Two-dimensional weak localization in combined perpendicular and parallel magnetic fields

N. Giordano and M. A. Pennington
Department of Physics, Purdue University, West Lafayette, Indiana 47907
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We have studied the magnetoresistance of thin films of Au, Ag, and Au(Fe) (Au doped with small concentrations of Fe) at low temperatures. Magnetic fields both perpendicular and parallel to the plane of the film were employed, and in some experiments these two fields were applied simultaneously and independently. Our results are compared in detail with the predictions of weak-localization theory. The agreement between theory and experiment for purely perpendicular fields is reasonably good, but some of the behavior in parallel fields, especially for Au(Fe), cannot be reconciled with the usual theory. Our results can be qualitatively understood if one assumes that the spin-orbit-scattering length is field dependent, or that the phase-breaking length becomes shorter in large parallel fields. Either of these results would be contrary to conventional beliefs.

I. INTRODUCTION

It has been more than a decade since the phenomenon of weak localization (WL) was first identified, and subsequent work has established that it plays a major role in the low-temperature transport properties of disordered metals.[1–4] One property upon which WL has a pronounced effect is the magnetoresistance, and it turns out that WL is often the dominant contributor to the magnetoresistance in low fields.[2,4] In two dimensions (2D) this magnetoresistance is very anisotropic, being substantially larger when the field is directed perpendicular, as opposed to parallel, to the plane of the film. Most experiments have employed perpendicular fields, since the magnetoresistance is largest in this case, and the measurements do not require precise alignment of the field, as is usually necessary with parallel fields. The overall agreement between experiment and theory has been found to be reasonably good, especially for the perpendicular case.[2,4] However, these experiments have, with one exception,[5] all employed either purely perpendicular or purely parallel fields; in the present work we have studied the behavior with perpendicular and parallel fields applied *simultaneously* and *independently*. On general grounds, one might expect that such measurements may yield more information than experiments involving only one field orientation. Indeed, we will see from our results that this approach has yielded some interesting results, which are not yet completely understood.

To understand what motivated our work, it is necessary to recall that the effects of WL are functions of several electron scattering times.[1–3] These include the elastic-scattering time, τ_e, the inelastic-scattering time, τ_i, the spin-scattering time, τ_s, and the spin-orbit-scattering time, τ_{so}. The elastic time is the average time between events such as boundary scattering and scattering from static impurities, and dominates the resistivity at low temperatures, while τ_i is the time between inelastic events, such as electron-phonon or electron-electron scattering. These processes act to destroy the electron phase coherence, which is crucial to WL. In spin scattering the spin of an electron is flipped through the interaction with a localized "impurity" spin. Even though this process does not necessarily change the energy of the electron, it can still destroy phase coherence. τ_{so} arises from scattering through the spin-orbit interaction, a process that changes the spin state of the electron in a manner which preserves phase coherence. It is often useful to refer to the phase coherence time, also known as the phase breaking time, $\tau_\phi^{-1} = \tau_i^{-1} + 2\tau_s^{-1}$.

An interesting open question concerns the effect of a magnetic field on spin scattering. The currently accepted model of this process envisages a localized spin whose direction fluctuates rapidly with time, due to its interaction with the lattice, etc. In a quasiclassical picture, different electron partial waves encounter this localized spin in different states, and therefore the interaction between the electron and the impurity spin, which depends on the relative orientation of the two, is a source of phase randomization. A magnetic field could conceivably affect τ_s in several different ways. (1) A large field will align the impurity spins, so that the electrons always encounter the same impurity spin state. This should prevent spin scattering from contributing to phase breaking. (2) If the g factors of the electrons and the impurity spins are different, a nonzero field will make spin scattering inelastic, in that it will change the kinetic energy of the conduction electron. (3) A field may affect the dynamics of the impurity spins thereby influencing τ_s.[6] These questions have been discussed at various lengths by previous workers, but to the best of our knowledge there have been no clear experimental observations of such effects.

A primary goal of our work was to study the effect of a field on spin scattering, with the expectation that one might observe a reduction in the spin-scattering rate when the magnetic field is sufficiently large. As will be seen below, we have not yet been able to obtain clear evidence for such behavior. However, we did discover some unexpected results, some of which suggest that the

spin-orbit time may be field dependent; this would be in contrast to previous work (both theoretical and experimental), which has assumed that τ_{so} is independent of both field and temperature.

II. THEORY AND BACKGROUND

Our experiments consisted of measurements of the magnetoresistance in both perpendicular and parallel fields. In a perpendicular field (the only case we will consider here is 2D) the theory predicts[1,2]

$$\frac{\Delta R(H_\perp)}{R} = \frac{e^2 R_\Box}{2\pi^2\hbar}\left[\psi\left(\frac{1}{2}+\frac{H_1}{H_\perp}\right) - \psi\left(\frac{1}{2}+\frac{H_2}{H_\perp}\right) + \frac{1}{2}\psi\left(\frac{1}{2}+\frac{H_3}{H_\perp}\right) - \frac{1}{2}\psi\left(\frac{1}{2}+\frac{H_2}{H_\perp}\right)\right], \quad (1)$$

where ψ is the digamma function, and the fields H_n are defined by

$$H_1 = H_0 + H_{so} + H_s, \quad (2)$$
$$H_2 = \tfrac{4}{3}H_{so} + \tfrac{2}{3}H_s + H_i, \quad (3)$$
$$H_3 = 2H_s + H_i \quad (4)$$

with

$$H_n\tau_n = \hbar/4eD\ . \quad (5)$$

The subscripts here indicate elastic scattering, "0," inelastic scattering, "i," spin-orbit scattering, "so," and spin scattering, "s."

When the field is parallel to the plane of the film, the prediction is[1,7] (see also Refs. 8–11)

$$\frac{\Delta R(H_\parallel)}{R} = -\frac{e^2 R_\Box}{2\pi^2\hbar}\left[\frac{3}{2}\ln\left(1+\frac{L_2^2}{L_\parallel^2}\right) - \frac{1}{2}\ln\left(1+\frac{L_3^2}{L_\parallel^2}\right)\right], \quad (6)$$

where we define the lengths L_2 and L_3 in analogy with H_2 and H_3, by

$$L_n^2 = D\tau_n = \hbar/4eH_n\ , \quad (7)$$

where we have also used (5). The parallel field length $L_\parallel$ takes different forms depending on the relative magnitudes of the elastic mean free path, ℓ, and the film thickness, t.[7] In the "dirty" limit one has[1]

$$L_\parallel = \sqrt{3}\hbar/eH_\parallel t\ , \quad (8)$$

while in the clean limit[12]

$$L_\parallel \equiv \sqrt{\frac{16\ell}{3t}}\frac{\hbar}{eH_\parallel t}\ . \quad (9)$$

For our samples the clean limit is appropriate, and (9) will be used exclusively below. However, for our values of ℓ and t, the two results (8) and (9) differ by only about 20%; compared to other uncertainties, this difference is negligible for our purposes.

For a given field strength, the parallel magnetoresistance is much smaller than the perpendicular effect; this can be understood in terms of the well-known arguments relating WL to the interference of quasiclassical electron partial waves.[2,13] A measurement of the resistance can in general be thought of as an interference experiment, in which electron waves are emitted from a "source" at the origin; the resistance is then proportional to how efficiently these waves are localized in a region near the source. In a calculation of this efficiency, trajectories which return to the origin have a special significance. Associated with each such returning path is a "time-reversed" trajectory, in which the path is traversed in the opposite direction. In the absence of inelastic, spin-orbit, or spin scattering, and in zero field, waves which follow these two paths interfere constructively, yielding an increase in the probability of returning to the origin relative to what would be found if interference effects were ignored, and this in turn leads to an increase in the resistance. Through its action on the spin part of the wave function, spin-orbit scattering can, if it is sufficiently strong, reverse the sign of the interference term, giving decreased backscattering, and resulting in antilocalization as mentioned above. With this picture in mind, the effect of a magnetic field can easily be understood. For a perpendicular field one can view the two time-reversed trajectories as defining a closed loop; the flux through this loop produces a relative phase shift of the two partial waves, thereby affecting the interference amplitude. Since there are many such pairs of paths, with different loop areas, the net effect of the field will be to destroy the interference at the origin, and thus quench the contribution of WL. In a perfectly 2D system, in which the electrons move strictly in a plane, a field parallel to the plane will make no contribution to the flux through the trajectory loops, and there will be no parallel magnetoresistance. In a metal film with a nonzero thickness, the trajectories will have a nonzero component perpendicular to the film, and a parallel field will contribute some flux. However, for a field of a given magnitude, the parallel flux will be much less than the perpendicular flux, making the parallel magnetoresistance much smaller than the perpendicular one.

The predictions (1) and (6) apply for purely perpendicular and parallel fields respectively. For arbitrary field directions it has been proposed[5] that one can simply combine these expressions in a manner which is motivated by the arguments given above. The perpendicular magnetoresistance is given by (1), and is a function of the inelastic field, H_i. When a nonzero parallel field is present, the associated parallel flux reduces the effective phase coherence by an amount measured by $L_\parallel$, which plays a role analogous to that of the effective fields (2)–(4), and their associated lengths.[14] The phase breaking from this source can be accounted for by replacing L_i by a new length scale, L_i', which is a combination of L_i

and $L_{\|}$. The form of L_i' can be obtained by noting that since scattering rates combine by addition, the associated lengths must combine according to an "inverse square" rule, so that

$$L_i'^{-2} = L_i^{-2} + L_{\|}^{-2} . \tag{10}$$

The magnetoresistance in the presence of both perpendicular and parallel fields is then given by (1) with L_i' replacing L_i (or equivalently, H_i). That is, if we use (1) to define a function f as

$$f(H_\perp, L_i, L_s, L_{\rm so}) \equiv \Delta R(H_\perp)/R , \tag{11}$$

then for combined parallel and perpendicular fields we have

$$\Delta R(H_\perp, H_{\|})/R = f(H_\perp, L_i', L_s, L_{\rm so}) . \tag{12}$$

This form for the magnetoresistance $\Delta R(H_\perp, H_{\|})$ has been shown to work well for the 2D electron gas in Si inversion layers,[5] and we will use it to analyze our results below.[15] We should add, however, that to the best of our knowledge, this result has not yet been derived from a quantitative calculation.

The magnetoresistance arising from electron-electron interactions is typically much smaller than that due to WL,[3] and is usually ignored. In the general case the interaction magnetoresistance depends in a complicated manner on a number of factors. However, it has been found that as far as the experiments are concerned,[11] this magnetoresistance can be described by the predicted form

$$\frac{\Delta R_\Box}{R} = \frac{e^2 R_\Box F}{4\pi^2 \hbar}(0.84 y^2) , \tag{13}$$

where

$$y \equiv g\mu_B H/k_B T . \tag{14}$$

This form applies only for weak fields ($y < 1$), but will be sufficient for our purposes. We should also note that a magnetic field can have a substantial effect on any Kondo contribution to the resistivity. A field quenches the Kondo contribution to ρ, as one would expect, leading to a purely negative magnetoresistance.[16,17]

III. EXPERIMENTAL METHOD

The samples were thin films of Ag, Au, and Au(Fe), all of which were produced by thermal evaporation. The purity of the Ag was 99.9999%, while that of the Au was 99.999%; according to the supplier the latter contained approximately 1 ppm Fe, and lower concentrations of other magnetic impurities. These films will be referred to as "pure" in the following. The Au(Fe) films were prepared by flash evaporation of a portion of the 99.999% Au, together with a measured amount of Au–0.07 at. % Fe alloy material, with the resulting concentration of Fe, c, being in the range 2–120 ppm. Previous studies of similarly prepared films did not show any indications of clustering of the Fe.[18,19]

The films were all ≈ 140 Å thick, and had low temperature resistivities of ≈ 5 $\mu\Omega$ cm, with sheet resistances near 4 Ω. Using free electron theory we estimate an elastic mean free path of 100 Å, and a diffusion constant $D = 50$ cm^2/s. The samples were patterned with photolithography and lift off to form long strips, which were 60 cm $\times$ 80 μm. The magnetic fields were produced by two magnets, a superconducting solenoid and a split-coil magnet wound of copper. The two fields were arranged to be perpendicular to each other, and for the experiments in which both were used, the plane of the sample was oriented approximately parallel to the field of the superconducting magnet, and perpendicular to that of the split-coil magnet. In the parallel field measurements, the misalignment of the sample plane with respect to the axis of the superconducting magnet varied from run to run, and was generally 0.5° or less. This misalignment could be monitored *in situ* from the measured symmetry of the magnetoresistance as a function of perpendicular field, and was corrected for in the analysis, as will be described below.

IV. RESULTS

In this section we begin by presenting results for the behavior in perpendicular fields; it will be seen that these results agree well with WL theory. Experiments of this kind have been described by many previous workers, so ordinarily there would be no need for us to also show such data. However, we will see later that our results for the parallel magnetoresistance, and for the behavior in combined parallel and perpendicular fields, reveal unexpected behavior. In order to critically analyze and appreciate those results, it is essential to determine independently the phase breaking and spin-orbit lengths, and this can be accomplished most readily from the perpendicular magnetoresistance. We therefore feel that it is essential for the overall analysis that all of the results, even those which are now fairly routine, be shown explicitly.

A. Perpendicular fields

1. Au and Au(Fe)

Figure 1 shows results for the perpendicular magnetoresistance of a Au sample, at several different temperatures. The solid curves in Fig. 1 are fits to (1), and it can be seen that the theory fits the data quite acceptably. It is known from previous studies[20,21] that the spin-orbit scattering in Au is quite strong, and in this field range the predicted magnetoresistance turns out to be independent of the precise value of $H_{\rm so}$. The only adjustable parameters in these fits are therefore the values of H_ϕ (i.e., L_ϕ) at each temperature. The results for L_ϕ are shown in Fig. 2; here we also give results for two Au(Fe)

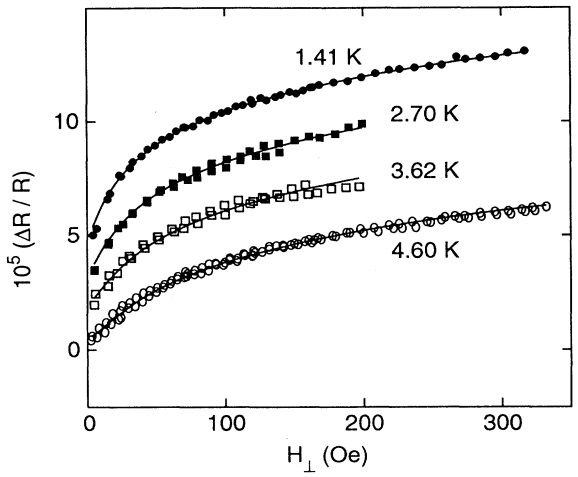

FIG. 1. $\Delta R/R$ as a function of $H_\perp$ for a pure Au film with $R_\square = 4.1\ \Omega$ at several temperatures. The solid curves are fits to the theory, (1), as described in the text. The zeros of the vertical scale are arbitrary and offset for clarity.

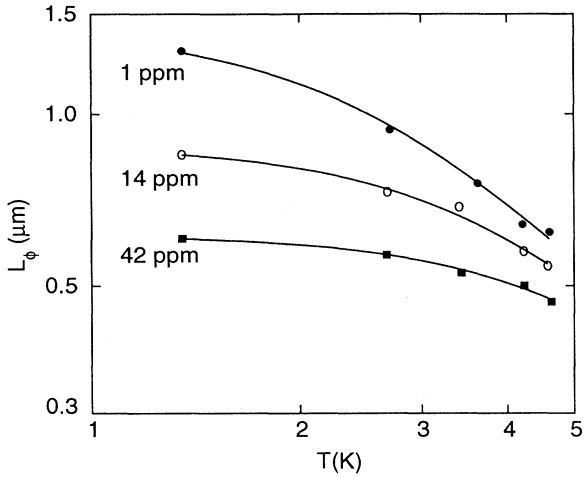

FIG. 2. Phase breaking length, L_ϕ, as a function of temperature, derived from the perpendicular magnetoresistance for several samples. The top curve shows results for the pure Au sample considered in Fig. 1, while the bottom curves show results for two Au(Fe) samples. The Fe concentrations, c, are indicated in the figure; here $c = 1$ ppm for the "pure" Au sample, since that is the residual Fe content. The sheet resistances were 3.7 Ω for the 14 ppm sample, and 3.2 Ω for the 42 ppm sample.

samples. The variation of L_ϕ with T is very similar to that found in previous studies,[2,4] and can be understood if one recalls that in general

$$\frac{1}{\tau_\phi} = \frac{1}{\tau_{e\text{-}e}} + \frac{1}{\tau_{e\text{-ph}}} + \frac{2}{\tau_s}\ , \tag{15}$$

where $\tau_{e\text{-}e}$ is the inelastic-scattering time due to electron-electron scattering, and $\tau_{e\text{-ph}}$ is corresponding time for electron-phonon scattering. It is known,[1,22,23] that in metal films $\tau_{e\text{-}e} \propto (R_\square T)^{-1}$, and $\tau_{e\text{-ph}} \propto T^{-2.5}$, while in this range of field and temperature we expect τ_s to be a constant.[24] Since $L_\phi \equiv \sqrt{D\tau_\phi}$, the combination (15) yields a phase breaking length that increases as T is lowered, until the spin scattering dominates, at which point L_ϕ approaches a constant. This is precisely the behavior seen in Fig. 2.

Quantitatively the theory predicts that the electron-electron phase breaking time is given by[1,22]

$$\tau_{ee} = \frac{2\pi\hbar^2}{e^2 k_B T R_\square \ln(\pi\hbar/e^2 R_\square)} \approx 2.4\times 10^{-8}(R_\square T)^{-1}\ \text{s} \tag{16}$$

(with $R_\square$ measured in Ω, and T in K), a result which has been confirmed by numerous experiments. For $\tau_{e\text{-ph}}$ theory predicts, and experiments support, the result[23]

$$\tau_{e\text{-ph}} \approx 1.6\times 10^{-9} T^{-2.5}\ \text{s}\ , \tag{17}$$

for Au films with values of $R_\square$ similar to ours. Evaluating (16) and (17) for $R_\square = 4\ \Omega$, as appropriate for the samples in Fig. 2, we find that at $T = 2$ K, $\tau_{e\text{-}e} = 3\times 10^{-9}$ s, and $\tau_{e\text{-ph}} = 3\times 10^{-10}$ s. Thus, electron-phonon scattering should be dominant in the temperature range we have studied, although if we allow for uncertainties in these estimates, electron-electron scattering may make a small contribution at the lowest temperatures. From (15) we have

$$L_\phi^{-2} = A_1 + A_2 T^{-1} + A_3 T^{-2.5}\ , \tag{18}$$

where A_1 is a measure of the strength of the spin scattering, and is proportional to the spin scattering rate, A_2 is proportional to the electron-electron scattering rate, and A_3 to the electron-phonon rate. The temperature dependences in (18) derive from (16) and (17), and we assume the spin-scattering rate to be temperature independent. The solid curves in Fig. 2 were obtained with $A_2 = 0$; this amounts to assuming that electron-electron scattering is much weaker than electron-phonon scattering, which is the prediction of the theory for our case. The value of A_3 so obtained was about a factor of 2 larger than the theoretical prediction (17). Given the uncertainties in both the experimental and theoretical values, we consider this to be good quantitative agreement.

These fits also yielded values of τ_s, i.e., A_1. In Table I we list the values of τ_s^{-1}/c obtained for several samples. Normalizing the spin-scattering rates by the concentration should yield a quantity which is independent of c, and we see that the results for the larger values of c are in reasonable agreement with this expectation. The sample with the smallest concentration, $c \approx 1$ ppm, which is the

TABLE I. Results for τ_s^{-1}/c (in units of s^{-1}at.%) for Au(Fe).

1 ppm	14 ppm	42 ppm	120 ppm
1.0×10^{13}	2.1×10^{12}	1.7×10^{12}	2.3×10^{12}

nominally pure sample, has a larger value of τ_s^{-1}/c. In this case c was obtained from the manufacturer's analysis, and it is conceivable that it could be in error by one or two ppm, or that perhaps other impurities (which were present at the sub-ppm level) may make significant contributions to τ_s. It is also possible that we introduced a few ppm of Fe in the fabrication, but this seems unlikely, since even when we took great pains to avoid contamination (by, e.g., extra cleaning of the evaporator, or careful shielding of the sample substrates from all but the evaporation source) similar values of L_ϕ were obtained. In our opinion, the most likely cause of the (relatively) large value of τ_s^{-1}/c for the purest sample is that electron-electron scattering makes a small contribution to τ_ϕ at the lowest temperatures. This would yield the beginning of a "roll-over" of the curve in Fig. 2, similar to that caused by spin scattering. In addition, electron-electron scattering should be most important in the lowest concentration samples, since they have the largest values of τ_ϕ. In any event the results in Table I again suggest that there is no problem with clustering of the Fe, since that would lead to a much different dependence of τ_s on c.

The results in Figs. 1 and 2 all involved fields small compared to the spin-orbit field, H_{so}, and as noted above, the theoretical prediction (1) is independent of the precise value of H_{so} in this limit. We have also examined the behavior in large perpendicular fields, and some typical results are shown in Fig. 3, where we see that $\Delta R/R$ displays a maximum near $H_\perp = 3$ kOe. This arises due to the interplay of $H_\perp$ and H_{so},[2] and permits us to determine the value of H_{so}, and therefore τ_{so}. The solid curves in Fig. 3 are fits to (1), using the value of L_ϕ determined from the low-field behavior (Fig. 2), and with τ_{so} as the only adjustable parameter. The fits at both temperatures yielded $L_{so} = 550$ Å; combining this with our estimate $D = 50$ cm^2/s gives $\tau_{so} = 6 \times 10^{-13}$ s, which is in reasonable agreement with the values reported previously for Au.[20,21]

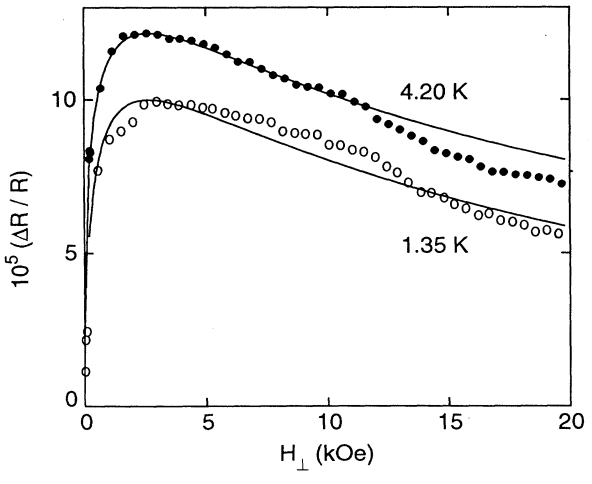

FIG. 3. $\Delta R/R$ as a function of $H_\perp$ at two temperatures, for the pure Au sample considered in Fig. 1. The solid lines are fits to the theory (1), as described in the text. The zeros of the vertical scale are arbitrary and offset for clarity.

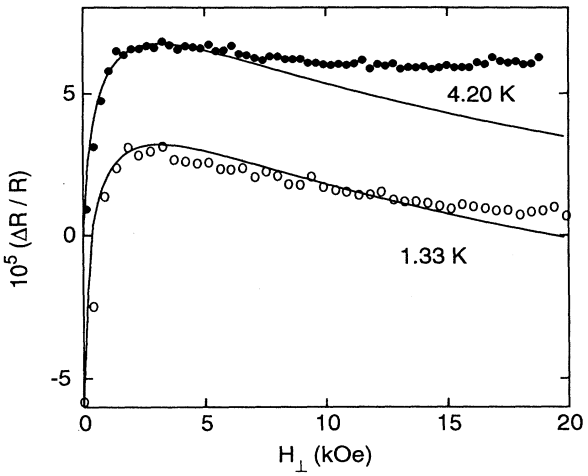

FIG. 4. $\Delta R/R$ as a function of $H_\perp$ for two temperatures, for the Au(Fe) sample with $c = 42$ ppm considered in Fig. 2. The solid lines are fits to the theory (1), as described in the text. The zeros of the vertical scale are arbitrary and offset for clarity.

Results for Au(Fe) in large perpendicular fields are shown in Fig. 4, along with fits to the theory (1). Again we have used the values of L_ϕ obtained from the low-field results, so that the fits in Fig. 4 involve only one adjustable parameter, τ_{so}. The behavior at 1.33 K agrees well with the theory, while the agreement at 4.20 K is not as good. Similar results were found for other Au(Fe) samples, which suggests that there may be another source of high-field magnetoresistance in Au(Fe), a point we will return to below. Nevertheless, the value found for τ_{so}, which in the fits is determined by the field at which the maximum of $\Delta R/R$ occurs,[2] is essentially the same as that found for pure Au.

2. Ag

Figure 5 shows results for Ag in perpendicular fields, along with fits to the theory (1). The qualitative behavior is seen to be similar to that of Au (Fig. 3), but with the maximum of $\Delta R/R$ occurring at a smaller field. This is expected, since the spin-orbit scattering is known to be much weaker in Ag than in Au. It is seen from Fig. 5 that the theory describes the results for Ag quite well, and the two parameters, L_i and L_{so} (or equivalently, τ_i and τ_{so}) can both be determined accurately. We find $L_i = 13\,000$ and $20\,000$ Å at 4.20 and 1.33 K respectively, yielding $\tau_i = 3 \times 10^{-10}$ and 8×10^{-10} s at these temperatures. For L_{so} we find 5000 Å at both temperatures, yielding $\tau_{so} = 5 \times 10^{-11}$. These values of τ_i are in accord with the electron-phonon scattering times for similar materials[23] [for comparison, the predicted electron-phonon times (17) are 2×10^{-10} and 6×10^{-10} s, respectively]. The value found for τ_{so} is a bit longer than found previously for Ag,[4] for reasons that are not clear. In any event, the behavior of the Ag films in small perpendicular fields is well described by WL theory.

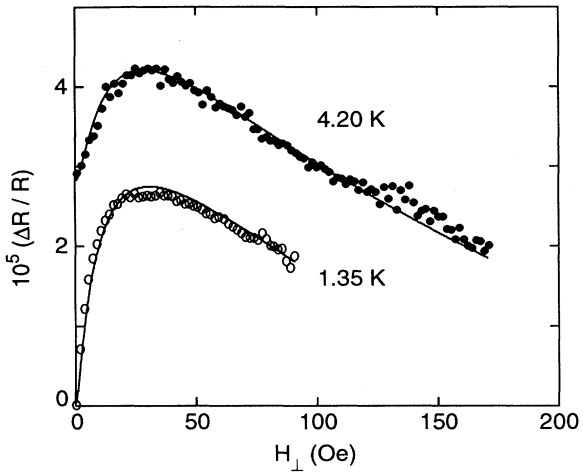

FIG. 5. $\Delta R/R$ as a function of $H_\perp$ for a Ag sample with $R_\square = 3.1\ \Omega$. The solid lines are fits to the theory (1). Note that the zeros of the vertical scale are arbitrary and offset for clarity.

B. Parallel fields

1. Au and Au(Fe)

Next we consider the magnetoresistance for purely parallel fields, beginning with Au and Au(Fe). In Fig. 6 we show $\Delta R/R$ as a function of $H_\parallel$ at 4.2 K for the Au sample considered in Fig. 1. As noted in Sec. II, and as can also be seen by comparing Figs. 1 and 6, the magnetoresistance at low fields is quite anisotropic. A misalignment of $H_\parallel$ by only 1° with $H = 10^4$ Oe yields a contribution to the perpendicular field of $H_\perp \sim 150$ Oe; this can cause a substantial change in $\Delta R/R$, and thus the problem of field alignment is particularly important. We addressed this problem in the following way. The sample was first aligned so that the field from the superconducting magnet was approximately parallel to the plane of the film. This approximately parallel field was then set at the desired value, and held fixed. The resistance was next measured as a function of the field produced by the split coil magnet, which was oriented with its axis perpendicular to the film plane; we will term this field $H'_\perp$. The "true" perpendicular field has contributions from both $H'_\perp$, and $H_\parallel$ due to the latter's slight misalignment with respect to the film plane. That is, $H_\perp \approx H'_\perp + H_\parallel \sin\theta$, where θ is the angle of misalignment. Since the resistance exhibits a minimum when the $H_\perp$ is zero we could use the location of this minimum to determine the value of $H'_\perp$ at which $H_\perp = 0$. This is how the results for $\Delta R/R$ as a function of $H_\parallel$ were obtained. Hence, in these measurements the field generated by the split coil magnet served only as a "trim" field. In the measurements of $\Delta R/R$ as a function of $H_\perp$ with $H_\parallel \neq 0$, which will be considered below, this field played a more central role.

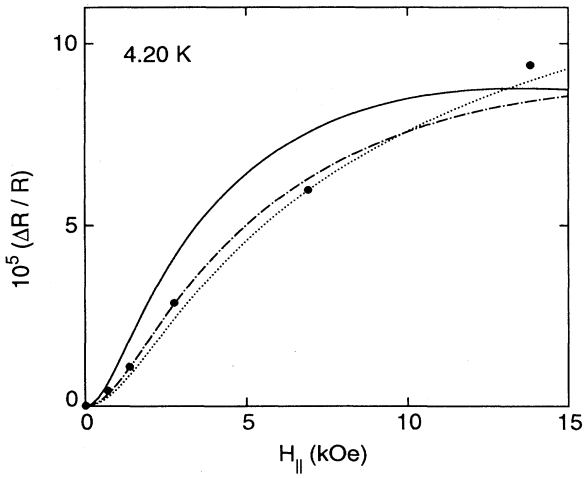

FIG. 6. $\Delta R/R$ as a function of $H_\parallel$ for the pure Au film considered in Figs. 1, 2, and 3, at 4.20 K. The zero of the vertical scale has been taken as the value measured for $H_\parallel = 0$ at each temperature. The curves are the theory (6), evaluated using parameter values discussed in the text.

Figure 6 shows results for a Au sample at 4.20 K, along with the theory (6), evaluated using three different parameter sets. In all three cases we used the value of L_ϕ determined independently from the fits to the perpendicular magnetoresistance, Fig. 2. The solid curve was evaluated using the spin-orbit length obtained from the perpendicular measurements ($L_{so} = 550$ Å), and the thickness was taken to be 140 Å, our best estimate. Note also that here, and in all similar cases below, the clean limit value of $L_\parallel$ (9) has been employed. The agreement with the data in Fig. 6 is not very good, so we tried several other parameter sets. The dot-dashed curve in Fig. 6 was obtained using the same values of L_{so} and L_ϕ, but with a somewhat smaller value of the film thickness, $t = 110$ Å. The agreement is now acceptable, although not perfect. This value of t is somewhat smaller than the value measured during film deposition; however, the latter corresponds to the average thickness, and it seems quite likely that the films are not perfectly uniform in thickness. This inhomogeneity could be responsible for the need to use a reduced value of t in evaluating the theory. The best agreement with the theory was obtained with values of L_{so} which were somewhat smaller than the estimates obtained from the perpendicular magnetoresistance. This is shown by the dotted curve in Fig. 6, which was obtained with $L_{so} = 300$ Å, and $t = 100$ Å. However, this value of L_{so} is a bit outside the range of values which is consistent with the perpendicular results, Fig. 3.

Results similar to Fig. 6 were obtained for the same sample at other temperatures, for other Au samples, and for Au(Fe) films with Fe concentrations below about 10 ppm. The agreement with WL theory is thus acceptable, with the proviso that the effective film thickness is taken to be about 25% smaller than the average value, perhaps because of inhomogeneities in the thickness. However, this may well be an indicator of more serious difficulties.

We next consider the parallel magnetoresistance found for the more concentrated Au(Fe) samples. Some typical results are shown in Fig. 7 (similar results were found at

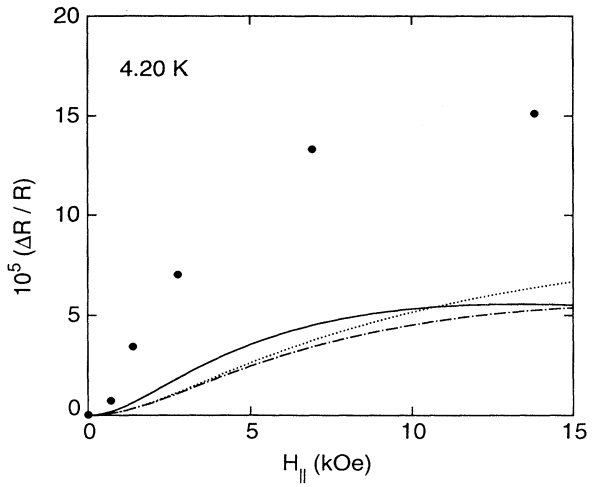

FIG. 7. $\Delta R/R$ as a function of $H_{||}$ for the Au(Fe) film (c = 42 ppm) considered in Fig. 4, at 4.2 K. The zero of the vertical scale has been taken as the value measured for $H_{||} = 0$ at each temperature. The curves are the theory (6), evaluated using parameter values discussed in the text.

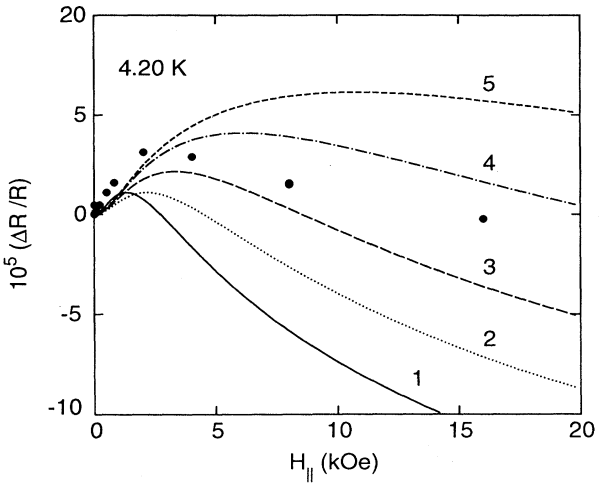

FIG. 8. $\Delta R/R$ as a function of $H_{||}$ at 4.2 K for a Ag sample ($R_\square$ = 3.1 Ω) considered in Fig. 5. Here the zero of the vertical scale is taken as the value measured for $H_{||} = 0$ at each temperature. The curves are the theory (6), as described in the text. Curve 1: L_{so} = 5000 Å, t = 140 Å; curve 2: L_{so} = 5000 Å, t = 100 Å; curve 3: L_{so} = 3500 Å, t = 100 Å; curve 4: L_{so} = 2000 Å, t = 100 Å; curve 5: L_{so} = 1150 Å, t = 100 Å. In all cases L_ϕ = 13 000 Å.

other temperatures); the curves are again the theory (6) obtained in the same way as in Fig. 6. The solid curve corresponds to L_{so} = 550 Å, and t = 140 Å, our best estimates obtained from the perpendicular measurements, the dot-dashed curve to L_{so} = 550 Å, and t = 110 Å, and the dotted curve to L_{so} = 300 Å, and t = 100 Å; in all cases the values of L_ϕ obtained from the perpendicular measurements (Fig. 2) have been used. In contrast to the results for pure Au, there is now sharp disagreement between theory and experiment for *all* of these parameter sets, as the observed magnetoresistance is substantially larger than predicted by WL theory. It is also interesting to note that the parallel magnetoresistance of Au(Fe) is substantially *larger* than that found for Au.

This discrepancy between the calculated and observed magnetoresistance was observed for all Au(Fe) samples with c greater than about 20 ppm, and the discrepancy became larger as c was increased. In all cases the observed magnetoresistance was always larger (and more positive) than predicted. We also considered adjusting the value of L_ϕ, that is, using a value different from that measured independently from the magnetoresistance in perpendicular fields, but it still was not possible to obtain acceptable agreement between theory and experiment for the samples with high Fe concentrations. Below we will consider what parameter adjustments are necessary to bring the theory into agreement with these results.

2. Ag

Figure 8 shows results for the parallel magnetoresistance of a Ag sample at 4.2 K. The curves are the theory evaluated for different values of L_{so} and t; in all cases L_ϕ has been held fixed at the value found from the perpendicular magnetoresistance for this sample. The curve labeled "1" was obtained with our best estimates of these parameters, and it is seen that it does not compare at all well with the results. The other parameter sets (whose values are given in the caption to Fig. 8) do a better job of reproducing the qualitative trends in the data, but none of them are in terribly good agreement. Also, the agreement is generally worse at 1.35 K than at 4.20 K. These discrepancies will be discussed further below.

C. Perpendicular magnetoresistance in the presence of nonzero parallel fields

Our analysis of the parallel magnetoresistance has hinged on the values of L_ϕ, which we have obtained from the perpendicular magnetoresistance in low fields. However, the parallel measurements involved large fields, and it is worthwhile to consider if L_ϕ or perhaps one of the other relevant length scales might be field dependent. With this in mind, we have investigated the perpendicular magnetoresistance *in the presence* of a nonzero parallel field.

Figure 9 shows results for the pure Au sample considered in Figs. 1, 2, and 6; here we plot $\Delta R/R$ as a function of $H_\perp$ for different fixed values of $H_{||}$, at a fixed temperature. Here $H_\perp \approx H'_\perp + H_{||} \sin\theta$, where θ, the angle of misalignment of the superconducting solenoid (see above), was determined from the location of the minimum of $\Delta R(H'_\perp)$, and $H'_\perp$ and $H_{||}$ were defined previously. Since $H_{||}$ was fixed for a given scan, this simply amounts to a constant shift along the horizontal axis in Fig. 9. This is why the data in Fig. 9 sometimes extend to both polarities of $H_\perp$, and in a few cases $H_\perp = 0$ was not reached.[25] Typical values of θ were $\lesssim 1°$, and the correction was accurate to a small fraction of this; we

believe that $H_\perp = 0$ was located to typically 10 Oe when $H_\parallel = 10^4$ Oe, corresponding to an effective alignment of $\approx 0.04°$. Finally, we also note that $\Delta R/R$ in Fig. 9 is measured relative to the value found for $H_\perp = H_\parallel = 0$ at each temperature.

The result for $H_\parallel = 0$ in Fig. 9 is just the perpendicular magnetoresistance considered in Fig. 2, and is well described by the theory (1) with the phase-breaking lengths already discussed (Fig. 2). As $H_\parallel$ was gradually increased, the behavior was qualitatively similar, but the width of the dip near $H_\perp = 0$ became broader, as expected, since the parallel field leads to a shorter effective phase-breaking length, L_i'. These results are compared with the theory (12) in Fig. 10. Here, for the sake of clarity, and consistency with the presentation of the results for other samples below, the data and theory have been offset as described in the caption. Hence, in Fig. 10 the relative positions of the results for different values of $H_\parallel$ are arbitrary. The theoretical curves require a value of $L_\parallel$, which we have computed using (9) and the parameters $L_{so} = 550$ Å, and $t = 110$ Å; these values were already found to account well for both the purely

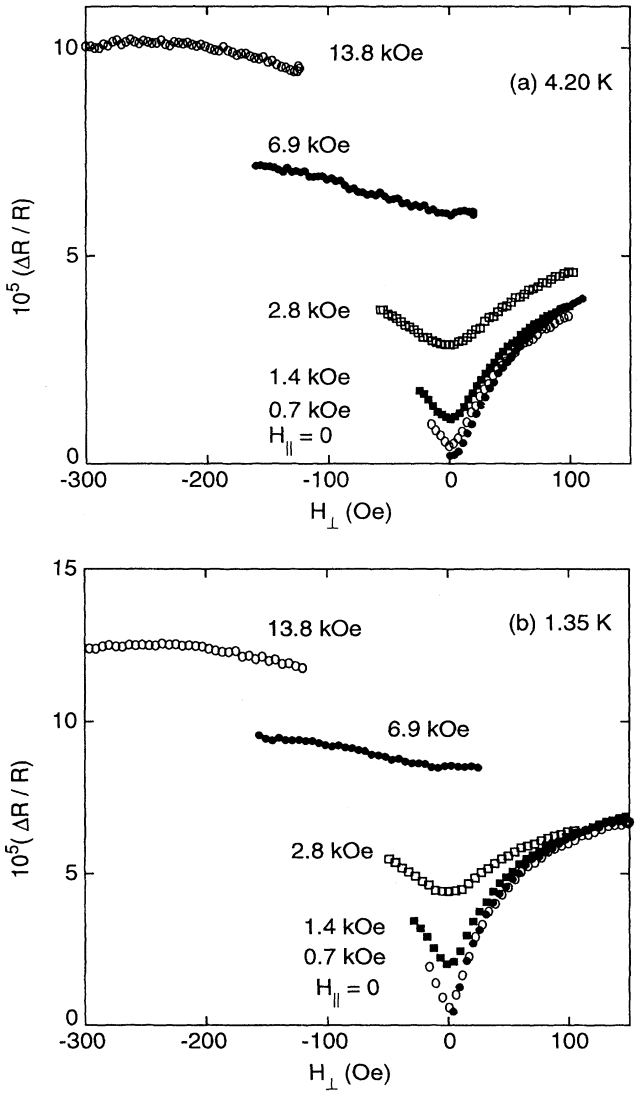

FIG. 9. $\Delta R/R$ as a function of $H_\perp$ for different values of $H_\parallel$, for the pure Au sample considered in Figs. 1, 3, and 6. The values of $H_\parallel$ are indicated in the figure. Here the zero of the vertical scale is taken as the value measured for $H_\parallel = H_\perp = 0$ at each temperature. (a) $T = 4.20$ K; (b) $T = 1.35$ K.

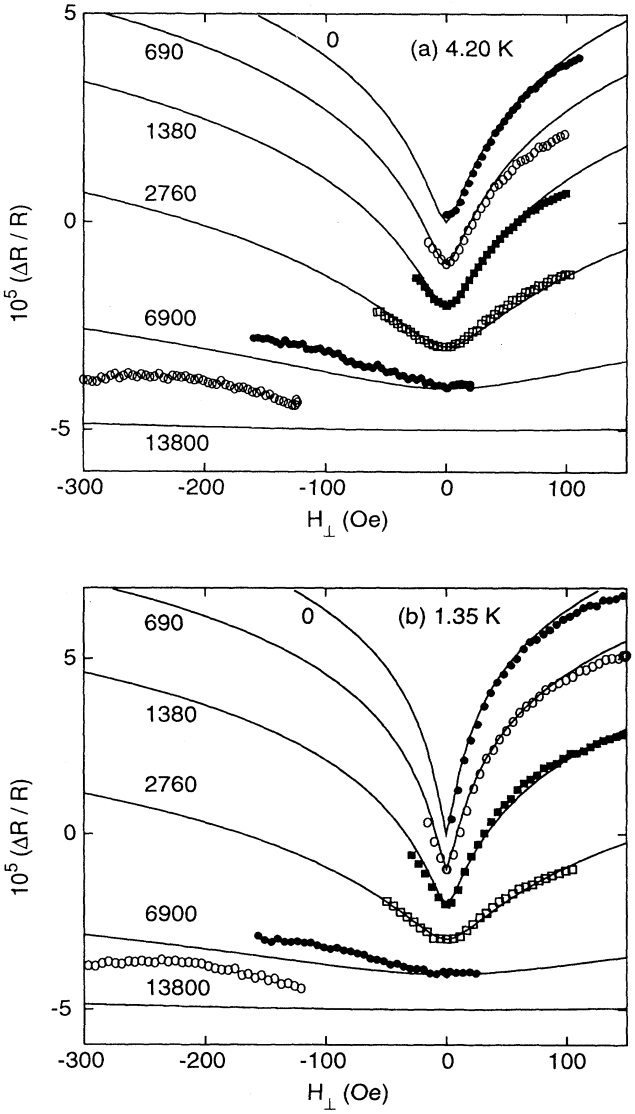

FIG. 10. $\Delta R/R$ as a function of $H_\perp$ for a Au sample ($R_\square = 4.1\ \Omega$). The numbers in the figure are the values of $H_\parallel$. This is the same data as in Fig. 9, but with the results for different values of $H_\parallel$ offset to systematically lower values as $H_\parallel$ increases. This offset makes it easier to compare these results with those for the Au(Fe) and Ag samples considered in Figs. 12 and 14. The solid curves are the theory (10), as described in the text. We have taken $L_\phi = 7000$ and 12 000 Å at 4.20 and 1.35 K, respectively, with $L_{so} = 550$ Å and $t = 110$ Å at both temperatures. These values were obtained from the measurements in purely perpendicular fields. (a) $T = 4.20$ K; (b) $T = 1.35$ K.

perpendicular and the purely parallel magnetoresistance. The phase-breaking length has been obtained from the low-field perpendicular measurements, so there are *no* adjustable parameters involved in the theoretical predictions. It is seen from Fig. 10 that the theory works well for $H_{\|}$ below about 3000 Oe. However, at higher parallel fields the theory predicts a somewhat weaker dependence on $H_{\perp}$ than is observed. In particular, at $H_{\|} = 13\,800$ Oe the magnetoresistance is predicted to be essentially independent of $H_{\perp}$, but this was definitely *not* observed. Such behavior was found at both 4.20 K and 1.35 K, and was also observed for Au(Fe) and Ag; it will be discussed further below. We conclude at this point that while the theory for $\Delta R(H_{\perp}, H_{\|})$ works fairly well for parallel fields below 3000 Oe, there is a small but qualitatively significant discrepancy at higher fields.

We next consider the behavior of the Au(Fe) samples. Figure 11 shows results for a sample with 42 ppm Fe. Here the results are plotted with $\Delta R/R$ taken as zero when $H_{\perp} = H_{\|} = 0$. Comparing these results with the theory requires that we estimate $L_{\|}$, and in doing so we are faced with a problem not encountered with the pure Au samples. For Au(Fe) the parallel magnetoresistance was larger than predicted by WL theory, which may indicate that $L_{\|}$ is much different than predicted by (9), but it is also quite conceivable that the problem may not

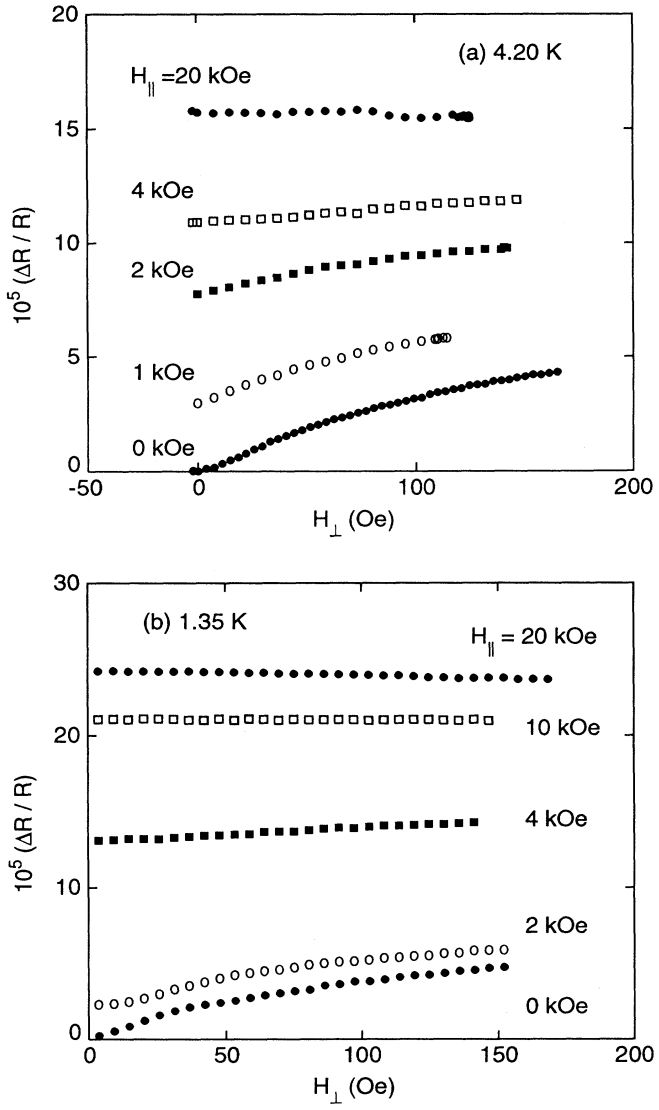

FIG. 11. $\Delta R/R$ as a function of $H_{\perp}$ for different values of $H_{\|}$, for the Au(Fe) sample considered in Figs. 4 and 7. The values of $H_{\|}$ are given in the figure. Here the zero of the vertical scale is taken as the value measured for $H_{\|} = H_{\perp} = 0$ at each temperature. (a) $T = 4.20$ K; (b) $T = 1.35$ K.

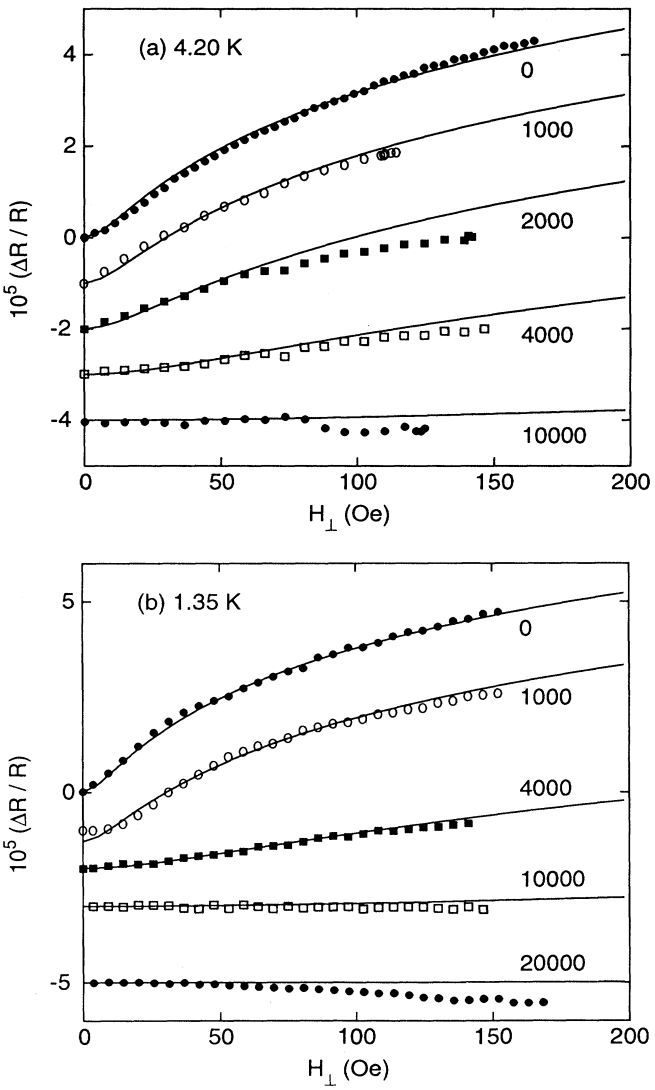

FIG. 12. $\Delta R/R$ as a function of $H_{\perp}$ for a Au(Fe) sample. The numbers in the figure are the values of $H_{\|}$. This is the same data as in Fig. 11, but with the results for different values of $H_{\|}$ offset to systematically lower values as $H_{\|}$ increases. The solid curves are the theory as described in the text. We have taken $L_{\phi} = 6000$ and 7000 Å at 4.20 and 1.35 K, respectively, with $L_{so} = 550$ Å and $t = 110$ Å at both temperatures. These values were as obtained from the measurements in purely perpendicular fields. (a) $T = 4.20$ K; (b) $T = 1.35$ K.

be related to $L_{\|}$ at all. It is thus not clear what value of $L_{\|}$ is appropriate in evaluating (12). We have chosen to use the theory (9) to compute $L_{\|}$, using the values of L_ϕ found from the perpendicular measurements with $H_{\|} = 0$, and taken the other parameter values to be the same as used in the analysis of the pure Au sample in Fig. 10; $L_{so} = 550$ Å and $t = 110$ Å. (In the next section we will consider the use of other parameter sets.) The results are shown along with the data in Fig. 12, where we have offset the data for clarity (as in Fig. 10). We should add that it is the failure of the theory to account for the purely parallel magnetoresistance for this sample which is the reason we have chosen to offset each data set in Fig. 12. We have already seen that the theory fails to describe the magnetoresistance when $H_\perp = 0$; our goal here is to determine if the theory is able to account for just the variation of $\Delta R/R$ with $H_\perp$. Returning to Fig. 12, we see that the theory (12) works fairly well for $H_{\|} \lesssim 4000$ Oe, but at larger parallel fields it is unable to reproduce the weak, but distinct *maximum* in $\Delta R/R$ found at $H_\perp = 0$ (see especially the data at 1.35 K). The behavior of Au(Fe), and the discrepancies with the theory, in Fig. 12 are in many respects quite similar to the results found for pure Au (Fig. 10). It does not appear that WL theory with the assumption of field independent scattering rates can account for this behavior; this will be discussed further below.

Finally, we consider the results for Ag in Fig. 13, where, as in Figs. 9 and 11, we take $\Delta R/R$ to be zero when $H_\perp = H_{\|} = 0$; Fig. 14 shows the same results offset for clarity, along with the theory (12). The theoretical predictions were calculated using the values of L_ϕ and L_{so}, obtained from the fits when $H_{\|} = 0$: $L_\phi = 13\,000$ and $20\,000$ Å at 4.20 K and 1.35 K, respectively, and $L_{so} = 5000$ Å. It is again seen that the theory works fairly well for small $H_{\|}$, but when $H_{\|} \gtrsim 4000$ Å the theory predicts much less

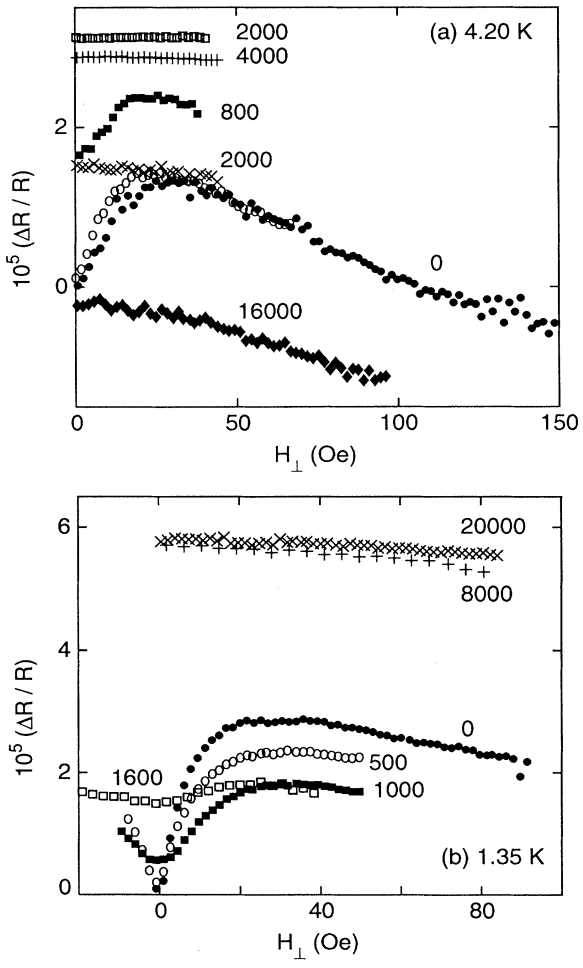

FIG. 13. $\Delta R/R$ as a function of $H_\perp$ for different values of $H_{\|}$, for the Ag sample considered in Figs. 5 and 8. The numbers in the figure are the values of $H_{\|}$ for the adjacent data; for the open circles $H_{\|} = 80$ Oe. (a) $T = 4.20$ K; (b) $T = 1.35$ K.

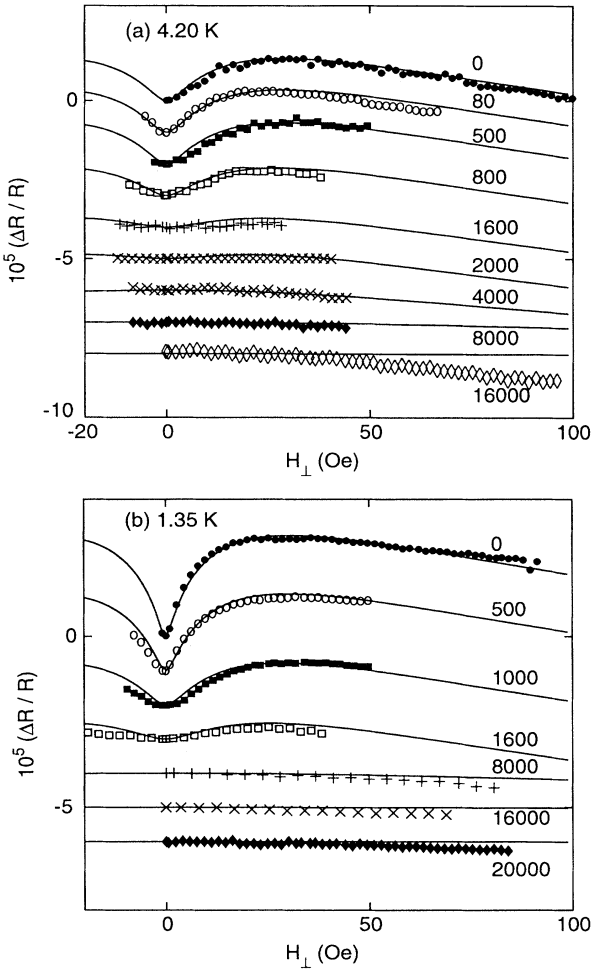

FIG. 14. $\Delta R/R$ as a function of $H_\perp$ for a Ag sample ($R_\square = 3.1\ \Omega$). The numbers in the figure are the values of $H_{\|}$. This is the same data as in Fig. 13, but with the results for different values of $H_{\|}$ offset to systematically lower values as $H_{\|}$ increases. The solid curves are the theory evaluated as described in the text. In these evaluations we have taken $L_\phi = 13\,000$ and $20\,000$ Å at 4.20 and 1.35 K, respectively, with $L_{so} = 5000$ Å and $t = 110$ Å at both temperatures. (a) $T = 4.20$ K; (b) $T = 1.35$ K.

dependence on $H_\perp$ than is observed; this is very similar to what was found for Au and Au(Fe).

V. DISCUSSION

In our analysis to this point we have only considered the effects of WL, and have found that several important features of our results cannot be explained, even qualitatively, in terms of this theory. Let us therefore consider other possible sources of magnetoresistance, with regards to the parallel magnetoresistance of Au(Fe). The interaction magnetoresistance (13) is positive, which is of at least the correct sign required to account for our extra magnetoresistance. However, evaluation of (13) yields the prediction $\Delta R/R \sim 1 \times 10^{-5}$ at H =10 kOe and T = 1.4 K. This is more than an order of magnitude smaller than our discrepancy for Au(Fe), so it appears that this is not the source of our extra magnetoresistance. Moreover, even if it was somehow large enough to account for the discrepancy, one would still have to explain why it is important for Au(Fe), but not for pure Au.[26] Indeed, calculations[27] indicate that the presence of strong spin or spin-orbit scattering should reduce this contribution to the magnetoresistance.

Let us next consider contributions related to the Kondo effect. So far as we know, all of these mechanisms yield a negative magnetoresistance,[28] since they correspond to a quenching of the Kondo contribution; this is of the wrong sign with respect to our discrepancy. In addition, the magnetoresistance associated with spin-disorder scattering is negative.[29,17,16,30] and thus cannot account for our results. Our best estimates indicate that these effects are not only of the wrong sign, but are also somewhat smaller in magnitude than our extra parallel magnetoresistance for Au(Fe). However, we should also note that current theories of the Kondo magnetoresistance are not able to account for the recent observation[31,32,18] that the Kondo effect is suppressed in two dimensions when the film thickness is reduced. It is conceivable that a magnetic field may restore at least part of this resistance, yielding a positive magnetoresistance, which would be qualitatively consistent with our observations. However, this is only a speculation at present.

The above discussion suggests that contributions other than WL cannot account for our results. Of course, there may be other mechanisms which we have not considered, so we cannot use such arguments to conclude that our effects must be due to WL. However, since WL is able to account fairly well for the behavior we find for pure Au, the problem, at least for Au(Fe), seems to be connected with the presence of the Fe spins (we will return to the other discrepancies later). With this in mind, we now consider how (or if) WL theory might be made consistent with our experiments, beginning with the parallel magnetoresistance.[33]

In Fig. 6 we found that the theory agreed fairly well with the results for the parallel magnetoresistance of Au using t = 110 Å and L_{so} = 550 Å (together with the value of L_ϕ measured in low perpendicular fields), but that these values did not yield agreement for Au(Fe) (Fig. 7). We now consider how different values of L_ϕ, t, and L_{so} affect the level of (dis)agreement. First, if we vary only t, holding L_ϕ and L_{so} fixed at their "best" values (i.e., our best estimates based on the perpendicular magnetoresistance), we find that *no* choice of t can account for the discrepancy in Fig. 7. The effect of changing t in the range 110–140 Å is not large, so we will henceforth keep t fixed at 110 Å, the value that worked well for Au. If we then vary L_ϕ to obtain agreement, we find that rather large values of L_ϕ, $\sim$ 25 000 Å, are required. This seems quite unreasonable, since it is much larger than found from either the perpendicular measurements for Au(Fe) (4000 Å), or for Au. Finally, adjusting L_{so} in either direction also fails to bring agreement; the theory is always a factor of 3 or more smaller than the results.

In our analysis so far we have generally assumed that the various length scales are independent of field, and that L_ϕ is temperature dependent only to the extent implied by the perpendicular magnetoresistance at low fields (Fig. 2). We next consider the effect of relaxing these assumptions. In this context, it is interesting to consider the temperature dependence of the parallel magnetoresistance of Au(Fe) (Fig. 7). One would ordinarily expect the only temperature dependence here to come from that of L_ϕ. For this Au(Fe) sample L_ϕ varies little (from $\approx$4000 to 5000 Å) in going from 4.20 to 1.35 K, so there should be correspondingly little variation in $\Delta R/R$. Surprisingly, the Au(Fe) results exhibited a sizable temperature dependence especially at the larger fields. As discussed in the Introduction, one might naturally expect that L_ϕ will be field dependent when $g\mu_B H \gtrsim k_B T$, since the spin scattering should be frozen out by a large field. However, such behavior cannot explain our results. Evaluation of the theory (6) indicates that letting L_ϕ become larger as H increases does not significantly increase the magnetoresistance. For example, if we assume that L_ϕ increases from 4000 to 6000 Å as the field is increased from zero to 10 000 Oe [which corresponds to quenching *all* of the spin scattering found in Au(Fe)] the parallel magnetoresistance would increase by only $\Delta R/R \approx 1 \times 10^{-5}$, which is about a factor of 10 less than required to explain the results in Fig. 7.

The only length scale left to consider is then L_{so}. It turns out that $\Delta R/R$ is quite sensitive to L_{so}, and permitting it to be field dependent can easily lead to behavior in agreement with Fig. 7; we only need to assume that L_{so} becomes larger as $H_\parallel$ increases. If we require that L_{so} = 550 Å in zero field (the value found in the perpendicular measurements), then L_{so} must increase to about 1000 Å to account semiquantitatively for the magnetoresistance seen in Fig. 7. It is not possible to be quantitative here, since we have no model for how L_{so} might vary with $H_\parallel$. In any event, such behavior could account, at least qualitatively, for the large parallel magnetoresistance of Au(Fe). However, we have no explanation for why L_{so} should be field dependent. While many previous experiments have been successfully interpreted by assuming that L_{so} is independent of field, we should hasten to note that essentially all of those experiments have employed only perpendicular fields (a case for which we

too found agreement with the theory). Also, if L_{so} is field dependent for Au(Fe), then this should presumably also be the case for Au, but for Au the results can be explained well without this assumption. Although it seems likely that they could also be explained with a field-dependent L_{so}, it is still not clear why the presence of Fe leads to the different behavior seen with Au(Fe).

We next consider the parallel magnetoresistance of Ag (Fig. 8). The different curves in Fig. 8 show the theory evaluated with a variety of different values for the length scales that enter the theory (6). The results in Fig. 8 indicate that the only way to obtain acceptable agreement with the experiments is to assume that L_{so} is somewhat smaller than found in small fields; this is illustrated by curve c in Fig. 8. It is not clear how to (or if one can) reconcile this result with those for Au(Fe), but with Ag the fact that the variation of ΔR with $H_{\|}$ is nonmonotonic [due to the reduced spin-orbit scattering as compared to Au and Au(Fe)] makes it more difficult to draw unambiguous conclusions.

Let us next consider the behavior of the perpendicular magnetoresistance in the presence of large parallel fields. For Au, Au(Fe), and Ag we found that in the largest parallel fields, the resistance was more strongly dependent on $H_{\perp}$ than predicted (Figs. 10, 12, and 14). While the field dependence we observed was not large, it was observed in many different runs, with a number of different samples. In the usual measurement of the perpendicular magnetoresistance, the size of the change in $\Delta R/R$ near $H = 0$ is determined by the magnitude of L_{ϕ}, with larger values of L_{ϕ} yielding a greater field dependence. However, in our case it is very difficult to explain the enhanced field dependence by simply assuming a larger L_{ϕ}, since when $H_{\|}$ is large it dominates the phase breaking, making the theoretical prediction almost completely insensitive to L_{ϕ}. The only way the theory can exhibit a greater dependence on $H_{\perp}$ is if the value of $L_{\|}$ is increased. For example, for Au at 4.20 K and $H_{\|} = 6900$ Oe, one must assume that $L_{\|} = 3000$ Å in order to match the field dependence seen in Fig. 10. However, this is not consistent with the value $L_{\|} = 1800$ Å obtained from the behavior with a purely parallel field (Fig. 6). Moreover, the value $L_{\|} = 1800$ Å also extrapolates from the values of $L_{\|}$ which give good agreement at smaller $H_{\|}$ in Fig. 10. This extrapolation assumes only that the field dependence of $L_{\|}$ is described by (9), and it is very hard to see why this would fail. The reason for this discrepancy may well be the theory itself; as noted above, the theory (12) which we have used to analyze the data for combined perpendicular and parallel fields[5] has not, to the best of our knowledge, been obtained from the sort of quantitative calculation used to derive (1) and (6). For this reason, it may be best to defer concern over the "enhanced" dependence of $\Delta R/R$ on $H_{\perp}$ at the highest values of $H_{\|}$.

Close examination of the results for both Au and Au(Fe) (Figs. 10 and 12) reveals another intriguing result; at the highest values of $H_{\|}$ it is seen that $\Delta R/R$ can sometimes *decrease* with increasing $H_{\perp}$. According to the theory, such nonmonotonic behavior with $H_{\perp}$ can only happen when the spin-orbit scattering is of intermediate strength.[2] If we use the value $L_{so} = 550$ Å found from the perpendicular measurements, then the theory (12) does *not* predict such behavior for any reasonable value of $L_{\|}$ or $H_{\|}$. According to (12), this behavior can only occur if L_{so} is increased to at least 2000 Å, or if L_{ϕ} is decreased so as to be comparable to L_{so}. It is hard to see how this discrepancy could be due simply to a failure of the theoretical expression we have used for $\Delta R(H_{\perp}, H_{\|})$. The interplay of $H_{\perp}$, L_{so}, and L_{ϕ}, and the effect on $\Delta R/R$ is well understood and well documented. While it would not be terribly surprising if the expression we have used turns out to be quantitatively incorrect, we would be much more surprised if it were so grossly in error. We therefore believe that the negative magnetoresistance at large parallel fields is suggestive of a field dependence of L_{so} or L_{ϕ}, or both.

Examination of the results for Ag leads to a similar conclusion; the relatively pronounced dependence of $\Delta R/R$ on $H_{\perp}$, when $H_{\|}$ is large, can only be accounted for if $L_{\|}$ is larger than expected based on extrapolations from small $H_{\|}$, and, according to the theory, also requires that L_{so} be increased from the value found from the low-field measurements.

VI. SUMMARY

We have presented the results of extensive magnetoresistance measurements of Au, Ag, and Au(Fe) films. In all cases the results for the purely perpendicular magnetoresistance are well described by the established theory of the WL magnetoresistance. However, the parallel magnetoresistance, especially for Au(Fe) and Ag, cannot be accounted for by the theory, unless we presume that the spin-orbit-scattering length is field dependent. The measurements in combined parallel and perpendicular fields also reveal unexpected behavior, especially when $H_{\|}$ is large. Detailed analysis suggests either that large parallel fields reduce the strength of the spin-orbit scattering, (i.e., L_{so} is increased) relative to the values found in measurements with smaller perpendicular fields, or that L_{ϕ} becomes smaller in a parallel field, or both. It is not clear that this very tentative conclusion concerning the possible field dependences of L_{so} or L_{ϕ} is consistent with the large body of previous work in this field. However, the vast majority of the prior experiments have employed only perpendicular fields. It is possible that the behavior in parallel fields is substantially different than predicted by the theory, or expected based on past results. Of course, it is also possible that some heretofore unanticipated mechanism makes a dominant contribution to the magnetoresistance. It is hard to see how this could explain the results we have obtained in combined parallel and perpendicular fields, but it is a possibility that should certainly not be ruled out.

ACKNOWLEDGMENTS

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[1] B. L. Al'tshuler, A. G. Aronov, D. E. Khmel'nitskii, and A. I. Larkin, in *Quantum Theory of Solids*, edited by I. M. Lifshitz (Mir, Moscow, 1982), p. 130.
[2] G. Bergmann, Phys. Rep. **107**, 1 (1984).
[3] P. A. Lee and T. V. Ramakrishnan, Rev. Mod. Phys. **57**, 287 (1985).
[4] C. Van Haesendonck, M. Gijs, and Y. Bruynseraede, in *Localization, Interaction, and Transport Phenomena*, edited by B. Kramer, G. Bergmann, and Y. Bruynseraede (Springer, Berlin, 1985), p. 221.
[5] P. M. Menz and R. G. Wheeler, Phys. Rev. B **35**, 2844 (1987).
[6] V. I. Fal'ko, Pis'ma Zh. Eksp. Teor. Fiz. **53,** 325 (1991) [JETP Lett. **53,** 340 (1991)].
[7] V. K. Dugaev and D. E. Khmel'nitskii, Zh. Eksp. Teor. Fiz. **86**, 1784 (1984) [Sov. Phys. JETP **59**, 1038 (1984)].
[8] M. E. Gershenzon, B. N. Gubankov, and Yu. E. Zhuraviev, Zh. Eksp. Teor. Fiz. **83**, 2348 (1982) [Sov. Phys. JETP **56**, 1362 (1982)].
[9] Yu. F. Komnik, E. I. Bukhshtab, V. V. Andrievskii, and A. V. Butenko, J. Low Temp. Phys. **52**, 315 (1983).
[10] E. I. Bukhshtab, A. V. Butenko, Yu. F. Komnik, and V. V. Pilipenko, Solid State Commun. **53**, 347 (1985).
[11] R. Rosenbaum, Phys. Rev. B **32,** 2190 (1985).
[12] There are actually two additional regimes depending on the relative magnitudes of t, the elastic mean-free-path, and other length scales. The limit we refer to here as "clean" is the one relevant to our experiments.
[13] D. E. Khmel'nitskii, Physica **126B**, 235 (1984).
[14] Here and throughout this paper we consider the effective fields, lengths, and scattering times (e.g., H_ϕ, L_ϕ, and τ_ϕ, etc. for inelastic, spin, and spin-orbit scattering) interchangeably, since they are all related by (5) and (7).
[15] Expressions (10) and (11) also appear to be the same as those employed in Ref. 10.
[16] M.-T. Béal-Monod and R. A. Weiner, Phys. Rev. **170**, 552 (1968).
[17] H. Rohrer, Phys. Rev. **174**, 583 (1968).
[18] M. A. Blachly and N. Giordano, Phys. Rev. B **46**, 2951 (1992).
[19] C. Van Haesendonck, J. Vranken, and Y. Bruynseraede, Phys. Rev. Lett. **58**, 1968 (1987).
[20] G. Bergmann, Z. Phys. B **48**, 5 (1982).
[21] T. Kawaguti and Y. Fujimori, J. Phys. Soc. Jpn. **52**, 722 (1983).
[22] B. L. Al'tshuler and A. G. Aronov, in *Electron-Electron Interactions in Disordered Systems*, edited by A. L. Efros and M. Pollak (Elsevier, Amsterdam, 1985).
[23] D. Belitz and S. Das Sarma, Phys. Rev. B **36,** 7701 (1987).
[24] These temperatures are well above the Kondo temperature of Au(Fe) ($T_K \approx 0.2$ K), and thus τ_s should be approximately independent of T.
[25] This analysis yields a value of the misalignment angle for each value of $H_\parallel$. As expected, for a given run (and hence sample orientation) these values were found to be independent of $H_\parallel$.
[26] Note that the same is true of the classical magnetoresistance. Our estimates indicate that it should be negligible in our case, but even if it could explain the discrepancy found for Au(Fe), its contribution should also be present in Au, but it evidently is not.
[27] A. Millis and P. A. Lee, Phys. Rev. B **30**, 6170 (1984).
[28] F. J. Ohkawa and H. Fukuyama, J. Phys. Soc. Jpn. **53**, 2640 (1984).
[29] T. van Peski-Tinbergen and A. J. Dekker, Physica **29**, 917 (1963).
[30] We note that the discrepancy with the parallel magnetoresistance of Au(Fe) is not seen in the perpendicular case. Hence, a mechanism which yields an isotropic magnetoresistance (such as the classical magnetoresistance or spin-disorder scattering) is probably not responsible for the extra parallel magnetoresistance. This would also suggest a WL based mechanism as the most likely source of our discrepancy.
[31] G. Chen and N. Giordano, Phys. Rev. Lett. **66**, 209 (1991).
[32] J. F. DiTusa, K. Lin. M. Park, M. S. Isaacson, and J. M. Parpia, Phys. Rev. Lett. **68**, 1156 (1992).
[33] Note that we have ignored the effect of spin precession [S. Maekawa and H. Fukuyama, J. Phys. Soc. Jpn. **50**, 2516 (1981); J. C. Price, Phys. Rev. B **44**, 8997 (1991)]. This effect should only be important when $\omega_p \equiv g\mu_B H/\hbar \gtrsim \tau_{so}^{-1}$, τ_s^{-1}. For our Au and Au(Fe), ω_p at $H = 1$ T is about a factor of 10 smaller than τ_{so}^{-1}, so the effect should be negligible. For Ag, ω_p and τ_{so}^{-1} are comparable at a field of ≈ 2000 Oe, so it may conceivably be important; however, the WL magnetoresistance becomes roughly field independent at this and higher fields, so we will also ignore spin precession in the analysis Ag samples. It does not appear that this effect could account for the unexpected behavior we have observed.