Effects of collision retardation on hot-electron transport in a two-dimensional electron gas

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The effect of collision retardation on hot-electron transport in a two-dimensional electron gas is examined using an ensemble Monte Carlo simulation. We find that collision retardation (i.e., a nonzero collision duration) tends to make the electrons hotter by suppressing energy-relaxing collision events. Collision retardation also increases the steady-state drift velocity and high-field mobility by suppressing momentum-relaxation events. Finally, it also increases velocity overshoot somewhat.

Hot-electron transport within the semiclassical formalism has traditionally been modeled by the Boltzmann transport equation (BTE). The assumption made in applying the BTE is that the *duration* of individual collision events suffered by electrons is vanishingly small compared to quasiparticle lifetimes or mean times between successive collisions.[1,2] If this assumption is to be avoided, then one must either resort to the full quantum kinetic equation[3] (such as the Kadanoff-Baym-Keldysh equation, which is much more difficult to solve than the BTE) or incorporate the effects of a finite collision duration (collision retardation) in the BTE in some heuristic but appropriate manner.

Recently, the effects of collision retardation were incorporated phenomenologically in the BTE.[4] The BTE is solved by Monte Carlo simulation, and in the simulation, a scattering event is treated as a true scattering event only if a uniform random number in the interval [0,1] is larger than the quantity $\exp[-t/\tau_d]$, where t is the time that elapsed since the previous collision and τ_d is the collision duration time for the event. If the random number is smaller than this quantity, then the event is considered a self-scattering event. The collision duration time τ_d is assumed to be equal to $h/(E-E_{\mathrm{th}})$, where E is the initial energy of the colliding electron and E_{th} is the threshold energy for the scattering process. This expression for τ_d is derived from Landau's model for metals[5] (Fermi-liquid theory). In addition, Lipavsky *et al.*[6] have shown that τ_d, calculated from this expression, is identical with the quasiparticle formation time associated with the single-particle propagator. Using this expression to calculate τ_d (as was done in Ref. 4), we have studied the effects of a finite collision duration on hot-electron transport in a two-dimensional electron gas, using the algorithm proposed in Ref. 4.

The test system that we chose for our simulation is a rectangular quantum well of length 1 μm, width 10 μm, and well thickness 100 Å. The confining potentials in both transverse directions are infinite (hardwall boundary conditions). The well material is GaAs and the lattice temperature is assumed to be 40 K. Electrons are injected at the left contact from a Maxwellian distribution and the simulation proceeds just as described in Ref. 7. There are, however, two differences between our approach and that of Ref. 7. We do not include space-charge effects by solving the Poisson equation at every time step in the Monte Carlo simulation, and instead of using a full-band Monte Carlo, we chose an approximate analytical model for the band structure of GaAs, which gives the energy dispersion relation as

$$\frac{\hbar^2 k^2}{2m^*}=E(1+\alpha E)\ , \tag{1}$$

where E and k are the energy and wave vector, respectively, m^* is the effective mass at the band bottom, and α is the nonparabolicity factor. The parameters m^* and α are different for the three different conduction-band valleys in GaAs and their values are chosen from Ref. 8. Since the electric field in our simulation is quite low (only 500 V/cm), we believe that the above approximate analytical relation for the band structure is adequate for our purpose. Note that it is necessary to keep the electric field low in order to ensure that the collision retardation time τ_d is typically much smaller than the mean time between collisions. This situation is necessary for the algorithm of Ref. 4 to be valid.[9]

In the simulation, we considered intrasubband and intersubband nonpolar acoustic-phonon scattering, intrasubband and intersubband polar optical-phonon scattering, electron-electron scattering, and intervalley scattering. Piezoelectric (polar acoustic-phonon) scattering, nonpolar optical-phonon scattering, and remote ionized impurity scattering were neglected since they are not very important in modulation-doped GaAs quantum wells at the lattice temperature of 40 K. Also, plasmon scattering[10] was not included. Electron-electron scattering is modeled after Goodnick and Lugli,[11] who have calculated the rates for two-dimensional electron gases. Phonon scattering was treated by Ridley's model[12] for quantum wells, which assumes the phonon modes to be bulk modes rather than confined slab modes and neglects surface modes altogether. This is not a bad approximation. Since the amplitudes of the slab modes decay at the interface while those of the surface modes increase at the interface, the sum of all modes will appear approximately bulklike.[10] In fact, the scattering rates calculated by using bulk modes do not differ greatly[13] from those calculated by using more sophisticated models (including mi-

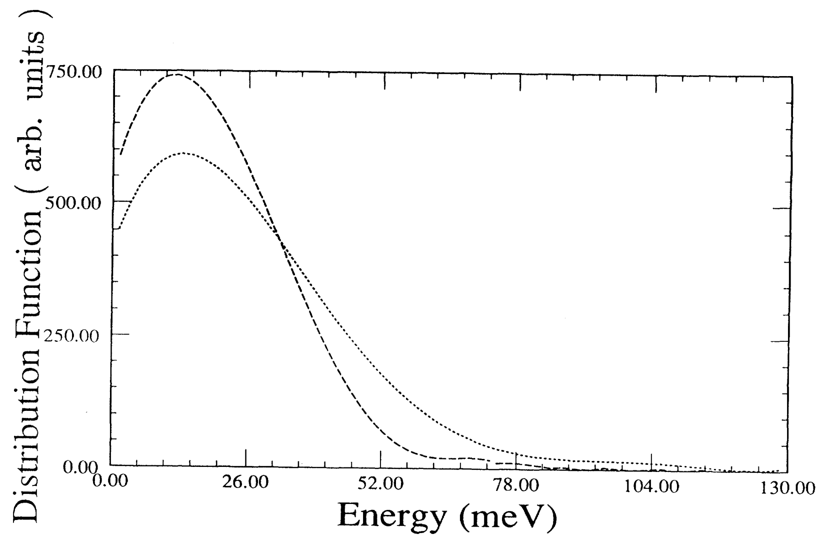

FIG. 1. The steady-state electron-distribution functions in energy for an electric field of 500 V/cm. The short-dashed line corresponds to the case when collision retardation is included and the long-dashed line corresponds to the case when collision retardation is neglected.

croscopic models).[14] Finally, we assume that the phonon modes are decoupled from plasmon modes, which is a good approximation[15] at the low carrier concentration of $10^{11}/cm^2$. We also neglect hot-phonon effects, the role of the Pauli exclusion principle,[16] self-consistent (space-charge) effects,[7] and many-body effects (exchange/correlation)[17] in the simulation.

In Fig. 1, we show the steady-state electron distribution functions in energy for an applied electric field of 500 V/cm with and without collision retardation. Collision retardation shifts electrons from low-energy states to the high-energy tail, thereby causing a relative depopulation of low-energy states. Both distribution functions are approximately drifted Maxwellians but with very different

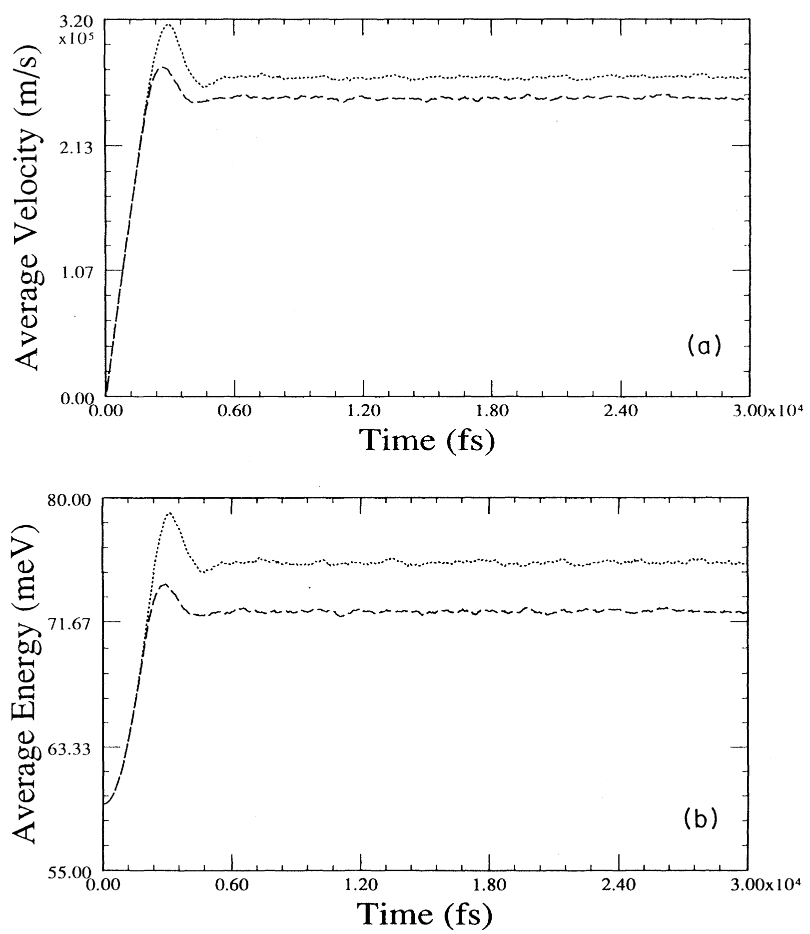

FIG. 2. (a) Velocity and (b) energy vs time for an electric field of 500 V/cm. The short-dashed line and the long-dashed line correspond, respectively, to the situations when collision retardation is included and neglected.

temperatures. When collision retardation is neglected (long-dashed line), the electron temperature is calculated to be ~98 K (the lattice temperature is 40 K). However, when collision retardation is included (short-dashed line), the electron temperature rises to ~147 K. Therefore, collision retardation makes the electron distribution much hotter. This is obviously due to the fact that collision retardation suppresses scattering by rejecting many scattering events near the thresholds. The scatterings that are suppressed are those for which the collision duration exceeds the elapsed time since the previous collision. For instance, retardation completely suppresses optical-phonon emission at just above the emission threshold, since the collision duration for such an event is infinitely long. This robs the electron ensemble of many energy-relaxing scattering events and makes the distribution hotter.

In Figs. 2(a) and 2(b), we show the transient response of the ensemble average velocity and the average energy to an applied electric field of 500 V/cm. Collision retardation (short-dashed line) *increases* the velocity and energy overshoot somewhat, and also *increases* the steady-state velocity and energy. The increase in the steady-state velocity is obviously caused by the suppression of momentum-relaxing collisions due to retardation, and the increase in energy is caused by the suppression of energy-relaxing events. It is interesting to note that collision retardation has the beneficial effects of increasing both the steady-state velocity and the velocity overshoot, which have serious implications for high-speed device applications. However, the increase is only slight; it is merely ~10%.

In conclusion, we have studied the effects of collision retardation on hot-electron transport in quantum-well samples. The results show that retardation increases the steady-state drift velocity, average energy, and the high-field mobility. These have important implications for high-speed devices.

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