrt{1+\alpha^2(1+1/2\gamma)}]\,, \tag{32}$$

where $K(x)$ is the complete elliptic integral of the first kind and $\gamma=c/2a_B$ with $a_B=\epsilon_i/e^2m^*$ as the effective Bohr radius. From (32) it follows that the decrease of the interlayer distance c decreases the interelectron Coulomb repulsion parameter μ, which assumes the limiting value for a small interlayer distance ($c \ll 2a_B$):

$$\mu \cong \tfrac{1}{2}\sqrt{c/a_B}\ .$$

Perhaps this is why all of the high-T_c superconductors have layered structures, where even for the phonon mechanism of superconductivity, we would have an increase of T_c due to decrease of the Coulomb repulsion parameter.

IV. SUPERCONDUCTIVITY IN A LAYERED 2D SYSTEM

Now we consider the plasmon mechanism for superconductivity. First, we consider the kernel of electron-plasmon interaction for a layered 2D system $S_{\rm pl}(\Omega,\omega')$. Using (23) in (12), we have

$$S_{\rm pl}(\Omega,\omega')=\frac{\alpha\Omega^2}{4\pi^2\omega_c^2}\int_{Y(\omega')/\omega_c}^{x_c}[\Phi_-(x,\omega')+\Phi_+(x,\omega')]\times\psi\left[x,\frac{\Omega}{\omega_c}\right]dx\,, \tag{33}$$

where

$$\Phi_\pm(x,\omega')=\left[x^2-\left[\frac{\alpha}{2}x^2\pm\frac{Y(\omega')}{\omega_c}\right]^2\right]^{-1/2} \tag{34}$$

and

$$\Psi(x,y)=\left[\left[\frac{x}{\tanh\gamma x}-y^2\right](y^2-x\tanh\gamma x)\right]^{-1/2}. \tag{35}$$

Here $x_c=k_c a_B>1$, $\gamma=c/2a_B$, and $\omega_c=e^2p_F/\epsilon_i$. The parameter γ in (35) characterizes the strength of interlayer plasmon coupling. For instance, at large interlayer distance ($\gamma \gg 1$), ω_+ approaches ω_- and we have the case of the weak interlayer plasmon coupling. In this case the function $\psi(x,y)$ takes the form[10]

$$\psi(x,y)\cong\pi\delta(y^2-x)$$

and expression (33) for the kernel of electron-plasmon interaction in the case of the high-density limit ($\alpha \ll 1$; i.e., we assume that the strength of the electron-electron interaction is small, and henceforth we will consider only this case) takes the form

$$S_{\rm pl}(\Omega,\omega')\cong\frac{\alpha\Omega^2}{2\pi}\frac{1}{\sqrt{\Omega^4-\omega_c^2Y^2(\omega')}}\,,\quad \sqrt{\omega_c|Y(\omega')|}\le\Omega\le\omega_{\max}\,, \tag{36}$$

where $\omega_{\max}$ is the maximum plasmon excitation energy in a pure 2D case and is given by $\omega_{\max}=\omega_p(k_c)$ for $k_c<2p_F$. In the opposite limit when the interlayer distance c is smaller than $2a_B$ (i.e., $\gamma \ll 1$), integration over x in (33) gives

$$S_{\rm pl}(\Omega,\omega')\cong\frac{\alpha\Omega}{2\pi^2}\frac{1}{\sqrt{\omega_{\rm pl}^2-\Omega^2}}\ln\left[\frac{4\Omega}{\sqrt{\gamma}|Y(\omega')|}\right],\quad \sqrt{\gamma}|Y(\omega')|\le\Omega\le\omega_{\rm pl}\,, \tag{37}$$

where $\omega_{\rm pl}=(4\pi e^2n_s/\epsilon_i m^*c)^{1/2}$ is the bulk plasmon frequency for a layered 2D system. It follows from (36) and (37) that the main contribution to the electron-plasmon interaction constant will be defined by a small ω'.

Now we can obtain the expression for the renormalization factor $Z(\omega)$ from (13). It can be shown after straightforward manipulations of (13) that the expression for $Z(\omega)$ can be represented (for $T>0$) in the form

$$Z(\omega)=1+\frac{\pi}{\omega}\int_0^\infty d\omega'\,{\rm Re}\left\{\frac{\omega'}{\sqrt{\omega'^2-\Delta^2(\omega')}}\right\}\times S'(\omega',\omega')\left[\tanh\frac{\omega+\omega'}{2T}+\tanh\frac{\omega-\omega'}{2T}\right], \tag{38}$$

where $S'(\omega,\omega')$ is connected with $S(\omega,\omega')$ by the Kramers-Kronig relation

$$S'(\omega,\omega')=\frac{2}{\pi}\int_0^\infty\frac{\Omega d\Omega}{\Omega^2-\omega^2}S(\Omega,\omega')\ .$$

Neglecting the frequency ω dependence in this expression, i.e., using $S'(\omega,\omega')|_{\omega\to0}$ and introducing

$$\bar S(\omega')=4\int_0^\infty\frac{d\Omega}{\Omega}S(\Omega,\omega')\,, \tag{39}$$

we can rewrite the expression for the renormalization parameter $Z(\omega)$ in the form

$$Z(\omega)\cong1+\frac{1}{2\omega}\int_0^\infty d\omega'\,\bar S(\omega')\left[\tanh\frac{\omega+\omega'}{2T}+\tanh\frac{\omega-\omega'}{2T}\right]. \tag{40}$$

In (40) we assume $\Delta=0$; i.e., we consider the case of $T \geq T_c$. The case of $T=0$ will be considered later in this paper.

Using (36) and (37) in (39), we have, within logarithmic accuracy,

$$\bar{S}(\omega) \cong \frac{\alpha}{\pi} \ln \frac{C\Omega_0}{\omega} , \tag{41}$$

where

$$\Omega_0 \cong \begin{cases} \omega_{\max}, & c \gg 2a_B , \\ \varepsilon_F, & c \ll 2a_B , \end{cases}$$

$$C \propto \begin{cases} \dfrac{\omega_{\max}}{\omega_c}, & c \gg 2a_B , \\ \dfrac{\alpha}{\gamma}, & c \ll 2a_B . \end{cases}$$

Inserting this expression for $\bar{S}(\omega)$ in (40), the renormalization parameter at the finite temperature can be represented as

$$\begin{aligned} Z_{\rm pl}(\omega) &= 1+\lambda_{\rm pl}(\omega) \cong 1+\frac{\alpha}{\pi} \ln \frac{C\Omega_0}{\omega}, \quad \omega \gg T , \\ Z_{\rm pl}(T) &= 1+\lambda_{\rm pl}(T) \cong 1+\frac{\alpha}{\pi} \ln \frac{2\gamma_E C\Omega_0}{\pi T}, \quad \omega \gg T . \end{aligned} \tag{42}$$

Here $\lambda_{\rm pl}$ is the electron-plasmon interaction constant and γ_E is the Euler constant. It follows from (42) that the electron-plasmon interaction constant has a logarithmic divergence as $\omega \to 0$ and $T \to 0$, which is similar to the behavior in the marginal Fermi-liquid model,[11] even though it has been obtained in the framework of standard Fermi-liquid approach without any assumption about the polarizability of the electron gas.[11,12] It will be seen later that the logarithmic divergence in Eq. (42), which is connected with the electron-plasmon interaction in a 2D system, is not an obstacle for superconductivity.

Before going into the calculation of the critical temperature for superconducting transition, let us make a digression and show that the quasiparticle damping calculated in Ref. 10 can also be obtained from the present formulation. Using (13), the expression for damping can be represented in the form

$$\begin{aligned} \Gamma(\omega) &= \omega \, {\rm Im} Z(\omega) \\ &= 2\pi \int_0^\infty d\Omega \, {\rm Re} \left\{ \frac{\Omega}{\sqrt{\Omega^2-\Delta^2(\Omega)}} \right\} S(\Omega,\Omega) \\ &\quad \times \left\{ \coth \frac{\Omega}{2T} - \frac{1}{2} \tanh \frac{\Omega+\omega}{2T} - \frac{1}{2} \tanh \frac{\Omega-\omega}{2T} \right\} . \end{aligned} \tag{43}$$

Substituting the expressions for $S(\Omega,\omega')$ [see (12)] for any electron-boson interaction in (43), we can obtain expressions for damping in a layered 2D metal [we will assume $\Delta=0$ in (43)]. For instance, using expression (24) for the Green's function which describes the single-particle excitation in a layered 2D system, in (12) and then in (43) we can obtain the well-known result for quasiparticle damping in a pure 2D case,[17]

$$\Gamma(\omega) \cong \frac{\varepsilon_F}{8\pi} \left[\frac{\omega}{\varepsilon_F} \right]^2 \ln \frac{\varepsilon_F}{\omega} \quad \text{at } \omega \gg T ,$$

$$\Gamma(T) \cong \frac{\pi}{4} \varepsilon_F \left[\frac{T}{\varepsilon_F} \right]^2 \ln \frac{\varepsilon_F}{T} \quad \text{at } \omega \ll T \quad (c \gg 2a_B) ,$$

and for a layered 2D system,[10]

$$\Gamma(\omega) \cong \frac{\alpha}{4\sqrt{2}} \omega_{\rm pl} \left[\frac{\omega}{\omega_{\rm pl}} \right]^2 \ln \frac{\varepsilon_F}{\omega} \quad \text{at } \omega \gg T ,$$

$$\Gamma(T) \cong \frac{\pi\alpha}{2\sqrt{2}} \omega_{\rm pl} \left[\frac{T}{\omega_{\rm pl}} \right]^2 \ln \frac{\varepsilon_F}{T} \quad \text{at } \omega \ll T \quad (c \ll 2a_B) .$$

Substituting (36) and (37) in (43), we obtain expressions for electron-plasmon damping in a layered 2D system ($c \gg 2a_B$),[10]

$$\Gamma(\omega) \cong \alpha \, {\rm Re} \sqrt{\omega^2-\omega_c^2} \quad \text{at } \omega \gg T ,$$

$$\Gamma(T) \cong 2\alpha T \ln \frac{2T}{\omega_c} \quad \text{at } \omega < \omega_c \ll T ,$$

and

$$\Gamma(T) \propto \alpha \omega_c \exp \left[-\frac{\omega_c}{T} \right] \quad \text{at } \omega \ll T < \omega_c .$$

Here $\omega_c = e^2 p_F/\epsilon_i$ is the finite-energy threshold for electron-plasmon damping and, in the case of strong interlayer plasmon exchange ($c \ll 2a_B$),[10]

$$\Gamma(\omega) \cong \frac{\alpha}{4\pi} \ln \left[\frac{32a_B}{c} \right] \omega_{\rm pl} \left[\frac{\omega}{\omega_{\rm pl}} \right]^2 \quad \text{at } \omega \gg T ,$$

$$\Gamma(T) \cong \frac{\pi\alpha}{2} \ln \left[\frac{32a_B}{c} \right] \omega_{\rm pl} \left[\frac{T}{\omega_{\rm pl}} \right]^2 \quad \text{at } \omega \ll T ,$$

and for the intermediate case of the interlayer plasmon exchange ($c \cong 2a_B$),[10]

$$\Gamma(\omega) \cong \frac{\alpha}{2} [\omega_c - {\rm Re}\sqrt{\omega_c^2-\omega^2}] + \alpha \, {\rm Re}\sqrt{\omega^2-\omega_c^2} \quad \text{at } \omega \gg T ,$$

$$\Gamma(T) \cong \frac{\pi^2\alpha}{2} \omega_c \left[\frac{T}{\omega_c} \right]^2 \quad \text{at } T < \omega_c ,$$

and

$$\Gamma(T) \cong \frac{\pi\alpha}{2} T + 2\alpha T \ln \frac{2T}{\omega_c} \quad \text{at } T > \omega_c .$$

As seen from these expressions for the "electron-plasmon damping," the inelastic processes involving excitation of the plasmon can lead to the linear frequency and temperature dependences of the quasiparticle damping.

Now we turn to the evaluation of the superconducting transition temperature due to plasmon excitation. Let us consider the gap equation (11) in conjunction with (29), the contribution of the Coulomb repulsion. Following Leavens and Carbotte,[18] we use a two-step model for the gap,

$$\Delta=\begin{cases}\Delta_0, & 0<\omega<\omega_0 ,\\ \Delta_\infty, & \omega_0<\omega<\omega_\infty ,\end{cases}$$

where ω_∞ is the cutoff frequency of the Coulomb repulsion, which according to (9) and (29) is of order $\sim\varepsilon_F$, and ω_0 is the cutoff frequency of the electron-boson interaction. Taking into account the expression for the electron-plasmon interaction constant (42) and requiring that $\lambda_{\rm pl}(\omega)>0$, we can define the cutoff frequency of the electron-plasmon interaction as $\omega_0\approx C\Omega_0$ (where the constant C will be ~1). Using the same approach as Ref. 15 and after some manipulations, we can write T_c in the form

$$T_c\cong\frac{2\gamma_E}{\pi}\omega_0\exp\left[-\frac{1}{\lambda_{\rm eff}}\right], \tag{44}$$

where

$$\frac{1}{\lambda_{\rm eff}}\cong-(A-\tilde{\mu}-1)+\sqrt{(A-\tilde{\mu}-1)^2+2\pi/\alpha+2A+a^2-b}\ , \tag{45}$$

with

$$A=\ln\frac{C\Omega_0}{\omega_0},\quad \tilde{\mu}=\frac{\pi}{\alpha}\mu^*\left[1+\frac{\alpha}{\pi}\delta\right],$$

$$a=\ln\frac{\pi}{4\gamma_E},\quad b=\int_0^\infty\frac{\ln^2x}{\cosh^2x}dx\ ,$$

$\delta\sim1$ is some constant, and

$$\mu^*=\mu\Big/\left[1+\mu\ln\frac{\omega_\infty}{\omega_0}\right]$$

is the effective Coulomb repulsion parameter. Using the condition $\lambda_{\rm pl}(\omega)>0$ [see (42)], we can assume $\omega_0\sim\Omega_0$; since $C\sim1$, we can replace in the expression for T_c [Eq. (44)] the value ω_0 by Ω_0 and x_0 by

$$\frac{1}{\lambda_{\rm eff}}=1+\frac{\pi}{\alpha}\mu^*+\left[\left[1+\frac{\pi}{\alpha}\mu^*\right]^2+\frac{2\pi}{\alpha}+a^2-b\right]^{1/2}.$$

Thus we can see that the expression for the transition temperature [Eq. (44)] along with (45) differs from well-known McMillan expression[15] and this is due to the fact that in our model the kernel of electron-plasmon interaction for layered 2D system has a logarithmic divergence in frequency. In addition, we can see that for a layered 2D metal with strong interlayer plasmon exchange (i.e., $c\ll2a_B$), we have $\omega_0\sim\Omega_0\sim\omega_\infty\sim\varepsilon_F$, $\mu^*\sim\mu$, and for $\alpha\ll1$,

$$T_c\cong1.13\varepsilon_F\exp\left[-\left\{1+\frac{\pi}{\alpha}\mu+\left[\left[1+\frac{\pi}{\alpha}\mu\right]^2+\frac{2\pi}{\alpha}\right]^{1/2}\right\}\right]. \tag{46}$$

This implies that for a layered 2D metal with strong interlayer plasmon exchange ($c\ll2a_B$), T_c could be proportional to ε_F. Thus, as seen from (44)–(46), the logarithmic divergence of the renormalization parameter $Z(\omega)\to\infty$ as $\omega\to0$ due to the electron-plasmon interaction in a layered 2D system is not an obstacle for a transition to superconductivity at $T=0$. Nevertheless, according to (42), the electron-plasmon interaction constant is not defined. Now we show that at $T=0$ the renormalization parameter $Z(\omega)$, and hence the electron-plasmon interaction constant, is defined in terms of the superconducting gap. With this in mind, let us now consider the low-temperature ($T=0$) case. The expression for the renormalization parameter $Z(\omega)$ in this case can be written as

$$Z_{\rm pl}(0)=1+4\int_\Delta^\infty\frac{\omega'd\omega'}{\sqrt{\omega'^2-\Delta^2}}\int_0^\infty\frac{S_{\rm pl}(\Omega,\omega')}{(\omega'+\Omega)^2}d\Omega\ .$$

Substituting (36) and (37) in this equation and carrying out the integration, it can be shown that

$$Z_{\rm pl}(0)=1+\lambda_{\rm pl}(0)\cong1+\frac{\alpha}{\pi}\ln\frac{C\Omega_0}{\Delta}\ , \tag{47}$$

where C and Ω_0 are the same as in (41). Thus the renormalization parameter in our model is finite at $T=0$ and is defined in terms of the superconducting gap. This result, which is obtained in the framework of the standard Fermi-liquid approach, is similar to the result obtained by Littlewood and Varma[12] on the phenomenology of the superconducting state in the framework of the marginal Fermi-liquid hypothesis. Here we should note that the result in (47) has been derived without consideration of the renormalization plasmon Green's function due to the development of the superconducting state. It is understood that the energy scale of the variation of the plasmon Green's function is much greater than the energy gap Δ. Therefore (47) gives the electron-plasmon interaction constant for $T=0$ with high precision. We should note that our model, in fact, is closely related to the phenomenology of the superconducting state of a marginal Fermi liquid.[12] Use of the plasmon Green's function with the appropriate kernel of the electron-plasmon interaction of the form (41) corresponds to the "base-line" model A and whereas the incorporation of the correlations of the form given in Eq. (8b) of Littlewood and Varma would lead to the model B of their paper. Thus our model based on the electron-plasmon coupling in a layered 2D Fermi liquid can be considered to provide a microscopic basis of the marginal Fermi-liquid hypothesis. In this paper we did not consider in detail the development of the superconducting state, i.e., the case when $0<T<T_c$, and also the low-energy collective modes which may exist in layered superconductors[19] and left it for future study. Finally, we would like to comment on two facts that follow from (47). First, the fact that $\Delta=v_F/\pi\xi_0$ (where ξ_0 is the coherence length) indicates that the main contribution to the Cooper pairing comes from the plasmon with $k\sim\xi_0^{-1}$. Second, (47) gives a measure of the strength of the superconducting transition due to a plasmon mechanism. For example, at $T=0$ it is impossible to destroy superconductivity by applying

a magnetic field since a decrease of Δ will increase $\lambda_{\rm pl}(\Delta)$ [see (47)], which in its turn leads to enhancement of Δ.

V. DISCUSSION

In this paper we have developed a general approach to the pairing theory of superconductivity in a layered 2D metal system. First, we obtained the Eliashberg equations for a layered 2D metal. Second, we showed that the interelectronic Coulomb repulsion parameter μ, which weakens superconductivity, is connected with both the bare Coulomb interaction and the single-particle excitation which defines the screening process in the electron liquid. We have calculated this parameter for a layered 2D metal and have shown that while it remains positive, it decays with the decrease of interlayer distance. This result, in our opinion, has been experimentally confirmed indirectly in La-Sr-Cu-O compounds where an increase of T_c has been observed with an increase of pressure,[20] i.e., a decrease in interlayer distance. The second very important result is that only those excitations with energy scale comparable or less then the Fermi energy contribute to superconductivity; i.e., the cutoff energy for superconductivity should be on the order of the Fermi energy. We have also proved that the wide band of plasmon excitations in a layered 2D metal system provides the attractive electron-electron interaction which can lead to high-T_c superconductivity. If the interlayer distance between metallic sheets, c, is comparable or less than $2a_B$ (Bohr radius), there is strong interlayer plasmon coupling, leading to the existence of the wide plasmon excitation band, and in this situation the bulk plasmon frequency exceeds the Fermi energy. This implies that in this situation one has to use the Fermi energy as a cutoff frequency, and not the bulk plasmon frequency. This important result also has experimental confirmation. In most of the high-T_c superconductors where the bulk plasmon frequency $\omega_{\rm pl}$ exceeds the Fermi energy ε_F,[21] T_c increases linearly with doping (charge carrier density n_s), i.e., with the Fermi energy since it is proportional to n_s. Then one might ask a legitimate question: Why does the plasmon not contribute to superconductivity in an isotropic three-dimensional system? There are essential differences between an isotropic 3D system and a layered 2D system. First, the plasmon frequency in the 3D case has a weak dispersion on k leading to the narrow peak in the strength of the electron-plasmon interaction. Second, the plasmon frequency for most low-temperature superconductors is greater than the Fermi energy, and third, there is very small-k region where plasmon is a well-defined excitation for the 3D case. Our estimation shows that the plasmon contribution to superconductivity in the 3D case is negligibly small. In contrast, in a layered 2D system the plasmon has a wide excitation band starting at $k=0$, which leads to the fact that all the electrons within the Fermi surface feel an attractive electron-electron interaction due to plasmon exchange. We have also shown that a layered 2D Fermi liquid with an electron-plasmon interaction has a behavior similar to that of the marginal Fermi liquid: The renormalization factor $Z(\omega)$ for the electron-plasmon interaction has a logarithmic divergence on frequency that leads to a logarithmic divergence of the electron-plasmon interaction parameter and of the effective mass on both frequency and temperature, whereas in an ordinary Fermi-liquid model it remains finite as $T\rightarrow 0$ and is defined by the superconducting gap Δ. Also, it has been shown in this paper and before[10] that as in the marginal Fermi-liquid case the quasiparticle damping $\Gamma(\omega)$ due to an inelastic electron-plasmon interaction has a linear dependence in both frequency and temperature beyond a finite threshold energy. Thus, in contrast to the hypothetical marginal Fermi-liquid model, we have used the standard Fermi-liquid approach without any assumptions about the polarizability of electron gas and have shown that the electron-plasmon interaction in a layered 2D system can lead to results for the normal and superconducting states which are similar to those of the marginal Fermi-liquid hypothesis.

ACKNOWLEDGMENTS

The work of Y.M.M. and J.D.F. has been supported in part by the U.S. Air Force Office of Scientific Research under Grant No. AFOSR-91-0337 DEF. The work of S.M.B. and P.L. has been supported in part by NATO Research Grant No. CRG 91096. The work of P.L. has also been supported by the Fonds National de la Recherche Scientifique, Belgium; and also partially supported by the Joint Project 92/159 of the Ministry of Research of the French Community of Belgium.

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