Phase coherence and nonadiabatic transition at a level crossing in a periodically driven two-level system

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(Received 14 September 1992)

Some aspects of the Landau-Zener-type transition of a periodically driven two-level system with dephasing are clarified. Under the condition that the transition is well localized around the level-crossing region, the transition dynamics for the typical cases is described by simple formulas; namely, by the transfer-matrix method for the long-range coherence, by the application of the incoherent Landau-Zener formula for the short-range coherence, and by the strong-dephasing formula in the limit of complete decoherence.

The transition dynamics of two-level systems driven by an externally controlled parameter has long been a subject of considerable interest. There are many topics that can be classified in this category: the two-level atoms in the intense electromagnetic fields,[1-3] the so-called adiabatic rapid passage in magnetic resonance[4] or its optical analog,[5-7] the quantum tunneling in double-well potentials modulated by external fields,[8,9] to name but a few. Specifically, recent progress in microfabrication techniques has brought about a renewed interest in this issue in connection with the modulated tunneling in mesoscopic or macroscopic quantum systems.[10]

One of the central problems in this subject is the nonadiabatic transition at level crossings. As a prototype model, consider a two-level system $|1\rangle$ and $|2\rangle$, the energies $\epsilon_1(t)$ and $\epsilon_2(t)$ of which are driven by some time-dependent parameter and undergo multiple crossings. Assume that the two states are coupled with each other through an off-diagonal coupling constant $J(>0)$. In the case where the transition is well localized around the level-crossing region, the adiabaticity is characterized by the ratio $J^2/\hbar|v|$, where $v\equiv\partial\{\epsilon_1(t)-\epsilon_2(t)\}/\partial t$ is the velocity of the change of the diabatic energy difference measured at the crossing. The probability of the transition $|1\rangle\rightarrow|2\rangle$ after the crossing is given by the Landau-Zener formula[11]

$$P_{\mathrm{LZ}}=1-\exp(-2\pi J^2/\hbar|v|)\ , \tag{1}$$

in those cases where the crossing region can be regarded as being swept with a constant velocity.

In condensed-matter physics, it often occurs that the system is subject to dephasing (phase relaxation) due to the stochastic fluctuation of the energy caused by the perturbation of the surrounding medium. The effect of the energy fluctuation and dephasing on the transition probability at level crossing has been investigated by the present author for a single crossing.[12,13] The purpose of the present paper is to clarify some aspects of the phase coherence in the long-time behavior of the two-level system that undergoes multiple crossings. We focus our attention to the case where the transition occurs effectively within a short-time interval around the crossing times. It will be shown below that, under this condition, the transition dynamics are well described by simple formulas in some typical cases.

The Hamiltonian is written as

$$H(t)=\tfrac{1}{2}\epsilon(t)(|1\rangle\langle 1|-|2\rangle\langle 2|)+J(|1\rangle\langle 2|+|2\rangle\langle 1|)\ , \tag{2}$$

without loss of generality. For the sake of definiteness, we assume a sinusoidal time dependence $\epsilon(t)=A\cos\omega t$ for the diabatic energy difference. The two diabatic states $|1\rangle$ and $2\rangle$ cross with each other at the time $t_n=(n-\frac{1}{2})\pi/\omega$, $n=1,2,3,\ldots$, with the velocity of the change of the energy difference $|v|=A\omega$. Hereafter, the unit $\hbar=1$ is adopted. We impose the condition

$$A/J\gg 1\ , \tag{3}$$

which guarantees the assumption that the transition is well localized around the times $t=t_n$ as compared with the time interval π/ω between the successive crossings, since the duration of time that the system resides in the transition region is given by $J/|v|(=J/A\omega)$ in the order of magnitude.

We assume a simple Markoffian dephasing. The transition dynamics is then described by the equation of motion for the density matrix:

$$\rho(t)=\sum_{i,j=1,2}\rho_{i,j}(t)|i\rangle\langle j|\ , \tag{4}$$

as

$$i\frac{d\rho}{dt}=[H(t),\rho]+i\tilde{\Gamma}\rho\ , \tag{5}$$

where

$$\{\tilde{\Gamma}\rho\}_{i,j}\equiv(\delta_{i,j}-1)\Gamma\rho_{i,j}\ . \tag{6}$$

Here, Γ is the phase decay constant and is the inverse of the dephasing time T_2.

Let us calculate the probability $P(t)$ that the system is in $|2\rangle$ at time t with the initial condition that it starts from $|1\rangle$ at time $t=0$. Under the assumption (3), the time dependence of $P(t)$ can be decomposed into two components; one is the abrupt change due to the transition at level crossings, and the other is the rapid oscillation with small amplitude around the stationary point of $\epsilon(t)$. By smoothing out this rapid oscillation, we can define the averaged probability P_n to find the system in $|2\rangle$ after the nth crossing.

We note that there are three typical regimes according to the relative length of the three characteristic time scales, π/ω, $J/|v|$, and Γ^{-1}.

Long-range coherence

In the case $\Gamma^{-1} \gg \pi/\omega$, the successive crossings are regarded essentially as a coherent process. The transition at a level crossing is described by the transfer matrix M given by[14]

$$M = \begin{bmatrix} \sqrt{q} & \sqrt{1-q}\,e^{-i\phi} \\ -\sqrt{1-q}\,e^{i\phi} & \sqrt{q} \end{bmatrix} , \tag{7}$$

where $q \equiv \exp(-2\pi\delta)$, with $\delta \equiv J^2/|v|$. The (i,j) element represents the transition $|j\rangle \to |i\rangle$. Note that we have defined the *transition* not between the adiabatic states but between the diabatic states $|1\rangle$ and $|2\rangle$. The above M corresponds to the case where $|1\rangle$ crosses $|2\rangle$ from the lower-energy side. In the opposite case, M should be replaced by its transpose $\tilde{M}$. The phase factor ϕ is called the Stokes phase and is given by

$$\phi = \pi/4 + \arg\Gamma(1-i\delta) + \delta(\ln\delta - 1) , \tag{8}$$

where $\Gamma(z)$ is the Γ function. The Stokes phase is a monotonously decreasing function of δ and takes the following limiting values in the adiabatic $(\delta\to\infty)$ and the diabatic $(\delta\to 0)$ limits: $\phi(\delta\to\infty)=0$ and $\phi(\delta\to 0)=\pi/4$.

The outgoing state at the nth crossing and the incoming state at the $(n+1)$th crossing are connected by the propagator

$$G_n = \begin{bmatrix} \exp[-(-1)^n i\alpha] & 0 \\ 0 & \exp[(-1)^n i\alpha] \end{bmatrix} , \tag{9}$$

when $\alpha \equiv 2\int_0^{\pi/2\omega}\sqrt{\{\epsilon(t)/2\}^2+J^2}\,dt\,(\simeq A/\omega)$. The state vector $\mathbf{x}_n$ after the nth crossing is obtained by operating M, $\tilde{M}$, and G_n successively on the initial state $\mathbf{x}_0 \equiv \binom{1}{0}$ as $\mathbf{x}_1 = \tilde{M}\mathbf{x}_0$, $\mathbf{x}_2 = MG_1\mathbf{x}_1$, $\mathbf{x}_3 = \tilde{M}G_2\mathbf{x}_2$, etc., aside from an irrelevant common phase factor. After some manipulation of 2×2 matrices, we find the formula for P_n as

$$P_{2n-1} = |\beta_{2n-2} + \sqrt{1-q}\,e^{-i(\alpha+\phi)}\beta_{2n-1}|^2 ,$$
$$P_{2n} = q|\beta_{2n}|^2 , \quad n=1,2,3,\ldots, \tag{10}$$

where β_n is defined by the recursive relation

$$\beta_{n+1} = -2i\{\sqrt{1-q}\,\sin(\alpha+\phi)\}\beta_n + \beta_{n-1} , \tag{11}$$

with $\beta_0=0$, $\beta_1=1$. Probability P_1 after the first crossing is given by the Landau-Zener formula (1), but the interference between the paths to reach $|2\rangle$ becomes very important for multiple crossings.

In Fig. 1(a), an example of the numerical calculation of $P(t)$ is shown for $J/\omega=3$, $A/\omega=45$, and $\Gamma=0$. The dashed line is the prediction by formula (10). The agreement is fairly good if the rapid oscillation around the stationary point is smoothed out. Note that the interference between the two paths $|1\rangle\to|2\rangle\to|2\rangle$ and $|1\rangle\to|1\rangle\to|2\rangle$ to reach $|2\rangle$ at $t\simeq 2\pi/\omega$ works destructively and strongly suppresses P_2 for these parameter values. In general, the sequence $\{P_n\}$ shows a rather

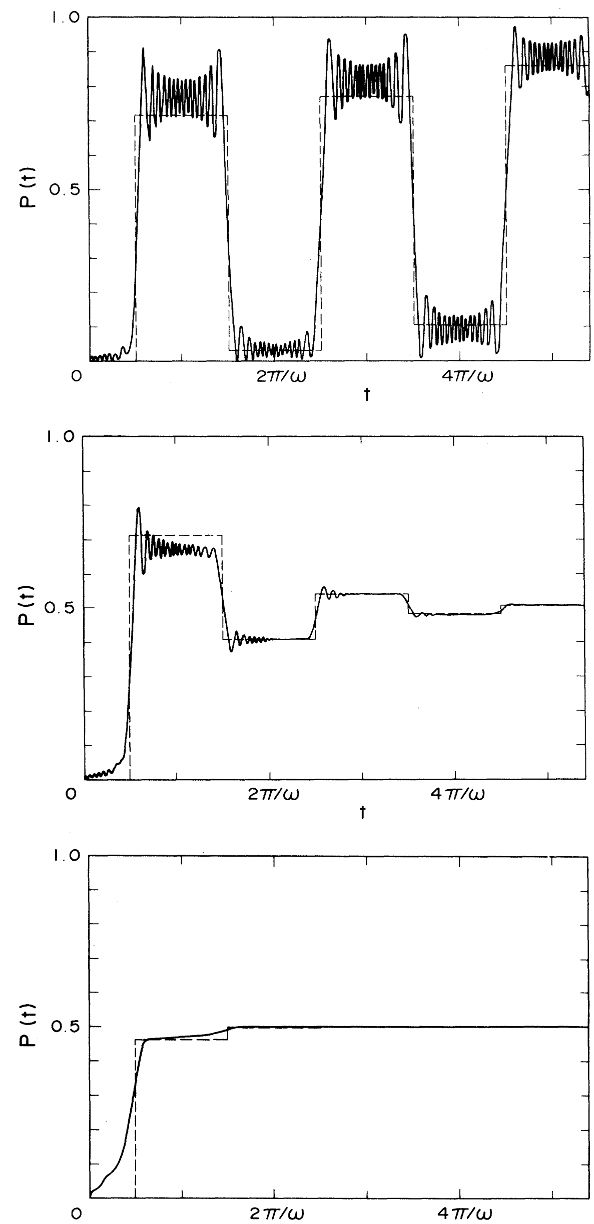

FIG. 1. (a) The probability $P(t)$ for $J/\omega=3$, $A/\omega=45$, and $\Gamma=0$. The dashed line is the value of P_n by formula (10). (b) The same as (a) for $J/\omega=3$, $A/\omega=45$, and $\Gamma/\omega=0.7$. The dashed line is the value of P_n by formula (12). (c) The same as (a) for $J/\omega=3$, $A/\omega=45$, and $\Gamma/\omega=10$. The dashed line is the value of P_n by formula (15).

complicated behavior for the coherent case. In the case that Γ is very small but finite, the coherence is gradually lost for $t \gtrsim \Gamma^{-1}$ and $P(t)$ slowly tends to $\frac{1}{2}$ for $t \to \infty$.

Short-range coherence

If the parameter values satisfy the inequality $\pi/\omega \gg \Gamma^{-1}$, the phase memory is lost during the propagation between the successive crossings. Let the probability of transition $|1\rangle \to |2\rangle$ for a signal crossing under the influence of dephasing be P_Γ. Since only the diagonal terms of the density matrix $\rho(t)$ survive during the propagation, the probability P_n is calculated by applying P_Γ and $1-P_\Gamma$ to each incoming state $|1\rangle$ and $|2\rangle$ at the nth crossing, respectively, and by taking the weighted sum of the probability to find the system in $|2\rangle$ after the crossing. By solving a recursion equation, we find $P_n = \{1-(1-2P_\Gamma)^n\}/2$. Most interesting is the case when the inequality $\Gamma^{-1} \gg J/|v|$ is satisfied in addition to the one above. Then each crossing event is still regarded as a coherent process, as shown in Ref. 13. We may put $P_\Gamma \simeq P_{LZ}$ in this case and find

$$P_n = \tfrac{1}{2}\{1-(1-2P_{LZ})^n\} \ . \tag{12}$$

Note that P_n tends to $\frac{1}{2}$ in the limit $n \to \infty$ as a monotonously increasing function for $0 < P_{LZ} < 0.5$, but as an oscillating function for $0.5 < P_{LZ} < 1$.

In Fig. 1(b), the numerical value of $P(t)$ is shown by the solid line for $\Gamma/\omega = 0.7$, with the same values of J and A as in Fig. 1(a). The exact value is well reproduced by formula (12), which is shown by the dashed line. Note that the interruption of the destructive interference dramatically increases the value of $P(t)$ at $t \simeq 2\pi/\omega$, as compared with the case $\Gamma = 0$ shown in Fig. 1(a).

Strong decoherence

In the case of strong dephasing $J/|v| \gtrsim \Gamma^{-1}$, the whole process becomes incoherent. The off-diagonal terms of $\rho(t)$ are strongly damped at each moment. Under this condition, the equation of motion (5), or equivalently the Bloch equation, can be solved approximately to yield

$$P(t) = \frac{1}{2}\left\{1 - \exp\left[-4J^2\Gamma \int_0^t \{\epsilon(\tau)^2 + \Gamma^2\}^{-1} d\tau\right]\right\} . \tag{13}$$

This is an extension of the well-known formula for the diffusion limit of the transition robability of the *static* two-level system. In the case where the level-crossing region can be regarded as being swept with a constant velocity, we put $\epsilon(t) = vt$ and replace the integral domain by $(-\infty, \infty)$ in the above formula. The transition probability P_{SD} for a single crossing in the strong-dephasing limit is then obtained as[12]

$$P_{SD} = \tfrac{1}{2}\{1 - \exp(-4\pi J^2/|v|)\} \ . \tag{14}$$

This formula should be contrasted with P_{LZ}. It has been shown that generally the inequality $P_{LZ} \geq P_\Gamma \geq P_{SD}$ holds for a fixed value of $J^2/|v|$. The probability P_n in the strong-dephasing limit is given by

$$P_n = \tfrac{1}{2}\{1 - \exp(-4n\pi J^2/|v|)\} \ . \tag{15}$$

In Fig. 1(c), the calculated $P(t)$ is shown for $\Gamma/\omega = 10$ with the same value of J and A as in Figs. 1(a) and 1(b). The dashed line is the value by formula (15). See how the decoherence changes the transition dynamics of the level-crossing system to the diffusionlike process as Γ is increased in the sense that $P(t)$ tends to $\frac{1}{2}$ for $t \to \infty$.

In the present work, the effect of dephasing on the transition dynamics of a periodically driven two-level system has been investigated by the most simplified model. In many cases, homogeneous dephasing is caused by the random fluctuation of the energy levels due to perturbation by elementary excitations such as phonons.[15] The effect of the dissipative interaction on the level-crossing system has been a subject of theoretical interest.[10,12,16,17] It has been shown that at high temperature and in weak coupling, the dissipative interaction can be regarded as mainly causing the fluctuation in energy, while at low temperature and in strong coupling, the energy relaxation becomes important.[12] The theoretical prediction presented here would be examined experimentally for two-level systems weakly coupled with the environment. The effect of dephasing on the nonadiabatic transition has been studied experimentally for the optical adiabatic rapid passage in localized centers in solids,[6] and in the gas phase[7] for a single crossing. More recently, Spreeuw *et al.*[18] demonstrated that a classical analog of the Landau-Zener transition can be realized by using an optical-ring resonator. It will be worthwhile to clarify such a fundamental quantum-mechanical process in detail by using highly controlled optical techniques.

I thank Professor S. Takagi and Professor T. Tsuzuki for valuable discussions. This work was partially supported by the Grant-in-Aid from the Ministry of Education of Japan.

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