Magnetic relaxation in small-particle systems: $T \ln(t/\tau_0)$ scaling

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(Received 13 April 1993)

Although most of the experimental work dealing with the magnetic relaxation of systems characterized by the existence of an energy-barrier distribution have been analyzed in terms of the so-called logarithmic approximation, this is not a good approximation because obviously it cannot describe the behavior for the whole range of temperatures and times. An alternative method called $T \ln(t/\tau_0)$ scaling is proposed. This scaling method allows one to extrapolate the relaxation behavior at times that are experimentally completely unaccessible. From this scaling it is also possible to determine the attempt frequency $1/\tau_0$, and if a certain distribution of energy barriers is assumed the width σ and the mean energy barrier is also obtained. The validity of the logarithmic approximation is critically discussed.

I. INTRODUCTION

There is a broad variety of physical systems that show a time-dependent behavior of some of their physical properties because of the existence of energy barriers separating local minima corresponding to different equilibrium states of the system. Many of the efforts in this field have been devoted to the study of magnetic relaxation in systems with different degrees of disorder. For example: spin glasses where the distribution of energy barriers is due to the frustration of the competing magnetic interactions among the individual spins;[1,2] type-II superconductors in which vortices are submitted to pinning by defects and dislocations, and where the application of a magnetic field creates metastable states in the vortex lattice;[3,4] small-particle systems with a volume distribution and random orientation of easy axes which show blocking phenomena depending on the experimental time window;[5,6] and thin films and other magnetic materials for which the existence of energy barriers is a consequence of the competition between anisotropy and exchange energies.

However, most of the experimental data available in this field have been analyzed by using rough approximations or not well justified semiempirical expressions.[7–10] The main reason for this fact is often related to the relatively short interval of time during which relaxation data are recorded, two or three time decades, in the best situation. When this is the case, it is very difficult to distinguish among different relaxation laws.[11] Moreover, when the magnetization evolves slowly with the ellapsed time, the small curvature of the data in the observation time interval, can erroneously lead one to analyze the data in terms of oversimplified expressions, as for example, the so-called logarithmic approximation. There are two possibilities to avoid this problem. The most obvious solution is to extend the observation time interval, but this is almost impossible in most of the experimental setups. The second alternative is to search for a scaling hypothesis that allows one to bring all the relaxation data, obtained at different temperatures, in a unique master curve. By using the last procedure it is possible to cover a large number of time decades, since in all thermally activated problems, any change in the temperature of the system corresponds to a change in the time scale of the magnetic relaxation.

The simplest guess for the scaling law of a relaxing system governed by thermally activated processes is the so-called $T \ln(t/\tau_0)$ master curve, previously proposed by Prejean and Souletie[12–13] and co-workers[14] and already used for the study of the magnetic relaxation in spin-glass systems, which are characterized by the existence of a broad distribution of energy barriers hierarchically organized and high degree of magnetic frustration. Recently, a similar approach has been used by Barbara and Gunther to describe the magnetic relaxation close to the logarithmic regime, introducing what they call "barrier plot."[15] In this paper, we want to discuss the validity of the $T \ln(t/\tau_0)$ scaling law for the study of the magnetic relaxation in a disordered system consisting of an assembly of non- or weakly interacting entities, as is the case in the majority of single-domain particle systems, where the disorder is only due to the existence of a distribution of energy barriers which are blocking the direction of the magnetization vector of each entity. We also discuss how it can be used to obtain information about the time dependent behavior at long times that are usually experimentally unaccessible.

On the other hand, we discuss the validity of the commonly used "logarithmic approximation" to characterize and analyze the time-dependent behavior of systems with a distribution of relaxing entities. We also show how the inaccuracy of this method can give place to misunderstandings and misinterpretations in the analysis of the experimental data.

II. PHENOMENOLOGICAL BACKGROUND FOR THE $T \ln(t/\tau_0)$ SCALING

The decay of the magnetization of a distribution of single-domain particles is given by the relaxation law:[5]

$$M(t) = \int_0^\infty dE\, e^{-t/\tau(E)} f(E)\,, \tag{1}$$

where $f(E)$ is the distribution function of energy barriers

that have to be overcome by thermal fluctuations, in order to change the equilibrium magnetization direction of the particles. The exponential factor is the classical Boltzmann probability for a particle to change its equilibrium magnetization value and $\tau(E)$ is the relaxation time used in Néel's theory[16] and is given by

$$\tau(E)=\tau_0 e^{E/k_B T} , \tag{2}$$

where $1/\tau_0$ is an attempt frequency of the order 10^8–10^{12} s^{-1}.

Let us introduce the function $p(t,E)$, which is defined by

$$p(t,E)=e^{-(t/\tau_0)\exp(-E/k_B T)} . \tag{3}$$

Taking into account that $p(t,E)$, for a given time t varies abruptly from 0 to 1, as the energy barrier E increases, the usual simplification[17] consists on approximating $p(t,E)$ by a step function whose discontinuity $E_c(t)$ moves to higher values of E as time elapses. As a consequence, the integral is "cutoff" at the lower limit by the value of $E_c(t)$, which is the only time-dependent parameter, and the expression (3) is approximated by

$$M(t)\simeq \int_{E_c(t)}^{\infty} dE\, f(E) . \tag{4}$$

$E_c(t)$ corresponds to the energy-barrier value for which the function $p(t,E)$ has the inflection point and is given by

$$E_c(t)=k_B T \ln(t/\tau_0) . \tag{5}$$

From Eq. (5) it can be deduced that the remnant magnetization $M(t)$ obtained after integration over the energy barriers E is a function of the parameter $E_c(t)=k_B T\ln(t/\tau_0)$. The existence of this scaling variable implies that measuring the magnetization as a function of the temperature at a given time is equivalent to measure the magnetization as a function of the $\ln(t)$ at a fixed temperature. This time-temperature correspondence is characteristic of activated processes governed by the Arrhenius law.

The approximation of $p(t,E)$ by a step function cutting off the integral in (4) can be improved by taking into account the essential temperature and time dependence of $M(t)$. It is clear from (3) that the interval of energies for which $p(t,E)$ differs significantly from 0 and 1, for a given value of the time, will depend on the temperature. To have an idea of the influence of this region on $M(t)$ let us approximate $p(t,E)$ by (see Fig. 1)

$$p(t,E)\simeq \begin{cases} 0, & E\le E_1^* \\ r(E), & E_1^*\le E\le E_2^* \\ 1, & E\ge E_2^* , \end{cases} \tag{6}$$

where we have approximated $p(E)$ by a straight line passing by the inflection point of this function, which is placed at an energy E_c, with $p(E_c)=1/e$, a constant independent of time, and with slope equal to the derivative of $p(E)$ at the inflection point. This straight line has the following equation:

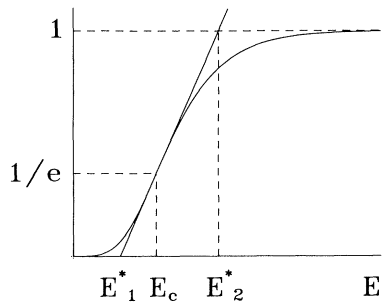

FIG. 1. The function $p(t,E)$ defined by Eq. (3), at an arbitrary time and temperature as a function of the energy barrier. The straight line represents the approximation described in the text.

$$r(E)=\frac{1}{e}\left[\frac{E}{k_B T}+\left[1-\ln\frac{t}{\tau_0}\right]\right] . \tag{7}$$

Only for $T=0$ does Eq. (7) reduce to the cutoff approximation. Then we can characterize $p(t,E)$ by its width, $\lambda=E_2^*-E_1^*=k_B Te$, which is time independent. Thus, as time elapses, $p(t,E)$ moves to higher values of the energy without changing its shape.

It is evident that for a given value of the scaling variable the inflection point of $p(t,E)$ is placed at a constant energy with respect to the distribution function, but with a width that increases linearly with the temperature. For this reason the validity of the scaling hypothesis is only determined by the validity of the cutoff approximation. As a result, the scaling will be fulfilled as long as the width of $p(t,E)$, which is approximately given by $k_B Te$, is small as compared to the width of the energy-barrier distribution function. Even though this condition seems to be very restrictive, this is the usual situation encountered in experimental observation of the magnetic relaxation. For instance, if a normal distribution of energy barriers centered at the reduced value 1 (reduced values defined as the energy divided by the value of the energy at the maximum of the distribution) is assumed for small-particle systems, the characteristic half width of this distribution ranges approximately from $\sigma=0.2$ to 1.0 in real systems.[18,19] The experimental observation of the magnetic relaxation in these systems is usually performed far below the mean blocking temperature of the particle magnetization, which for dc magnetization measurements is approximately $1/25k_B$ if the mean energy barrier is taken as one. For these temperatures the width of $p(t,E)$ is much less than $e/25$, which is small as compared to the total width of the energy distribution ($\sim 2\sigma$).

In order to verify all these assumptions we have calculated the decay of the magnetization for a small-particle system with a logarithmic-linear distribution of energy barriers (which is a more realistic distribution than the normal one) by numerical integration of expression (1). With the aim to obtain more general results, we will define reduced variables as follows: a dimensionless magnitude T_0 as $T_0=E_0/k_B$, where E_0 is the energy corresponding to the peak of the distribution, the reduced tem-

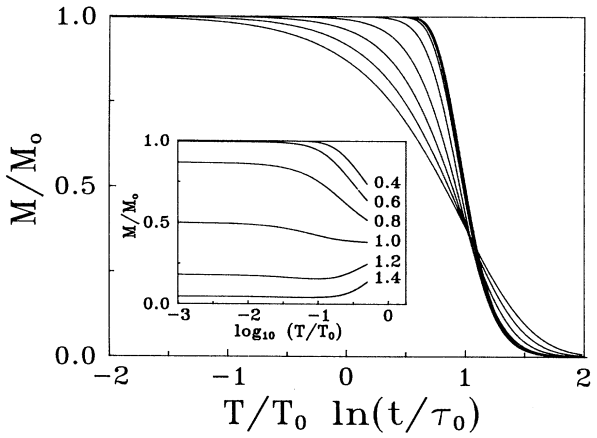

FIG. 2. The scaling plot of the calculated data by numerical integration of Eq. (1) with a logarithmic-linear distribution of energy barriers of dimensionless width, $\sigma=0.2$. The reduced temperatures T/T_0 corresponding to each curve are as follows: 0.001, 0.005, 0.01, 0.02, 0.04, 0.1, 0.2, 0.3, 0.4, and 0.5. The lowest reduced temperature corresponds to the highest curve. In the inset, the reduced magnetization as a function of the reduced temperature for constant value of the scaling variable E_c is shown. Each curve corresponds to a different value of E_c, indicated to the right of each curve. The log scale of the T/T_0 axis has been chosen for better observation of the behavior at temperatures below the blocking temperature.

perature of the system as T/T_0, and the reduced time as t/τ_0. The results of these calculations for three values of the σ parameter of the logarithmic-linear distribution, and reduced temperatures ranging from 0.001 to 0.5, are shown in Figs. 2, 3, and 4. The insets of these figures show the dependence of the magnetization for a constant value of the scaling variable E_c as a function of the reduced temperature. Clearly the scaling is valid in the range of temperatures for which a constant behavior of the magnetization is observed in the insets of Figs. 2, 3, and 4. From these figures, the obvious fact can be verified that the range of validity of the scaling increases as the width of the distribution increases. It is important to note, that in these figures, curves corresponding to

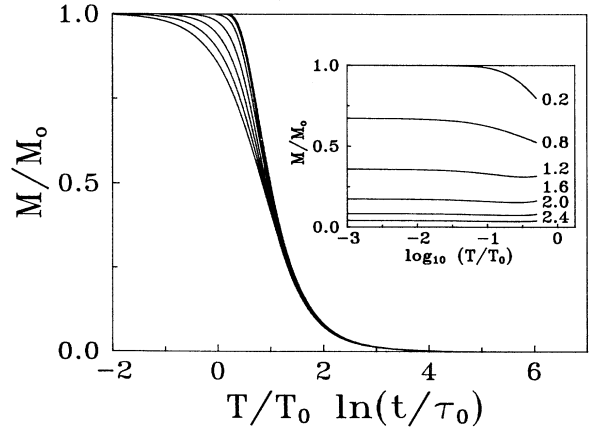

FIG. 3. Same as Fig. 2 but for $\sigma=0.5$.

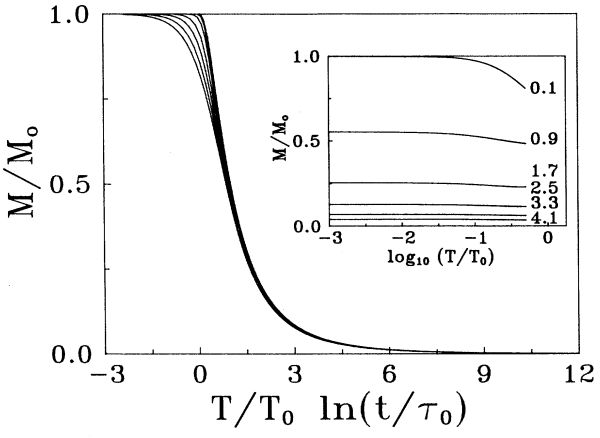

FIG. 4. Same as Fig. 2 but for $\sigma=0.8$.

temperatures much above the mean blocking temperature, which in reduced units is approximately 0.04, have been included and that at temperatures below or around the mean blocking temperature scaling is almost fulfilled in the whole range of the scaling variable $T/T_0 \ln(t/\tau_0)$ for the three widths of the studied energy distributions.

III. EXPERIMENTAL VERIFICATION OF THE $T \ln(t/\tau_0)$ SCALING

In order to verify the validity of the $T\ln(t/\tau_0)$ scaling law in real small-particle systems, the magnetic relaxation data obtained for three different samples were analyzed within the scope of the scaling hypothesis. The first two samples are both ferrofluids of Fe_3O_4 small particles dispersed in a hydrocarbon oil.[20] The average particle diameter ranges from 50 to 60 Å. Although the degree of dilution is not large enough to completely remove the interaction among particles, it was estimated to be very small.[21] The third sample consists on FeC small particles in stable dilution with a hydrocarbon oil that freezes at 19 °C.[22] The average diameter of these particles was measured to be 36 Å.[19] For this latter sample the results of the experiments on the magnetic relaxation are published elsewhere.[23] The procedure to measure the relaxation of the magnetization was the same for samples 1 and 3. It consists on cooling down the sample from a temperature above the blocking temperature, down to the measurement temperature, that is always lower than the blocking temperature, under an applied field of 10 Oe. Once the temperature is stabilized, the field was switched off and magnetization data versus time were recorded. The superconducting quantum interference device magnetometer used for this purpose was a commercial one with a modified procedure of measurement which is described in Ref. 24. Sample 2 was zero-field cooled from room temperature down to the measurement temperature. After stabilizing it, a field of 10 Oe was applied and the values of the magnetization versus time were recorded. So, in this latter case, it is not the magnetization decay vs time that is measured, but the establishment of an equilibrium magnetization value when a field is applied to a sample with initial zero magnetization.

According to the scaling hypothesis discussed in Sec. II of this paper, for each sample, all the different curves corresponding to different temperatures, would have to scale onto one single master curve when plotted as a function of the scaling variable $T\ln(t/\tau_0)$. In order to verify the validity of this model we try to scale the relaxation data of the referred samples. The procedure used for this purpose consists on plotting the relaxation curves in a M vs $\ln(t)$ plot and trying to connect each of them continuously with the adjoining curves corresponding to the nearest measured temperatures. To do that we shifted the experimental curves in the $T\ln(t)$ axis by an amount equal to $T\ln(\tau_0)$, where τ_0 is a characteristic time which governs the relaxation processes on an atomic scale. For each sample, τ_0 is the same for all of the measured temperatures and it was chosen to be the best in bringing all the curves into one. These values are given in Table I. The values obtained for samples 2 and 3 lie in the range of expected values in small-particle systems.[25] Moreover, in the case of sample 2, τ_0 is within the error limits corresponding to the value reported in Ref. 19, which was obtained by other techniques. However, the value of τ_0 for sample 1 is somewhat larger than the expected one. This fact may be related to the lower degree of dilution of sample 1 with respect to sample 3, which increases the strength of the dipolar interactions among particles. In any case, it is not clear how these interactions would affect the atomic scale constant τ_0 and how the relaxation law would be modified.

Due to the inaccuracy in the determination of the initial value of the magnetization (for each temperature, the value of M at $t=0$), it was also necessary to normalize the experimental data dividing them by an arbitrary reference magnetization value M_0.

In Figs. 5, 6, and 7 the results of this scaling for the three samples are shown. One of the most interesting aspects of these results is that, in fact, measuring the relaxation at a given temperature is completely equivalent to measure it at a different temperature but shifting the observation time window according to the law $T\ln(t/\tau_0)$. In this sense, the method enables us to obtain the relaxation curve at a certain temperature, in a time range that is not experimentally accessible, by simply dividing the $T\ln(t/\tau_0)$ axis by this temperature. For instance, in the case of sample 3, we can obtain the relaxation curve at the lowest measured temperature of 1.8 K at times as large as 10^{119} s, which is obviously an experimentally inaccessible time. For sample 1, where the highest temperature that we have measured was 37 K, we are observing the relaxation curve corresponding to 2 K at times as large as 10^{173} s.

TABLE I. Values of the parameters obtained by fitting the experimental scaled curves to Eq. (1) with a logarithmic-linear distribution of energy barriers.

Sample	τ_0 (s)	σ	T_0 (K)
1	$(4.7\pm5)\times10^{-7}$	0.85±0.05	245±50
2	$(9.4\pm5)\times10^{-13}$	1.02±0.05	627±80
3	$(3.5\pm5)\times10^{-11}$	0.44±0.05	287±50

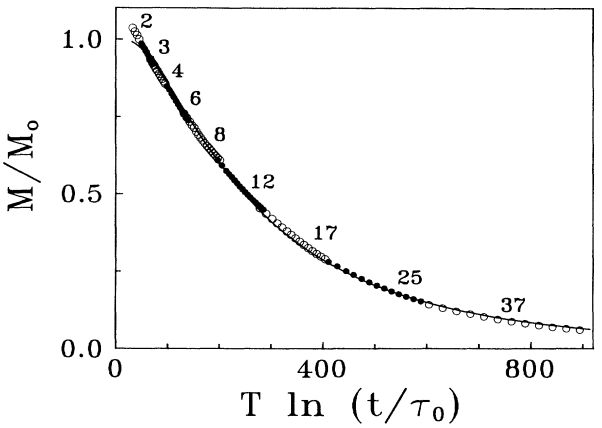

FIG. 5. The resulting scaling for a ferrofluid sample composed of Fe_3O_4 small particles is shown. The figure shows the reduced magnetization as a function of the scaling variable. Open and full circles correspond alternatively to adjoining temperatures, that are indicated above the corresponding interval. The solid line is the theoretical curve calculated by using Eq. (1) and the values of the fitting parameters indicated in Table I.

One way to test the validity of these scaling plots consists in trying to reproduce the experimental curves shown in Figs. 5, 6, and 7 by using a reasonable model for the magnetic relaxation of these systems. As was done in Sec. II, we can assume that the evolution toward equilibrium of a system consisting of an assembly of randomly oriented particles is governed by the Arrhenius law with a given distribution of energy barriers. The particular mathematical form of this distribution is not very important as far as it agrees with some reasonable conditions, and usually a logarithmic-linear distribution is assumed. Consequently we have fitted the experimental master curves of Figs. 5, 6, and 7 to the theoretical one calculated from Eq. (1) at an intermediate temperature and using a logarithmic-linear distribution of energy barriers. The free parameters for the fitting were σ and T_0, which have the same meaning as in Sec. II, and the obtained values are summarized in Table I, while solid lines in Figs. 5, 6, and 7 represent the corresponding theoretical curves. In

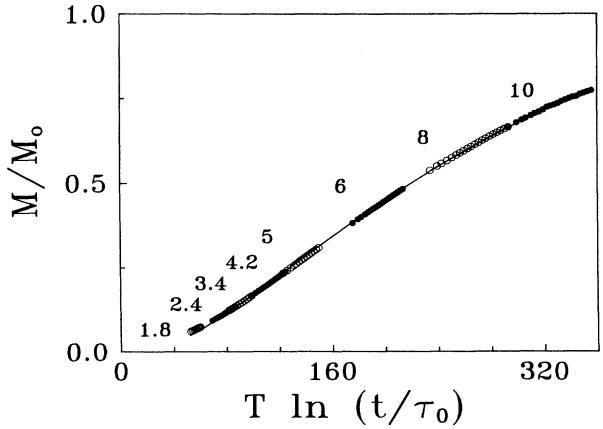

FIG. 6. Same as Fig. 5 for the case of sample 2.

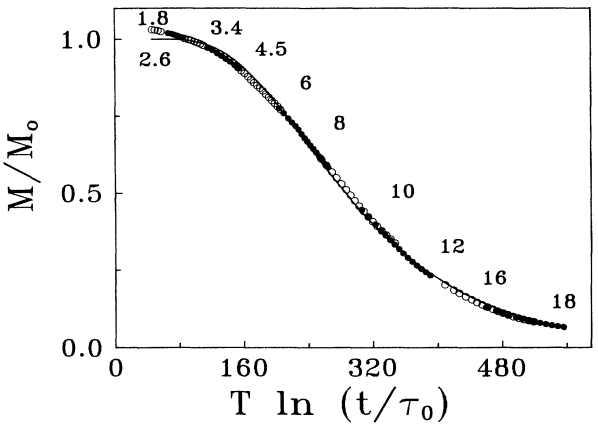

FIG. 7. Same as Fig. 5 for the FeC small-particle sample.

all the cases, reasonably good agreement is achieved and the only significant departures between theoretical and experimental data are observed for small values of the scaling variable which for a given temperature corresponds to short times. In our case, this small discrepancy could indicate that the logarithmic-linear distribution is not very realistic for the lowest energy barriers (which are overcome at shortest times) and underestimates their contribution to the total relaxation process of the system. Recently, some authors[23,26–28] have also suggested that below a certain crossover temperature, nonthermal processes (quantum tunneling) dominate the evolution of the magnetization towards equilibrium in zero-dimensional systems with high or moderate degree of anisotropy as is the case of the small-particle systems studied in this paper. The main effect of this quantum contribution is to increase the rate of variation of the magnetization below the crossover temperature, with respect to the thermal value. This fact could explain the small discrepancies observed for small values of the scaling variable, taking into account that this interval of the master curve corresponds to the relaxation curves measured at the lowest temperatures.

Samples 1 and 3 were measured in the same way, with no applied field while recording the data, so that what one is measuring is the decay of the magnetization after field cooling the sample and removing the field. So in these two cases the existence of a distribution of energy barriers is only due to the distribution of particle sizes, which usually can be well approximated by a logarithmic-linear distribution. As a consequence, the fitting values for the parameter σ can be compared with those obtained by other more direct granularimetric methods. For instance, in the case of sample 3, σ was also determined to be 0.45 ± 0.03 from the distribution of blocking temperatures,[19] which is in very good agreement with that deduced by the scaling procedure. From the values of the anisotropy constant and the mean size of the particles, reported in Ref. 19, one can evaluate the value of the parameter T_0 for the FeC particles to be 320 ± 70 K, which also compares well with the corresponding value reported in Table I. For sample 2, on the contrary, we deduce a very large value of σ which may seem unrealistic. This large value of σ can be justified taking into account that, in this case, we are not measuring the decay of the magnetization at zero magnetic field but the establishment of a new equilibrium when a field is applied after zero-field cooling the sample. The presence of this nonzero magnetic field, while relaxation data are recorded, modifies the shape of the energy-barrier distribution in a complicated manner, decreasing the height of those barriers that correspond to particles for which the easy direction of magnetization forms an angle close to 45 degrees with the magnetic-field vector. If one tries to fit this distorted distribution to a logarithmic-linear function a larger value of the distribution width than the corresponding particle size distribution value is obtained.

IV. ON THE LOGARITHMIC APPROXIMATION

If $f(E)$ is nearly constant in the interval of energy barriers which can be overcome during the observation time, then Eq. (4) can be approximated by[29]

$$M(t)\simeq k_B T f(E_c)\ln(t/\tau_0)\ , \tag{8}$$

which is the so-called logarithmic approximation.

For a better understanding of the limits of validity of this logarithmic approximation, let us consider the simple case of a flat distribution of energy barriers: $f(E)=1/\sigma$ for $E_1\le E\le E_2$ with $\sigma=E_2-E_1$ and $f(E)=0$, otherwise, and let us look more carefully at the function $p(t,E)$.

At least, for a uniform distribution of energy barriers, the range of validity of the logarithmic decay of the magnetization, at a fixed temperature, is given by the instants of time at which the function $p(t,E)$ crosses the beginning E_1 and end E_2 of the distribution function of energies. At every instant of time t the center of $p(t,E)$ is at $E_c(t)=k_B T\ln(t/\tau_0)$, and this means that it will arrive at the beginning of the distribution E_1 at an instant of time

$$\ln\left[\frac{t_1}{\tau_0}\right]=\frac{E_1}{k_B T} \tag{9}$$

and to the end at

$$\ln\left[\frac{t_2}{\tau_0}\right]=\frac{E_2}{k_B T}\ . \tag{10}$$

Given that we have approximated $p(E)$, in the range $[E_1^*,E_2^*]$, by a straight line (see Sec. II), we can estimate the widths $\lambda_1=E_c-E_1^*$ and $\lambda_2=E_2^*-E_c$ of $p(t,E)$ to be

$$\lambda_1=k_B T;\quad \lambda_2=k_B T(e-1)\ . \tag{11}$$

The logarithmic law will be only valid when the whole $p(t,E)$ curve is inside the region $[E_1,E_2]$; i.e., in the range of times

$$\frac{E_1+\lambda_1}{k_B T}<\ln\left[\frac{t}{\tau_0}\right]<\frac{E_2-\lambda_2}{k_B T}\ . \tag{12}$$

Then the logarithmic law is valid for the whole range of times only in the case of an uniform and infinite distribution of energy barriers,[17] which of course, is absolutely

nonphysical.

If the width of the energy distribution σ is much larger than that of $p(t,E)$ λ at a given temperature, the logarithmic approximation is valid in a wide range of times. In the opposite case, when $\lambda \ll \sigma$, the range of validity can be very small (the logarithmic approximation could be no longer valid), not only because $p(t,E)$ does never really lie completely inside the peak of the distribution, but also because the approximation of $p(E)$ by a straight line is poor. It is worth noting at this point that, even in the case of the uniform energy-barrier distribution, which is the most favorable candidate for a logarithmic law, the validity of the logarithmic approximation depends crucially on the temperature of the system, and not only on the width or the shape of the distribution.

For the case of a general distribution of energy barriers, such as a logarithmic-linear or similar distributions found in real systems, the logarithmic approximation is, strictly speaking, nowhere valid, and it is possible to choose other laws for $M(t)$ (such as potential or stretched exponential laws common in the literature, or even a single exponential[11]) that fit experimental data with equal or similar success compared to the logarithmic law, which assumes a uniform distribution of energies. The only region for which a logarithmic law is a reasonable approximation is the range of times corresponding to energy barriers near the maximum of the distribution function, as long as it can be considered uniform around this value. So we can estimate the range of times for which logarithmic approximation will be approximately accomplished simply substituting E_1 and E_2 in Eq. (12) by the position of the inflection points of the energy-barrier distribution. For instance, in the case of a normal distribution these positions are $E_0-\sigma$ and $E_0+\sigma$, where σ is the standard deviation of the distribution and E_0 is the position of the maximum. If a logarithmic-linear distribution is assumed the inflection points are located at

$$E_1;E_2=E_0\exp[-1.5\sigma^2\pm\sqrt{0.25\sigma^4+\sigma^2}]\ , \tag{13}$$

where σ is the dimensionless width of the logarithmic-linear distribution.

As an example, if we take the values of the parameters σ and T_0 obtained in the analysis of the FeC system (see Table I), the region where logarithmic decay is a reasonable approximation in the master curve, from Eqs. (13) and (12), to be $140<\ln(t/\tau_0)<340$, which is in good agreement with the data shown in Fig. 7. This last result has very important experimental implications. If magnetic relaxation data are recorded at different temperatures only in a few time decades (for example, from 10 to 10^4 s) many of the obtained curves may seem logarithmic, due to the limited extension of the experimental time window. In the case of the FeC particles almost all the relaxation curves corresponding to the range of temperatures between 1.8 and 18 K follow approximately linear behavior when they are plotted altogether in a logarithmic scale of time, even though only those corresponding to temperatures ranging between 6–10 K follow approximately true logarithmic decay in the experimental time window, as can be demonstrated from the scaling plot (Fig. 7). Of course, all of these considerations can be applied to the rest of samples studied in the preceding section and, as far as we are concerned, to many of the results published in the literature. So, in conclusion, the logarithmic decay is only an accurate approximation in a time window whose extension is temperature dependent. For a given sample the extension and position of this time window can be estimated from the mean height of the energy barriers and the width of the energy distribution by using Eq. (12). Attempting to fit experimental data to a logarithmic law outside this region will clearly result in fitting parameters that have no physical sense at all, as has been previously remarked by other authors.[11]

V. CONCLUSIONS

The so-called $T\ln(t/\tau_0)$ scaling, previously used in describing the magnetic relaxation in spin-glass systems, has been applied in the case of systems composed of an assembly of noninteracting or weakly interacting entities characterized by the existence of an energy-barrier distribution separating local minima. One of the most important benefits of this method is that as a consequence of the existence of the scaling variable E_c, measuring the magnetization as a function of the temperature at a given time is completely equivalent to measuring the magnetization as a function of $\ln(t)$ at a given temperature. Therefore, this method enables one to extrapolate the relaxation behavior of the system towards equilibrium at times completely inaccessible from the experimental point of view and also it gives the value of the attempt frequency $1/\tau_0$, and if a certain distribution of energy barriers is assumed, the values of T_0 and the width σ are also obtained.

The validity of the logarithmic approximation has been also discussed as a function of observation time window, temperature, and the shape of the energy-barrier distribution. A phenomenological criterion has been proposed to estimate the range in which the approximation is accurate enough. In any case, the information obtained by using the logarithmic approximation is poorer than that obtained from the scaling hypothesis because from the former only the value of the distribution at E_c can be determined. Moreover, one must always bear in mind, that the logarithmic law only characterizes the part of the system with energy barriers lying in the experimental time window and it is not characterizing the time-dependent behavior of the whole system of relaxing entities. In this way, systems with very different distribution functions but with similar shape around the values of the energy barriers, which are relaxing at a given observation time, may be characterized by the same parameters of the relaxation function, despite the fact that the structure of the barriers is very different, the only thing in common between them being the behavior in the time window where the logarithmic law is valid for every one of them.

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