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# Disorder effects in transport properties of weak itinerant-electron ferromagnets

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The electric and thermal conductivity of weak itinerant-electron ferromagnets in the presence of spin fluctuations and disorder is calculated and the validity of the Wiedemann-Franz law in these systems is discussed. We used the Keldysh-diagram technique to obtain the expression for the conductivities of the clean system as well as the correction due to disorder. The electric conductivity shows a maximum at a low temperature, which characterizes the crossover behavior from the disorder-dominated region to the spin-fluctuation region. The thermal conductivity ($\kappa/T$) also has a maximum at the crossover temperature. The contribution due to spin fluctuations and the quantum correction due to disorder violate the Wiedemann-Franz law while the Drude term restores it as disorder increases. The experimental results for conductivities in disordered weak itinerant ferromagnets in a very low-temperature range are not available. However, we find that our results agree well with the resistivity of amorphous $Ni_{75}P_{16}B_6Al_3$, which considered to be a nearly ferromagnetic metal.

## I. INTRODUCTION

Weak itinerant-electron ferromagnets such as $Ni_3Ga$, $Ni_3Al$, $(NiPd)_3Al$, $ZrZn_2$, $Sc_3In$, $Ti(BeCu)_2$, etc., have been known for decades.[1-6] These materials have a number of distinctive properties in common. They undergo a transition from highly paramagnetic to weakly ferromagnetic metals as the composition of the system is varied or by doping the system with impurities near to their stoichiometric composition. For example, $Ni_{0.75+x}Ga_{0.25-x}$ is highly paramagnetic for $x<0.006$ and is a weak ferromagnet for $x>0.006$.[1] These alloys have a low transition temperature $T_c<50$ K and a low saturation moment compared with conventional ferromagnetic materials, such as Fe, Co, and Ni. They have a large specific-heat coefficient and their low-temperature resistivity and magnetization show a quadratic temperature dependence with a relatively large coefficient. The transition from highly paramagnetic to weakly ferromagnetic phase can be expressed in terms of Stoner parameter $\alpha_0 \equiv (1-U\rho_d)$ changing from positive value to negative value.[7] Here $\rho_d$ is the $d$-electron density of state at Fermi level $\epsilon_d$ and $U$ is the on-site Coulomb interaction. When the composition or concentration of impurities is varied in these materials, the density of states at the Fermi level $\rho$ changes and the value of $\alpha_0$ determines the ground state of these materials at zero temperature. The discussion in this paper applies to systems with $|\alpha_0| \ll 1$. For the pure case, the temperature variation of various physical properties of these materials has been explained qualitatively and in some cases quantitatively using the spin-fluctuation theory. The reason for the success for such a theory is that because of low saturation moment and low $T_c$, these systems can always be considered to be in the vicinity of magnetic phase transition. The amplitude of transverse and longitudinal spin fluctuations with small momentum $q$ and frequency $\omega$ is enhanced due to the Stoner enhancement factor $1/\alpha_0$. The temperature variation of various quantities is then expected to be governed by these fluctuations. The dynamics of these fluctuations is, therefore important to help explain the various observables. A brief description of this theory is given in a recent review by one of us [8] and for more details we refer the readers to an earlier monograph by Moriya.[9]

However, for impure systems or systems away from the stoichiometric composition a clear description of various physical properties of these alloys is lacking. For example, the spin-fluctuation theory predicts a $T^2$ dependence for spin susceptibility at the low temperatures in the paramagnetic phase of these materials. This has been observed in many systems. But in some cases, for example, in $Ni_3Ga$, $Ni_3Al$,[1,10] and in $(NiPd)_3Al$,[2] the spin susceptibility at low temperatures deviates from this $T^2$ behavior. In these alloys, it is observed that the deviation is systematic and it reduces as the Ni concentration of the system approaches towards the stoichiometric composition (i.e., 75% of Ni). It is also observed that the residual resistivity of these alloys has the lowest value at the stoichiometric composition and it increases as the composition moves away from the stoichiometric one.[2,11] Residual resistivity is a measure of the disorder present in the system. These observations suggest that the deviation seen in the spin susceptibility is due to the disorder and there is, therefore, a need to modify the spin-fluctuation theory to take into account the effect of disorder. This has become possible recently because of a substantial modification in our understanding of disordered metals during the past decade.[12] It has been observed that the quantum nature of impurity scattering has profound consequences on the low-temperature electronic and magnetic properties of these metals. The characteristic parameter which describes this quantum nature of impurity scattering is $1/\epsilon_F\tau$, where $\tau$ is the lifetime of electrons in the presence of impurities. A perturbative expansion in $1/\epsilon_F\tau$ has been realized to consider the effects of low-lying diffusive modes (diffusions) and particle-particle backscattering (cooperons) on transport and various oth-

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er properties. Motivated by these observations, in our earlier work[13] we have calculated the effect of disorder on various magnetic properties such as spin susceptibility, magnetization, Arrott plots, etc., of these alloys in the presence of spin fluctuations. We found that the spin susceptibility vary as $\sqrt{T}$ for temperatures $T \ll \tau_d^{-1}$, where $\tau_d$ is the average collision time of $d$ electrons due to impurities. This explains the deviation seen in the spin susceptibility of these alloys at low temperatures.[14] The agreement with the experimental results of $Ni_3Ga$, $Ni_3Al$, and $(NiPd)_3Al$ is good. So it is reasonable to ask how the transport properties also are modified by disorder in the low-temperature region. Keeping this in mind we calculate electric and thermal conductivity in the presence of both disorder and spin fluctuations. For simplicity, we consider only the effect of diffusions on the transport properties. It is possible to calculate the effect of cooperons also, that will, however, change various temperature coefficients but will not introduce a new temperature variation.

The magnetism in these alloys is basically due to the presence of the unfilled $d$ shells of Ni, Sc, Zr, Ti, etc. But these $d$ shells do not contribute to the transport because the mobility of these electrons are small compared to that of conducting $s$ electrons. Mills and Lederer[15] have introduced a simple model consisting of $s$ and $d$-band electrons and the conduction electrons are scattered by the spin fluctuation of the $d$ electrons via the $s$-$d$ exchange interaction. In weak itinerant ferromagnets it has been observed that the resistivity reduces with the application of magnetic field.[16] This is due to the fact that the magnetic field suppresses the spin fluctuations of the $d$ electrons. This negative magnetoresistance seen in these systems confirms that the main contribution for the resistivity is due to the spin fluctuations besides the impurities. As a model for the impurities, we take a random potential $v(\mathbf{r})$ with zero mean value ($\langle v \rangle = 0$) and a correlator $\langle v(\mathbf{r})v(\mathbf{r}')\rangle = N_i v^2 \delta(\mathbf{r}-\mathbf{r}')$, where $\langle\ \rangle$ denotes the averaging over various realizations of impurity positions. $N_i$ is the concentration of the impurities and $v$ is its strength. The $d$ electron spin fluctuation has been calculated in our earlier paper[8,14] using a self-consistent approach in the presence of disorder.

The plan of the paper is as follows. In Sec. II we obtain the kinetic equation for distribution function following Keldysh's nonequilibrium diagram technique.[17] In this technique the Green's functions and self-energies are represented by $2\times 2$ matrices. The off-diagonal component of the Green's function (Keldysh component) contains the information of the nonequilibrium distribution function. The collision integral is expressed in terms of self-energy and it is obtained using a standard many-body technique. This technique has been applied to include the effect of disorder by Altshuler.[18] He obtained the temperature dependence of conductivity of impure systems by taking into account the interference between the phonons and impurity scattering mechanism. We extend this technique to include the effect of spin fluctuation and disorder and obtain the expression for conductivities for the pure systems and the correction due to the impurities. In Sec. III, the self-energy in the presence of disorder and spin fluctuation is calculated. The disorder renormalizes the electron-spin-fluctuation vertex and its importance is considered by taking the effect of diffusions on the vertex. This makes the vertex function diffusive for $ql \ll 1$ and $\omega\tau \ll 1$. In this limit it is possible to write the self-energy and the vertex function as a sum of pure spin-fluctuation contribution and the disorder correction. This enables us to obtain the expression for the conductivity in the clean limit as well as the correction term due to disorder from the same self-energy diagram. This is the advantage of the present approach over the standard procedure applied to the Boltzmann equation.[20] The expressions for the conductivities are derived in Sec. IV. In the clean limit, we reproduce the results of Ueda and Moriya.[19] They obtain the same expressions using the standard procedure.[20] The disorder corrections are important only for temperature $T \ll \tau^{-1}$, and in this region the electric conductivity varies as $\sqrt{T}$ and it has a positive temperature coefficient. This means that on adding the pure and disorder correction for conductivity, a maximum is obtained at low temperature. In the disorder region the thermal conductivity varies as $T^{3/2}$, the Wiedemann-Franz law is violated, and the Lorentz number becomes temperature dependent. In Sec. V, we analyze the results of our calculation numerically and discuss its implications for various alloys and amorphous systems.

## II. THE KINETIC EQUATION

In the Keldysh-diagram technique[17] for nonequilibrium process, the electron propagator $\hat{G}$ and its self-energy $\hat{\Sigma}$ are represented by matrices,

$$\hat{G} = \begin{bmatrix} G^r & G^k \\ 0 & G^a \end{bmatrix} \quad \text{and} \quad \hat{\Sigma} = \begin{bmatrix} \Sigma^r & \Sigma^k \\ 0 & \Sigma^a \end{bmatrix} \tag{1}$$

where the superscripts $r$, $a$, and $k$ represent retarded, advanced, and Keldysh component, respectively. In general the diagonal components of $\hat{G}$ characterize the states and the Keldysh component gives the information on the occupation of these states. In the absence of external fields, the system is in equilibrium and the Keldysh component of the equilibrium Green's function is given by

$$G_0^k(p) = h_0(\epsilon)[G_0^r(p) - G_0^a(p)] \ , \tag{2}$$

where $h_0(\epsilon) \equiv 1 - 2f_0(\epsilon)$ and $f_0(\epsilon)$ is the equilibrium Fermi distribution function. The Keldysh component in general is proportional to the nonequilibrium distribution function $h(p)$. Here and in what follows we use a four-component notation for $p \equiv \{\mathbf{p}, \epsilon\}$ and consider $\hbar = k_B = 1$.

In general, the external perturbation destroys the translational invariance in space and time coordinates and one then introduces mixed or Wigner representation. In the presence of external fields, one makes an ansatz for the Keldysh component of the Green's function and in the Wigner representation it is written as[18,21]

$$G^k(p) = h(p)[G^r(p) - G^a(p)] + \frac{i}{2}\{G^r(p) + G^a(p); h(p)\} \ . \tag{3}$$

The second term in Eq. (3) represents the generalized Poisson bracket. In the present problem, in the presence of electric field $\mathbf{E}$ and temperature gradient $\nabla T$, the Poisson bracket is given by

$$\{A;B\}=e\mathbf{E}\cdot\left[\frac{\partial A}{\partial\epsilon}\nabla_p B-\frac{\partial B}{\partial\epsilon}\nabla_p A\right]+\nabla T\cdot\left[\frac{\partial A}{\partial T}\nabla_p B-\frac{\partial B}{\partial T}\nabla_p A\right]. \tag{4}$$

The kinetic equation in the presence of electric field and temperature gradient is then written as

$$\mathbf{v}\cdot\left[\nabla T\frac{\partial}{\partial T}+e\mathbf{E}\frac{\partial}{\partial\epsilon}\right]h(p)=I[h], \tag{5}$$

where $I[h]$ is the collision integral, which can be expressed in terms of the self-energy. In general, the self-energy is a functional of the distribution function $h(p)$ explicitly through Eq. (3) and hence it contains the Poisson bracket terms. To simplify the notation, we separate the self-energy $\hat{\Sigma}=\hat{\Sigma}_0+\delta\hat{\Sigma}$ and the collision integral $I[h]=I_0[h]+\delta I[h]$, where $\delta\hat{\Sigma}$ and $\delta I[h]$ are the terms of the self-energy and the collision integral, which contain the Poisson brackets, respectively. Then the collision integrals are given in term of the self-energy by

$$\begin{aligned}I_0[h]&=i\Sigma_0^k(p)-ih(p)[\Sigma_0^r(p)-\Sigma_0^a(p)],\\ \delta I[h]&=i\delta\Sigma^k(p)-ih(p)[\delta\Sigma^r(p)-\delta\Sigma^a(p)]\\&\quad+\tfrac{1}{2}\{\Sigma_0^r(p)+\Sigma_0^a(p);h(p)\}.\end{aligned} \tag{6}$$

To further simplify the notation we henceforth drop the subscript 0 from the self-energy and the collision integral. Once the Green's function and its self-energies are obtained, the nonequilibrium distribution function can be determined by solving Eqs. (3)–(6).

The Green's function in the presence of spin fluctuations and disorder is obtained by considering the self-energies given in Fig. 1. The total self-energy is written as $\hat{\Sigma}=\hat{\Sigma}_{\text{imp}}+\hat{\Sigma}_{\text{in}}$, where the first term $\hat{\Sigma}_{\text{imp}}$ is the self-energy due to the impurities [Fig. 1(a)] and the second term $\hat{\Sigma}_{\text{in}}$ due to the inelastic scattering of electrons from the spin fluctuations of the $d$ electrons [Fig. 1(b)]. In order to simplify the calculation we expand the Green's function to the linear order in the self-energy as

$$G^a(p)=G_0^a(p)+G_0^{a2}(p)\Sigma_{\text{in}}^a(p). \tag{7}$$

Here

$$G_0^a(p)=[G_0^r(p)]^*=(\epsilon-\epsilon_p-\mu-i/2\tau)^{-1}$$

is the electron propagator in the presence of the impurity scattering. The electron lifetime due to elastic impurity scattering [Fig. 1(a)] is given by $\tau^{-1}=2\pi\rho_s N_i v^2$.

FIG. 1. Self-energy (a) due to impurity scattering (— — —×— — —) and (b) due to spin fluctuations (wavy line) in the presence of disorder renormalized vertex.

Nonequilibrium distribution function is obtained by solving the kinetic equation Eq. (5). This is done iteratively by expanding

$$h(p)=h_0(\epsilon)+\phi_0(p)+\phi_1(p), \tag{8}$$

where $\phi_0(p)$ is the correction to $h_0(\epsilon)$ due to the impurity scattering [Fig. 1(a)] and $\phi_1(p)$ due to the scattering of electrons from the spin fluctuations of $d$ electrons [Fig. 1(b)]. The expression for $\phi_0(p)$ and $\phi_1(p)$ are obtained from the corresponding collision integrals and are given by

$$\phi_0(p)=-\tau I_{\text{imp}}^a[h] \tag{9}$$

and

$$\phi_1(p)=\tau(I_{\text{imp}}^b[h]+I_{\text{in}}[h]+\delta I_{\text{in}}[h]). \tag{10}$$

The expression for collision integrals are obtained from the self-energy from Eq. (6) and given in Sec. III.

In the presence of weak electric field and temperature gradient the electric and thermal current to the linear order in fields are given by

$$\mathbf{J}=L_{EE}\mathbf{E}+L_{ET}\nabla T$$

and

$$\mathbf{U}=L_{TE}\mathbf{E}+L_{TT}\nabla T, \tag{11}$$

respectively. From Eq. (11), the electrical conductivity $\sigma$ and thermal conductivity $\kappa$ follows immediately;

$$\sigma=L_{EE}$$

and

$$\kappa=-\left[L_{TT}-\frac{L_{TE}L_{ET}}{L_{EE}}\right]. \tag{12}$$

In order to obtain different components of the response functions $L_{EE}$, etc., the currents carried by the electron with a given spin are expressed in terms of Green's function and nonequilibrium distribution function by

$$\mathbf{J}=-e\int dp\,\mathbf{v}h(p)\operatorname{Im}G^a(p)$$

and

$$\mathbf{U}=-\int dp(\epsilon-\mu)\mathbf{v}h(p)\operatorname{Im}G^a(p), \tag{13}$$

where $dp\equiv d^3p\,d\epsilon/(2\pi)^4$, which will be calculated in the following.

## III. SELF-ENERGY

For simplicity we consider the scattering of the $s$ electrons by the spin-density fluctuation of the $d$ electrons via $s$-$d$ exchange interaction;

$$H_{s-d}=g\sum_{\mathbf{q},\mathbf{k}}[a^{\dagger}_{\mathbf{k}+\mathbf{q}\downarrow}a_{\mathbf{k}\uparrow}S_{+}(q)+a^{\dagger}_{\mathbf{k}+\mathbf{q}\uparrow}a_{\mathbf{k}\downarrow}S_{-}(q)+(a_{\mathbf{k}+\mathbf{q}\uparrow}a_{k\uparrow}-a^{\dagger}_{\mathbf{k}+\mathbf{q}\downarrow}a_{\mathbf{k}\downarrow})S_z(q)]\ , \tag{14}$$

where $a_{\mathbf{k}\sigma}$ is the annihilation operator for the $s$ electron with momentum $\mathbf{k}$ and spin $\sigma$ and $S_{+}(q)$ and $S_{-}(q)$ are the spin raising and lowering operator for the $d$ electron spin and $S_z(q)$ is its $z$ component. In what follows we assume the system to be in the paramagnetic phase, then the different spin-fluctuation propagator components obey the relation $\hat{\chi}_{+-}=\hat{\chi}_{-+}=2\hat{\chi}_{zz}\equiv\hat{\chi}$. The extension into the ferromagnetic phase is straightforward, where each component of the spin susceptibility should be considered separatively. This will not be discussed here in what follows except for some comments in the discussion section.

In the self-energy diagram (Fig. 1), the dashed line with a cross represents the impurity scattering and the wavy line stands for the spin-fluctuation propagator,

$$\hat{\chi}=\begin{bmatrix}\chi^r & \chi^k\\ 0 & \chi^a\end{bmatrix}\ . \tag{15}$$

To simplify the calculation we assume that the applied fields do not affect the $d$ electrons. Therefore, they are at equilibrium and the Keldysh component of the spin-fluctuation propagator obey an expression similar to Eq. (2),

$$\chi^k(q)=[2N(w)+1][\chi^r(q)-\chi^a(q)]\ , \tag{16}$$

where $N(w)=(e^{w/T}-1)^{-1}$ and $q\equiv\{\mathbf{q},\omega\}$.

The spin-fluctuation propagator in the presence of spin fluctuation and disorder has been calculated earlier[14,22–24] and it is given by

$$\chi^r(q)=\rho_d\left[\alpha(T)+\delta\frac{q^2}{k_d^2}+i\pi\gamma\frac{\omega}{v_d q}\right]^{-1}\quad\text{for } ql_d>1$$

$$=\rho_d\frac{D_d q^2}{\alpha(T)D_d q^2-i\omega}\quad\text{for } ql_d\ll 1\ , \tag{17}$$

where $\delta=\frac{1}{12}$ and $\gamma=\frac{1}{2}$ (for free Fermi gas). In order to distinguish between $s$ electrons and $d$ electrons here we use the following notations. $\rho_d$ ($\rho_s$), $k_d$ ($k_s$), and $v_d$ ($v_s$) are the density of state at Fermi energy $\epsilon_d$ ($\epsilon_s$), Fermi momentum, and Fermi velocity of $d$ electrons ($s$ electrons) and $D_d$ ($D_s$) diffusion constant with $\tau_d$ ($\tau_s$) being the relaxation time and $l_d$ ($l_s$) being the mean free path of $d$ electrons ($s$ electrons), respectively. In Eq. (17), $\alpha(T)$ is the temperature-dependent Stoner parameter. In the presence of spin fluctuation and disorder, it is given by[14]

$$\alpha(T)=\alpha_0-\lambda_p(T)-\lambda_d(T)\ , \tag{18}$$

where the pure spin-fluctuation contribution $\lambda_p(T)$ (Ref. 25) is given by

$$\lambda_p(T)=-\frac{5}{3\pi^2}(U\rho_d)^2\int_0^1 dq\frac{q^3}{2y+12y^2}+\frac{32}{3\pi^2}(U\rho_d)^3\int_0^1 dq\frac{1}{(1+6y)^2}\ , \tag{19}$$

where

$$y=\frac{[\alpha(T)+q^2/12]2q}{\pi^2 T}\ ,$$

and the disorder correction term in $\alpha(T)$ is

$$\lambda_d(T)=\frac{3\sqrt{3}(U\rho_d)^2}{(\epsilon_d\tau_d)^2}[1-\sqrt{2\pi T\tau_d}]\ , \tag{20}$$

for $T\ll\tau_d^{-1}$ and it vanishes otherwise. Equation (18) should be solved self-consistently to obtain $\alpha(T)$.

Now we calculate the self-energy term explicitly. Figure 1(a) represents the impurity self-energy

$$\Sigma^{ij}_{\text{imp}}(\epsilon)=N_i v^2\int\frac{d^3k}{(2\pi)^3}G^{ij}(k)\ , \tag{21}$$

where $k\equiv\{\mathbf{k},\epsilon\}$ and $i,j=1,2$. The collision integral corresponding to this self-energy follows immediately from Eq. (6) and is given by $I_{\text{imp}}[h]=I^a_{\text{imp}}[h]+I^b_{\text{imp}}[h]$, where

$$I^a_{\text{imp}}[h]=2N_i v^2\int\frac{d^3k}{(2\pi)^3}[h(k)-h(p)]\text{Im}G^a_0(k) \tag{22}$$

and

$$I^b_{\text{imp}}[h]=2N_i v^2\int\frac{d^3k}{(2\pi)^3}[h(k)-h(p)]\times\text{Im}G^{a2}_0(k)\Sigma^a_{\text{in}}(k)\ . \tag{23}$$

There is no term in the collision integral which involves Poisson brackets, i.e., $\delta I_{\text{imp}}[h]=0$.

Figure 1(b) gives the self-energy due to spin fluctuation in the presence of disorder renormalized vertex. It can be written as

$$\Sigma^{ij}_{\text{in}}(p)=ig^2\int dq\,\chi^{kk'}[\Gamma^k_{ii'}(-q,\epsilon)G^{i'j'}(p-q)\tilde{\Gamma}^{k'}_{j'j}(q,\epsilon)] \tag{24}$$

and

$$\delta\Sigma^{ij}_{\text{in}}(p)=ig^2\int dq\,\chi^{kk'}\Bigg[\{\delta\Gamma^k_{ii'}(-q,\epsilon)G^{i'j'}(p-q)\tilde{\Gamma}^{k'}_{j'j}(q,\epsilon)+\Gamma^k_{ii'}(-q,\epsilon)G^{i'j'}(p-q)\delta\tilde{\Gamma}^{k'}_{j'j}(q,\epsilon)\}$$
$$+\left[\frac{i}{2}\Gamma^k_{ii'}(-q,\epsilon)\{G^{i'j'}(p-q);\tilde{\Gamma}^{k'}_{j'j}(q,\epsilon)\}+\frac{i}{2}\{\Gamma^k_{ii'}(-q,\epsilon);G^{i'j'}(p-q)\}\tilde{\Gamma}^{k'}_{j'j}(q,\epsilon)\right]$$
$$+\text{Poisson bracket terms of}[\Gamma^k_{ii'}(-q,\epsilon)G^{i'j'}(p-q)\tilde{\Gamma}^{k}_{j'j}{}'(q,\epsilon)]\Bigg]\ , \tag{25}$$

where $i,j,k=1,2$. Here and in what follows we assumed a summation over the repeated indices. The last expression of Eq. (25) stands for the terms of the expression given in the bracket which contain the Poisson brackets. $\tilde{\Gamma}$ and $\hat{\hat{\Gamma}}$ are the disorder renormalized vertex functions corresponding to absorption and emission of spin fluctuation, respectively. In the Keldysh technique they are not identical. In the above equations we split the vertex function as $\hat{\Gamma}=\hat{\Gamma}_0+\delta\hat{\Gamma}$ and drop the subscript 0 from the equation as we did in the case of self-energy and collision integrals.

It is difficult to obtain the exact vertex function with respect to disorder. However, it is seen that the most important contribution comes from the diffusive nature of the vertex function[12] which arises due to the repeated scattering of the electron from the impurities (diffusions). In that case the vertex function can be expressed as in Fig. 2, and is given by

$$\Gamma^k_{ij}(-q,\epsilon)=\gamma^k_{ij}+N_i v^2\int\frac{d^3k}{(2\pi)^3}G^{ii'}(k)\Gamma_{i'j'}(-q,\epsilon)G^{j'j}(k-q)$$

and

$$\delta\Gamma^k_{ij}(-q,\epsilon)=N_i v^2\int\frac{d^3k}{(2\pi)^3}\left[G^{ii'}(k)\delta\Gamma_{i'j'}(-q,\epsilon)G^{j'j}(k-q)+\frac{i}{2}G^{ii'}(k)\{\Gamma_{i'j'}(-q,\epsilon);G^{j'j}(k-q)\}\right.$$
$$\left.+\frac{i}{2}\{G^{ii'}(k);\Gamma_{i'j'}(-q,\epsilon)\}G^{j'j}(k-q)\right] . \tag{26}$$

Here $\hat{\gamma}$ is the vertex function corresponding to the clean limit and its values are given by $\gamma^1_{ij}=[1/\sqrt{2}]\delta_{ij}$ and $\gamma^2_{ij}=[1/\sqrt{2}]\tau^1_{ij}$; where

$$\tau^1=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix} .$$

is the Pauli spin matrix. A similar equation can be written for $\tilde{\Gamma}^k_{ij}$. Solving Eq. (26), we obtain

$$\hat{\Gamma}^1=\frac{1}{\sqrt{2}}\begin{bmatrix}1 & \dfrac{\eta^{rk}+\eta^{ka}}{1-\eta^{ra}}\\ 0 & 1\end{bmatrix} , \tag{27}$$

where

$$\eta^{\alpha\beta}=N_i v^2\int\frac{d^3k}{(2\pi)^3}G^{\alpha}_0(k)G^{\beta}_0(k-q)$$

with $\alpha,\beta=r,a,k$. The solutions for the other components of the vertex function are given in the Appendix. In the limit $N_i\rightarrow 0$, the vertex function $\hat{\Gamma}^k\rightarrow\hat{\gamma}^k$ and $\delta\Gamma^k\rightarrow 0$. This means that the vertex function can be written as the sum of two terms, the first being $\hat{\gamma}^k$, the vertex function due to the spin-fluctuation scattering in the clean limit, and the second term comes as a correction to the first one due to the crossing of impurity lines in the vertex which vanishes in the limit $N_i\rightarrow 0$. Considering this fact, the self-energy can be calculated in a straightforward manner and separated as $\hat{\Sigma}_{\text{in}}(p)=\hat{\Sigma}_{\text{sf}}(p)+\hat{\Sigma}_{\text{sf-imp}}(p)$, where the first terms represent the pure self-energy and the second term is the correction to the first one which vanishes in the limit $N_i\rightarrow 0$.

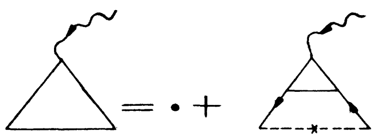

FIG. 2. Disorder renormalized vertex function. ● represents the free-vertex function.

The diffusive nature of the vertex function is seen from Eq. (27). In the limit $ql_s\ll 1$ and $\omega\tau_s\ll 1$, we have

$$(1-\eta^{ra})=\tau_s(D_s q^2-i\omega) . \tag{28}$$

In the opposite limit, i.e., $ql_s\gg 1$, $\eta^{ra}\simeq 1/ql_s$, which is very small compared to the diffusive region contribution. In the calculation below in obtaining the disorder correction term of the self-energy $\hat{\Sigma}_{\text{sf-imp}}(p)$, the most important contribution will be the one which is most singular as $\omega\rightarrow 0$, $q\rightarrow 0$. Considering these facts the self-energy expressions are given explicitly in the Appendix. Using these expressions the collision integrals and $\phi_1(p)$ are obtained from Eqs. (6) and (10), respectively, and the conductivity is calculated below.

## IV. ELECTRIC AND THERMAL CONDUCTIVITIES

The currents due to the electric and temperature gradient field are calculated from Eq. (13). In order to discuss various contributions, the total current is written as $\mathbf{J}=\mathbf{J}_0+\mathbf{J}_{\text{sf}}+\mathbf{J}_{\text{sf-imp}}$ and a similar expression for the thermal current. The first term corresponds to the current in the absence of spin fluctuations, which is given by

$$\mathbf{J}_0=-2e\int dp\;\mathbf{v}\phi_0(p)\text{Im}G^a_0(p)$$

and

$$\mathbf{U}_0=-2\int dp(\epsilon-\mu)\mathbf{v}\phi_0(p)\text{Im}G^a_0(p) . \tag{29}$$

In Eq. (29) the factor 2 accounts for the spin degeneracy. From Eqs. (5) and (9),

$$\phi_0(p)=-\tau_s\mathbf{v}\cdot\left[\nabla T\frac{\partial}{\partial T}+e\mathbf{E}\frac{\partial}{\partial\epsilon}\right]h_0(\epsilon) . \tag{30}$$

Considering the temperature dependence of the chemical potential of the form $\mu(T)=\epsilon_s(1-\pi^2T^2/12\epsilon_s^2)$, which arises out of the usual Sommerfeld expansion, we have

$$\frac{\partial}{\partial T}h_0(\epsilon)=-\left[\frac{\epsilon-\epsilon_s}{T}-\frac{\pi^2 T}{6\epsilon_s}\right]\frac{\partial h_0(\epsilon)}{\partial\epsilon}\ . \tag{31}$$

Applying Eqs. (30) and (31) in Eq. (29), we obtain the Drude result for the conductivities $\sigma_0=ne^2\tau_s/m_s$ and $\kappa_0=n\tau_s\pi^2T/3m_s$. The Wiedemann-Franz law is obeyed and the Lorentz number $L_0=\kappa_0/\sigma_0T=\pi^2/3e^2$. The corrections to this Drude expression in the presence of spin fluctuations are discussed below. $\mathbf{J}_{\rm sf}$ and $\mathbf{J}_{\rm sf\text{-}imp}$ correspond to pure spin-fluctuation contribution and the quantum corrections due to disorder, respectively. In the following we examine these two terms separately.

First we calculate $\mathbf{J}_{\rm sf}$ and $\mathbf{U}_{\rm sf}$, which are obtained using Eqs. (8), (10), and (13). The expression for $\mathbf{J}_{\rm sf}$ is explicitly given by

$$\mathbf{J}_{\rm sf}=-e\int dp\ \mathbf{v}\{\phi_{1,\rm sf}(p)\,{\rm Im}G_0^a(p)+\phi_0(p)\,{\rm Im}G_0^{a2}(p)\Sigma_{\rm sf}^a(p,h_0(\epsilon))+h_0(\epsilon)\,{\rm Im}G_0^{a2}(p)[\Sigma_{\rm sf}^a(p,\phi_0)+\delta\Sigma_{\rm sf}^a(p,h_0(\epsilon))]\}\ , \tag{32}$$

where the $\phi_{1,\rm sf}(p)$ is obtained from Eq. (10) by taking only the pure spin-fluctuation collision integrals. The corresponding self-energies are given in the Appendix. We notice that in Eq. (32), the contributions to the current coming from the first term of Eq. (10) and the second term of Eq. (32) cancel each other. The only nonzero term contributes to the $\mathbf{J}_{\rm sf}$ is that coming from the second term of Eq. (10), which is given as

$$\mathbf{J}_{\rm sf}=-e\tau_s\int dp\ \mathbf{v}I_{\rm sf}[h]\,{\rm Im}G_0^a(p)\ ,$$

where

$$I_{\rm sf}[h]=24g^2\int dq\ {\rm Im}\chi^a(q)\,{\rm Im}G_0^a(p-q)R_p^q\ ,$$

with

$$R_p^q=[1+N(\omega)]f(p)[1-f(p-q)]-N(\omega)f(p-q)[1-f(p)]\ . \tag{33}$$

In the collision integral account for the scattering of the electron from all the spin-fluctuation components has been taken. Expanding $R_p^q$ to the linear order in $\phi_0(p)$ (which correspond to linear order in $\mathbf{E}$ and $\boldsymbol{\nabla}T$) using the Eq. (8) and performing the $p$ integration, the current $\mathbf{J}_{\rm sf}$ is obtained. A similar procedure is applied for obtaining $\mathbf{U}_{\rm sf}$. Below we summarize results for the different components of the response function coming from $\mathbf{J}_{\rm sf}$ and $\mathbf{U}_{\rm sf}$:

$$L_{EE}^{\rm sf}=-\frac{c_1e^2}{T}\int_0^{2k_s}dq\int_{-\infty}^{\infty}d\omega\, q^3\,{\rm Im}\chi^a(q,\omega)N(\omega)I_0(\omega)\ ,$$

$$L_{ET}^{\rm sf}=\frac{c_1e}{T^2}\int_0^{2k_s}dq\int_{-\infty}^{\infty}d\omega\, q\ {\rm Im}\chi^a(q,\omega)N(\omega)\times\left[q^2I_1(\omega)-\frac{\pi^2T^2q^2}{6\epsilon_s}I_0(\omega)+(2k_s^2-q^2)\omega I_0(\omega)\right]\ ,$$

$$L_{TE}^{\rm sf}=-\frac{c_1e}{T}\int_0^{2k_s}dq\int_{-\infty}^{\infty}d\omega\, q^3\,{\rm Im}\chi^a(q,\omega)N(\omega)I_1(\omega),$$

and

$$L_{TT}^{\rm sf}=\frac{c_1}{T^2}\int_0^{2k_s}dq\int_{-\infty}^{\infty}d\omega\, q\ {\rm Im}\chi^a(q,\omega)N(\omega)\times\left[q^2I_2(\omega)-\frac{\pi^2T^2q^2}{6\epsilon_s}I_1(\omega)+(2k_s^2-q^2)\omega I_1(\omega)\right]\ , \tag{34}$$

where

$$I_n(\omega)=\int d\epsilon[1-f_0(\epsilon)]f_0(\epsilon-\omega)(\epsilon-\mu)^n$$

and $c_1=\tau_s^2g^2/8\pi^4$. Following Ziman (page 389, Ref. 20), these integrations can be done easily to obtain $I_0=\omega[1+N(\omega)]$, $I_1=(\omega^2/2)[1+N(\omega)]$, and $I_2=\frac{1}{3}[1+N(\omega)][\omega^3+\pi^2\omega T^2]$. Using these expressions in Eq. (34), we notice that the integrand of $L_{TE}^{\rm sf}$ is an odd function of $\omega$. Therefore it vanishes on integration over $\omega$. Electric and thermal conductivity is then obtained using Eq. (12),

$$\sigma_{\rm sf}=-\tfrac{3}{4}\sigma_0\tilde{g}^{\,2}C_p\epsilon_d\tau_s\bar{\sigma}_{\rm sf}(T)\ ,$$

$$\kappa_{\rm sf}=-\frac{9}{4\pi^2}\kappa_0\tilde{g}^{\,2}C_p\epsilon_d\tau_s[\bar{\kappa}_{\rm sf}^1(T)+\bar{\kappa}_{\rm sf}^2(T)]\ , \tag{35}$$

where

$$\bar{\sigma}_{\rm sf}(T)=\frac{1}{T}\int_0^{2k_s/k_d}dq\int_{-\infty}^{\infty}d\omega\, q^3\omega\,{\rm Im}\chi^a(q,\omega)\times N(\omega)[1+N(\omega)]\ ,$$

$$\bar{\kappa}_{\rm sf}^1(T)=\frac{1}{T^3}\int_0^{2k_s/k_d}dq\int_{-\infty}^{\infty}d\omega\, q\omega^3\,{\rm Im}\chi^a(q,\omega)\times N(\omega)[1+N(\omega)]\times\left[\left[\frac{k_s}{k_d}\right]^2-\frac{q^2}{6}\right]\ ,$$

and

$$\bar{\kappa}_{\rm sf}^2(T)=\frac{\pi^2}{3}\bar{\sigma}_{\rm sf}(T)\ , \tag{36}$$

where $\tilde{g}\equiv g\rho_d$ and $C_p=(\rho_s/\rho_d)(k_d/k_s)^4$. Reduced units $(q/k_d\rightarrow q, T/\epsilon_d\rightarrow T, \omega/\epsilon_d\rightarrow\omega)$ are used to express $\bar{\sigma}_{\rm sf}(T)$ and $\bar{\kappa}_{\rm sf}(T)$, which are dimensionless quantities that express the temperature dependence of electrical and thermal conductivity. Rewriting the above expression as resistivities using the relation $R_{\rm sf}=-\sigma_{\rm sf}/\sigma_0^2$ and $W_{\rm sf}=-\kappa_{\rm sf}/\kappa_0^2$, the results of Ueda and Moriya[19] are reproduced. These results show the equivalence of the present method with the one used by Ueda and Moriya. The advantage of the present approach over the standard approach is that it can be extended to include the quantum corrections due to the disorder, as will be discussed below.

Similar to Eq. (32), the disorder correction term for the currents $\mathbf{J}_{\text{sf-imp}}$ and $\mathbf{U}_{\text{sf-imp}}$ are obtained using Eqs. (8), (10), and (13) by considering the disorder correction part of the self-energy $\hat{\Sigma}_{\text{sf-imp}}$, which is explicitly given in the Appendix.

$$\mathbf{J}_{\text{sf-imp}}=-e\int dp\,\mathbf{v}\{\phi_{1,\text{sf-}imp}(p)\mathrm{Im}G_0^a(p)+\phi_0(p)\mathrm{Im}G_0^{a2}(p)\Sigma^a_{\text{sf-imp}}(p,h_0(\epsilon))$$
$$+h_0(\epsilon)\mathrm{Im}G_0^{a2}(p)[\Sigma^a_{\text{sf-imp}}(p,\phi_0)+\delta\Sigma^a_{\text{sf-imp}}(p,h_0(\epsilon))]\}\ , \tag{37}$$

where $\phi_{1,\text{sf-imp}}$ is the disorder correction term from Eq. (10) by taking only the disorder part of the collision integral. A similar expression can be written for $\mathbf{U}_{\text{sf-imp}}$. As in the case of $\mathbf{J}_{\rm sf}$, the current coming from the first term of Eq. (10) and the second term of Eq. (37) cancel each other. Different components of the response function can be calculated easily and the results are summarized below:

$$L^{\text{sf-imp}}_{EE,a}=\frac{3}{2\pi T}\sigma_0\tilde{g}^{\,2}\mathrm{Im}\int dq\,d\omega\,q^2\chi^r(q)\Lambda^2(q,\omega)[S_0(\omega)+S_0(-\omega)-2S_0(0)]\ ,$$

$$L^{\text{sf-imp}}_{ET,a}=-\frac{3}{2e\pi T^2}\sigma_0\tilde{g}^{\,2}\mathrm{Im}\int dq\,d\omega\,q^2\chi^r(q)\Lambda^2(q,\omega)\left[[S_1(\omega)+S_1(-\omega)-2S_1(0)]-\frac{\pi^2T^2}{6}[S_0(\omega)-S_0(-\omega)-2S_0(0)]\right]\ ,$$

$$L^{\text{sf-imp}}_{TE,a}=\frac{3}{2e\pi T}\sigma_0\epsilon_s\tilde{g}^{\,2}\mathrm{Im}\int dq\,d\omega\,q^2\chi^r(q)\Lambda^2(q,\omega)[S_1(\omega)+S_1(-\omega)-2S_1(0)+\omega S_0(\omega)]\ ,\tag{38}$$

$$L^{\text{sf-imp}}_{TT,a}=-\frac{3}{2e^2\pi T^2}\sigma_0\epsilon_s\tilde{g}^{\,2}\mathrm{Im}\int dq\,d\omega\,q^2\chi^r(q)\Lambda^2(q,\omega)\left[[S_2(\omega)+S_2(-\omega)-2S_2(0)+\omega S_1(\omega)]-\frac{\pi^2T^2}{6}[S_1(\omega)+S_1(-\omega)-2S_1(0)+\omega S_0(\omega)]\right]$$

and

$$L^{\text{sf-imp}}_{EE,b}=-\frac{2}{\pi T}\sigma_0\tilde{g}^{\,2}\mathrm{Im}\int dq\,d\omega\,D_sq^4\chi^r(q)\Lambda^3(q,\omega)[S_0(\omega)-S_0(0)]\ ,$$

$$L^{\text{sf-imp}}_{ET,b}=\frac{2}{e\pi T^2}\sigma_0\tilde{g}^{\,2}\mathrm{Im}\int dq\,d\omega\,D_sq^4\chi^r(q)\Lambda^3(q,\omega)\left[[S_1(\omega)-S_1(0)]-\frac{\pi^2T^2}{6}[S_0(\omega)-S_0(0)]\right]\ ,$$

$$L^{\text{sf-imp}}_{TE,b}=-\frac{2}{e\pi T}\sigma_0\epsilon_s\tilde{g}^{\,2}\mathrm{Im}\int dq\,d\omega\,D_sq^4\chi^r(q)\Lambda^3(q,\omega)[S_1(\omega)-S_1(0)]\ ,\tag{39}$$

$$L^{\text{sf-imp}}_{TT,b}=\frac{2}{e^2\pi T^2}\sigma_0\epsilon_s\tilde{g}^{\,2}\mathrm{Im}\int dq\,d\omega\,D_sq^4\chi^4(q)\Lambda^3(q,\omega)\left[[S_2(\omega)-S_2(0)+\omega S_1(\omega)]-\frac{\pi^2T^2}{6}[S_1(\omega)-S_1(0)]\right]\ ,$$

where $S_n(\omega)=\int d\epsilon(\epsilon-\mu)^n[1-f_0(\epsilon+\omega)]f_0(\epsilon)[1-f_0(\epsilon)]$, which can be obtained following Ziman (page 389, Ref. 20) and $\Lambda(q,\omega)=(D_sq^2-i\omega)^{-1}$. $S_n(\omega)$ is an odd (even) function of $\omega$ depending upon if $n$ is an even (odd) integer. This immediately implies that the integrand of $L^{\text{sf-imp}}_{TE,a}$ and $L^{\text{sf-imp}}_{TE,b}$ is an odd function of $\omega$ and they are identically zero. Since $S_0(\omega)$ is an odd function of $\omega$, $L^{\text{sf-imp}}_{EE,a}$ vanishes, but the corresponding thermal conductivity term $L^{\text{sf-imp}}_{TT,a}$ does not go to zero. Using Eq. (12), disorder correction to the conductivities are given by

$$\sigma_{\text{sf-imp}}=L^{\text{sf-imp}}_{EE,b}=-\frac{2\sigma_o\tilde{g}^{\,2}}{\pi}\mathrm{Im}\int dq\int_{-\infty}^{\infty}d\omega\,D_sq^4\chi^r(q)\Lambda^3(q,\omega)\left[1+2N(\omega)+2\frac{\omega}{T}N(\omega)N(-\omega)\right]\tag{40}$$

and

$$\kappa^{a}_{\text{sf-imp}} = L^{\text{sf-imp}}_{TT,a} = -\frac{9\kappa_0\tilde{g}^{2}}{2\pi^3}\text{Im}\int dq \int_{-\infty}^{\infty} d\omega\, q^2\chi^r(q)\Lambda^2(q,\omega)\left[\frac{\omega}{T}\right]^3 N(\omega)N(-\omega)\;,$$

$$\begin{aligned}\kappa^{b}_{\text{sf-imp}} &= L^{\text{sf-imp}}_{TT,b}\\ &= -\frac{2\kappa_0\tilde{g}^{2}}{\pi}\text{Im}\int dq\int_{-\infty}^{\infty} d\omega\, D_s q^4\chi^r(q)\Lambda^3(q,\omega)\\ &\qquad\times\left[1+2N(\omega)+2\frac{\omega}{T}N(\omega)N(-\omega)-\frac{1}{\pi^2}\left[\frac{\omega}{T}\right]^3 N(\omega)N(-\omega)\right]\;. \end{aligned} \tag{41}$$

After doing the integration over $q$ and $\omega$ we obtain

$$\begin{aligned}\sigma_{\text{sf-imp}} &= -\frac{3\sqrt{3}\sigma_0\tilde{g}^{2}C_d A_1}{\pi(\epsilon_d\tau_s)^2}[1-a_1\sqrt{T\tau_s}],\\ \kappa^{1}_{\text{sf-imp}} &= -\frac{3\sqrt{3}\kappa_0\tilde{g}^{2}C_d A_1}{\pi(\epsilon_d\tau_s)^2}\left[\frac{9A_2}{4A_1}-1\right]\frac{a_2}{2\pi^2}\sqrt{T\tau_s}\;,\\ \kappa^{2}_{\text{sf-imp}} &= -\frac{3\sqrt{3}\kappa_0\tilde{g}^{2}C_d A_1}{\pi(\epsilon_d\tau_s)^2}[1-a_1\sqrt{T\tau_s}]\;,\end{aligned} \tag{42}$$

with total thermal conductivity correction $\kappa_{\text{sf-imp}} = \kappa^{1}_{\text{sf-imp}} + \kappa^{2}_{\text{sf-imp}}$. Here

$$\begin{aligned}A_1 &= \text{Im}\int_0^{\infty} dq \frac{q^6}{(q^2-i)^3}\frac{1}{[\alpha(T)(D_d/D_s)q^2-i]}\;,\\ A_2 &= \text{Im}\int_0^{\infty} dq \frac{q^4}{(q^2-i)^2}\frac{1}{[\alpha(T)(D_d/D_s)q^2-i]}\;,\\ a_1 &= \int_0^{\infty}\frac{dy}{\sqrt{y}}\left[\frac{d}{dy}\frac{y}{1-e^y}\right] \simeq 2.5\;,\\ a_2 &= -\int_0^{\infty} dy\, y^{5/2}N(y)N(-y) \simeq 4.46\;,\\ C_d &= \left[\frac{\epsilon_d}{\epsilon_s}\right]^{3/2}\left[\frac{m_s}{m_d}\right]^{3/2}\frac{D_d}{D_s}\;.\end{aligned} \tag{43}$$

The total electrical and thermal conductivities are then given by

$$\sigma = \sigma_0 + \sigma_{\text{sf}} + \sigma_{\text{sf-imp}}$$

and

$$\kappa = \kappa_0 + \kappa_{\text{sf}} + \kappa_{\text{sf-imp}}\;, \tag{44}$$

respectively. The Lorentz number $L=\kappa/\sigma T$. The leading order behavior of the Lorentz number follows from Eq. (36) and Eq. (42), which is given by

$$\frac{L}{L_0} = 1 + \kappa^{1}_{\text{sf}}(T) + \kappa^{1}_{\text{sf-imp}}(T)\;, \tag{45}$$

which shows that the Wiedemann-Franz law is violated in the present problem due to the presence of $\kappa^{1}_{\text{sf}}$ and $\kappa^{1}_{\text{sf-imp}}$.

## V. DISCUSSION AND CONCLUSION

In the previous section we obtained the expression for electric and thermal conductivity in the presence of both disorder and spin fluctuations. In this section we analyze these results numerically and apply them to explain the resistivity behavior of some alloys and amorphous systems. For the sake of simplicity in what follows we do not differentiate between the $s$ and $d$ electrons, which means $C_p$ and $C_d$ are unity in Eq. (36) and Eq. (42), respectively.

As we saw in Sec. IV, the disorder corrections $\sigma_{\text{sf-imp}}$ and $\kappa_{\text{sf-imp}}$ are relevant only in the low-temperature region, where $T \ll \tau^{-1}$. In this low-temperature regime the main contribution of the $\omega$ integration in Eq. (36) comes from the region where $\omega^{1/3} \ll \alpha^{1/2}(T)$, which corresponds to $T \ll \alpha_0^{3/2}$. Here the imaginary part of the spin susceptibility can be approximated as $\text{Im}\chi^a(q,\omega) \sim \pi\omega/2q[\alpha(T)+\delta q^2]^2$ and the $\omega$ and $q$ integration can be performed easily to obtain

$$\begin{aligned}\sigma_{\text{sf}} &= -\frac{3\sqrt{3}\pi^2}{8}\sigma_0\tilde{g}^{2}\epsilon_F\tau\frac{T^2}{\sqrt{\alpha(T)}}\;,\\ \kappa_{\text{sf}} &= -\frac{3\sqrt{3}\pi^2}{8}\kappa_0\tilde{g}^{2}\epsilon_F\tau\frac{T^2}{\sqrt{\alpha(T)}}\left[\frac{1}{20\alpha(T)}+1\right]\;,\end{aligned} \tag{46}$$

where the first and second term in $\kappa_{\text{sf}}$ comes from the contribution of $\kappa^{1}_{\text{sf}}(T)$ and $\kappa^{2}_{\text{sf}}$, respectively. If the system is very close to the ferromagnetic instability, then $\alpha_0 \geq 0$, and the first term in $\kappa_{\text{sf}}$ dominates over the second. Then the thermal conductivity ratio $\kappa_{\text{sf}}/\kappa_0 \sim T^2/\alpha_0^{3/2}$. On the other hand if the system is not very close to the ferromagnetic instability we can neglect the first term and $\kappa_{\text{sf}}/\kappa_0 \sim \sigma_{\text{sf}}/\sigma_0$.

In the expression for disorder correction Eq. (42), $A_1$ and $A_2$ are temperature dependent through $\alpha(T)$ and its temperature variation for $\epsilon_F\tau = 20$, 10, and 8, and $\alpha_0 = 0.1$ are shown in Fig. 3. The temperature variation of $\alpha(T)$ in the presence of both spin fluctuation and disorder is given by Eq. (18). As the disorder increases, $\alpha(T)$ reduces sharply from its value calculated within spin-fluctuation theory and this is reflected in a sharp increase of $A_1$ and $A_2$ at low temperature for large disorder. In the absence of the disorder correction in $\alpha(T)$, the temperature dependence of $A_1$ and $A_2$ are very weak. Thus to get a self-consistent picture of the disorder corrections in the conductivities, disorder effects on the spin susceptibility should be included as well, and this will add to the

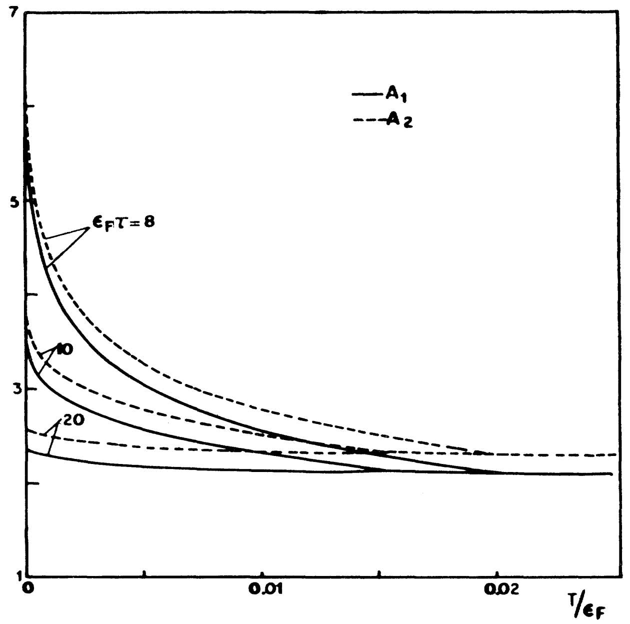

FIG. 3. Temperature variation of $A_1$ and $A_2$ for different $\epsilon_F\tau$ and $\alpha_0=0.1$.

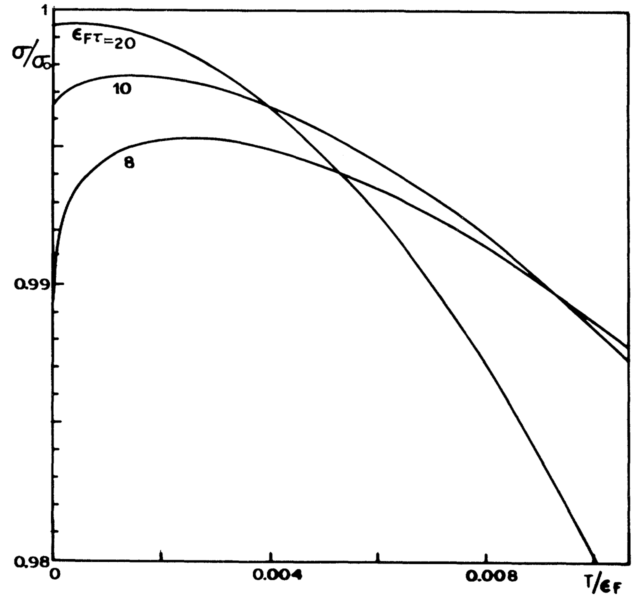

FIG. 4. Electric conductivity $(\sigma/\sigma_0)$ vs temperature $(T/\epsilon_F)$ for different $\epsilon_F\tau$ and $\alpha_0=0.1$.

correction coming from the vertex renormalization [see Fig. 1(b)].

The disorder correction for both electric and thermal conductivity has a positive temperature coefficient, whereas the spin-fluctuation term has a negative temperature coefficient. This means that as temperature increases the disorder correction increases, but the spin-fluctuation term decreases and a maximum is obtained in total electric conductivity. The total electric and thermal conductivity for different disorder parameter $\epsilon_F\tau$ are shown in Figs. 4 and 5, respectively. To emphasize the importance of disorder we have plotted $\sigma/\sigma_0$ and $\kappa/\kappa_0$ versus temperature instead of plotting $\sigma$ and $\kappa$. The maximum occurs at some temperature $T_m$ at which the conductivity behavior crosses over from the disorder correction term to the spin-fluctuation contribution. An estimate of the value of $T_m$ can be made in terms of basic parameters of the system $\alpha_0$ and $\epsilon_F\tau$. Neglecting the temperature dependence of $\alpha(T)$ we obtain $T_m \sim (\alpha_0/\epsilon_F\tau^5)^{1/3}$, which implies that $T_m$ will be small for the systems close to the ferromagnetic instability. This reflects the fact that due to the presence of $\alpha_0$ in the expression for conductivities [Eq. (46)], reduction in conductivities due to spin fluctuation is larger than the increase of disorder correction term. The temperature coefficient of electric and thermal conductivity are different due to the presence of $\kappa^1_{\mathrm{sf}}$ and $\kappa^1_{\mathrm{sf\text{-}imp}}$ in the thermal conductivity. This leads to a different $T_m$ for $\sigma/\sigma_0$ and $\kappa/\kappa_0$. This is due to the fact that the electron scattering from the spin fluctuation affects the electrical and thermal conductivity differently, which is reflected in Lorentz number as discussed below.

The Wiedemann-Franz law is violated because of the presence of $\kappa^1_{\mathrm{sf}}$ and $\kappa^1_{\mathrm{sf\text{-}imp}}$ in the thermal conductivity Eqs. (36) and (42). This is due to the fact that when an electron scatters from the spin fluctuations, it can lose or gain heat energy and it can change from a hot electron to a cold electron or vice versa. This process substantially reduces the heat conduction, while affecting the charge conduction very little. This implies that the thermal conductivity is suppressed more than the electrical conductivity due to the scattering of electrons from the spin fluctuations. The Lorentz number becomes temperature dependent as shown in Fig. 6. As disorder increases the Lorentz number approaches unity. This is due to the fact that the Drude term dominates over the spin-fluctuation

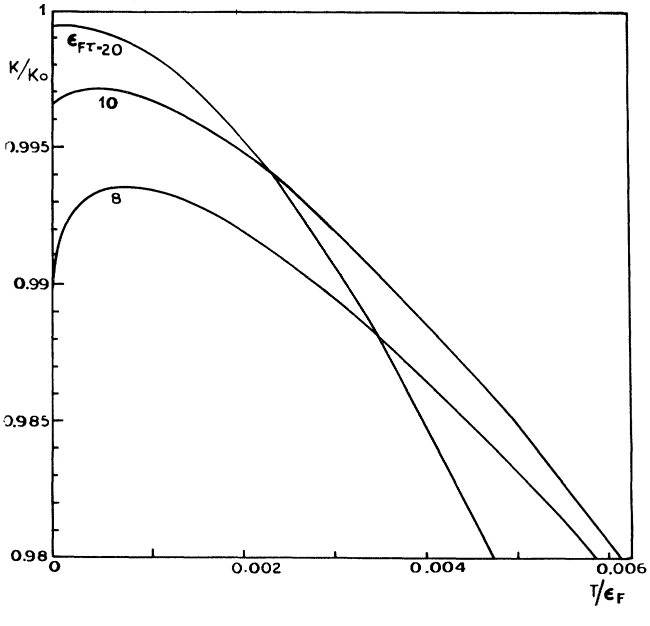

FIG. 5. Thermal conductivity $(\kappa/\kappa_0)$ vs temperature $(T/\epsilon_F)$ for different $\epsilon_F\tau$ and $\alpha_0=0.1$.

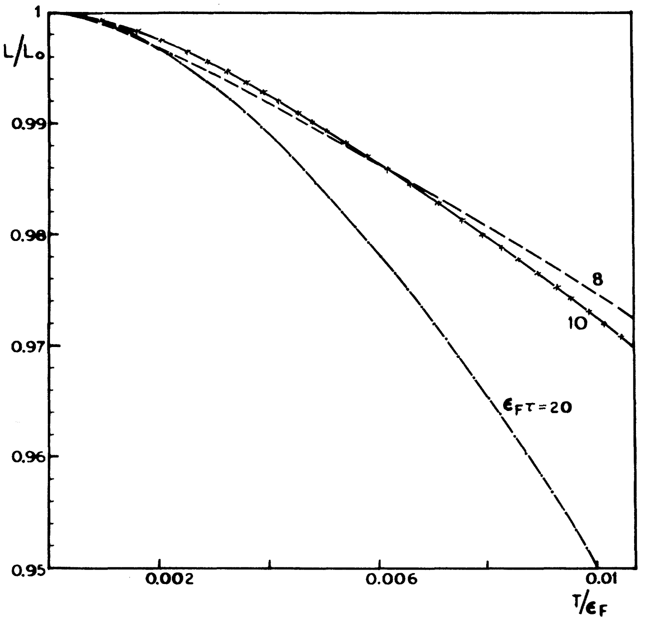

FIG. 6. Lorentz number ($L/L_0$) vs temperature ($T/\epsilon_F$) for different $\epsilon_F\tau$ and $\alpha_0=0.1$.

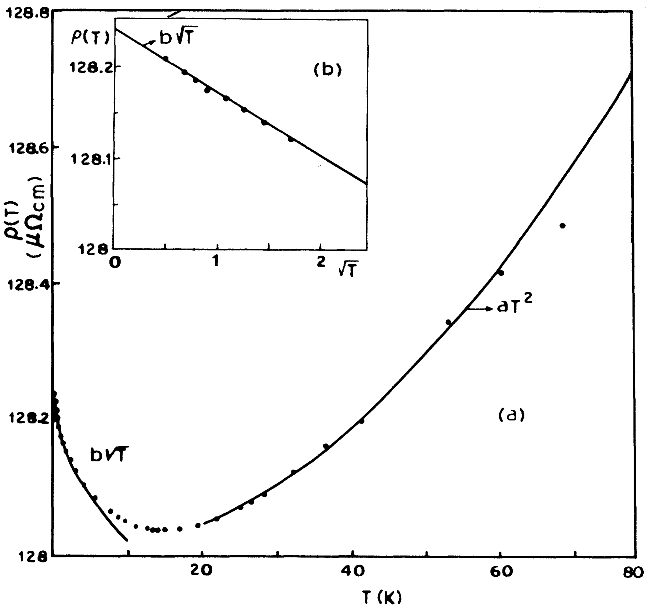

FIG. 7. (a) Electric resistivity of amorphous $Ni_{75}P_{16}B_6Al_3$ system as a function of temperature and (b) same low-temperature data as a function of $\sqrt{T}$. Data (●) are from Ref. 30.

scattering as disorder increases.

The above results can be applied to various alloys and amorphous systems. In our earlier paper[14] we explained the $\sqrt{T}$ behavior seen in the paramagnetic spin susceptibility of $Ni_3Ga$ and $Ni_3Al$ in terms of disorder effect. Applying the present result to these systems, for example to $Ni_{73}Ga_{27}$, which has $\alpha_0=0.058$ and $\epsilon_F\tau\simeq 25$, predicts the value of $T_m$ about 1−2 K. Measurements on resistivity of these systems at low temperatures, say up to few mK will be relevant in this context. Measuring the magnetization of amorphous Ni-Fe-based alloys, Schneider *et al.*, have suggested that amorphous Ni-rich $(Fe_xNi_{1-x})_{80}P_{10}B_{10}$ alloys are weak itinerant ferromagnets.[26] This system shows a ferromagnetic instability as $x$ increases above some critical value $x_c$ close to zero. For $x<x_c$ these systems can be considered nearly ferromagnetic. A similar conclusion can be drawn for the amorphous system $(Fe_xNi_{1-x})_{75}P_{16}B_6Al_3$. In these systems the transition temperature drops to zero in the range $0<x<0.2$.[27] Since the amorphous systems are highly disordered, it is not clear whether we can apply the present perturbative results to these systems. Nevertheless, the magnetization measurement on amorphous Ni-Fe-based alloys near the critical concentration $x_c$ (Ref. 26) shows a similarity in their magnetic properties with the weak itinerant ferromagnetic alloys. Motivated by these observations we assume $Ni_{75}P_{16}B_6Al_3$ to be a highly paramagnetic system and apply our results to this system. The resistivity of $Ni_{75}P_{16}B_6Al_3$ is given in Fig. 7. It shows a minimum around 13 K. Above this minimum, the resistivity rises as $aT^2$ with $a=0.000\,12$ and below the minimum, it can be fitted as $b\sqrt{T}$ with $b=0.07$. A theoretical estimate of $a$ and $b$ can be obtained from Eqs. (46) and (42), respectively. To eliminate the $s$-$d$ coupling constant $\tilde{g}$, we compare the ratio between $a$ and $b$. Experimental value shows $b/a=5.8\times10^2$. Assuming the system has $\alpha_0=0.1$, $\epsilon_F=1$ eV and $\epsilon_F\tau=15$, we obtain $b/a=5\times10^2$ and $T_m\sim10$ K, which reasonably agree with the experimental values. Measurement on the paramagnetic spin susceptibility of this system is highly desirable in order to clarify this point and confirm whether spin susceptibility at low temperature shows $\sqrt{T}$ behavior.

There have been several attempts to explain the origin of resistivity minimum in amorphous metallic systems over the last twenty years.[28,29] Out of these, the Kondo model and the two-level-system model predict a $\ln(T)$ variation of resistivity below a certain temperature, and adding to it the contribution from scattering with phonons gives minimum in the resistivity. In the present model the resistivity in the entire low-temperature region is due to spin fluctuation which is fully electronic in origin. The resistivity minimum occurs at a temperature at which the crossover takes place between disorder correction and the pure spin-fluctuation contribution. In our earlier paper[14] we have seen that a similar kind of effect happens in various properties such as paramagnetic spin susceptibility, magnetization, NMR relaxation time, thermal expansion, etc. To understand the importance of the disorder quantum corrections in the amorphous systems more clearly, it will be of great relevance to measure the low-temperature behavior of above-mentioned properties in these systems. Since the Ni-rich amorphous Fe-Ni-based alloys near to the critical concentration $x_c$ is a weak ferromagnet,[26] it is a good system with which to carry out such investigations.

This work can be easily extended to the ferromagnetic phase also. There the spin waves will contribute to the resistivity as well as the spin fluctuations. For the clean limit it has been done earlier by Ueda and Moriya,[19] where they found that the electric resistivity vary as $T^2$ at low temperatures and as $T^{5/3}$ near to both sides of the transition temperature $T_c$. As we saw in previous sections, the disorder correction term comes basically from the modification of electron-spin-fluctuation vertex. This means that the disorder correction for conductivity in the ferromagnetic phase will have the same temperature variation as in the paramagnetic phase. So the disorder correction for electric conductivity for ferromagnetic systems vary as $\sqrt{T}$ for $T \ll \tau^{-1}$ and the thermal conductivity as $T^{3/2}$.

## APPENDIX

In this Appendix expressions for different components of self-energies and vertex functions are given. Equation (26) is solved to obtain different component of the vertex function and $\hat{\Gamma}^1$ is given by Eq. (27). Other components of the vertex function are given below:

$$\hat{\Gamma}^2 = \frac{1}{\sqrt{2}(1-\eta^{ar})} \begin{bmatrix} \eta^{rk} & \dfrac{1-\eta^{ar}+\eta^{rk}\eta^{rk}+\eta^{kk}+\eta^{ka}\eta^{ak}}{1-\eta^{ra}} \\ 1 & \eta^{ak} \end{bmatrix} , \tag{A1}$$

and an expression for $\tilde{\Gamma}$ can be written with the following substitution,

$$\begin{aligned} \hat{\tilde{\Gamma}}^1(q,\epsilon) &= \hat{\Gamma}^2(-q,\epsilon)\big|_{\eta\to\tilde{\eta}} , \\ \hat{\tilde{\Gamma}}^2(q,\epsilon) &= \hat{\Gamma}^1(-q,\epsilon)\big|_{\eta\to\tilde{\eta}} , \end{aligned} \tag{A2}$$

where

$$\tilde{\eta}^{\alpha\beta} = N_i v^2 \int \frac{d^3k}{(2\pi)^3} G_0^{\alpha}(k-q) G_0^{\beta}(k) .$$

The $\delta\hat{\Gamma}$ part of the vertex function [Eq. (26)] contributes only to $\delta\hat{\Sigma}$, which contain Poisson brackets. Since the Poisson bracket is already proportional to electric field and temperature gradient, we approximate $h(p) \simeq h_0(\omega)$ in this expression. Then the solution for $\delta\hat{\Gamma}$ becomes simple and the nonvanishing components are given by

$$\begin{aligned} \Delta\Gamma^1_{12} &= -(\delta\tilde{\Gamma}^2_{12})^* \\ &= \frac{i}{2\sqrt{2}(1-\eta^{ra})^2} N_i v^2 \int \frac{d^3k}{(2\pi)^3} [G_0^r(k)\{h_0(\epsilon)-h_0(\epsilon-\omega);G_0^a(k-q)\} \\ &\qquad + \{G_0^r(k);h_0(\epsilon)-h_0(\epsilon-\omega)\}G_0^a(k-q)] \end{aligned} \tag{A3}$$

and

$$\begin{aligned} \delta\Gamma^2_{12} &= (\delta\tilde{\Gamma}^1_{12})^* \\ &= \frac{i}{2\sqrt{2}(|1-\eta^{ra}|)^2} N_i v^2 \int \frac{d^3k}{(2\pi)^3} [h_0(\epsilon-\omega)G_0^r(k)\{h_0(\epsilon);G_0^a(k-q)\} + h_0(\epsilon)G_0^r(k)\{h_0(\epsilon-\omega);G_0^a(k-q)\} \\ &\qquad + h_0(\epsilon-\omega)G_0^a(k-q)\{h_0(\epsilon);G_0^r(k)\} + h_0(\epsilon)G_0^a(k-q)\{h_0(\epsilon-\omega);G_0^r(k)\}] . \end{aligned} \tag{A4}$$

Using the above expressions for the vertex functions in Eqs. (24) and (25), the self-energies are obtained. The total self-energy is the sum of the spin-fluctuation part ($\hat{\Sigma}_{\text{sf}}+\delta\hat{\Sigma}_{\text{sf}}$) and the disorder correction term ($\hat{\Sigma}_{\text{sf-imp}}+\delta\hat{\Sigma}_{\text{sf-imp}}$). The explicit expression for each of these self-energies is given below.

$$\begin{aligned} \Sigma^a_{\text{sf}}(p) &= \frac{i}{2} g^2 \int dq \{\chi^a(q) h(p-q)[G_0^r(p-q) - G_0^a(p-q)] + \chi^k(q) G^r(p-q)\} , \\ \Sigma^k_{\text{sf}}(p) &= \frac{i}{2} g^2 \int dq \{[\chi^r(q) G_0^r(p-q) + \chi^k(q) h(p-q) G_0^r(p-q)] + (\text{H.c})\} , \\ \delta\Sigma^a_{\text{sf}}(p) &= -\frac{1}{4} g^2 \int dq\, \chi^a(q) \{G_0^r(p-q) + G_0^a(p-q); h_0(\epsilon-\omega)\} , \\ \delta\Sigma^k_{\text{sf}}(p) &= -\frac{1}{4} g^2 \int dq [\chi^k(q)\{G_0^r(p-q); h_0(\epsilon-\omega)\} - (\text{H.c})] . \end{aligned}$$

The disorder correction term of the self-energy is given by

$$\begin{aligned} \Sigma^a_{\text{sf-imp}} &= \frac{i}{2} g^2 \int dq \frac{\chi^a(q) G_0^r(p-q)}{(1-\eta^{ar})^2} (\eta_1^{kr} + \eta_1^{ak}) , \\ \Sigma^k_{\text{sf-imp}} &= \frac{i}{2} g^2 \int dq \left[ \frac{\chi^r(q) G_0^a(p-q)}{(1-\eta^{ra})^2} \{(\eta_1^{rk} + \eta_1^{ka}) \eta_1^{ka}\} + (\text{H.c}) \right] , \end{aligned} \tag{A6}$$

where $\eta_1^{\alpha\beta}$ is obtained from $\eta^{\alpha\beta}$ neglecting the Poisson bracket term from Eq. (3).

The disorder correction term, which contains the Poisson brackets, has three terms. Writing $\delta\Sigma^{\alpha}_{\text{sf-imp}}=\delta\Sigma^{\alpha}_{1}+\delta\Sigma^{\alpha}_{2}+\delta\Sigma^{\alpha}_{3}$ with $\alpha=r,a,k$, each of these terms are explicitly given below. We keep only the most singular terms as $\omega\rightarrow 0$, $q\rightarrow 0$. $\Delta\Sigma^{\alpha}_{1}$ comes from the last expression of Eq. (25) and is given by

$$\delta\Sigma^{a}_{1}=\frac{g^2}{4}\int dq\, N_i v^2\int\frac{d^3k}{(2\pi)^3}\frac{\chi^{a}(q)G^{r}_{0}(p-q)}{(1-\eta^{ar})^2}[G^{a}_{0}(k)\{G^{r}_{0}(k-q);h_0(\epsilon-\omega)\}+\{G^{a}_{0}(k);h_0(\epsilon)\}G^{r}_{0}(k-q)]\;,$$

$$\begin{aligned}\delta\Sigma^{k}_{1}=-\frac{g^2}{4}\int dq\, N_i v^2\int\frac{d^3k}{(2\pi)^3}&\frac{\chi^{r}(q)G^{a}_{0}(p-q)}{(1-\eta^{ra})^2}\\ &\times[([G^{r}_{0}(k)\{G^{a}_{0}(k-q);h_0(\epsilon-\omega)\}+\{G^{r}_{0}(k);h_0(\epsilon)\}G^{a}_{0}(k-q)]h_0(\epsilon)\\ &\quad+\{G^{r}_{0}(k);h_0(\epsilon)\}G^{a}_{0}(k-q)[h_0(\epsilon)-h_0(\epsilon-\omega)])-(\text{H.c})]\;. \end{aligned}\tag{A7}$$

The self-energy contribution coming from the third and fourth terms of Eq. (25) are expressed by $\delta\Sigma^{\alpha}_{2}$ and given by

$$\Sigma^{a}_{2}(p)=-\frac{g^2}{4}\int dq\, N_i v^2\int\frac{d^3k}{(2\pi)^3}\frac{\chi^{a}(q)G^{r}_{0}(p-q)}{(1-\eta^{ar})^2}\{G^{r}_{0}(p-q);[h_0(\epsilon-\omega)-h_0(\epsilon)]\}\;,$$

$$\begin{aligned}\delta\Sigma^{k}_{2}=-\frac{g^2}{4}\int dq\, N_i v^2\int\frac{d^3k}{(2\pi)^3}&\frac{\chi^{r}(q)G^{a}_{0}(p-q)}{(1-\eta^{ra})^2}\\ &\times[([h_0(\epsilon)-h_0(\epsilon-\omega)]\{G^{a}_{0}(p-q);h_0(\epsilon)\}\\ &\quad+\{[h_0(\epsilon)-h_0(\epsilon-\omega)];G^{a}_{0}(p-q)\}h_0(\epsilon))-(\text{H.c})]\;, \end{aligned}\tag{A8}$$

and the term coming from the first two expressions are given by $\delta\Sigma^{\alpha}_{3}$, where

$$\delta\Sigma^{a}_{3}=\frac{ig^2}{\sqrt{2}}\int dq\frac{\chi^{a}(q)G^{r}_{0}(p-q)}{1-\eta^{ar}}\delta\tilde{\Gamma}^{2}_{12}\;,\quad \delta\Sigma^{k}_{3}=\frac{ig^2}{\sqrt{2}}\int dq\left[\frac{\chi^{r}(q)G^{a}_{0}(p-q)}{1-\eta^{ra}}\delta\Gamma^{1}_{12}h_0(\epsilon)+(\text{H.c})\right]\;. \tag{A9}$$

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