Dipolar antiferromagnetism in the spin-wave approximation

L. R. Corruccini and Steven J. White
Department of Physics, University of California, Davis, California 95616-8677
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The spin-wave Hamiltonian for a two-sublattice antiferromagnet with purely dipolar interactions is diagonalized. The simple-cubic (sc) lattice is solved as an example, and is found to exhibit zero-point-motion corrections to the Néel state that are much larger than those of the nearest-neighbor sc Heisenberg antiferromagnet. The two-dimensional square dipolar lattice is found not to exhibit long-range order at finite temperature in this approximation.

I. INTRODUCTION

It is well known that the magnetic dipole-dipole interaction can give rise to spin-wave excitations in ordered magnetic systems, just as the exchange interaction does. These are frequently ignored in the analysis of real dipolar magnets, many of which are anisotropic. When no anisotropy is present, however, dipolar spin waves can lead to substantial quantum effects. Ferromagnets with purely dipolar interactions have been studied in the spin-wave approximation by Cohen and Keffer,[1] who found, in the bcc and fcc lattices, appreciable zero-point corrections to the energy of the classically expected ground state.

In this paper, we analyze a purely dipolar antiferromagnet in the Néel two-sublattice model, using the spin-wave theory. General results for the spin-wave spectrum and zero-point corrections to the ground-state energy and sublattice magnetization are obtained. These results are applied to a simple cubic (sc) lattice of magnetic dipoles, which has a classical antiferromagnetic ground state. We find that the zero-point corrections in this case are more than double those found in the nearest-neighbor sc Heisenberg antiferromagnet, suggesting that dipolar magnets are more quantum mechanical than generally suspected. We also examine the question of fluctuations in two-dimensional dipolar lattices. These are compared to those in the two-dimensional square lattice Heisenberg antiferromagnet, which (for the case $S=1/2$) has recently been the focus of interest in possible connection with high-temperature superconductivity.[2]

A somewhat different treatment has been given earlier[3] for the "truncated" dipolar Hamiltonian appropriate for nuclear magnetic resonance. This Hamiltonian, which contains only those dipolar parts which commute with the Zeeman energy, leads to much smaller quantum corrections.

II. SPIN-WAVE SPECTRUM

The dipolar Hamiltonian is

$$\mathcal{H}_{dd}=\frac{1}{2}\sum_{l\neq m}\left[\frac{\boldsymbol{\mu}_l\cdot\boldsymbol{\mu}_m}{r_{lm}^3}-\frac{3(\boldsymbol{\mu}_l\cdot\mathbf{r}_{lm})(\boldsymbol{\mu}_m\cdot\mathbf{r}_{lm})}{r_{lm}^5}\right].$$

In the spin-wave approximation, for a two-sublattice antiferromagnet with two spins in a primitive unit cell and polarization axis $\hat{z}$, this takes the form[4]

$$\mathcal{H}_{dd}=\mathcal{H}_d^0+\mathcal{H}_d' ,$$

where

$$\mathcal{H}_d^0=C+\sum_k[a_k^\dagger a_k(E_k^{aa}+2E_0^{aa}-2E_0^{ab}) +b_k^\dagger b_k(E_k^{bb}+2E_0^{bb}-2E_0^{ab})] \tag{1}$$

and

$$\mathcal{H}_d'=\frac{1}{2}\sum_k[2a_k^\dagger b_k^\dagger E_k^{ab}+a_k^\dagger a_{-k}^\dagger(B_k^{aa})^*+b_k^\dagger b_{-k}^\dagger B_k^{aa} +a_k b_{-k}^\dagger(B_{-k}^{ab}+B_k^{ab})+\text{H.c.}] . \tag{2}$$

For magnetic moments μ having at least axial symmetry, the numerical coefficients above are defined as follows:

$$C=-2S^2g_\parallel^2\left[\sum_{l\neq l'}E_{ll'}+\sum_{m\neq m'}E_{mm'}-2\sum_{lm}E_{lm}\right] ,$$

$$E_{lm}=-\frac{\mu_B^2}{4r_{lm}^3}\left[1-3\left[\frac{z_{lm}}{r_{lm}}\right]^2\right] ,$$

$$E_k^{aa}=2Sg_\perp^2\sum_{l'}E_{ll'}\exp(i\mathbf{k}\cdot\mathbf{r}_{ll'})=-\frac{1}{2}Sg_\perp^2\mu_B^2A_{aa}^{zz}(\mathbf{k}) ,$$

$$E_0^{aa}=2Sg_\parallel^2\sum_{l'}E_{ll'} , \tag{3}$$

$$E_k^{ab}=2Sg_\perp^2\sum_m E_{lm}\exp(i\mathbf{k}\cdot\mathbf{r}_{lm})=-\frac{1}{2}Sg_\perp^2\mu_B^2A_{ab}^{zz}(\mathbf{k}) ,$$

$$E_0^{ab}=2Sg_\parallel^2\sum_m E_{lm} ,$$

$$B_k^{aa}=4Sg_\perp^2\sum_{l'}B_{ll'}\exp(i\mathbf{k}\cdot\mathbf{r}_{ll'}) =\tfrac{1}{2}Sg_\perp^2\mu_B^2[A_{aa}^{xx}(\mathbf{k})-A_{aa}^{yy}(\mathbf{k})-2iA_{aa}^{xy}(\mathbf{k})] ,$$

$$B_k^{ab}=4Sg_\perp^2\sum_m B_{lm}\exp(i\mathbf{k}\cdot\mathbf{r}_{lm}) =\tfrac{1}{2}Sg_\perp^2\mu_B^2[A_{ab}^{xx}(\mathbf{k})-A_{ab}^{yy}(\mathbf{k})-2iA_{ab}^{xy}(\mathbf{k})] ,$$

where

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$$B_{lm}=-\frac{3}{8}\frac{\mu_B^2}{r_{lm}^3}\left[\frac{x_{lm}-iy_{lm}}{r_{lm}}\right]^2,$$

a and b (l and m) refer to the two sublattices, S is spin, and $g_\parallel$ and $g_\perp$ are the components of the g factor parallel and normal to the z axis. The definition of the $A_{ij}^{\alpha\beta}(\mathbf{k})$ is chosen to conform to that of Refs. 5–7:

$$A_{ij}^{\alpha\beta}(\mathbf{k})=-\frac{\partial^2}{\partial x^\alpha \partial x^\beta}{\sum_n}'\frac{e^{i\mathbf{k}\cdot(\mathbf{R}_n+\mathbf{r}_i-\mathbf{r}_j)}}{|\mathbf{R}_n+\mathbf{r}_i-\mathbf{r}_j-\mathbf{r}|}\Bigg|_{\mathbf{r}=0} \tag{4}$$

where R_n is the location of primitive cell n, and $\mathbf{r}_i$ and $\mathbf{r}_j$ are the locations of two spins within the unit cell.

This Hamiltonian can be expressed in matrix form as

$$\mathcal{H}_{dd}=C-\sum_k\epsilon_k+\frac{1}{2}\sum_k\mathbf{a}_k^\dagger[M_k]\mathbf{a}_k\ , \tag{5}$$

where $\mathbf{a}_k^\dagger=(a_k^\dagger,b_k,b_{-k}^\dagger,a_{-k})$, the 4×4 matrix $[M_k]$ is

$$[M_k]=\begin{bmatrix}\epsilon_k & E_k^{ab} & (B_{-k}^{ab})^* & (B_k^{aa})^*\\ (E_k^{ab})^* & \epsilon_k & (B_k^{aa})^* & (B_k^{ab})^*\\ B_{-k}^{ab} & B_k^{aa} & \epsilon_k & (E_k^{ab})^*\\ B_k^{aa} & B_k^{ab} & E_k^{ab} & \epsilon_k\end{bmatrix},$$

and $\epsilon_k=E_k^{aa}+2E_0^{aa}-2E_0^{ab}$. The Hamiltonian can be diagonalized[8] in closed form, yielding

$$\mathcal{H}_{dd}=C-\sum_k\epsilon_k+\sum_k\left[\alpha_k^\dagger\alpha_k+\frac{1}{2}\right]\omega_k^+ +\sum_k\left[\beta_k^\dagger\beta_k+\frac{1}{2}\right]\omega_k^-\ , \tag{6}$$

with eigenvalues ω given by

$$(\omega_k^\pm)^2=\epsilon_k^2+|E_k^{ab}|^2-|B_k^{aa}|^2-\tfrac{1}{2}(|B_k^{ab}|^2+|B_{-k}^{ab}|^2)\pm\tfrac{1}{2}\Delta\ ,$$

where

$$\Delta=\{(|B_k^{ab}|^2-|B_{-k}^{ab}|^2)^2+16\epsilon_k^2|E_k^{ab}|^2+4|B_k^{aa}(B_{-k}^{ab})^* +(B_k^{aa})^*B_k^{ab}|^2-4|E_k^{ab}B_{-k}^{ab}-(E_k^{ab})^*B_k^{ab}|^2 -8[\epsilon_k B_k^{aa}([E_k^{ab}]^*[B_{-k}^{ab}]^*+E_k^{ab}[B_k^{ab}]^*)+\text{c.c.}]\}^{1/2}\ . \tag{7}$$

If the underlying lattice (ignoring spin) is a Bravais lattice, then all the matrix elements will be even functions of $\mathbf{k}$, and the E_k^{ab} will be real. In this situation the eigenvalues simplify considerably:

$$(\omega_k^\pm)^2_{\text{Bravais}}=(\epsilon_k\pm E_k^{ab})^2-|B_k^{aa}\pm B_k^{ab}|^2\ . \tag{8}$$

Because $\epsilon_k=E_k^{aa}+2(E_0^{aa}-E_0^{ab})$, the sum $\epsilon_k+E_k^{ab}$ can be expressed in terms of lattice sums E_k^{BL} over the entire Bravais lattice, corresponding to one spin per primitive cell instead of two:

$$\begin{aligned}\epsilon_k+E_k^{ab}&=(E_k^{aa}+E_k^{ab})+2(E_0^{aa}-E_0^{ab})\\ &=E_k^{\text{BL}}+2\frac{g_\parallel^2}{g_\perp^2}E_{k_0}^{\text{BL}}\\ &=-\tfrac{1}{2}S\mu_B^2[g_\perp^2A^{zz}(\mathbf{k})+2g_\perp^2A^{zz}(k_0)]\\ &\equiv A_k\ ,\end{aligned} \tag{9}$$

where k_0 is the wave vector of the antiferromagnetic structure, and $A^{\alpha\beta}(\mathbf{k})$ is defined as in Eq. (4) with $\mathbf{r}_i=\mathbf{r}_j$. Similarly,

$$\begin{aligned}&\epsilon_k-E_k^{ab}=E_{k_0-k}^{\text{BL}}+2(g_\parallel^2/g_\perp^2)E_{k_0}^{\text{BL}}\equiv A_{k_0-k}\ ,\\ &B_k^{aa}+B_k^{ab}=B_k^{\text{BL}}\\ &\qquad=\tfrac{1}{2}Sg_\perp^2\mu_B^2[A^{xx}(\mathbf{k})-A^{yy}(\mathbf{k})-2iA^{xy}(\mathbf{k})]\\ &\qquad\equiv B_k\ ,\\ &B_k^{aa}-B_k^{ab}=B_{k_0-k}^{\text{BL}}\equiv B_{k_0-k}\end{aligned} \tag{10}$$

so that the eigenvalues take the familiar form

$$\begin{aligned}\omega^+_{\text{Bravais}}&=\sqrt{A_k^2-|B_k|^2}\ ,\\ \omega^-_{\text{Bravais}}&=\sqrt{A_{k_0-k}^2-|B_{k_0-k}|^2}.\end{aligned} \tag{11}$$

Unlike the nearest-neighbor Heisenberg antiferromagnet, the two branches of the spin-wave spectrum are nondegenerate. These results for ω_{Bravais} are very similar to those obtained in the ferromagnetic case,[1] the only difference being the replacement of $A^{zz}(0)$ in the latter by $A^{zz}(k_0)$.

III. ZERO-POINT ENERGY AND SPIN DEVIATION

The ground-state energy of the system is

$$W_0=C-\sum_k\epsilon_k+\frac{1}{2}\sum_k(\omega_k^++\omega_k^-)\ . \tag{12}$$

The constant C is the classical Néel ground-state energy, which can be written

$$\begin{aligned}C&=-4S^2g_\parallel^2\left[\sum_{l\neq l'}E_{ll'}-\sum_{lm}E_{lm}\right]\\ &=-4S^2g_\parallel^2\sum_l\sum_n E_{ln}e^{i\mathbf{k}_0\cdot\mathbf{r}_{ln}}\\ &=-2SN_0\frac{g_\parallel^2}{g_\perp^2}E_{k_0}\ ,\end{aligned} \tag{13}$$

where E_{k_0} represents a summation over all spins n in the lattice, and N_0 is the number of two-spin primitive cells in the lattice. The second two terms in Eq. (12) constitute the quantum zero-point energy. For any lattice, $\sum_k E_k^{aa}=0$ since $\sum_k e^{i\mathbf{k}\cdot\mathbf{r}_l}=\delta_{r_l,0}$. Hence

$$\sum_k\epsilon_k=2N_0\frac{g_\parallel^2}{g_\perp^2}(E_0^{aa}-E_0^{ab})=2N_0\frac{g_\parallel^2}{g_\perp^2}E_{k_0}\ , \tag{14}$$

and the ground-state energy becomes

$$W_0=-2SN_0\frac{g_\parallel^2}{g_\perp^2}E_{k_0}-2N_0\frac{g_\parallel^2}{g_\perp^2}E_{k_0}+\frac{1}{2}\sum_k(\omega_k^+ +\omega_k^-)\ . \tag{15}$$

For purposes of calculating the zero-point spin deviation in the ground state, an anisotropy field term $\mathcal{H}_\mathcal{A}$ is added to the Hamiltonian:

$$\begin{aligned}\mathcal{H}_\mathcal{A}&=-H_a\sum_l\mu_l^z-H_b\sum_m\mu_m^z\\&=g_\parallel\mu_B H_a\left[\sum_k a_k^\dagger a_k-N_0S\right]\\&\quad-g_\parallel\mu_B H_b\left[\sum_k b_k^\dagger b_k-N_0S\right]\end{aligned}$$

in the spin-wave approximation, where H_a and H_b are oppositely directed anisotropy fields. This leads to modified eigenvalues $\Omega_k^\pm$ given by

$$\begin{aligned}(\Omega_k^\pm)^2&=\tfrac{1}{2}(\epsilon_k^{a2}+\epsilon_k^{b2})+|E_k^{ab}|^2-|B_k^{aa}|^2-\tfrac{1}{2}(|B_k^{ab}|^2+|B_{-k}^{ab}|^2)\\&\pm\tfrac{1}{2}((\epsilon_k^{a2}-\epsilon_k^{b2})^2+(|B_k^{ab}|^2-|B_{-k}^{ab}|^2)^2+4|E_k^{ab}|^2(\epsilon_k^a+\epsilon_k^b)^2-2(\epsilon_k^a-\epsilon_k^b)^2(|B_k^{ab}|^2+|B_{-k}^{ab}|^2)\\&\quad+4|B_k^{aa}(B_{-k}^{ab})^*+(B_k^{aa})^*B_k^{ab}|^2-4|E_k^{ab}B_{-k}^{ab}-(E_k^{ab})^*B_k^{ab}|^2\\&\quad-4(\epsilon_k^a+\epsilon_k^b)\{B_k^{aa}[(E_k^{ab})^*(B_{-k}^{ab})^*+E_k^{ab}(B_k^{ab})^*]+\text{c.c}\})^{1/2}\ .\end{aligned} \tag{16}$$

The sublattice magnetization deviation at $T=0$ is

$$\Delta M_s(T=0)\equiv\pm M_{a,b}-(N_0/V)g_\parallel\mu_B S\ .$$

It is found to be[9,10]

$$\Delta M_s(T=0)=\pm\frac{1}{V}\frac{\partial}{\partial H_{a,b}}\sum_k\frac{1}{2}(\epsilon_k^a+\epsilon_k^b-[\Omega_k^+ +\Omega_k^-])\ , \tag{17}$$

where $\epsilon_k^{a,b}=\epsilon_k\pm g_\parallel\mu_B H_{a,b}$. The corresponding spin deviation at $T=0$ is

$$\Delta S^z=V\Delta M_s(T=0)/N_0g_\parallel\mu_B\ .$$

In the limit $H_{a,b}\to 0$ these results yield

$$\Delta M_s(T=0)=\frac{g_\parallel\mu_B}{2V}\left[N_0-\frac{1}{2}\sum_k\left[\frac{\epsilon_k}{\omega_k^+}(1+\delta)+\frac{\epsilon_k}{\omega_k^-}(1-\delta)\right]\right], \tag{18}$$

where

$$\delta=\Delta^{-1}(4|E^{ab}|^2-\epsilon_k^{-1}\{B_k^{aa}([E_k^{ab}]^*[B_{-k}^{ab}]^*+E_k^{ab}[B_k^{ab}]^*)+\text{c.c}\})\ .$$

In the case of a Bravais lattice, this simplifies to $\delta=E_k^{ab}/\epsilon_k$, with the more compact result

$$\Delta M_s^{\text{Bravais}}(T=0)=-\frac{g_\parallel\mu_B}{4V}\left[\sum_k\left[\frac{A_k}{\omega_k^+}-1\right]+\sum_k\left[\frac{A_{k_0-k}}{\omega_k^-}-1\right]\right]. \tag{19}$$

At finite temperatures an additional temperature-dependent contribution $\Delta M_s(T)$ exists, which vanishes as $T\to 0$ but not as $H_{a,b}\to 0$:[9,10]

$$\begin{aligned}\Delta M_s(T)&=\pm\frac{1}{\beta V}\frac{\partial}{\partial H_{a,b}}\ln\prod_k(1-\exp[-\beta\Omega_k^+])^{-1}\\&\quad\times(1-\exp[-\beta\Omega_k^-])^{-1}\ ,\\\lim_{H_{a,b}\to 0}\Delta M_s(T)&=-\frac{g_\parallel\mu_B}{2V}\sum_k\left[\frac{\epsilon_k(1+\delta)}{\omega_k^+(\exp[\beta\omega_k^+]-1)}+\frac{\epsilon_k(1-\delta)}{\omega_k^-(\exp[\beta\omega_k^-]-1)}\right],\end{aligned} \tag{20}$$

with simplifications as above for the Bravais lattice case.

IV. EXAMPLES

A. Three dimensions

Amongst the primitive cubic lattices, only the simple cubic (sc) has a classical dipolar antiferromagnetic ground state.[1,11] It consists of (110) planes of spins alternately polarized in the $\pm z$ directions, so that $k_0=(\pi/a,\pi/a,0)$, where a is the cube edge. The lattice sums required for the $A^{\alpha\beta}(\mathbf{k})$ in the simple cubic case have been evaluated previously,[12] for a representative set of 512 values of k within the cubic Brillouin zone. We find that the spin-wave energies $\omega_k^\pm$ are positive and real throughout the zone. The spectrum is plotted in Fig. 1 for representative directions in the tetragonal Brillouin zone of the antiferromagnet, assuming an isotropic g factor. At $\mathbf{k}=\mathbf{0}$, the energies ω_k^+ display the nonuniform convergence characteristic of the dipolar sums $A^{\alpha\beta}(\mathbf{k})$. $\omega_{k=0}^+$ was evaluated using the small k expansion for $A^{\alpha\beta}(\mathbf{k})$ of Aharony and Fisher.[6] The fact that $\omega_{k=0}^+$ is not zero, as it is for the isotropic Heisenberg antiferromagnet, reflects the fact that the dipolar Hamiltonian is not invariant under a rotation of all spins. The spectrum has a zero in both branches at the corners of the Brillouin zone. Near these points the $\omega^\pm$ are linear in k.

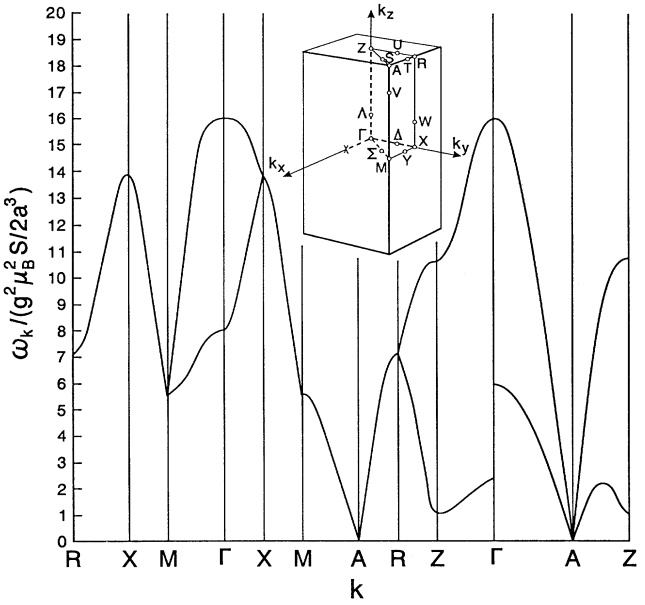

FIG. 1. Numerical evaluation of the dipolar antiferromagnetic spin-wave spectrum of the sc lattice, from Eq. (11). The convergence of ω_k^+ at $k=0$ is nonuniform. The upper curve corresponds to ω_k^- and the lower curve to ω_k^+. The two branches are degenerate on the x and y faces of the Brillouin zone.

This is an important difference with the case of the truncated dipolar Hamiltonian, whose spectrum exhibits a substantial gap[3] for all k. The gap is responsible for the much smaller zero-point effects predicted for that Hamiltonian.

At $T=0$, we obtain the zero-point energy by numerically carrying out the sum $\sum_k(\omega_k^+ + \omega_k^-)$ in Eq. (15). We obtain a ground-state energy

$$W_0 = N_0(g\mu_B S)^2\{A^{zz}(k_0) + (1/S)[A^{zz}(k_0) + 4.091a^{-3}]\}$$
$$= W_0^{\text{Classical}}(1+0.236/S)\ , \tag{21}$$

where $a^3 A^{zz}(k_0) = -5.352$ and a is the sc unit cell edge. The zero-point contribution $\Delta W = W_0 - W_0^{\text{Classical}}$ is rather large, amounting to 47% of $W_0^{\text{Classical}}$ for $S=\frac{1}{2}$. This is more than twice as large as the corresponding result for the nearest-neighbor sc Heisenberg antiferromagnet,[13]

$$\left[\frac{\Delta W}{W^{\text{Néel}}}\right]_{\text{Heis}} = \frac{\gamma}{zS} = \frac{0.097}{S}\ .$$

The magnitude of $\Delta W/W_0^{\text{Classical}}$ in cubic dipolar ferromagnets is similar to the dipolar antiferromagnet, but even larger:[1] $0.280/S$ (fcc), and $0.287/S$ (bcc). At $T=0$, the spin deviation ΔS^z is given by Eq. (19) as

$$\Delta S^z = -\frac{1}{4N_0}\sum_k\left[\left[\frac{A_k}{\omega_k^+}-1\right]+\left[\frac{A_{k_0-k}}{\omega_k^-}-1\right]\right]$$
$$= -0.252\ ,$$

after numerically carrying out the integration. This result is more than three times the spin reduction for the sc Heisenberg case,[13] $\Delta S^z_{\text{Heis}} = -0.078$. For $S=\frac{1}{2}$, the reduction amounts to slightly more than half the sublattice polarization in the Néel state. These zero-point deviations are unexpectedly large, and call into question the common perception that dipolar systems are inherently more classical than those based on exchange. The numerical results are summarized in Table I.

TABLE I. Zero-point contributions ΔW_0 to the ground-state energy and ΔS_z to the sublattice polarization, calculated in the spin-wave approximation for sc and square lattices, for dipolar and nearest-neighbor Heisenberg Hamiltonians.

Lattice	$\Delta W_0/W^{\text{Néel}}$	ΔS^z
sc dipolar	$0.236/S$	-0.252
sc Heisenberg	$0.097/S$[a]	-0.078[a]
Square dipolar, 3D field	$0.114/S$	-0.149
Square dipolar, 2D field	$0.076/S$	-0.141
Square Heisenberg	$0.158/S$[a]	-0.197[a]

[a]Reference 13.

B. Lower dimensionality

It is interesting that both ΔS^z and ΔW also exceed the corresponding spin-wave results for the two-dimensional square lattice Heisenberg antiferromagnet (Table I). Since long-range order becomes more problematical for $d<3$, it is of some interest to investigate lower-dimensional dipolar lattices to see if zero-point effects become larger. Dipolar interactions for dimensions other than three can be defined in more than one way. The simplest case to examine, and perhaps the only one which is physically realizable, is that of three-dimensional spins (and magnetic fields) on a lattice of reduced dimensionality. In one dimension, the classical ground state is ferromagnetic, with dipoles arranged head to tail. This is confirmed by imaginary spin-wave energies, for some values of k, in all the possible antiferromagnetic structures. In two dimensions we have studied only the square lattice, which we find to have a classical antiferromagnetic ground state composed of (10) chains of spins aligned alternatively up and down in the lattice plane, with $k_0=(\pi/a,0)$ oriented normal to the chains. The maximum Luttinger-Tisza eigenvalue[11] for this lattice is $-a^3A^{zz}(k_0)=5.099$, corresponding to a ground-state energy of $N_0(g\mu_B S)^2 A^{zz}(k_0)$. The other eigenvalues are 4.488 ($k_0=[0,0]$), -1.323 ($k_0=[\pi/a,\pi/a]$), and -6.034 ($k_0=[0,\pi/a]$) for spins lying in the plane; and 2.646 ($k_0=[\pi/a,\pi/a]$), -0.935 ($k_0=[\pi/a,0]$), and -8.978 ($k_0=[0,0]$) for spins normal to the plane. The states with $k_0=(0,0)$ correspond to ferromagnetic order. The spin-wave energies for the ground state are real and positive throughout the first Brillouin zone, with a zero in the spectrum of ω_k^+ at $k=(0,\pm\pi/a)$. Near these points the spectrum is linear in k. A numerical evaluation of the ground-state energy gives

$$W_0 = -N_0(g\mu_B S)^2[5.099+1/S(0.583)]a^{-3}$$
$$= W_0^{\text{Classical}}(1+0.114/S)\ ,$$

and the spin deviation is found to be

$$\Delta S^z = -0.149 \ .$$

From the standpoint of analyzing the dependence of fluctuations on spatial dimensionality, an alternate definition of the dipolar interaction is to require the dimension of spins and fields to be the same as that of the lattice. (This approach ignores inconsistencies which may arise from a tensor, rather than vector, definition of spin.[14]) An example is given by Aharony and Fisher,[6] who define the interaction between two dipoles to be

$$\epsilon_{dd} = \frac{\partial^2}{\partial R_1^\alpha \partial R_2^\beta} \left[\frac{S_1^\alpha S_2^\beta}{|\mathbf{R}_1 - \mathbf{R}_2|^{d-2}} \right] .$$

The magnetic field and the spins, as well as the lattice, are of dimension d. In two dimensions, $\ln|\mathbf{R}_1 - \mathbf{R}_2|$ replaces $|\mathbf{R}_1 - \mathbf{R}_2|^{2-d}$. For $d \leq 2$, this expression must be modified by a factor (-1) to avoid inversion of all energies with respect to $d > 2$. For $d = 1$, the interaction vanishes. In two dimensions, we find the square lattice again has the same classical antiferromagnetic ground state, with $k_0 = (\pi/a, 0)$ oriented normal to sublattice polarization. The maximum Luttinger-Tisza eigenvalue for this case is $-a^2 A^{zz}(k_0) = 3.437$. The other eigenvalues are 0 ($k_0 = [\pi/a, \pi/a]$), -3.437 ($k_0 = [0, \pi/a]$), and 0 ($k_0 = [0,0]$). The last corresponds to ferromagnetic order and has a minimum ground-state energy of $-N_0(g\mu_B S/a)^2\pi$, when the (two-dimensional) Lorentz field factor is included. For a spin-wave Hamiltonian defined as in Eqs. (1) and (2), we obtain a real spectrum qualitatively similar to the previous example, with ω_k^+ decreasing linearly to zero at $k = (0, \pm\pi/a)$. A numerical evaluation of the ground-state energy gives

$$W_0 = -N_0(g\mu_B S)^2[3.437 + (1/S)(0.262)]a^{-2}$$
$$= W_0^{\text{Classical}}(1 + 0.076/S) \ ,$$

and for the spin deviation we find

$$\Delta S^z = -0.141 \ .$$

Thus for both types of square lattice, the zero-point corrections are *smaller* than for the sc dipolar lattice. They are also smaller than those of the square Heisenberg model. In the second, strictly two-dimensional example, this may be a reflection on the inadequacy of the spin-wave approximation for two-dimensional spins, since one ordinarily expects fluctuations to increase as dimensionality is lowered. However, in both cases the integrals for $\Delta M_s(T)$ diverge, just as for the square lattice Heisenberg antiferromagnet, and carry the same implications about the absence of long-range order at finite temperature. It is interesting to speculate that this result may be general, and that there may exist an extension of the Mermin-Wagner theorem[15] to long-range interactions such as the dipole interaction.

V. CONCLUSIONS

We have obtained a general solution of the two-sublattice dipolar antiferromagnet in the spin-wave approximation. The results are particularly simple for Bravais lattices. In the case of the simple cubic lattice, zero-point corrections to the energy and sublattice magnetization are unusually large, more than double those of the nearest-neighbor sc Heisenberg antiferromagnet.

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