Magnetic relaxation over the Bean-Livingston surface barrier

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Magnetic relaxation over the Bean-Livingston surface barrier is considered in high-temperature superconductors at fields $H > H_p$ (where $H_p > H_{c1}$ is the first field for flux penetration) using the Clem model for a critical state with a surface barrier. The relaxation rates for vortex entry and exit are expressed through the basic thermodynamic characteristics of a superconductor. For the flux exit the magnetization $M(t)$ depends logarithmically on time whereas for the case of entry $M(\ln t)$ appears to be a strongly nonlinear function with downward curvature, as has been found experimentally. The initial relaxation rate, $dM/d\ln t$, proves to be much larger for flux entry than for exit, in contrast to the case of conventional bulk creep. The competing interplay between this surface relaxation and the usual bulk one, which results in a crossover in the $M(\ln t)$ curves, is discussed.

I. INTRODUCTION

Magnetic relaxation, being one of the key methods for investigation of irreversible properties in superconductors, provides important data on the pinning of the Abrikosov vortices, their dynamic properties, structure of the critical state, etc. Since the discovery of the irreversibility line[1,2] and the giant flux creep[3] in the high-temperature superconductors (HTSC), numerous experimental and theoretical investigations of the flux motion and magnetic relaxation in these compounds were carried out. Different original models, e.g., vortex glass,[4] vortex lattice melting,[5] and others were introduced to explain the unusually high relaxation rates in HTSC.

Present descriptions of magnetic relaxation are based conventionally on the Bean critical-state model[6] and the theory of the thermally activated flux creep.[7–9] The latter predicts the logarithmic decay of the magnetic moment M in time. Recent papers are mostly devoted to theoretical and experimental study of such a decay in $YBa_2Cu_3O_{7-x}$, $Bi_2Sr_2CaCu_2O_x$, and other HTSC compounds.[10–18] This approach is very fruitful for evaluation of the characteristic pinning energies U, their distribution, and, in turn, critical currents. The conventional Bean and Anderson-Kim schemes have been elaborated significantly to provide better fit to the experimental data. The former was generalized[19,20] for rather complicated dependencies of the critical current J_c on the local field B, which enables one to explain the peak in the dependence of the logarithmic relaxation rate, $dM/d\ln t$, on the external field H. The Anderson-Kim creep theory was modified in order to account for the possible nonlinear dependence of the activation energy U on J_c, collective creep phenomena, etc.[9] But, nevertheless, some important properties of relaxation phenomena in HTSC are far from consistent understanding yet. Thus, nonlogarithmic time dependencies of M have been reported recently[12–15] and the dependence of the relaxation rate on the external field H and temperature T appears to be rather complicated.[12] In Refs. 14 and 17 the presence of crossover in the $M(\ln t)$ dependence was reported. All these peculiarities imply that there should exist other mechanisms for magnetic relaxation rather than the conventional flux creep.

In this paper we will describe the surface contribution to the relaxation process and prove that at some conditions, especially at high temperatures $T \simeq T_c$, the surface relaxation becomes more important than the usual bulk creep in HTSC. As has been shown by Bean and Livingston,[21] at the surface of the type-II superconductors there appears a potential barrier that prevents the vortices from entering and leaving the sample. The Bean-Livingston (BL) barrier arises from the competition between the *repulsion* of a vortex from the surface due to its interaction with the exponentially decreasing external field (or, in other words, with the shielding current), and the Magnus hydrodynamic *attraction* to the surface. The latter force is usually described as an interaction of a vortex with its mirror imaged "antivortex." Such a barrier inhibits flux penetration inside the sample at the first critical field, H_{c1}, where penetration first becomes thermodynamically favorable. Instead of H_{c1}, penetration starts at the first field for flux penetration, $H_p > H_{c1}$, where the barrier disappears. In the case of an ideal surface, H_p is approximately equal to the thermodynamic field, $H_c \simeq \kappa H_{c1}/\ln(\kappa) \gg H_{c1}$, where $\kappa = \lambda/\xi$ is of order 100 in HTSC (λ is the penetration depth and ξ is the coherence length).

Because of these large values of κ, the BL barrier should be very pronounced in HTSC, as has already been discussed,[13,22–24] even if being diminished by surface imperfections. Some evidence for the BL barrier in HTSC was achieved last year. Namely, the magnetization curves in very clean untwinned samples near T_c were found[23,17] to be described well by the Clem model,[25] where the bulk pinning is totally neglected and only the surface barrier is responsible for the irreversible properties. According to that model, the magnetization loop proves to be very asymmetric, in contrast to the Bean model for bulk pinning. Particularly, the magnetization M should be negligibly small at the decreasing external field H, which has been observed experimentally.[23,24] The first penetration field, H_p, was proved to be larger than H_{c1} and to vary significantly within the interval $H_{c1} < H_p < H_c$ as a function of temperature,[23,24] which results in a strongly nonlinear dependence of H_p on T near T_c. And probably the most direct evidence for the BL barrier was achieved by an electron irradiation of the sample[23] after which H_p reduced drastically due to suppression of the barrier by surface damage introduced by the irradiation (the bulk T_c remains constant after the irradiation, which confirms that this is really a surface effect). The BL barrier was also reported to affect the relaxation data[14,17] and the low-temperature behavior of the H_{c1} field in $YBa_2Cu_3O_x$ (Ref. 26) and $Tl_2Ba_2CaCu_2O_x$ (Ref. 13).

The goal of this paper is to obtain the basic relations for the thermoactivated surmounting of the barrier by an Abrikosov vortex and thus to make predictions about the surface relaxation. We will consider the usual Abrikosov vortices, which should be relevant to $YBa_2Cu_3O_x$, and then compare the results with the case of two-dimensional "pancake"[27] vortices, which have already been discussed.[13,28] It will be proved that the flux entry in and exit from the sample (or, in other words, the relaxation "in" and "out," respectively) are very asymmetric. While the rate of flux exit, $R_{ex} = dM/d\ln t$ is constant throughout the whole relaxation process, that of flux entry, R_{en}, starts from very high values at the initial stage of the relaxation in and then continuously decreases. Since the experimental "time window" in most cases is enough to observe only this initial stage, we predict $R_{en} > R_{ex}$ in the experiment. Also the temperature dependence of R_{en}, R_{ex} is quite different. Finally we discuss the interplay between the bulk and surface relaxation, which results in the appearance of a crossover in the $M(\ln t)$ curves, which has been observed experimentally.[14,17]

It should be emphasized that our analysis differs from the recent calculations of relaxation, where some modifications of the Bean critical-state model[19,20] and the Anderson-Kim creep theory[12] were made. Instead of the Bean-like models, we use the Clem model, where the bulk pinning is neglected completely and the magnetization loop is determined by the BL surface barrier only. Then we discuss a formation of the vortex nucleus near the surface and its spreading over the barrier in a similar manner to that done by Petukhov and Chechetkin[29] and by Koshelev.[30] This approach has an apparent advantage that the activation energy U is not introduced as a given parameter, as in the bulk-creep models, but is calculated directly. Therefore, the answer is expressed through the basic thermodynamic characteristics of superconductor. It is worth mentioning that our consideration differs from the case where some enhancement of pinning near the surface of Nb was considered due to increase of the pinning site concentration there.[32]

II. MAGNETIZATION LOOP WITH THE SURFACE BARRIER

Magnetization loop $M(H)$, where H is the external field, embraces the area of metastable states $M(H)$ around the equilibrium Abrikosov magnetization curve $M_{eq}(H)$, see Fig. 1. The magnetization curve is given by the boundary values for flux entry, M_{en} (i.e., obtained in the increasing H) and that for flux exit, M_{ex} (for decreasing H) provided the experimental measurements are fast enough to ignore all the relaxation processes. In the classic Bean model[6] and its modifications,[19,20] M_{en} and M_{ex} are determined by the critical current $J_c(B)$, sample size and shape, history of the process, etc. In the M vs H phase diagram the relaxation means that a point $M(H)$ moves at given H towards the thermodynamically equilibrium value, M_{eq}. Thus the understanding of the relaxation requires detailed knowledge about the structure of the metastable states $M_{ex} < M < M_{en}$.

The case of negligible bulk pinning, where only the surface contribution is taken into account, was considered by Clem[25] (see also Ref. 33). We reproduce his consideration below in greater detail.

Consider the case where the mean distance d between the Abrikosov vortices is less than the penetration depth λ, so the field distribution in the sample can be described by the average local flux density $\Phi(x)$; see Fig. 2. This condition practically always holds and is used also in the Bean model, where $\Phi(x)$ form linear or more complicated profiles.[6,19,20] Clem[25] has shown that in the absence of bulk pinning the solution for $\Phi(x)$ being in force balance (but not in the thermodynamic equilibrium) with the exponentially decreasing external field, $H\exp(-x/\lambda)$, is

$$\Phi(x) = B\,\theta(x_f - x) \qquad (1)$$

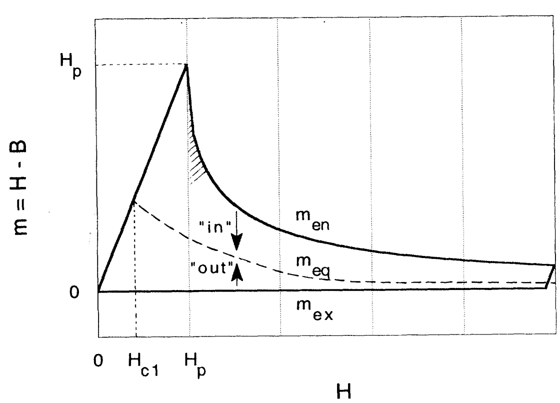

FIG. 1. The Clem magnetization loop for the surface irreversibility without any bulk pinning. Arrows indicate relaxation in (from m_{en} to m_{eq}) and out (from m_{ex}).

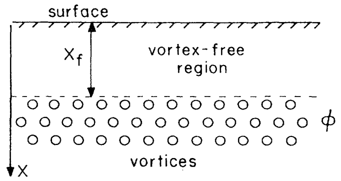

FIG. 2. Structure of the vortex lattice near the surface and formation of a vortex-free region of width x_f.

where θ is the Heavyside step function, B is the magnetic induction inside the sample, and x_f is the vortex-free region of the width

$$x_f = \lambda \cosh^{-1}(H/B). \quad (2)$$

Equation (2) can be derived in a straightforward manner from the condition that the superposition of the external field $H\exp(-x/\lambda)$, the field of vortices at $x > x_f$, and their mirror images at $x < -x_f$ form a constant field B at $x > x_f$, so the vortices are force balanced. The mirror images should be added, similarly to Ref. 21, to satisfy the boundary condition $h(0) = H$ where $h(x)$ is the local field. We perform averaging over the distances of order $d \ll \lambda$; therefore $h(x)$ differs from $\Phi(x)$, unlike the Bean models, where they coincide due to averaging over distances of order λ. At $x > x_f$ they are equal in our case too: $\Phi(x) = h(x) = B$, but at $0 < x < x_f$, where $\Phi(x) = 0$, one obtains

$$h(x) = B \cosh[(x_f - x)/\lambda]. \quad (3)$$

The condition of penetration of a new vortex from the surface into the bulk means that the force of interaction, $(\phi_0/4\pi)\, dh/dx$, of this test vortex with the field $h(x)$, which pushes it into the sample, dominates upon the vortex-image attraction, $(\phi_0^2/8\pi^2\lambda^3)\, K_1(2x/\lambda)$,[21,34] which prevents penetration (here ϕ_0 is the unit flux and K_1 is the modified Bessel function). The latter force increases sharply (as 1/x) at $x \approx \xi$, where dh/dx remains almost constant, so the condition of penetration is determined at the cutoff $x = \xi$ as $(dh/dx)_{x=0} \approx (\phi_0/2\pi\lambda^3)K_1(2\xi/\lambda) \approx \phi_0/4\pi\lambda^2\xi$. For the first penetration field, H_p [where there is no flux yet in the sample, so $B = 0$ and $h(x) = H\exp(-x/\lambda)$], we have $H_p \simeq \phi_0/4\pi\lambda\xi \simeq H_c$, where $H_c \approx \sqrt{H_{c1}H_{c2}} \approx \sqrt{2}\kappa H_{c1}/\ln(\kappa)$ is the thermodynamic critical field. In most cases the vortex-image attraction is diminished by the surface imperfections, so the condition $H_p \simeq H_c$ should be replaced by $H_{c1} < H_p < H_c$. At $H > H_p$, where $B \neq 0$, we derive, using the same condition for $(dh/dx)_{x=0}$ and Eqs. (2) and (3), the equation for the magnetization curve in the ascending external field (flux entry) (see Refs. 25 and 33),

$$H_{\rm en} = (H_p^2 + B^2)^{1/2}. \quad (4)$$

The condition $d \ll \lambda$ holds everywhere along the hyperbola (4), except the very vicinity of H_p, where $B \leq H_{c1}$ (dashed in Fig. 1). At $H > H_p$ one can use an approximation for Eq. (4):

$$m_{\rm en} = H - \sqrt{H^2 - H_p^2} \approx H_p^2/2H, \quad (5)$$

where $m = H - B = -4\pi M$ (M is the true magnetic moment, but we will use below the reduced value, m, for simplicity). This value should be compared with the equilibrium (Abrikosov) value $m_{\rm eq} = H - B_{\rm eq}$ to prove that our consideration is self-consistent. The latter has been discussed in detail recently[35] and the evaluation is

$$m_{\rm eq}(H) = \frac{\alpha H_{c1}}{2\ln\kappa} \ln\frac{\beta H_{c2}}{H}, \quad (6)$$

where $\alpha, \beta \simeq 1$. The dependence $m_{\rm eq}(H)$ is very smooth (logarithmic), so from Eq. (6) we have

$$m_{\rm eq}(H_p) \simeq H_{c1}/2$$

(since $H_p \simeq H_c$ and $H_{c2}/H_c \simeq \kappa$) and, using Eq. (5),

$$m_{\rm en}/m_{\rm eq} \simeq H_p^2/H_{c1}H \simeq H_{c2}/H \gg 1. \quad (7)$$

Consider now the flux exit, following Ref. 25. As H decreases, x_f decreases also [see Eq. (2)], whereas B remains constant because, while x_f is finite, vortices cannot leave the sample. Finally x_f vanishes at $H = B$, where vortices are free to leave. Consequently, for vortex exit we have $B = H$, or $m_{\rm ex} = 0$, as has been observed.[23,24] There exists a problem whereby at very small $m \ll H_{c1}$ the width of the vortex-free region x_f becomes smaller than the mean intervortex distance d at $x > x_f$. To solve it and expand this approach down to $m = 0$, Clem[25] considered separately the interaction of a test vortex at $x = x_f$ with its mirror image. He derived that the surface barrier exists and prevents flux exit at $m > m_0 = \phi_0/16\pi\lambda^2 \ll m_{\rm eq}$, so the picture of flux exit appears also to be self-consistent. The moment m_0 determines the descending branch, $m_{\rm ex}$, of the magnetization curve in Fig. 1. Since m_0 is much less than all the other moments in our problem, we can put $m_{\rm ex} = 0$, as above. It is worth mentioning that Ternovskii and Shekata,[33] who analyzed the same problem simultaneously with Clem, came to slightly different result for m_0 and have not emphasized the formation of the vortex-free region x_f.

III. FORMATION OF A VORTEX NUCLEUS AND SURMOUNTING THE BARRIER

Consider first the case where the external field, after a fast increase from 0 to H, is kept at this value, so the moment m decreases from its initial value $m_{\rm en}$ given by Eq. (5), where the barrier is absent, to $m_{\rm eq}$; see Fig. 1. This means that new vortices are to be created near the surface and to surmount the barrier, which grows as m decreases. The energy profile $V(x)$ for vortex nucleation at $x < x_f$ can be written as

$$V_{\rm en}(x) \approx \frac{\phi_0}{4\pi}\left[\frac{B}{2}\left(\frac{x_f - x}{\lambda}\right)^2 + m_{\rm eq} - m + S\right] \quad (8)$$

(see Fig. 3), where S includes the attraction of the nu-

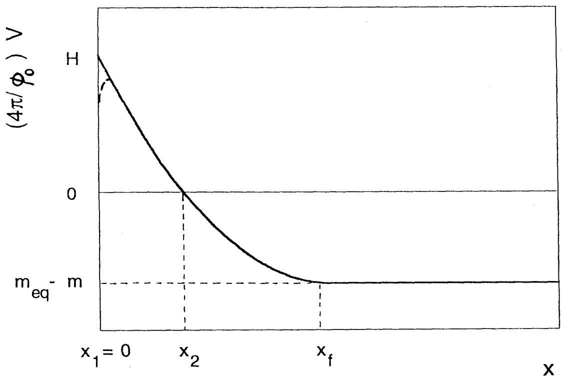

FIG. 3. The generalized Bean-Livingston potential for vortex entry, where x the depth nucleus penetration (in the isotropic case this is just the radius of the nucleus, see in the text). The dashed line accounts for the "mirror-image" interaction, S, at $x \simeq \xi$.

cleus to its own "mirror image," normalized by $S(x_f) = 0$, and we used the expansion of Eq. (3) at $x_f \ll \lambda$. At $x > x_f$ we get $V_{\rm en} \propto m_{\rm eq} - m$, so vortex penetration becomes thermodynamically unfavorable as soon as m decreases till $m = m_{\rm eq}$, that explains the normalization chosen in Eq. (8). The latter provides a natural generalization of the Bean-Livingston energy profile,[21] which was considered in their original work for the case $B = 0$ (i.e., at $H < H_p$).

Let us estimate S and show that it can be neglected at $x \gg \xi$. The vortex nucleus, which has just appeared near the surface and penetrated to the depth $a \ll \lambda$, has the shape of a semi-ellipsis (circle in the isotropic superconductor);[38,30] see Fig. 4. Thus S is the energy of interaction of this semi-ellipsis with its mirror image and can be calculated using a universal method developed by Brandt,[39] where the flux line is considered as a composition of the pointlike vector elements, see Fig. 4. In the isotropic case the nucleus is a semicircle of radius $a \ll \lambda$ and the interaction between its two elements, dr_i and dr_j, becomes

$$u_{ij} = (\phi_0/4\pi\lambda)^2 \cos\alpha \, \exp(r_{ij}/\lambda)/r_{ij},$$

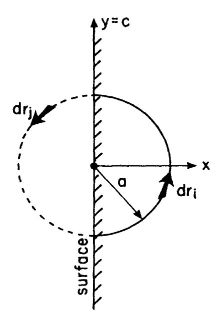

FIG. 4. Vortex nucleus (semicircle) with its mirror image (dashed) near the surface.

where r_{ij} and α are the distance and the angle between these two elements. If we take $r_{ij} \approx a$ and average over α, we get $S \approx \phi_0^2 a/8\pi^4\lambda^2$. The self-energy of the semicircle is $E = \varepsilon_0 \pi a$, where $\varepsilon_0 = (\phi_0/4\pi\lambda)^2 \ln\kappa$ is the energy of flux line per unit length; therefore $E/S \approx (\pi^3 \ln\kappa)/2 \gg 1$ and S proves not to be essential for our analysis. The anisotropy does not affect this estimation significantly.

The energy profile given by Eq. (8) forms a barrier, which a vortex has to surmount while entering or leaving the sample. This problem is similar to those where the string movement in external potential is considered, and the algorithm for its solution is as follows. First we have to find the equilibrium form of a nucleus, then determine the energy of the nucleus, which has reached the same energy level from which it started to propagate (this means that spreading of the nucleus over the barrier becomes energetically favorable), and finally estimate the rate of this spreading. This problem was considered by Petukhov and Chechetkin[29] at $H_{c1} < H < H_p$, who concluded that such a penetration is practically impossible because the characteristic activation energies U exceed kT by five orders of magnitude. Below we will prove that at $H > H_p$, as it follows from the results of the previous section, the barrier is much less which, together with large T_c in HTSC, result in observable relaxation rates.

Consider the case where the external field is parallel to the c axis. Let y be parallel to the sample surface along the c axis and x, as in the previous section, is directed into the sample; see Fig. 4. The energy of a curved vortex nucleus $x(y)$ placed into an external potential $V(x)$ can be written as[29–31]

$$U = \int \left\{ \frac{\phi_0 H_{c1}}{4\pi} \left[\sqrt{1 + \frac{1}{\gamma}\left(\frac{dx}{dy}\right)^2} - 1 \right] + V(x) \right\} dy, \tag{9}$$

where $\gamma = m_c/m_{ab}$ is the anisotropy parameter, m_c and m_{ab} are the effective masses along the c axis and in the ab plane. This expression accounts for the increase of the vortex energy through the increase of its length (which implies that the characteristic curvatures should be much less than $1/\xi$), partially compensated due to the anisotropy γ. Since γ is a large parameter ($\gamma \approx 25$ for $YBa_2Cu_3O_7$), the range of applicability of the Eq. (9) is very wide and covers most of possible vortex configurations; see a detailed discussion in Ref. 31. Varying Eq. (9) with respect to $x(y)$, we find the condition for the equilibrium form of nucleus, $x_0(y)$, and for the activation energy U; see Ref. 29:

$$U = \frac{2}{\sqrt{\gamma}} \int_{x_1}^{x_2} \sqrt{V(x)\left(\frac{\phi_0 H_{c1}}{2\pi} - V(x)\right)}\, dx, \tag{10}$$

where $V(x_1) = V(x_2)$, so for flux entry $x_1 = 0$ and x_2 is determined by $V(x_2) = 0$; for flux exit $x_2 = x_f$ and $V(x_1) = V(x_f)$.

Flux entry (relaxation "in"). For the case of flux entry into the sample we get [see Eq. (8) and Fig. 3] $x_1 = 0$

and

$$(x_f - x_2)/x_f = \sqrt{1 - m_{\rm eq}/m}. \qquad (11)$$

Then, using Eqs. (10) and (11), we obtain

$$U_{\rm en} = \frac{\phi_0 \lambda}{2\pi}\sqrt{\frac{1}{\gamma}\frac{H_{c1}}{B}}\left[\sqrt{m m_{\rm eq}} + \frac{1}{2}(m - m_{\rm eq})\ln\frac{\sqrt{m}-\sqrt{m_{\rm eq}}}{\sqrt{m}+\sqrt{m_{\rm eq}}}\right]. \qquad (12)$$

Since $m_{\rm en} \gg m_{\rm eq}$ [see Eq. (7)], we can expand Eq. (12) over $m_{\rm eq}/m$ in most of the relaxation region, $m_{\rm en} > m > m_{\rm eq}$, and get a more convenient expression:

$$U_{\rm en} \approx \frac{\phi_0 m_{\rm eq} \lambda}{3\pi}\left(\frac{1}{\gamma}\frac{H_{c1}}{B}\frac{m_{\rm eq}}{m}\right)^{1/2}. \qquad (13)$$

Thus, using the condition $B \simeq H \gg m_{\rm eq}$, one can see from Eq. (13) that the activation energy increases from

$$U_{\rm en}(m_{\rm en}) \simeq (\phi_0\lambda m_{\rm eq}/3\pi)(H_{c1}/H)^{1/2}(m_{\rm eq}/m_{\rm en})^{1/2} \approx 0$$

till

$$U_{\rm en}(m_{\rm eq}) = (\phi_0 \lambda m_{\rm eq}/2\pi)(H_{c1}/H)^{1/2}$$

as m decreases.

The activation energy U determines the main (exponential) term in the Arrhenius expression for the characteristic time τ for surmounting the barrier (per unit surface area)

$$1/\tau = \nu \exp(-U/kT), \qquad (14)$$

where ν has the meaning of the "attempt" frequency.[7,8] Its estimation found by Petukhov and Chechetkin,[29] who solved the Fokker-Planck equation for vortex spreading into the sample at $H \approx H_{c1}$, is

$$\nu \simeq \frac{1}{\lambda^4 \eta \sqrt{T}}\left(\frac{\phi_0 H_{c1}}{\lambda}\right)^{1/2},$$

where η is the viscosity coefficient for flux flow. We will not actually use this expression, since the crucial term in Eq. (14) is the exponent provided U/kT is sufficiently large, as in the case of the usual bulk creep.[7,8] From Eq. (14) we conclude that

$$dm/dt = \phi_0 (A_s/A_f)\, \nu_0 \exp(-U/kT) \qquad (15)$$

where A_s and A_f are the side and face areas of the sample (vortices penetrate through the side).

In the next section we will estimate the value of U/kT for the case of $YBa_2Cu_3O_x$ and show that the characteristic values are 10–100, so the relaxation can be observable in contrast with the predictions for $H < H_p$ obtained in Ref. 29. Nevertheless the exponent in Eq. (15) is small, and the solution of Eq. (15) for the relaxation rate, R, with the exponential accuracy reads

$$R = dm/d\ln t \simeq kT(dU/dm)^{-1}. \qquad (16)$$

As follows immediately from Eq. (16), the dependence of $\ln t$ as a function of m is very similar to that of U. Thus, for flux entry we get (entry)

$$\ln(t/t_0) = [U_{\rm en}(m(t)) - U_{\rm en}(m(t_0))]/kT, \qquad (17)$$

where $U_{\rm en}(m)$ is determined by Eq. (12) and t_0 is the initial time. This function is plotted in Fig. 5. We see that for flux entry the dependence $m(\ln t)$ proves to be strongly nonlinear if considered in the whole region $m_{\rm en} < m < m_{\rm eq}$. Such a behavior has been observed by Donglu Shi and Ming Xu[15] in $Bi_2Sr_2CaCu_2O_x$ at 40 K, though their explanation of the effect is rather different. Certainly the experimental "time window" can be too small to observe the whole curve in Fig. 5. We will estimate it in the next section and discuss the experiment[15] more.

Flux exit (relaxation "out"). The procedure of calculation of the activation energy $U_{\rm ex}$ for flux exit is quite similar to that for flux entry. We should use Eq. (8), where the expansion over $(x_f - x)$ already accounts for the circumstance that the interaction between the test vortex and its image is not important in this case as well as for flux entry. Then, similar to Eqs. (12) and (13), we obtain

$$U_{\rm ex} = \frac{\phi_0\lambda}{2\pi}\, m \left(\frac{1}{\gamma}\frac{H_{c1}}{B}\right)^{1/2}. \qquad (18)$$

As m increases from its initial value $m_{\rm ex}$ till $m_{\rm eq}$ (see Fig. 1), $U_{\rm ex}$ increases as well from $U_{\rm ex}(m_{\rm ex}) \approx 0$ till $U_{\rm ex}(m_{\rm eq}) = (\phi_0 m_{\rm eq}\lambda/2\pi)(H_{c1}/B)^{1/2}$, which coincides with $U_{\rm en}(m_{\rm eq})$ because at $m = m_{\rm eq}$ the limits in the integral in Eq. (10) are the same for both entry and exit: $x_1 = 0$ and $x_2 = x_f$. Using Eqs. (16) and (18), we immediately get (exit)

$$m(t) - m(t_0) = \frac{2\pi}{\phi_0\lambda}\left(\gamma\frac{B}{H_{c1}}\right)^{1/2} kT \ln(t/t_0). \qquad (19)$$

IV. ESTIMATION OF THE PARAMETERS

In the preceding section we have proved that the relaxation starts at $m_{\rm en}$ [see Eq. (5)] for flux entry and at

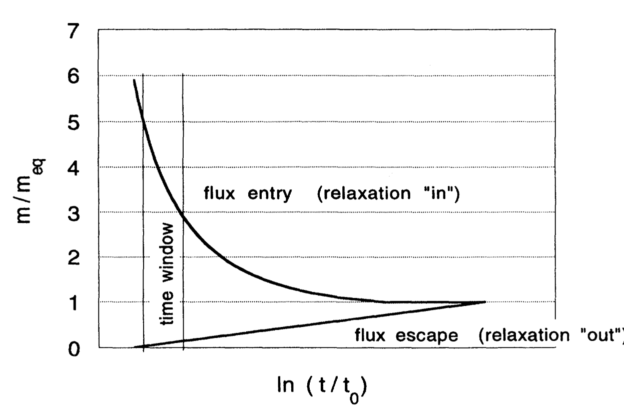

FIG. 5. Surface relaxation in and out.

$m_{ex} \simeq 0$ for flux exit, and then develops in both cases towards $m = m_{eq}$. The activation energy U grows from 0 in the beginning up to its maximum value, $U(m_{eq})$, which coincides for both entry and exit. As we see from Eq. (17), the number of time decades, which is enough to observe the whole relaxation process from m_{en} or m_{ex} till m_{eq} is of order $U(m_{eq})/kT$. For $YBa_2Cu_3O_x$, when H is parallel to the c axis, we can use $\lambda \simeq \lambda_0/\sqrt{\tau}$, where $\lambda_0 \simeq 1400$ Å (Ref. 36) and $\tau = (T_c - T)/T_c$. Substituting these values together with $\kappa \simeq 100$, $\gamma \simeq 25$ (Ref. 36) and $B \simeq H$ into Eq. (12) [or Eq. (18)], we have

$$\frac{U(m_{eq})}{kT} \simeq \frac{4 \times 10^4}{T} \left(\frac{H_p}{H}\tau\right)^{1/2} .$$

At $H/H_p = 10$, $\tau = 0.1$ we get $U(m_{eq})/kT \approx 50$, which is not so large ($\simeq 10^5$) as has been obtained for relaxation at $H = H_{c1}$.[29] In the layered superconductors, where γ is very large, the estimation can be an order of magnitude less, which could explain the observation of a whole curve $m(\ln t)$ from m_{en} till m_{eq} for relaxation in (see Fig. 5) in $Bi_2Sr_2CaCu_2O_x$.[15] Nevertheless, it looks very difficult (even in the very vicinity of T_c) to perform an experiment with the time window large enough to observe the whole relaxation process in $YBa_2Cu_3O_x$. Thus the experimental picture will be usually confined by the initial stage of the relaxation process, see Fig. 5, and we can just substitute $m = m_{en}$ into Eq. (17) and get for flux entry:

$$R_{en} = dm/d\ln t\,|_{m=m_{en}} \simeq -\frac{6\pi}{\phi_0\lambda}\left(\gamma\frac{H}{H_{c1}}\right)^{1/2}\left(\frac{m_{en}}{m_{eq}}\right)^{3/2} kT. \qquad (20)$$

If we compare the flux entry rate, R_{en}, with that for flux exit:

$$R_{ex} = \frac{2\pi}{\phi_0\lambda}\left(\gamma\frac{H}{H_{c1}}\right)^{1/2} kT \qquad (21)$$

[see Eq. (18)], which does not depend on m, we see that the relaxation over the surface barrier proves to be very asymmetric: the vortex entry at $m = m_{en}$ is faster than exit by factor $3(m_{en}/m_{eq})^{3/2} \simeq 3\kappa^{3/2}(H_p/H)^{3/2} \approx 100$ for reasonable values as $\kappa \simeq 100$ and $H/H_p \simeq 10$. This concerns only the beginning of the relaxation process because, as we see from Eqs. (12) and (17), the rate of vortex entry (as a function of $\ln t$) decreases and finally becomes smaller than that for exit, see Fig. 5. Nevertheless only the first stage, where the relaxation rates are very asymmetric, is observable in an experiment with the time window of a few decades. The reason for such an asymmetry is that the width of the barrier for entry, x_2, increases from $x_2(m_{en}) = 0$ till $x_2(m_{eq}) = x_f$, while for exit the width remains constant and equal to x_f. More accurately, the width varies slightly due to change in S; see Eq. (8), but this can be ignored. It is worth mentioning that relaxation in from m_{en} till m_{eq} takes approximately the same time (in logarithmic scale) as relaxation out from $m_{ex} \approx 0$ till m_{eq}, see Fig. 5. This follows from Eqs. (16) and (18), where $U_{ex}(m_{eq}) = U_{en}(m_{eq})$ and $U_{en}(m_{en}) \approx U_{ex}(m_{ex}) \approx 0$.

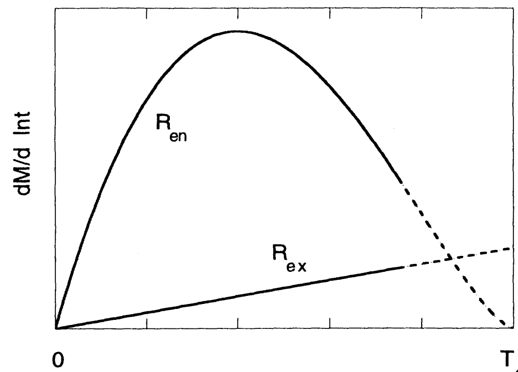

FIG. 6. Dependence of the rate of relaxation in (R_{en}) and out (R_{ex}) on temperature at constant H. Near T_c (dashed lines), where $H \sim H_{c2}$, our analysis is not applicable.

We get more difference between the relaxation in and out if we consider the temperature dependence of R_{en} and R_{ex} at given H; see Fig. 6. While R_{ex} appears to be just a linear function of T [note that in Eqs. (20) and (21) the product $\lambda H_{c1}^{1/2}$ is temperature independent], R_{en} show a maximum at $T \approx 0.4\,T_c$ due to the additional factor $(m_{en}/m_{eq})^{3/2}$.

We have shown that if the relaxation process is determined by the surface barrier only in the absence of any bulk pinning, its properties become very unusual and asymmetric if we compare relaxation in and out. Certainly the bulk irreversibility cannot be removed completely, and we now discuss the surface and bulk effects together.

V. INTERPLAY BETWEEN THE SURFACE BARRIER AND BULK PINNING

Consider the case where the surface barrier and the bulk pinning determine the irreversible properties together, as has been already discussed.[14,17] In Ref. 14 it was concluded that, since a vortex has to surmount first the surface barrier before it experiences the bulk pinning potential, anyway there should exist two successive processes determined by the surface barrier and the bulk pinning, respectively. In Ref. 17 the inverse sequence is proposed, i.e., bulk relaxation precedes the surface one. These two different regimes are characterized by different slopes, $dM/d\ln t$; thus there appears a crossover in the $M(\ln t)$ curves.[14,17] We will try to elaborate this scenario and show that one should compare the activation energies, U_{bulk} and U_{surf}, in order to conclude which kind of relaxation takes place first.

In Fig. 7(a), the initial state ($t = t_0$) for flux entry is presented. The magnetic induction is now spatially dependent: $B = B(x)$. We plotted the simplest Bean profile, where $|\, dB/dx \,| = (4\pi J/c) = \text{const}$, since the particular form of $B(x)$ is not important for our further qualitative consideration. At $t = t_0$, of course, $J = J_c$. It will be convenient to divide the total moment, M, into two parts: $M_{bulk}(J)$ and M_{surf}. The latter is de-

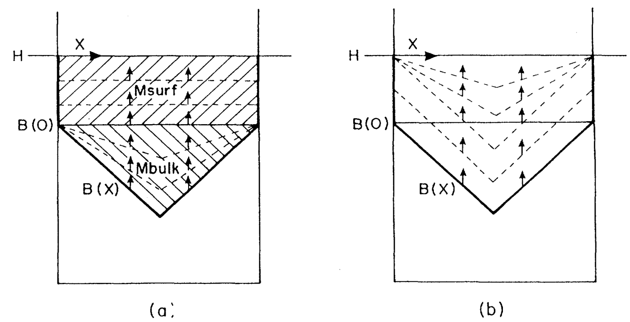

FIG. 7. Competition between the surface and bulk relaxation for the cases where the surface barrier is (a) stronger and (b) weaker than the bulk pinning.

termined by the finite step of the field at the surface: $M_{\rm surf} = 4\pi[H - B(0)]$, and varies from $M_{\rm en}$ to $M_{\rm eq}$; see Sec. II. Usually the variation of flux density due to bulk pinning is very small at distances of order λ, whereas only such distances from the surface are involved into our analysis of $M_{\rm surf}$ in previous sections. Thus we can apply all the results obtained above for the surface just by using $B = B(0)$. Then the relaxation in can be realized as follows: first a vortex surmounts a surface barrier $U_{\rm en}$, which is determined by Eq. (12) [i.e., by H and $B(0)$], and then it has to overcome the series of "bulk" barriers in order to leave the surface and penetrate into the bulk of the sample. The bulk barriers are related to the bulk activation energy $U_{\rm bulk}$ considered in the conventional flux-creep models,[7,8] which vanishes at $J = J_c$ and grows as J decreases. One should not expect a drastic difference in the pre-exponential factors for bulk and surface creep, since both are determined by the viscous flux-flow coefficient. Let us show that during the relaxation, the condition $U_{\rm en} \approx U_{\rm bulk}$ should hold. For instance, if at some moment $U_{\rm surf}$ is significant and $U_{\rm bulk}$ is small (i.e., $J \approx J_c$), then the vortices, which penetrate into the sample, will leave the surface area very quickly if compared with the rate of surmounting the surface barrier. Thus the next vortex, overcoming the barrier, is not affected by the previous, since the latter is already far away. Therefore $B(0)$, $M_{\rm surf}$ and, consequently, $U_{\rm surf}$ do not change, while J and $M_{\rm bulk}$ decrease and $U_{\rm bulk}$ grows. This continues until $U_{\rm bulk} = U_{\rm surf}$, where the rates of surface and bulk creep become equal and $U_{\rm surf}$ starts to increase simultaneously with $U_{\rm bulk}$. As a result, the condition $U_{\rm bulk} = U_{\rm surf}$ holds from the very initial state at $t = t_0$, where $U_{\rm bulk} = U_{\rm surf} = 0$ and start to grow together. Then two possible situations can be outlined.

(1) The surface activation energy is sufficiently larger than the bulk one:

$$U_{\rm surf}(M_{\rm surf} = M_{\rm en}) \gg U_{\rm bulk}(J = 0),$$

or, more exactly,

$$U_{\rm bulk}(J = 0) \ll M_{\rm bulk}(J = J_c) dU_{\rm surf}/dM_{\rm surf}(M_{\rm surf} = M_{\rm en}).$$

This means that the same change of both $U_{\rm bulk}$ and $U_{\rm surf}$ from 0 till $U_{\rm bulk}(J = 0)$ corresponds to total vanishing of $M_{\rm bulk}$ and to very little deviation in $M_{\rm surf}$ from its initial value, $M_{\rm en}$. Therefore, while $M_{\rm bulk}$ decreases, the change of the total moment, $M = M_{\rm bulk} + M_{\rm surf}$, is mainly due to $M_{\rm bulk}$, see Fig. 7(a). At this stage the relaxation rate is determined by that of bulk relaxation, $dM/d\ln t \approx dM_{\rm bulk}/d\ln t$; see Fig. 8. After the bulk relaxation is over, i.e., $J = 0$, $M_{\rm bulk} = 0$, and B =const throughout the sample, the surface relaxation starts [see Fig. 7(a)], so $dM/d\ln t \approx dM_{\rm bulk}/d\ln t$. The crossover between these two regimes can be observed as a change in slope in $M(\ln t)$ curves as has been found in experiments.[14,17]

(2) The bulk pinning dominates over the surface barrier: $U_{\rm surf}(M_{\rm en}) \ll U_{\rm bulk}(J_c = 0)$, or, more exactly, $U_{\rm surf}(M_{\rm surf} = 0) \ll M_{\rm en} dU_{\rm bulk}/dM_{\rm bulk}$, where $dU_{\rm bulk}/dM_{\rm bulk}$ can be expressed through $dU_{\rm bulk}/dJ$ using the geometrical parameters of the sample. Then, for the same reasons as above, we should expect that the initial stage is actually the surface relaxation, where $M_{\rm surf}$ decreases at approximately constant J; see Fig. 7(b). Once $M_{\rm surf} = M_{\rm eq}$, the slope in $dM/d\ln t$ changes (decreases) and the relaxation continues owing to the bulk one.

We see that the initial stage of relaxation is determined by the weakest one of two sources of the irreversibility: the bulk and the surface. Some samples show very small bulk irreversibility,[23,24,17] especially at high temperatures $T \simeq T_c$, if compared with the surface barrier. Therefore it seems reasonable that the first stage in relaxation in Ref. 17 and, probably, in Ref. 14 is due to the bulk. Then, after $M_{\rm bulk}$ is exhausted, the surface relaxation starts, so the relaxation rate after the crossover is determined by the surface barrier. It should be emphasized that $U_{\rm bulk}$ depends on temperature very strongly (exponentially), unlike $U_{\rm surf}$; therefore, as temperature is reduced, the bulk pinning becomes dominant and all the surface effects are suppressed.[23] This can provide an explanation for the asymmetry in the initial rate, $dM/d\ln t$, for relaxation in and out, which has been observed in Ref. 14 at low temperatures only, while at higher tempera-

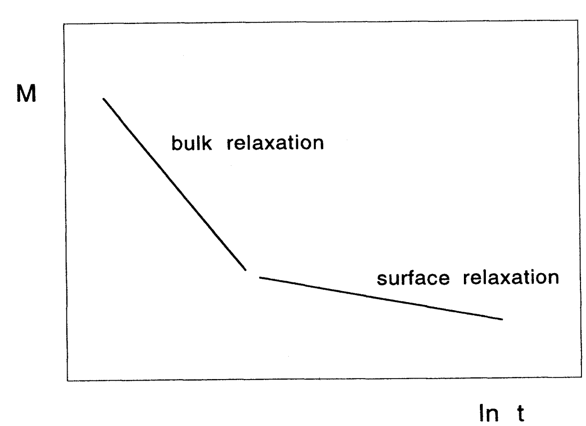

FIG. 8. Crossover in $M(\ln t)$ in the case where the surface barrier is stronger than the bulk pinning.

tures the rates are equal. At higher temperatures the bulk pinning is weak, so the bulk relaxation takes place first [see case (1) above] and the initial stage is quite symmetric for flux entry and exit, as usually happens for bulk relaxation. As temperature is reduced, $U_{\rm bulk}$ grows, and we come to case (2), where the initial relaxation is of surface nature, and, therefore, the difference between the rates of vortex entry and exit should be observed.

It is worth mentioning that the $M(\ln t)$ curves for relaxation in reported in Ref. 15 for $Bi_2Sr_2CaCu_2O_x$ show no interplay between bulk and surface but are evidence of a pure surface relaxation. This can be owing to a very weak bulk pinning in the sample, where $M_{\rm bulk} \ll M_{\rm surf}$.

VI. CONCLUSION

We have obtained the activation energies and relaxation rates, $R = dM/d\ln t$, for the flux relaxation over the Bean-Livingston surface barrier using the Clem model of the critical state in superconductors where all the irreversibility is due to such a barrier in the absence of any bulk pinning. The activation energies U and relaxation rates are expressed through the basic thermodynamic parameters of superconductor. The relaxation rate for flux exit, $R_{\rm ex} = dM/d\ln t$, appears to be constant throughout the relaxation process, while that for flux entry $R_{\rm en}$ depends strongly on M and exceeds $R_{\rm ex}$ significantly in the initial stage of relaxation. The temperature dependencies of $R_{\rm en}$ and $R_{\rm ex}$ are strongly different and can serve as a tool for distinguishing between the surface and bulk relaxation. If, as usually happens, the bulk pinning cannot be ignored and the bulk and surface irreversibilities interplay, then the relaxation at its initial stage is determined by the lowest of the activation energies, $U_{\rm bulk}$ and $U_{\rm surf}$. If, say, the bulk pinning is weak: $U_{\rm bulk} < U_{\rm surf}$, then the bulk relaxation should be observed first (with some rate $R_{\rm bulk}$), which means that J decreases from J_c to 0, while the step at the boundary almost does not change. Only after this process is over, the surface relaxation starts with the different rate, $R_{\rm surf} < R_{\rm bulk}$. If $U_{\rm bulk} > U_{\rm surf}$, the order is inverse. The change between these two regimes can be observed as a crossover in $M(\ln t)$ plots.[14,17]

To conclude, it is worth mentioning the following. The case considered in this paper is a superconducting cylinder with a field parallel to its axis and strong surface barrier. Experimentally we deal usually with thin plates, where the demagnetization factor is very important. For instance, the condition we used, $d \ll \lambda$, where d is the mean intervortex distance, can be broken due to demagnetization since flux penetration starts at very low external fields; see Ref. 23. Then, the barrier can be suppressed very significantly because of surface imperfections, so that H_p/H_{c1} is not large (we used above in most formulas $H_p \simeq H_c \gg H_{c1}$). These circumstances[40] can be rather important for description of the experimental results and should be considered further.

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