# Quantum theory of spin waves for magnetic-overlayer systems

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A quantum theory of spin-wave excitations in a magnetic-overlayer system is presented. The system considered in this paper consists of a set of overlayers and a substrate that are both ferromagnetic. The interactions between spins in both materials are assumed to be Heisenberg exchange interactions. A Green's-function formalism is employed to determine the spin-wave spectra of the system. Analytic expressions for the Green's functions in an overlayer system are obtained by solving Dyson's equations with an assumed coupling constant between a magnetic thin film (overlayers) and a surface system (substrate). The number of surface spin waves associated with the overlayer system is found to be quite different from that of the pure surface system, where at most one branch of surface wave may exist. In addition, resonant magnon states are found in the overlayer system. These states retain well-defined features in the local density of states and do not broaden when the interfacial coupling increases. Numerical calculations for a specific overlayer system are presented as an example of the analysis of spin waves in arbitrary systems.

## I. INTRODUCTION

There has been a growing interest in the magnetic properties of layered structures during the past few years. First, recent advances in experimental techniques for making thin films, such as molecular-beam epitaxy (MBE), sputtering, etc.,[1] have made it possible to fabricate high-quality layered structures for experimental research. Second, layered systems often present novel properties[2,3] that have no counterparts in the pure bulk systems.

A magnetic-overlayer system is a kind of layered structure in which a thin magnetic film of one material is deposited on the surface of another magnetic material. A simple overlayer system formed by two ferromagnetic materials is schematically shown in Fig. 1. The investigation of spin-wave excitations in such a system is of particular interest, because properties of isolated films can be deduced from the experiments on overlayer systems. One may notice in Fig. 1 that magnetic-overlayer system possesses both a surface and an interface between the two materials forming the system. Thus one might expect that the spin waves associated with the surface and the interface in such systems interact with each other, leading to interesting magnetic behaviors.

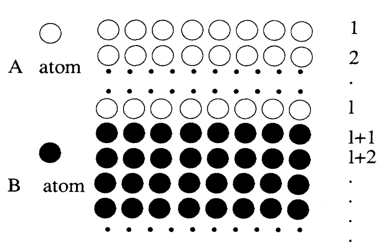


FIG. 1. The overlayer system considered in this paper with the first $l$ layers occupied by A atoms. B atoms occupy layers with $n \geq l+1$.

Two related structures are semi-infinite systems and interface systems formed by two magnetic materials. The former can be realized when the two materials in the overlayer system are identical while the later corresponds to the case when the thickness of the overlayers becomes infinite. The spin-wave properties of semi-infinite ferromagnetic systems are well understood at the present time.[4,5] Surface waves which are localized in the surface region but decay when going into the bulk are found to exist under certain circumstances, depending on the coupling strength between spins and pinning effect on the surface layer. The subject of interface spin waves in bimagnetic systems was also addressed by several authors.[6–8] For example, it has been found[6] that either 0, 1, or 2 branches of interface magnons may exist in a biferromagnetic system. On the other hand, Cottam and Kontos[9] have considered the spin waves of a finite thickness ferromagnetic slab with two equivalent surfaces and found that the effect of finite film thickness is to split the surface spin-wave mode into two branches. In such a case, the bulk magnon energy is quantized. This model has also been considered by Puszkarskii[10] and Lévy.[11]

Additionally, Camley[12] has studied the spin-wave properties of an overlayer system fomed by two ferromagnetic materials that are coupled antiferromagnetically across their interface. Spin-wave properties were examined within the context of semiclassical quantum theory for different sets of ground states in his work.

In this paper, we present a quantum theory for the spin-wave excitations of an overlayer system formed by two ferromagnetic materials. The coupling between the two materials is taken to be ferromagnetic so that the ground state of the system is well defined, i.e., all the spins align in the same direction. We employ the cleavage method developed by Kalkstein and Soven[13] to get the expressions for the Green's functions at each layer in the overlayer system. This method was initially developed to deal with surface electronic problems.

## II. MODEL

The system considered in this paper can be conceived as one thin ferromagnetic film grown eptaxially on top of another material. The geometry of the structure is illustrated in Fig. 1. The first $l$ atomic layers (from the 1st to the $l$th) are $A$ material whose local spin operator is denoted by $S_a$. The remaining layers are $B$ atoms with spins denoted by $S_b$. We assume for simplicity that each material has a simple cubic crystal structure and the interface between them is a (100) plane. The extension to include different kinds of crystal structures, such as bcc or fcc, and different interface planes can be made readily.

We then assume that the interactions between spins are of the Heisenberg exchange type with the Hamiltonian of the system expressed as

$$H=-\sum_{\langle ij\rangle} J_{ij}\mathbf{S}_i\cdot\mathbf{S}_j \ . \tag{1}$$

Here $\mathbf{S}_i$ is the local spin operator at lattice site $i$ and $J_{ij}>0$ is the exchange integral between nearest-neighbor spins. $J_{ij}$ may take different values corresponding to different types of bonds between the $i$th and $j$th atoms, i.e.,

$$J_{ij}=\begin{cases} J_a, & \text{if } 1\le i,j\le l \ ; \\ J_b, & \text{if } i,j\ge l+1; \\ J_{ab}, & \text{if } i=l,\ j=l+1; \ \text{ or } i=l+1, j=l \ . \end{cases} \tag{2}$$

Notice that we have neglected the changes of $J_{ij}$ in the vicinities of the surface and interface of the system. If it must be considered, it is straightforward to include such changes in the present formalism.

It is convenient to think about the overlayer system as being created by bringing together a thin film and semi-infinite system through a coupling potential. In this way, Dyson's equation can be used to determine the Green's function for the coupled system (overlayers) with known Green's functions of the individual systems (thin film and the substrate). Proceeding in this manner, we will first give the expressions of the Green's functions of spin waves for a semi-infinite system and a thin film.

### A. Semi-infinite system

In a previous work, Yaniv[6] obtained the Green's functions of spin-wave excitations for a semi-infinite system by introducing a decoupling potential to the corresponding bulk system and solving the Dyson's equations. Here we only outline the main steps of this cleavage method when applied to the corresponding bulk system.

The dispersion relation of the bulk magnons of a ferromagnetic system $B$ with simple cubic structure is given by the following well-known expressions;

$$E(\mathbf{k})=6T_b-\epsilon_b(\mathbf{k}) \ , \tag{3}$$

where $T_b=2J_bS_b$, $\epsilon_b(\mathbf{k})=2T_b[\cos(k_xa)+\cos(k_ya)]$, $a$ is the lattice constant, and $\mathbf{k}$ is a two-dimensional wave vector parallel to the surface plane, i.e., $\mathbf{k}=(k_x,k_y)$. The subscript $b$ here stands for the $B$ material. The Green's function for spin waves of a bulk system can be easily constructed in the mixed Wannier-Bloch representation,[13]

$$G^0(n)=\frac{i}{\mu_b}\left[\frac{\omega_b+i\mu_b}{-2T_b}\right]^n , \tag{4}$$

where $n$ is the atomic layer index number, and

$$\omega_b=E-6T_b+\epsilon_b(\mathbf{k}) \ , \tag{5}$$

$$\mu_b=\begin{cases} (4T_b^2-\omega_b^2)^{1/2}, & \text{for } \omega_b^2\le 4T_b^2; \\ i\ \mathrm{sgn}(\omega_b)(\omega_b^2-4T_b^2)^{1/2}, & \text{for } \omega_b^2>4T_b^2 \ . \end{cases} \tag{6}$$

Here $\mathrm{sgn}(\omega_b)$ denotes the sign of $\omega_b$. We can create a semi-infinite system by introducing a perturbation to the bulk system which breaks the bonds between the planes $n=l+1$ and $n=l$. Such a perturbation potential can be expressed as

$$V(l,l+1)=V(l+1,l)=T_b \ , \tag{7}$$

$$V(l,l)=V(l+1,l+1)=U_b \ , \tag{8}$$

where $V(l,l)$ and $V(l+1,l+1)$ represent the diagonal parts of the perturbation on the surfaces $n=l$ and $n=l+1$, respectively. For convenience in the discussion below we can let these diagonal parts have arbitrary values, but for a free-surface problem, it is obvious that their values are fixed, i.e., $V(l,l)=V(l+1,l+1)=-T_b$.

The surface Green's function can be obtained from the bulk Green's function and the perturbation through the following Dyson's equation:

$$G^s=G^0+G^0VG^s \ , \tag{9}$$

where $G^s$ and $G^0$ represent the surface and bulk Green's functions, respectively.

After substituting the expressions for $G^0$ and $V$, one finds that the surface Green's function is given by

$$G^s(m,n)=i\mu_b^{-1}\left[\left[\frac{\omega_b+i\mu_b}{2T_b}\right]^{|m-n|}+\left[\frac{\omega_b-i\mu_b}{2T_b}\right]^{|m+n-2(l+1)|}\times\frac{i\mu_b+(\omega_b-2U_b)}{i\mu_b-(\omega_b-2U_b)}\right] , \tag{10}$$

where $m$ and $n$ are layer index numbers with $m=l+1$ denoting the first layer of the substrate.

From Eq. (10), it is clear that in a surface system, in addition to the bulk state of spin waves given by $\mu_b=0$, there may exist a surface spin-wave state whose energy is determined by

$$\mathrm{sgn}(\omega_b)(\omega_b^2-4T_b^2)^{1/2}+\omega_b-2U_b=0 \ . \tag{11}$$

### B. Thin-film system

The Green's function for a thin-film system can be obtained in a similar way as that for a semi-infinite system. We start from a surface system with the surface layer being $n=1$ and introduce the same decoupling potentials described in Eqs. (7) and (8) to the planes between $n=l$ and $n=l+1$. At this time, the unperturbed system ($G^0$) is the surface system and the final system ($G^f$) is a thin film with surfaces being $n=1$ and $n=l$, respectively.

To obtain the film Green's function, it is necessary to write down Dyson's equation in matrix form as

$$G^f(m,n)=G^s(m,n)+\sum_{p,q} G^s(m,p)V(p,q)G^f(q,n) . \qquad (12)$$

Because $V(p,q)$ is nonvanishing only when the $p$ and $q$ refer to equal or adjacent planes, the above equation can be reduced to

$$G^f(m,n)=G^s(m,n)+G^s(m,l)V(l,l)G^f(l,n) + G^s(m,l+1)V(l+1,l)G^f(l,n) , \qquad (13)$$

where we have made use of the fact that $G^f(l+1,n)=0$.

Substituting the expressions of $G^s(m,n)$, $V(l,l)$, and $V(l+1,l)$ into the above equation and then setting $m=l$, we obtain the Green's function $G^f(l,n)$:

$$G^f(l,n)=G^s(l,n)[1-G^s(l,l)U_2-G^s(l,l+1)T_a]^{-1} . \qquad (14)$$

After substituting $G^f(l,n)$ back into Eq. (13), we finally obtain the film Green's function

$$\begin{aligned} G^f(m,n)=i\mu_a^{-1}\{F_a^{|m-n|}+(1/\Delta_a^2)[&T_a^2(F_a^{l-m-n+1}+F_a^{-l+m+n-1}-F_a^{l+m-n+1}-F_a^{-l-m+n-1}) \\ &+U_1U_2(F_a^{l-m-n+1}+F_a^{-l+m+n-1}-F_a^{l+m-n-1}-F_a^{-l-m+n+1}) \\ &+T_aU_1(F_a^{l+m-n}+F_a^{-l-m+n}-F_a^{l-m-n+2}-F_a^{-l+m-n-2}) \\ &+T_aU_2(F_a^{l+m-n}+F_a^{-l-m+n}-F_a^{l-m-n}-F_a^{-l+m+n})]\} , \end{aligned} \qquad (15)$$

where we defined

$$\begin{aligned} F_a&=\frac{\omega_a+i\mu_a}{-2T_a} , \\ \Delta_a&=F_a^{l-1}(T_aF_a-U_1)(T_aF_a-U_2) \\ &\quad -F_a^{-l+1}(T_aF_a^{-1}-U_1)(T_aF_a^{-1}-U_2) , \end{aligned} \qquad (16)$$

and $T_a=2J_aS_a$. The definitions for $\omega_a$ and $\mu_a$ are given by Eqs. (5) and (6), respectively, by changing the subscript $a$ to $b$. $U_1$ and $U_2$ are the diagonal elements of perturbation at surfaces $n=1$ and $n=l$, respectively. They may assume different values for a film with two unsymmetrical surfaces, as in the case of an overlayer system.

The element of the Green's function that is relevant to the density of states (DOS) is the diagonal one, which is given by

$$\begin{aligned} G^f(n,n)=i(\mu_a\Delta_a)^{-1}[&T_a^2(T_a^{-l+2n-1}+F_a^{l-2n+1}-F_a^{l+1}-F_a^{-l-1})+U_1U_2(F_a^{l-2n+1}+F_a^{-l+2n-1}-F_a^{l-1}-F_a^{-l+1}) \\ &+U_1T_a(F_a^{l}+F_a^{-l}-F_a^{l-2n+2}-F_a^{-l+2n-2})+U_2T_a(F_a^{l}+F_a^{-l}-F_a^{l-2n}-F_a^{-l+2n})] . \end{aligned} \qquad (17)$$

From the above equation, we can write down the matrix elements at the top and bottom surfaces of the film, i.e.,

$$G^f(1,1)=i(\mu_a\Delta_a)^{-1}[T_a^2(F_a^{-l+1}+F_a^{l-1}-F_a^{l+1}-F_a^{-l-1})+U_2T_a(F_a^{l}+F_a^{-l}-F_a^{l-2}-F_a^{-l+2})] \qquad (18)$$

and

$$G^f(l,l)=i(\mu_a\Delta_a)^{-1}[T_a^2(F_a^{-l+1}+F_a^{-l1}-F_a^{l+1}-F_a^{-l-1})+U_1T_a(F_a^{l}+F_a^{-l}-F_a^{l-2}-F_a^{-l+2})] . \qquad (19)$$

One may see that $G^f(1,1)$ and $G^f(l,l)$ are not equal when $U_1\neq U_2$.

The energy levels of spin waves in the film are given by the roots of the equation $\Delta_a=0$. It can be shown that there are total $l$ roots for this equation and each root spans into a subband when **k** varies. If we compare these energy levels with the bulk energy band, we may distinguish the volume modes from the surface modes in the film. The former has energy lying inside the bulk band, while the latter otherwise. Our numerical solutions to the equation $\Delta_a=0$ are in agreement with those of Ref. 9. We also found[14] that in the case of two surface states, the change of one of the $U$'s can only alter one of the surface states noticeably, leaving other states almost unaffected.

### C. Overlayer system

As we mentioned earlier in this paper, the overlayer system can be conceived of as a coupling between a thin film and a semi-infinite system. We now introduce this coupling potential $V$ and treat it as perturbation in

Dyson's equation. Because we have solved the problems for a film and a semi-infinite system with arbitrary values of diagonal matrix elements, we need only to write down the nondiagonal element of the coupling potential, i.e.,

$$V(m,n)=2J_{ab}(S_aS_b)^{1/2} \tag{20}$$

for $m=l$, $n=l+1$ or $m=l+1$, $n=l$.

From Dyson's equation, $G=G^0+G^0VG$, where $G^0$ is the Green's functions of the film and semi-infinite system and $G$ is that of the coupled system, we get the following equation:

$$G(m,n)=G^0(m,n)+G^0(m,l)V(l,l+1)G(l+1,n) +G^0(m,l+1)V(l+1,l)G(l,n) \; . \tag{21}$$

By letting $m=l$ and $l+1$, we get the coupled equations for $G(l,n)$ and $G(l+1,n)$, which can be solved directly. After substituting their expressions back into Eq. (21), we obtain the diagonal Green's functions for the overlayer system,

$$G(n,n)=G^s(n,n)+4J_{ab}^2S_aS_bG^f(l,l)G^s(n,l+1) \times G^s(l+1,n)[1-4J_{ab}^2S_aS_bG^f(l,l) \times G^s(l+1,l+1)]^{-1} \tag{22}$$

for $n\geq l+1$, and

$$G(n,n)=G^f(n,n)+4J_{ab}^2S_aS_bG^s(l+1,l+1)G^f(n,l) \times G^f(l,n)[1-4J_{ab}^2S_aS_bG^f(l,l) \times G^s(l+1,l+1)]^{-1} \tag{23}$$

for $0\leq n\leq l$.

The diagonal perturbations involved in forming the overlayer system can be easily evaluated by using Eqs. (1) and (2), i.e.,

$$U_1=-2J_aS_a \tag{24}$$

for the top layer of the film $n=1$,

$$U_2=-2J_aS_a+2J_{ab}S_b \tag{25}$$

for the interface layer in the film $n=l$, and

$$U_1^b=-2J_bS_b+2J_{ab}S_a \tag{26}$$

for the interface layer in the semi-infinite system $n=l+1$.

Substituting the expressions of $G^s$, $G^f$, and those of $U$'s into the Eqs. (22) and (23), the analytic Green's function at each layer for the overlayers system is uniquely determined. The local density of magnon states at point $\mathbf{k}$, on atoms in the $i$th layer, is given by

$$\rho_i(E;\mathbf{k})=-\frac{1}{\pi}\,\mathrm{Im}\,G(E;i,i;\mathbf{k}) \; . \tag{27}$$

We now turn to some discussions of the properties of spin waves in the overlayer system. Generally speaking, in a manner analogous to those in the semi-infinite system, the spin waves in an overlayer system can be classified into two categories. The first class has energy lying in the bulk bands of the substrate material. These waves can propagate throughout the whole crystal. We call these states bulk modes. Their energy satisfies

$$|E-6T_b+\epsilon_b(\mathbf{k})|<2T_b \; .$$

The second class of states has wave functions which are mainly localized in the overlayer region but decay exponentially when going into the substrate. These states have an energy which lies outside the bulk energy region of the substrate material. We call these states surface modes. Because the overlayers system possesses both a surface and an interface, the spin waves associated with them will interact with each other. This leads to the surface modes that are quite different from those of pure surface system. Equations (22) or (23) can be used to show that the energy of the surface modes in the overlayer system is given by the roots of the following equation:

$$1-4J_{ab}^2S_aS_bG^f(l,l)G^s(l+1,l+1)=0 \; , \tag{28}$$

while for a pure surface system, the surface state is given by Eq. (11).

Because of the complexity of the forms of $G^f$ and $G^s$, it is hard to determine an analytic solution to Eq. (28). Even the numerical analysis turns out to be complicated. The number of roots, as well as their locations is found to be dependent on the number of film layers, the strength of interface coupling and the $J$'s and $S$'s of the individual system. Fortunately, we found that the discussion could be greatly simplified if we start by comparing the spectrum of the uncoupled systems, the film and the substrate. We knew[4,9] that for a film or semi-infinite system, if the surface exchange integral is the same as that in the bulk, there would be no surface state in either of the systems. However, when the two systems are combined, there may appear surface states due to the mismatch of their spectra and the change of interface coupling. Below we present some discussions for a specific overlayer system, which can serve as an example for the analysis of the surface waves for arbitrary overlayer systems.

We first consider the case in which $J_a=J$, $S_a=2$, $J_b=J$, and $S_b=1$. The spin-wave spectra of the uncoupled film and the substrate material are shown in Fig. 2(a). It can be seen that two branches of spin waves of the film lie outside the bulk energy region of the substrate material. Thus we may expect that at least two branches of surface spin waves exist when the two systems are coupled. Our numerical results in Fig. 2(b) show that when $J_{ab}$ is small, there are indeed two branches of surface states which lie above the volume mode, i.e., they are optical branches. As the interfacial coupling $J_{ab}$ gets bigger, there may appear another branch of optical surface spin wave, which is truncated by the volume modes. It is found that there are at most three branches of surface waves existing for the system we chose. Another interesting feature we noticed in Fig. 2(b) is that the increase of the interfacial coupling $J_{ab}$ raises the branch with the highest energy more noticeably than it does on other branches with lower energy.

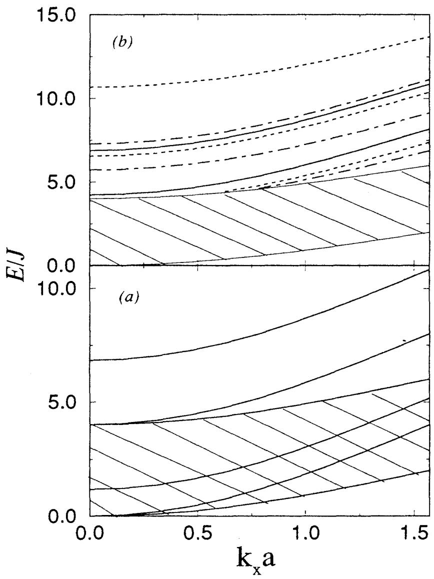


FIG. 2. (a) Spectra for the uncoupled film and substrate systems when $J_a=J$, $S_a=2$, $J_b=J$, and $S_b=1$. The number of layers for the film is $l=4$. Solid lines are the spectrum of the film modes. The energy region of the volume modes of the substrate material is shaded. Two branches of film modes are above the volume modes. (b) Surface modes for the overlayer system with the same parameters as in (a). Solid lines are obtained when $J_{ab}=0.5J$. Two branches of surface waves appear in this case. Dotted-dashed lines correspond to $J_{ab}=1.5J$ and dashed lines correspond to $J_{ab}=3J$. Three branches of surface states exist in both cases.

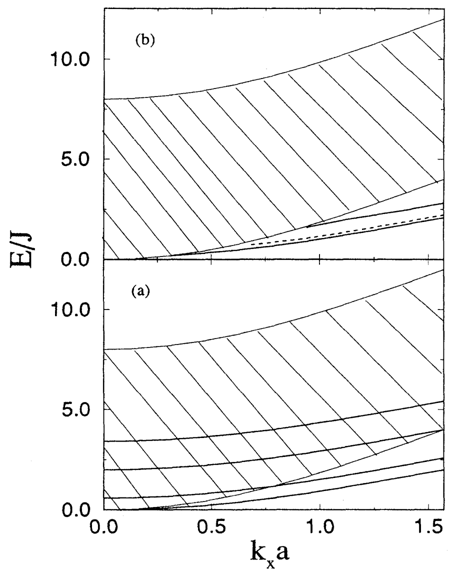


FIG. 3. (a) Spectra for the uncoupled film and substrate systems when $J_a=J$, $S_a=1$, $J_b=J$, $S_b=2$, and $l=4$. Opposite to the situation in Fig. 2(a), two branches of film modes are below the volume modes of the substrate. (b) Surface modes for the overlayer system with the same parameters as in (a). Solid lines are obtained when $J_{ab}=0.2J$. Two branches of surface waves exist in this case. Dashed line corresponds to $J_{ab}=2J$. Only one branch of spin wave exists.

We want to point out the difference in the number of surface modes between the overlayers system and the pure surface system. It is known[4] that only one branch of surface spin wave may exist in the pure surface system, depending on the relative strength between the couplings on the surface and in the bulk. However, in the overlayer system, more than one branch of spin wave may appear, which depends on the number of layers in the film, the strength of the interfacial coupling, and the relative positions of the film modes and those of the substrates.

We now consider the case in which $J_a=J$, $S_a=1$, $J_b=J$, and $S_b=2$. Contrary to the situation in the case discussed above, the film has two branches of spin wave which are below the bulk modes of the substrate, see Fig. 3(a). Thus we expect that when the interface coupling is small, there may exist two branches of acoustic surface waves. This is confirmed by our numerical calculation [see Fig. 3(b)]. In the same figure, we see that as the intrefacial coupling gets bigger, the truncated branch of surface wave is pushed into the volume modes, leaving only one acoustic branch of surface wave for the system.

In addition to the differences in the localized states between the overlayer systems and pure semi-infinite systems, there exist resonant modes in the overlayer system, which is absent in pure semi-infinite systems. These overlayer induced states manifest themselves by well-defined peaks in the local density of states (LDOS) within the bulk band. We show in Fig. 4 the LDOS on the surface atoms ($n=1$) at $\mathbf{k}=\mathbf{0}$ in an overlayer system with $l=8$. The whole system here corresponds to the case similar to the situation we considered in Fig. 2, i.e., the isolated film states are within the bulk bands of the substrate at $\mathbf{k}=\mathbf{0}$. Eight resonant modes were observed. One may recall that in an isolated film, the LDOS at a given point $\mathbf{k}$ is characterized by $\delta$ peaks in the spectrum. In the overlayer system, these states become well-defined resonant states whose positions are modified due to coupling between the film and the substrate. Furthermore, we found that these well-defined peaks do not broaden when the interfacial coupling between the film and the substrate increases. This suggests that resonant states can be observed experimentally in overlayers systems. In Fig. 5 we also show the case when the situation is similar to that in Fig. 3. The LDOS in this case consists of the contributions from both the localized states and the resonant

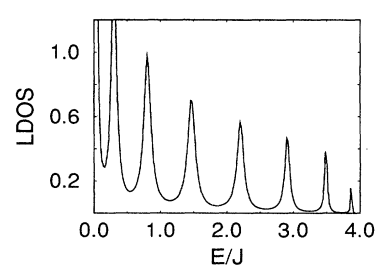


FIG. 4. Local density of magnon states on the surface atoms at $\mathbf{k}=0$ in the case of $l=8$. The parameters are $J_a=J_b=J$, $J=0.5J$, $S_a=1$, and $S_b=2$.

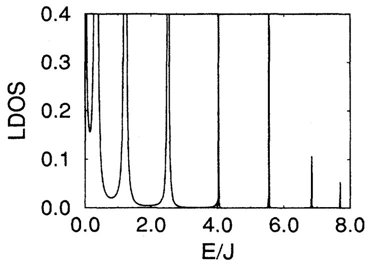


FIG. 5. Local density of magnon states on the surface atoms at $\mathbf{k}=0$ in the case of $l=8$. The parameters are $J_a=J_b=J$, $J=0.2J$, $S_a=2$, and $S_b=1$. The black bars are the contributions from the localized states.

states. The black bars in the figure correspond to the contributions from the localized states. Other peaks in the figure come from the resonant modes. Physically, these resonant states have wave functions with much larger amplitudes in the overlayers region than in the substrate.

The LDOS on atoms in other atomic layers has been calculated using Eq. (27). The results show the same features as the cases discussed above. The only difference is the intensities of the peaks are different in each case. The same discussions hold for the LDOS on atoms at any point $\mathbf{k}$.

In summary, we presented a quantum theory for the spin-wave excitations of a ferromagnetic overlayer system. Analytic expressions for the Green's function in such a system were expressed in terms of those of the film and surface systems. In spite of the similarities in the spin-wave propagation between overlayer systems and pure semi-infinite systems, there are overlayer-induced resonant states in the overlayer system. These states have well-defined features in the LDOS of magnons and could be observed in experiments. The energy spectra were calculated for a specific system which can serve as an example for the discussion of arbitrary magnetic-overlayer systems. We hope that our theory can stimulate further experiments on overlayer magnetic systems, especially the observation of the resonant states.

Finally, it is worth mentioning the limitations of the present formalism. Firstly, we employed the Heisenberg Hamiltonian with nearest-neighbor iterations to described magnetic systems. Though many systems, especially magnetic insulators, can be described by such a model, this model is not appropriate for metallic magnetic systems. Secondly, in the present formalism, we have kept our parameters at a minimum number by neglecting the changes of coupling constants in the surface and interface region. In addition, no pinning effect on the surface and interfaces was considered. In principle, there should be no problem to extend the current formalism to include these considerations.[15]

## ACKNOWLEDGMENT

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