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Spin-controlled resonances in the magnetotransport in quantum dots

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We discuss magnetoconductance oscillations arising from transmission and reflection of current via edge states which are confined within lateral quantum dots by entrance and exit port barriers. An interplay of several resonant processes involving spin-resolved edge states determines the evolution of the oscillations as a function of the conductance of the barriers.

I. INTRODUCTION

Submicrometer-sized quantum dots (QD's) with quantum-point-contact (QPC) entrance and exit ports can be defined within a two-dimensional electron gas using surface gates. Studies on these systems[1–6] have revealed a rich variety of magnetoconductance (MC) phenomena, including oscillations arising from discrete electron effects. In this paper we focus on the role of electron spin in the MC and present a physical picture to explain aspects of two recent experiments.[5,6]

Oscillations in the conductance $G_{\rm dot}$ (in units of e^2/h) of a QD arise as the energy spectrum is varied either by changing the applied magnetic field B,[1,2] or the electron density n_e by sweeping the backgate[4,6] or surface gate[3,5] voltages V_g and V_s. The conductance G_b (in units of e^2/h) across the individual QPC barriers can be tuned using the surface gates. Depending on G_b, n_e, and B, the system may have one or more edge states transmitted across the barriers. "Trapped" edge states, which are confined within the QD by the barriers, may also be present. These can contribute to $G_{\rm dot}$ by providing tunneling channels which couple either across the barriers to the edge states outside the QD or to the fully transmitted edge states (FTES) inside the QD. Within this picture, a single-particle (SP) model would predict MC oscillations of the Aharonov-Bohm type, yielding a SP spectroscopy as the levels pass through the Fermi energy E_F.

Coulomb interactions invalidate this SP picture and the MC becomes a probe of the "addition" spectrum—the change in free energy of the system on incrementing the number of electrons in the QD's confined states. Resonances then occur when this addition energy equals E_F. The recently observed period doubling[6] and further evolution of the MC profiles as a function of G_b are explained via resonant processes of the two spin channels, their Coulomb interactions which push them out of phase, and the progressive localization of one of the spin channels.

II. EXPERIMENTAL RESULTS

An interedge state resonant-reflection (RR) process, in which the current carried into the QD by a fully transmitted edge state is reflected via the confined edge states of the QD, was first invoked to interpret MC oscillations for QD's with diameters less than 0.5 μm.[2] This contrasted with the previously observed resonant transmission through the confined states of a larger dot.[1] In both experiments resonances were described within a SP model. However, whereas the total number of electrons in a QD remains constant during B sweeps, the number of electrons in trapped states does change. At resonance a previously unoccupied trapped state becomes populated and it is necessary to consider the associated cost in energy arising from electron interaction effects. The MC oscillations are thus a signature of an addition spectrum and their periodicity ΔB has recently been reinterpreted to accommodate this.[7] In this picture, ΔB is not simply related to the number of flux quanta enclosed by the trapped state, as in the simple Aharonov-Bohm approach. Instead it is dominated by electron interactions and the screening of this interaction by electrons in the fully transmitted edge states. In particular, for the data of Ref. 1, ΔB was shown to be inversely proportional to the number of fully transmitted edge states N.

We now consider a QD with a geometry similar to those of Ref. 2, and a diameter of 0.5 μm and n_e of 3.5×10^{11} cm^{-2}. Figure 1(a) shows the MC oscillations

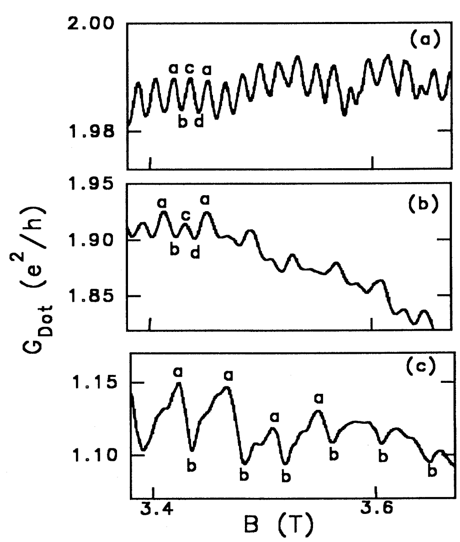

FIG. 1. The evolution of the MC oscillations as G_b is lowered (see text).

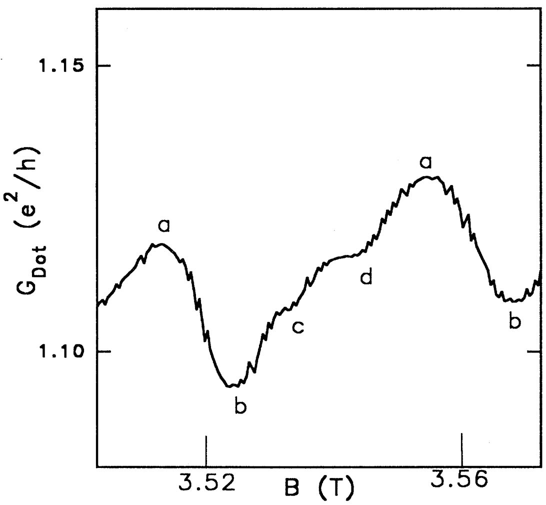

FIG. 2. A magnified plot of the triangular form seen in Fig. 1(c).

for $G_b=2$ (corresponding to $N=2$) and for which G_{dot} approximately equals G_b. Figures 1(b) and 1(c) show the evolution of the MC structure as G_b and N are lowered. The characteristic $G_{dot}<G_b$ of these experiments implies the existence of a coupling between the fully transmitted edge states and the confined states.[2] A comparison of traces 1(b) and 1(c) reveals an *apparent* period doubling of the MC oscillations, a phenomenon additional to the screening-induced changes in ΔB discussed above. The sequence starts with a uniform periodic oscillation for $G_{dot}=2$, seen in Fig. 1(a), followed by the suppression of every second oscillation seen in Fig. 1(b). For $G_{dot}\approx 1.1$, a triangular shape is observed in Fig. 1(c) followed by a region of conductance troughs. The triangular behavior has two superimposed minima, seen at points b and c in the magnified trace of Fig. 2. Finally, for $G_{dot}\leq 1$ the troughs disappear (not shown). The results of a recent experiment[5] are also consistent with this trend. See the B sweeps of Fig. 4 of Ref. 5, where ΔB for $G_{dot}\sim 1.5$ and 2.5 are approximately 26 and 13 mT, respectively. Also in Fig. 2 of Ref. 5, V_s sweeps exhibit similar qualitative behavior, and no explanation was given. We now present a consistent explanation for these observations.

III. SPIN-CONTROLLED RESONANT REFLECTION

Schematic edge-state configurations of the QD are shown in Fig. 3. Figure 3(a) illustrates the resonant-reflection mechanism for $G_b=2$, with two FTES in each QPC (one spin-up, u, and one spin-down, d, state). We denote these as χ_u and χ_d, with χ'_u and χ'_d corresponding to the returning edge states. The confined states are spin resolved and denoted by Φ_u and Φ_d. Spin is conserved in the RR process and therefore two separate tunneling processes (driven by the chemical potential difference between the χ and χ' states) reflect current through the confined states: $\chi_u\rightarrow\Phi_u\rightarrow\chi'_u$ and $\chi_d\rightarrow\Phi_d\rightarrow\chi'_d$. We define the associated reflection probabilities as R_u and R_d. At their respective resonances their values are nearly identical since they are essentially tunneling probabilities connecting edge and confined states, differing only in their spin. Furthermore, a simple calculation of the SP energy spectrum shows that consecutive resonances involve the reflection of electrons with opposite spin. The inclusion of Coulomb interaction effects, which determines the additional energy spectrum, does not change this sequence but adjacent resonances become exactly out of phase with one another even though opposite spins are involved, in order to optimize the mutual exclusion by the Coulomb interaction.[7] The picture is then of resonances equally spaced in B and corresponding to minima of essentially identical values of G_{dot}, as observed. The labels $a-d$, marked in Fig. 1(a), represent a complete cycle of the RR process, with Φ_u and Φ_d being populated at points b and d, respectively. We write $R_s=|r_s|^2$, with

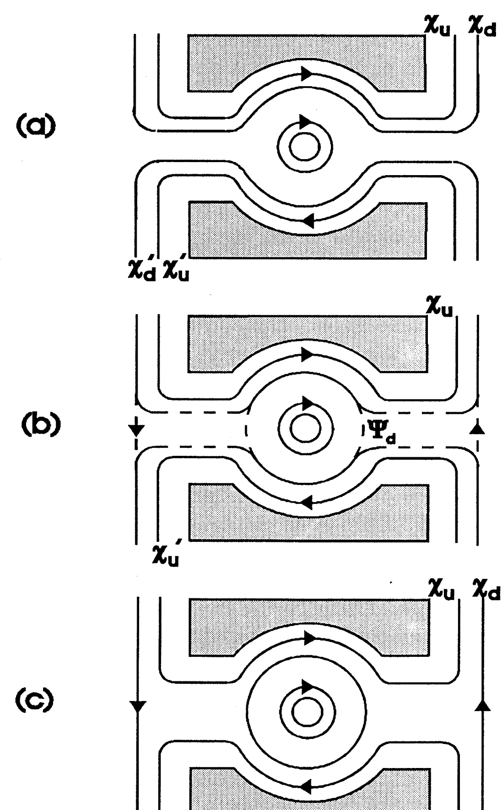

FIG. 3. A schematic representation of the edge states under different barrier conditions. In (a) $G_b=2$. Within the dot there are also confined (spin-resolved) states, Φ_u and Φ_d [this corresponds to the situation of Fig. 1(a)]. As G_b becomes <2 the extended state χ_d of diagram (a) becomes the mixed state Ψ_d of diagram (b) and Eq. 92) [corresponding both to Fig. 1(b) and the low-field side of 1(c)]. In (c) as G_b is lowered further the χ_d state is mostly reflected at the barriers (for clarity it is drawn fully reflected) and therefore all the resonances involving this state are inoperative [this corresponds to the high-field side of Fig. 1(c)].

$s=u$ or d, in the form

$$R_s=|\langle\chi_s(n_L,k_F)|T|\Phi_s(n_L^*,l)\rangle|^2 . \tag{1}$$

Here n_L and s identify the Landau-level index and the spin, while k_F approximately indicates the traveling wave nature of the fully transmitted edge state χ_s at E_F. Also, n_L^* is the Landau-level index for the confined state Φ_s of angular momentum l. T formally denotes an electron transfer operator for the process $\chi_s \rightarrow \Phi_s \rightarrow \chi_s'$.

When experimental parameters are tuned such that G_{dot} is reduced below 2, no edge state is fully transmitted across the QPC barriers.[6] However, the edge state with $s=d$ is preferentially reflected at the barriers, and in the following description we assume for simplicity that the $s=u$ state remains fully transmitted and described by χ_u [Fig. 3(b)]. Inside the QD the $s=d$ state is quasiconfined and is represented as a mixed state Ψ_d:

$$\Psi_d(n_L,k_F,\beta)=a\chi_d(n_L,k_F)+b\Phi_d(n_L,\beta)+\cdots , \tag{2}$$

where Φ_d and χ_d are the wave functions of the confined and extended components of the mixed state. $\Phi_d(n_L,\beta)$ is the confined state closest to E_F and is associated with the same Landau level as its extended partner but has a discrete angular momentum index β. The ellipsis in Eq. (2) indicates that other confined states also contribute to the mixed state but for simplicity we assume that $\Phi_d(n_L,\beta)$ is dominant. a^2 and b^2 represent the probabilities that a $s=d$ electron entering the QD appears in the extended or confined component of the mixed state, and their values are determined by boundary conditions at the first QPC barrier which match Ψ_d to the incoming edge state.[8] This does not, however, guarantee that the boundary conditions are matched for transmission at the second QPC. For selected boundary conditions, the net effect of the mixing in of $\Phi_d(n_L,\beta)$ is to decrease transmission across the QD. That is, in addition to the RR via the path of Eq. (1), we now have an additional reflection denoted by $R_{\text{int},d}$. Note that this reflection process, which reaches resonance at B fields corresponding to maxima in b^2, does not directly involve the inner confined levels $\Phi_d(n_L^*,l)$ invoked in Eq. (1). Now consider the RR processes of Eq. (2) which do link Ψ_d with $\Phi_d(n_L^*,l)$. We define $R_{\text{mix},d}$ such that

$$\begin{aligned}R_{\text{mix},d}&=|\langle\Psi_d(n_L,k_F,\beta)|T|\Phi_d(n_L^*,l)\rangle|^2\\&=a^2|r_d(k_F,l)|^2+b^2|r_d(\beta,l)|^2\\&\quad+\{ab^*r_d(k_F,l)r_d^*(\beta,l)+a^*br_d^*(k_F,l)r_d(\beta,l)\} .\end{aligned} \tag{3}$$

The first term describes RR from the extended component $\chi_d(n_L,k_F)$ to the inner confined states $\Phi_d(n_L^*,l)$, and is equivalent to the R_d process discussed for $G_{\text{dot}}=2$. The second term describes electron transfer from the confined component $\Phi_d(n_L,\beta)$ of the mixed state to the inner confined state $\Phi_d(n_L,l)$, producing a charge rearrangement *within* the dot.[4] The last term, featuring the coefficients ab^* and a^*b, describes an interference between RR and internal charge rearrangements.

With the above picture in mind, we return to the interpretation of the MC oscillations in Fig. 1. As the QPC barrier height is raised, the associated reduction in G_b is due to the partial reflection at the barrier of the current carried by the $s=d$ edge state. However, the change in conductance ΔG_b between Figs. 1(a) and 1(b) is such that for Fig. 1(b) $\Delta G_b \ll G_b$. The R_u and R_d routes (i.e., $\chi_u \rightarrow \Phi_u \rightarrow \chi_u'$ and $\chi_d \rightarrow \Phi_d \rightarrow \chi_d'$) which occur at the minima of the oscillations (marked b and d) are therefore relatively unperturbed compared to Fig. 1(a). Note that the minima b and d exhibit small yet identical variations in G_{dot} and we associate these with a B dependence in the coupling strength between the extended and confined wave functions. We also see a suppression of conductance maxima at c, indicating additional RR at every second maximum. We associate this RR effect with the $R_{\text{int},d}$ process. A similar resonance of a mixed state, resulting in a rise in transmission rather than reflection, has been previously reported for other QPC boundary conditions.[3,8] This second form of RR process is only evident at point c, and not at the equivalent point a, because for the $s=u$ process χ_u does not have a confined component mixed in it. Effects of the second and third terms of Eq. (3) seem to be minor in the region of conductance considered here.

When G_b is lowered further, all $s=d$ RR processes become smaller since a significant amount of the $s=d$ current is reflected at the QPC barriers. The minima at points d are then raised relative to those at point b, as seen in the magnified trace in Fig. 2. The confined component $\Phi_d(n_L,\beta)$ is now strongly established and the associated resonance becomes sharper. With this increased resolution the suppression of peak c is replaced by an incision into the peak, producing an extra minimum, clearly visible in Fig. 2. In Eq. (2), as b^2 increases, a^2 decreases. These effects produce the sawtooth-conductance behavior. As B is further increased, the $s=d$ processes at points c and d become inoperative [Fig. 3(c)]. Thus above $B \approx 3.6$ T, Fig. 1(c) then shows a sequence of *troughs* with a period approximately double that seen in the MC oscillations of Fig. 1(b). As G_b is lowered significantly below unity, current is no longer freely transmitted into the QD, and the traditional Coulomb blockade regime appears.[6]

IV. CONCLUSION

In quantum dots, MC oscillations arise from resonant-reflection or transmission processes via confined edge states and are controlled by electron interaction effects. These force the two spin channels to be mutually out of phase. A characteristic of MC oscillations, an *apparent* period doubling, is described. The "doubling" appears when one of the spin channels begins to behave differently from the other as G_b is varied. The detailed evolution of this characteristic can be understood in terms of the progressive confinement (i.e., localization) of one of the spin-resolved edge states as G_b is lowered. This confinement also induces an extra RR mechanism not available for fully transmitting edge states. The same physical model clarifies the MC phenomena described in Ref. 5.

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[7]M. W. C. Dharma-wardana *et al.*, Solid State Commun. **84**, 631 (1992).
[8]An intuitive way of setting the boundary condition (see Ref. 3) is to require that the electron in the confined state describe a closed path with an integral number of Fermi wavelengths, so that it matches the incoming wave of E_F and k_F near the first QPC. See also M. Yosefin and M. Kaveh, J. Phys. Condens. Matter **1**, 10 207 (1989).