Metals in a high magnetic field: A universality class of marginal Fermi liquids

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Parquet equations, describing the competition between superconducting and density-wave instabilities, are solved for a three-dimensional isotropic metal in a high magnetic field when only the lowest Landau level is filled. In the case of repulsive interaction between electrons, a phase transition to the density-wave state is found at finite temperature. In the opposite case of attractive interaction, no phase transition is found. With decreasing temperature T, the effective vertex of interaction between electrons renormalizes toward a one-dimensional limit in a self-similar way with the characteristic length (transverse to the magnetic field) decreasing as $\ln^{-1/6}(\omega_c/T)$ (ω_c is a cutoff). The residue of the one-particle Green function vanishes at the Fermi surface indicating the marginal character of the Fermi liquid.

I. INTRODUCTION

The behavior of an isotropic three-dimensional (3D) metal in a high magnetic field attracted the attention of physicists for a long time. In this system, the energy of an electron depends only on the momentum along the magnetic field. Thus, the system exhibits effects characteristic of one-dimensional (1D) metals, while intrinsically it is 3D. As an immediate consequence of this fact, it was suggested that the system could be unstable with respect to charge- or spin-density-wave (DW) formation.[1] Another suggestion was that the system could remain superconducting (SC) in an arbitrarily high magnetic field, since the 1D dispersion law still allows for a SC instability.[2] It was pointed out in Ref. 3 that both instabilities must be taken into account simultaneously in the so-called parquet approximation. The parquet equations were written correctly in Ref. 4, where it was found that the DW solution is indeed an asymptotic solution of the equations.[5] However, the equations were not solved numerically, thus the question of when the asymptotic solution develops remained open.

In this paper, the parquet equations for an isotropic 3D metal in a very high magnetic field, with only the lowest Landau level filled, are solved numerically. It is found that in the case of repulsive interaction, a phase transition to the DW state occurs, in agreement with the analysis given in Ref. 4. In the case of attraction, no phase transition occurs, and the system remains a nontrivially correlated metal to arbitrarily low temperatures. With decreasing temperature T, the system becomes progressively more one dimensional, with the characteristic interaction length perpendicular to the magnetic field decreasing as $\xi^{-1/6}$, where $\xi = \ln(\omega_c/T)$, and ω_c is a cutoff of the order of the cyclotron energy. However, the system never becomes strictly 1D. For this reason, various correlation functions have new forms neither characteristic of 1D nor of conventional 3D metals. Thus, the system represents a new nontrivial example of an unconventional Fermi liquid, neither of the Landau, nor of the Luttinger[6] type.

The paper has the following structure. In Sec. II, the parquet equations for a completely spin-polarized case are derived and solved. In Sec. III, the SC and the DW susceptibilities are calculated. In Sec. IV, the one-electron Green functions are calculated. In Sec. V, the consideration is generalized to the case, when electrons of both spin orientations are present, and to the cases of layered and quasi-one-dimensional systems. The issue of the high field superconductivity is discussed in Sec. VI. Conclusions are given in Sec. VII.

II. PARQUET EQUATIONS

Let us consider an isotropic 3D metal with an effective mass m and a parabolic dispersion law. We assume that magnetic field is so high that only the lowest Landau level is filled, and the spins of all electrons are completely polarized in the direction of magnetic field. Dispersion law has the form $\varepsilon = p_z^2/2m$, where p_z is the momentum along the field. The Fermi surface consists of the two Fermi points $p_z = \pm p_F = \pm 2\pi^2\hbar n l_H^2$, where n is the volume concentration of electrons, and $l_H = (\hbar c/eH)^{1/2}$ is the magnetic length. Coordinates X, Y in the plane perpendicular to the field will be measured in units of l_H. The wave functions of the lowest Landau level are[7]

$$\psi_x(X,Y) = \pi^{-1/4} l_H^{-1/2} \exp[ixY - (x-X)^2/2]. \quad (1)$$

Another representation can be obtained making a superposition of states (1):

$$\psi_y(X,Y) = (2\pi)^{-1/2} \int dx \exp(-ixy)\psi_x(X,Y) = \pi^{-1/4} l_H^{-1/2} \exp[iX(Y-y) - (y-Y)^2/2]. \quad (2)$$

To be distinguished from the running variables X and Y, the quantum numbers x and y, which label the wave functions (1) and (2), are denoted everywhere by the lowercase letters.

Let us introduce now the operators $\hat{a}^\dagger(x,p_z)$ and $\hat{b}^\dagger(y,p_z)$ which create electrons in the eigenstates (1) and (2) with p_z close to $+p_F$ and $-p_F$, respectively. Near the Fermi energy, the electron spectrum can be linearized in p_z, and the Hamiltonian of the model can be written in the form $\hat{H}=\hat{H}_0+\hat{H}_1$, where

$$\hat{H}_0 = 2\pi l_H \int \frac{dp_z}{2\pi\hbar} v_F \left[(p_z - p_F) \int dx\, \hat{a}^\dagger(x,p_z)\hat{a}(x,p_z) - (p_z + p_F) \int dy\, \hat{b}^\dagger(y,p_z)\hat{b}(y,p_z) \right], \qquad v_F = p_F/m, \tag{3}$$

$$\hat{H}_1 = g \int \frac{dp_1\, dp_2\, dp_3}{(2\pi\hbar)^3} dx\, dy\, dx' dy' \exp(-ixy' + iyx')\gamma_0(x-x', y-y')\hat{a}^\dagger(x,p_1)\hat{b}^\dagger(y,p_2)\hat{b}(y',p_3)\hat{a}(x',p_1+p_2-p_3), \tag{4}$$

$$\begin{aligned}\gamma_0(\mathbf{r}-\mathbf{r}') &= \exp(i\mathbf{r}\wedge\mathbf{r}')l_H^2 \int dX\, dY\, \psi_x^*(X,Y)\psi_y^*(X,Y)\psi_{y'}(X,Y)\psi_{x'}(X,Y) \\ &= \exp[-(\mathbf{r}-\mathbf{r}')^2/2].\end{aligned} \tag{5}$$

In Eq. (5), $\mathbf{r}=(x,y)$ and $\mathbf{r}'=(x',y')$, and $\mathbf{r}\wedge\mathbf{r}' = xy' - yx'$. In Eq. (4), the interaction amplitude is $g=g_2-g_1$, where g_2 and g_1 are the amplitudes of forward and backward scattering which are conventionally used to parametrize the interactions in 1D systems.[8] The combination g_2-g_1 is the interaction amplitude of electrons with parallel spins. We assume that the interaction has a certain dependence on the momentum p_z on the scale of $2p_F$, so that $g_1 \neq g_2$ and $g \neq 0$. *A priori*, g may be positive or negative[3]. The function γ_0 (5) is the form factor of the interaction between electrons in the representation given in (1) and (2). We assume that the spatial range of the original interaction is much shorter than l_H, so that the point interaction can be used in Eq. (5). It will be argued in Sec. II B that a deviation from expression (5) for the amplitude of interaction could not change results qualitatively. The interaction is assumed to be weak: $|g| \ll v_F l_H^2$.

The so-called parquet diagrams, which consist of the electron-hole and electron-electron loops inserted into each other in all possible ways, are the most important many-body corrections to the interaction vertex $\gamma_0(\mathbf{r})$.[3,4] The corrections form a series in powers of

$$\xi = \frac{|g|}{(2\pi)^3 v_F l_H^2} \ln\left[\frac{\omega_c}{\max(T,\omega,v_F||p_z|-p_F|)}\right], \tag{6}$$

where ω is an energy, since both of the one-loop diagrams are logarithmically divergent. The renormalized vertex of the interaction $\gamma(\mathbf{r},\xi)$ obeys the following equation, shown graphically in Fig. 1:

$$\frac{\partial\gamma(\mathbf{r},\xi)}{\partial\xi} = \int d^2r'\gamma(\mathbf{r}',\xi)\gamma(\mathbf{r}-\mathbf{r}',\xi)(1-e^{i\mathbf{r}\wedge\mathbf{r}'}), \tag{7}$$

$$\gamma(\mathbf{r},0) = \mathrm{sgn}(g)\gamma_0(\mathbf{r}). \tag{8}$$

Equation (7) can be obtained from Eq. (2) of Ref. 4 via a Fourier transformation over the variable k_x.

Another useful representation of the parquet equations, introduced in Ref. 4, can be obtained via two-dimensional (2D) Fourier transformation $\gamma(\mathbf{r},\xi) \to \Gamma(\mathbf{k},\xi)$ of Eq. (7):

$$\frac{\partial\Gamma(\mathbf{k},\xi)}{\partial\xi} = \Gamma^2(\mathbf{k},\xi) - \int \frac{d^2k_1\, d^2k_2}{(2\pi)^2}\Gamma(\mathbf{k}_1,\xi)\Gamma(\mathbf{k}_2,\xi)e^{i(\mathbf{k}_2\wedge\mathbf{k}+\mathbf{k}\wedge\mathbf{k}_1+\mathbf{k}_1\wedge\mathbf{k}_2)}, \tag{9}$$

$$\Gamma(\mathbf{k},0) = \mathrm{sgn}(g)\Gamma_0(\mathbf{k}), \quad \Gamma_0(\mathbf{k}) = 2\pi\exp(-\mathbf{k}^2/2). \tag{10}$$

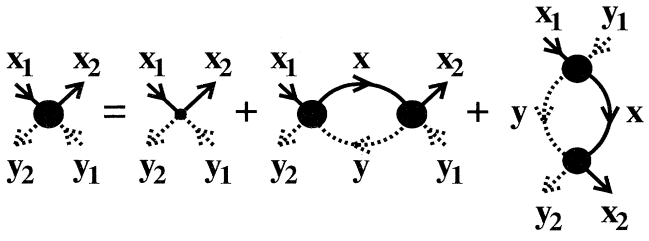

FIG. 1. Parquet equations for the vertex of interaction $\gamma(x_2-x_1, y_2-y_1, \xi)$ which is shown as a filled circle. The bare vertex $\gamma_0(x_2-x_1, y_2-y_1)$ is denoted as a dot. The Green functions of electrons with momenta p_z close to $\pm p_F$ are shown as solid and dotted lines, respectively.

The rhs of Eq. (7) [or Eq. (9)] is a difference of two terms. The first term is the contribution of the electron-hole loop, the second term of the electron-electron loop. If only one of these terms is retained, that corresponds to a ladder approximation instead of a parquet one, then the equations can be solved analytically.

A. Repulsive case

If one neglects the second term on the rhs, then Eq. (9) has a solution:[4]

$$\Gamma(\mathbf{k},\xi) = \frac{1}{\mathrm{sgn}(g)\Gamma_0^{-1}(\mathbf{k}) - \xi}. \tag{11}$$

In the case $g > 0$ (repulsion), solution (11) is called a "moving pole,"[9] because the position $\xi_p(\mathbf{k}) = \Gamma_0^{-1}(\mathbf{k})$ of the pole singularity in ξ depends on the value of $\mathbf{k}$. The vertex (11) becomes singular when ξ reaches the minimum value of $\xi_p(\mathbf{k})$ which, according to Eq. (10), is attained at $\mathbf{k}_c = 0$ and is equal to $\xi_c = 1/2\pi$. As it was shown by the calculation of the appropriate susceptibility in Ref. 4, the moving pole singularity (11) indicates a phase transition to a DW state, where the densities of the electron charge and spin are modulated along the magnetic field with the wave vector $2p_F/\hbar$, and are homogeneous in the perpendicular plane (since $\mathbf{k}_c = \mathbf{0}$). As shown in Ref. 4, once the moving pole (11) develops, the second term on the rhs of Eq. (9) indeed can be neglected, because this term contains the integrations over $\mathbf{k}_1$ and $\mathbf{k}_2$ which make it less singular than the first term. Thus, the moving pole is a possible self-consistent asymptotic solution of Eq. (9).

Solving numerically the full equation (9) at $g > 0$, we do find the moving pole singularity occurring at $\mathbf{k}_c = 0$ and $\xi_c = 1.3/2\pi$. The value of ξ_c is related to the transition temperature by the formula $T_c = \omega_c \exp[-(2\pi)^3 \xi_c v_F l_H^2/|g|]$ [see Eq. (6)]. Thus, the only effect of the second term on the rhs of Eq. (9) in the repulsive case is a certain decrease of the transition temperature. Otherwise, the ladder approximation gives qualitatively correct results.

B. Attractive case

If the first term on the rhs of Eq. (7) is neglected, then the resulting equation can be solved by performing the Fourier transformation $\gamma(x_1, y, \xi) \to \lambda(x_1, x_2, \xi)$ over the variable y and introducing the function $h(x_1, x_2, \xi) = \lambda(x_1 - x_2, x_1 + x_2, \xi)$ obeying the following equation:[10]

$$\partial h(x_1, x_2, \xi)/\partial \xi = -\int dx\, h(x_1, x, \xi) h(x, x_2, \xi). \tag{12}$$

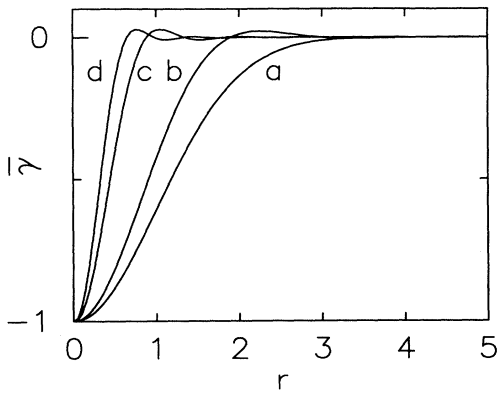

FIG. 2. The vertex of interaction $\bar{\gamma}(r, \xi)$ as a function of r at $2\pi\xi = 0$ (a), 5 (b), 500 (c), and 3200 (d).

With initial conditions (8), Eq. (12) has a solution:

$$h(x_1, x_2, \xi) = \frac{\sqrt{2\pi} \exp(-x_1^2 - x_2^2)}{\mathrm{sgn}(g) + \pi\xi}. \tag{13}$$

In the case $g < 0$ (attraction), Eq. (13) has a pole singularity at $\xi_c = 1/\pi$ that indicates a phase transition to a SC state.[11] Solution (13) is called a "standing pole,"[9] because the position of the pole in ξ does not depend on any continuous variable. For this reason, when expression (13) is substituted into the full equation (7), the first term has the same singularity as the second one, thus the SC ladder approximation can never be justified.[9] This fact explains why it is important to solve numerically the full equation (7) in the case $g < 0$.

Since the initial vertex $\gamma_0(\mathbf{r})$ (5) depends only on r, which is the absolute value of $\mathbf{r}$, then the same holds for $\gamma(\mathbf{r}, \xi)$. Eq. (7) can be rewritten for a new function $\bar{\gamma}(r, \xi) = \gamma(\mathbf{r}, \xi)$, which depends on one spatial argument r:

$$\partial\bar{\gamma}(r,\xi)/\partial\xi = 8\int_0^\infty r_1 dr_1 \int_{|r-r_1|}^{r+r_1} r_2 dr_2\, \bar{\gamma}(r_1,\xi)\bar{\gamma}(r_2,\xi) \frac{\sin^2\left([4(r_1r_2)^2 - (r_1^2 + r_2^2 - r^2)^2]^{1/2}/4\right)}{[4(r_1r_2)^2 - (r_1^2 + r_2^2 - r^2)^2]^{1/2}}, \tag{14}$$

$$\bar{\gamma}(r, 0) = \mathrm{sgn}(g) \exp(-r^2/2). \tag{15}$$

The numerical solution of Eqs. (14) and (15) in the case $g < 0$ is shown in Fig. 2 for several values of "time" ξ. After a short initial evolution, the function $\bar{\gamma}(r, \xi)$ attains the self-similar form $\bar{\gamma}_c[w(\xi)r]$, where $w(\xi)$ is a monotonically growing function of ξ. The ansatz $\bar{\gamma}(r, \xi) = \bar{\gamma}_c[w(\xi)r]$ is consistent with Eq. (14) provided w is sufficiently big. In this case, the sine in Eq. (14) can be replaced by its argument, and Eq. (14) decouples into two equations:

$$dw(\xi)/d\xi = Aw^{-5}(\xi), \tag{16}$$

$$2A\rho\partial\bar{\gamma}_c(\rho)/\partial\rho = \int_0^\infty \rho_1 d\rho_1 \int_{|\rho-\rho_1|}^{\rho+\rho_1} \rho_2 d\rho_2\, \bar{\gamma}_c(\rho_1)\bar{\gamma}_c(\rho_2)[4(\rho_1\rho_2)^2 - (\rho_1^2 + \rho_2^2 - \rho^2)^2]^{1/2}, \tag{17}$$

A is a constant. It follows from (16) that $w(\xi) = [A(\xi - \xi_0)]^{1/6}$. This dependence indeed was found numerically with $\xi_0 = 0$. The function $\bar{\gamma}_c(\rho)$ is also known numerically: with the convention $A = 1$, $\bar{\gamma}_c(\rho) = \bar{\gamma}(\rho\xi^{-1/6}, \xi)$, where for the latter function one can take any of the plots in Fig. 2, except the plot corresponding to $\xi = 0$.

In summary, the solution of the parquet equations in the case $g < 0$ has the self-similar form

$$\gamma(\mathbf{r}, \xi) = \gamma_c(\mathbf{r}\xi^{1/6}), \tag{18}$$

where we introduced a function of two variables $\gamma_c(\mathbf{r}) = \bar{\gamma}_c(r)$. Equation (18) is neither a moving nor a standing pole; it is rather a "squeezing" solution: the effective range of interaction contracts as $\ln^{-1/6}(\omega_c/T)$, thereby making the system increasingly one dimensional as temperature is reduced.

It is rather difficult to study analytically the stability of the solution since the function $\bar{\gamma}_c(\rho)$ is known only numerically. However, the solution persists in the computer up to the biggest employed value of $\xi = 1600/\pi$ without any sign of instability. Since some noise is always present in numerical computations, we conclude that the asymptotic solution is stable and, presumably, has a finite basin of attraction. This means that even if the primary interaction is not local, and the initial vertex differs from Eq. (5), the solution of Eq. (7) may still flow to the same asymptotic form.

III. SUSCEPTIBILITIES

In order to calculate susceptibilities, let us add to the Hamiltonian (3) and (4) the fictitious external fields, $f_{\rm SC}$ and $f_{\rm DW}$, which create electron-electron and electron-hole pairs:

$$\hat{H}_2 = \int \frac{dp_z}{2\pi} dx\, dy\, [f_{\rm SC}(x,y)\hat{a}^\dagger(x,p_z)\hat{b}^\dagger(y,-p_z) + f_{\rm DW}(x,y)\exp(-ixy)\hat{a}^\dagger(x,p_z)\hat{b}(y,p_z-2p_F)] + \text{H.c.} \tag{19}$$

Let us start with the SC susceptibility.[11] According to the parquet rules,[4] it is necessary to calculate first a vertex $\Psi(x,y,\xi)$, which is determined by the graphic equation shown in Fig. 3(a):

$$\frac{\partial\Psi(\mathbf{r},\xi)}{\partial\xi} = -\int dr'\,\gamma(\mathbf{r}-\mathbf{r}',\xi)\Psi(\mathbf{r}',\xi)e^{-i\mathbf{r}\wedge\mathbf{r}'}, \tag{20}$$

$$\Psi(\mathbf{r},0) = f_{\rm SC}(\mathbf{r}). \tag{21}$$

To solve this equation, let us make a Fourier transformation over the variable y: $\Psi(x_1,y,\xi) \to \Lambda(x_1,x_2,\xi)$ and $f_{\rm SC}(x_1,y) \to F_{\rm SC}(x_1,x_2)$; then, introduce the new variables: $\Xi(\bar{x},x,\xi) = \Lambda(\bar{x}-x,\bar{x}+x,\xi)$ and $\Phi_{\rm SC}(\bar{x},x) = F_{\rm SC}(\bar{x}-x,\bar{x}+x)$. In the new variables, Eq. (20) reads

$$\frac{\partial\Xi(\bar{x},x,\xi)}{\partial\xi} = -\int dx'\,\Xi(\bar{x},x',\xi)h(x',x,\xi), \tag{22}$$

$$\Xi(\bar{x},x,0) = \Phi_{\rm SC}(\bar{x},x), \tag{23}$$

where the function h was introduced in Sec. II B. Equation (22) is diagonal and degenerate with respect to $\bar{x}$, the center-of-mass coordinate of a Cooper pair, so this variable can be omitted. Taking into account Eq. (18), one can rewrite Eq. (22) in the form

$$\frac{\partial\Xi(x,\xi)}{\partial\xi} = -\int \frac{dx'}{\xi^{1/3}}\lambda_c\left(x',\frac{2x}{\xi^{1/6}}+\frac{x'}{\xi^{1/3}}\right) \times\Xi\left(x+\frac{x'}{\xi^{1/6}},\xi\right), \tag{24}$$

where $\lambda_c(x_1,x_2)$ is the Fourier transform of $\gamma_c(x_1,y)$ over the variable y. An approximate solution of Eq. (24) is

$$\begin{aligned}\Xi(x,\xi) &= \Phi_{\rm SC}(x)\exp\left\{-\int_0^\xi d\zeta\left[\int d^2r\,\gamma(r,\zeta) - x^2\int d^2r\,\mathbf{r}^2\gamma(\mathbf{r},\zeta)\right]\right\} \\ &= \Phi_{\rm SC}(x)\exp\left(\frac{3\Gamma_0\xi^{2/3}}{2} - 3\Gamma_1x^2\xi^{1/3}\right), \\ \Gamma_0 &= -\int_0^\infty 2\pi r\bar{\gamma}_c(r)dr = 3.1, \quad \Gamma_1 = -\int_0^\infty 2\pi r^3\bar{\gamma}_c(r)dr = 2.2.\end{aligned} \tag{25}$$

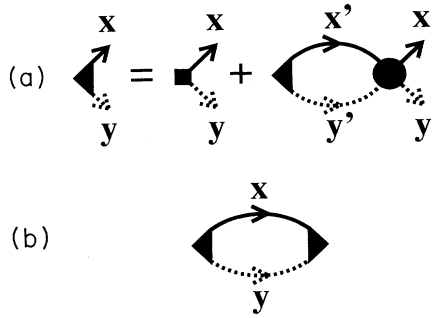

FIG. 3. (a) Parquet equations for the vertex $\Psi(x,y,\xi)$ shown as a filled triangle. The external field $f_{\rm SC}(x,y)$ is denoted as a square. (b) Contribution to the free energy $\delta_1 F$ due to the field $f_{\rm SC}$.

The contribution to the free energy $\delta_1 F$ due to the external field $f_{\rm SC}$ is shown graphically in Fig. 3(b):

$$\delta_1 F = \frac{2}{|g|}\int_0^\xi d\zeta\int dx\,|\Xi(x,\zeta)|^2. \tag{26}$$

Substituting Eq. (25) into Eq. (26), one finds the susceptibility with respect to the creation of an electron pair with the relative distance between electrons equal to x:

$$\chi(x,\xi) = \frac{\xi^{1/3}}{|g|\Gamma_0}\exp(3\Gamma_0\xi^{2/3} - 6\Gamma_1x^2\xi^{1/3}). \tag{27}$$

When $\xi \to \infty$ ($T \to 0$), the susceptibility diverges following an unusual law $\xi^{1/3}\exp(\text{const}\,\xi^{2/3})$. This behavior can be understood qualitatively in the following

way. The total amplitude of scattering to all possible channels,

$$\gamma_t(\xi) = \int d^2r\, \gamma(\mathbf{r},\xi) = -\frac{\Gamma_0}{\xi^{1/3}}, \tag{28}$$

goes to zero with increasing ξ. This means that the system renormalizes toward the noninteracting limit. However, the interaction decreases rather slowly, so that the integral $-\int_0^\xi \gamma_t(\zeta)d\zeta$, which appears in the exponents of (25) and (27), diverges as $\xi^{2/3}$. At the same time, the characteristic distance between electrons in a Cooper pair in (27) squeezes as $\xi^{-1/6}$, which indicates the increasingly 1D character of the system.

The behavior of the susceptibility is different in the case of genuine 1D Tomonaga-Luttinger model. In this model, γ does not depend on ξ since the electron-hole and electron-electron loop diagrams completely cancel each other in the parquet equations.[8] Thus,

$$\chi_{1D} \sim \exp(-2\gamma\bar{\xi}) = \left(\frac{\omega_c}{T}\right)^{|g|/\pi v_F}, \tag{29}$$

where the variable $\bar{\xi} = (2\pi l_H)^2\xi$ was introduced which does not contain the factor $2\pi l_H$ specific to the magnetic field case [see Eqs. (3) and (6)].

In our case, due to the 3D nature of the system, the cancellation of the two terms in the rhs of Eq. (7) takes place only at $\mathbf{r} = 0$. Otherwise, the DW channel dominates and pushes $\gamma(\mathbf{r} \neq 0, \xi)$ to zero. For this reason, the total interaction (28) decreases in absolute value, and susceptibility (27) contains a fractional power of ξ in the exponent instead of the first power in (29). When $\xi \to \infty$, susceptibility (27) is less singular than in the 1D case (29). Unlike (29), Eq. (27) is not a power-law function of temperature and represents a new universality class of Fermi liquid correlation functions. The powers 2/3 and 1/3 of ξ in Eq. (27) are universal constants.

Following an analogous procedure, one can find a contribution to the free energy due to f_{DW}:

$$\delta_2 F = \int_0^\xi \frac{d\zeta}{|g|} d^2k |F_{DW}(\mathbf{k})|^2 \exp\left[2\int_0^\zeta \Gamma_c\left(\frac{\mathbf{k}}{\eta^{1/6}}\right)\frac{d\eta}{\eta^{1/3}}\right],$$

where $F_{DW}(\mathbf{k})$ and $\Gamma_c(\mathbf{k})$ are the 2D Fourier transforms of $f_{DW}(\mathbf{r})$ and $\gamma_c(\mathbf{r})$. For all values of $\mathbf{k}$ the DW susceptibilities have finite limits at zero temperature.

IV. ONE-ELECTRON GREEN FUNCTIONS

The one-electron Green functions $G_\pm$, where the indices $\pm$ refer to the $\hat{a}(x,p_z)$ and $\hat{b}(y,p_z)$ fermions, respectively, are diagonal in the transverse indices x or y and do not depend on these indices even if the interaction is taken into account exactly. In the framework of the renormalization group (RG) approach,[12] the Green functions can be written as

$$G_\pm(\omega,p_z) = Z(\xi)G_\pm^{(0)}(\omega,p_z), \tag{30}$$

where $G_\pm^{(0)}(\omega,p_z) = [\omega \mp v_F(|p_z| - p_F)]^{-1}$ are the Green functions of noninteracting electrons, and $Z(\xi)$ is a renormalization factor which depends on parameter (6). The RG equation for the factor Z can be written taking into account the lowest-order logarithmic diagram shown in Fig. 4:

$$\frac{\partial \ln Z(\xi)}{\partial\xi} = -\frac{|g|}{2(2\pi)^3 v_F l_H^2}\int d^2r\, \gamma^2(\mathbf{r},\xi), \qquad Z(0) = 1. \tag{31}$$

Equation (31) is the same as in the 1D case,[12] except that the integration over the transverse variable $\mathbf{r}$ is added. Substituting Eq. (18) into Eq. (31), one finds

$$Z(\xi) = \exp[-3\Gamma_2\xi^{2/3}|g|/4(2\pi)^3 v_F l_H^2], \qquad \Gamma_2 = \int_0^\infty 2\pi r\bar{\gamma}_c^2(r)dr = 1.9. \tag{32}$$

It follows from Eq. (32) that, when $\omega \to 0$ ($\xi \to \infty$), the "residue" $Z(\xi)$ vanishes at the Fermi surface ($|p_z| = p_F$) at $T = 0$. This means that the considered system belongs to the class of the so-called marginal Fermi liquids.[13] They are distinguished from the conventional Fermi liquids where Z has a finite value at $\omega \to 0$. The only well-studied marginal Fermi liquid is the so-called Luttinger liquid[6] which exists in the 1D Tomonaga-Luttinger model. In the latter model, Eq. (31) does not have integration over $\mathbf{r}$, and γ does not depend on ξ, thus[12]

$$Z_{1D} = \exp\left(-\frac{|g|\bar{\xi}}{4\pi v_F}\right) = \left(\frac{\omega}{\omega_c}\right)^{g^2/8\pi^2 v_F}. \tag{33}$$

An agreement between Eq. (33) and the exact solution of the Tomonaga-Luttinger model[14] confirms the validity of the RG approach in the 1D case. We believe, that the RG approach is applicable to the considered 3D system in magnetic field too, because this system is essentially one dimensional, though highly degenerate.

The rhs of the RG equations (7) and (31) are written using the lowest-order diagrams: the one loop in the case of (7) and the two loop in the case of (31). Higher-order contributions, which contain more loops, can be neglected because they contain higher powers of the small constant g and higher powers of the amplitude γ which decreases in absolute value upon evolution. The latter point is crucial: the system does not renormalize toward the strong interaction limit, thus the perturbational approach is applicable. This statement is also supported by the fact that, when Eqs. (7) and (31) are stripped

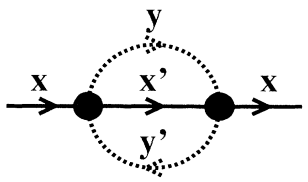

FIG. 4. The lowest-order logarithmic correction to the one-electron Green function.

of the transverse variable $\mathbf{r}$, they describe correctly the behavior of the 1D Tomonaga-Luttinger model, which is known from the exact solution.[14] Thus, we believe that the results, obtained in this paper, are true and are not artifacts of approximations. The results are nontrivial because the RG equations effectively describe a summation of an infinite number of specially selected diagrams.

Comparing expressions (32) and (33), one notes a difference between the one-electron Green functions of the considered model and of the Luttinger liquid. Equation (32) is not a power-law function of ω, and vanishes more slowly than (33) when $\omega \to 0$. Thus, the considered system represents a new universality class of marginal Fermi liquids. The index 2/3 of the fractional power of ξ in Eq. (32) is a universal constant.

V. SOME GENERALIZATIONS

A. Spins

Let us consider now the case when electrons of both spin orientations are present and occupy the lowest Landau level only. This is possible when the spin g factor is less than 2. In this case, the electrons with spins up and down have different Fermi momenta equal to $p_F \pm \Delta_Z/v_F$, where Δ_Z is the Zeeman energy. In addition to the amplitude of interaction between electrons with parallel spins, represented in Eq. (4), there is also an amplitude of interaction between electrons with antiparallel spins. Due to the difference of the Fermi momenta, the process, where the $\hat{a}$ and $\hat{b}$ fermions exchange their opposite spins, involve momenta far from the Fermi surface. Thus, this process does not contribute to the logarithmically divergent corrections and can be neglected. The remaining amplitudes of interaction between the $\hat{a}$ and $\hat{b}$ fermions with the parallel and the antiparallel spins do not mix in the parquet equations. The equations have the same form (7), while initial conditions (8) and the definitions of ξ (6) contain $g = g_2 - g_1$ and g_2 for the amplitudes of interaction between the parallel and the anti-parallel spins, respectively. (See also discussion of this subject in Ref. 10.)

The behavior of the system depends on the signs of g and g_2. If at least one of them is positive, then the electron-hole pairing (DW) with the parallel or the antiparallel spins appears in the system. If both amplitudes are negative, then the system is a marginal Fermi liquid as described above.

If $\Delta_Z \ll \omega_c$, then in the intermediate energy range $\Delta_Z \ll \omega \ll \omega_c$ the Zeeman splitting may be neglected, and the parquet equations have to be modified. We will not study this case here.

B. Layered systems

The results, described above, are valid also for an anisotropic system with a parabolic dispersion law perpendicular to the magnetic field and any dispersion law, which is reasonable to linearize, along the magnetic field. Particularly, the results are valid for a layered system in the magnetic field perpendicular to the layers provided the bandwidth of the electron motion across the layers is much higher than the amplitude of interaction.

If the latter condition is not fulfilled, the system is strongly correlated, and the perturbational approach, employed here, is not valid. This happens because in a strong magnetic field electrons have no kinetic energy of the motion inside the layers, and the only kinetic energy comes from the motion between the layers. To decide whether the system is weakly or strongly correlated, one has to compare the kinetic energy and the interaction (see also Ref. 15).

The approach of the paper is inappropriate, particularly, for a strictly 2D system, where there is no kinetic energy at all. It was emphasized in Ref. 16, that in the 2D case there is no logarithmic parameter to justify the selection of the parquet diagrams.

C. Quasi-one-dimensional systems

Parquet equations, similar to Eq. (9), also describe the behavior of a 2D metal with quasi-1D anisotropy in a strong magnetic field.[10] In that case, the main kinetic energy comes from the 1D motion along the chains, and the magnetic field affects the electron motion between the chains. Numerical solution[10] shows that the phase transitions to the DW states take place in both cases of repulsive and attractive interaction between electrons. Even if initially the interaction is attractive (favorable for superconductivity), in the course of renormalization the sign of the interaction is effectively changed giving rise to a nontrivial DW state exhibiting the integer quantum Hall effect.[17]

VI. THE ISSUE OF THE HIGH FIELD SUPERCONDUCTIVITY

Several groups discuss theoretically the possibility of existence of superconductivity in very high magnetic fields: see the most recent papers[18–21] and references herein. Here, we limit the discussion to the orbital effects only. The effects of spins are considered in detail in Ref. 22.

The authors of all the above-mentioned references employ a ladder or a mean-field approximation with respect to the SC channel only, thus neglecting the influence of the competing DW channel. As shown in the present paper, such an approach is certainly wrong at least in the case, discussed mostly in Ref. 18, when only the lowest Landau level is filled. Other groups[19–21] pay more attention to the case when several Landau levels are filled. In this case, the competition with the DW channel is more complicated, and a further investigation is necessary.

Another warning is that the authors of Refs. 19–21 do not pay enough attention to the difference in physical behavior of 3D and 2D systems in magnetic field. Mathematically, the mean-field equations are similar in the 2D and 3D cases. However, physically, there is a big difference. As discussed in Sec. V B, in the 3D case there is a kinetic energy of the electron motion along magnetic

field. As a consequence, the perturbational approach can be applied, and the logarithmic parameter justifies the selection of the parquet or the ladder diagrams. In the 2D case, there is no kinetic energy, thus the system is strongly correlated, so the perturbational approach and the selection of the ladder diagrams cannot be justified.[16]

However, the results of the present paper should not be interpreted as a "no-go" theorem for the high field superconductivity. In a 3D system, periodically modulated perpendicular to the magnetic field, the degeneracy of the Landau wave functions is lifted. Due to the additional energy dispersion, the SC and the DW channels are decoupled and do not suppress each other. The mechanism of decoupling is the same as in quasi-1D conductors with the interchain electron hopping.[8,9]

An extreme case of such modulation can be found in layered systems when magnetic field is applied parallel to the layers.[23] A particular case is represented by quasi-1D organic superconductors $(TMTSF)_2X$ (where TMTSF is tetramethyltetraselenfulvalene, X is a monovalent anion), which also have a layered structure.[24] In this geometry, the magnetic field can suppress the electron hopping between the layers, however, it has no effect on the electron motion inside the layers. Thus, the magnetic field should not suppress superconductivity.[23,24] It is important that the electrons in the layers have a 2D, not 1D, dispersion law, thus the competition with the DW channel is suppressed.

VII. CONCLUSIONS

An isotropic 3D metal in a high magnetic field is described as an infinite set of coupled 1D systems. A self-similar "squeezing" solution of the corresponding renormalization group equations is found for the first time. The solution describes a new type of the marginal Fermi liquid[13] where the residue Z of the one-electron Green function vanishes at the Fermi surface. Correlation functions in this system have unusual non-power-law forms indicating that the system represents a new universality class of Fermi liquids. This solution enriches our intuition beyond the Landau and the Luttinger schemes in consideration of what happens when many 1D channels interact.[25,26]

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