Orbital magnetism of mesoscopic metals: Extension to the nonperturbative regime

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We extend our investigation of orbital magnetic response of mesoscopic metallic systems to the limit of temperature and inelastic level broadening being comparable to or less than the average interlevel spacing. We address the role of level repulsion and derive a level density correlation function that interpolates between the perturbation result and the Wigner-Dyson statistics. We plot out the magnetic-field dependence of the magnetic moment of the system for the entire range of experimentally relevant temperatures, including the limiting dependence at near-zero temperature. We address the relation of our results to the response of ideal rings and classically integrable systems in the context of their respective level structures. We emphasize the semblance between the response of small metals and (compound) atomic and nuclear systems, in particular, the interplay of Curie orientational paramagnetism, Van-Vleck polarization paramagnetism, and Langevin precession diamagnetism. We also touch upon the related issues in the subject of quantum chaos.

I. INTRODUCTION

For many years the quantum nature of the orbital magnetic response remained a gripping topic in condensed-matter physics. In metals, the orbital magnetism[1] has long been associated with the notions of diamagnetic Landau susceptibility and the de Haas–van Alphen (dHvA) oscillations. The latter are specific to the Landau quantization whose observation requires strong magnetic fields; otherwise, both the disorder and the thermal distribution act to suppress the dHvA oscillations exponentially. On the other hand, the magnitude of the Landau susceptibility is only insignificantly affected by the temperature and disorder. Henceforth, it was identified with the skewing of the electron orbits by the magnetic field.[2]

As the quantum coherent phenomena, such as electron localization, came to prominence over the past decade[3] it was realized that the Landau susceptibility can be strongly renormalized due to the electron scattering off impurities and its effect on the electron-electron interactions.[4] The emergence of the mesoscopic field[5] has propelled a body of research on persistent currents in metallic rings subject to the Aharonov-Bohm flux. Although the accumulation of experimental data required truly remarkable advancements[6,7] in magnetic sensitivity, it is still limited and is seemingly in contradiction with the present state of the theory. As of this writing, there exist a number of theories[8] which cover various aspects of the orbital response of small metals. These include the mesoscopic fluctuations, the interaction corrections (both of Coulombic origin and induced by scattering off Kondo impurities),[9] and magnetism of systems with the fixed thermodynamical average number of electrons. The latter clearly represents a sharp departure from the conventional grand-canonical-ensemble description of bulk metals whereby a fixed chemical potential is assumed.

In a preceding paper,[8] which hereafter will be referred to as I, we made an emphasis on the congruous treatment of persistent currents in narrow rings and the orbital magnetism of grains by means of expressing either response in terms of the Landau susceptibility. We pointed to the difference in flux scales of different effects. In particular, we showed that at low temperature the typical flux scale for the Kondo-induced interaction[9] and the canonical-ensemble response is much smaller than the flux quantum which, in turn, defines the typical flux scale for the usual electron-electron interaction and mesoscopic fluctuations. We argued that the existence of the former scale is directly related to the absence of phase and energy relaxation at the sample boundary in the absence of electron reservoirs, such as the current leads, attached to the sample.[10]

Building on the fact that the magnitude of the orbital response of mesoscopic metallic systems far exceeds the Landau susceptibility (and that the electrons in such systems retain quantum coherence), we proposed its interpretation using the nomenclature ordinarily reserved for atomic and nuclear objects, namely, as the competition between the Lanevin precession diamagnetism and the Van Vleck polarization paramagnetism. In this picture the electrons fill up the states, up to the Fermi energy, of the effective combined potential of the impurities and sample walls. We extended this analogy[11] to the feedback of the orbital motion into spin degrees of freedom in the presence of spin-orbit scatterers and argued that this same mechanism responsible for the renormalization of the electron g factor in magnetic atoms[12] should lead to the orders-of-magnitude fluctuations of the electron g factor in mesoscopic systems.[13] We applied our picture to the analysis of the magnetic response of semiconductor quantum dot structures as well, where the Curie orientational paramagnetism and the analog of the atomic Hund's rules play a central role.[14]

In this paper we continue our investigation of the average orbital response of disordered metallic systems[8,15] using the model of noninteracting electrons in a random potential and assuming no contact with electron baths. We will extend our analysis to the nonperturbative[16] regime where the temperature and/or the inelastic level broadening become comparable or less than the mean interlevel spacing. We discuss the sign of the response in connection with the numerical work on the disordered and integrable systems, as well as the analytical results for the quasi-one-dimensional rings. This manuscript is structured in the following sequence. In Sec. II we give the derivation of the density-of-states correlation function, which provides the interpolation between the perturbative regime and the regime with the pronounced level repulsion. The relation to the problem of the "analytic bootstrap" of the density of states[17] in the theory of quantum chaos is discussed. We also derive a very general transformation establishing the link between the responses of a narrow ring and a disk of the same circumference. In Sec. III we review the results derived with the help of the standard Green's-function perturbation technique and make several clarifications to I. In Sec. IV we derive the formula for the orbital response in the nonperturbative regime and plot out its magnetic-field dependence for a range of relevant temperatures as well as the limiting dependence at near-zero temperatures. We also obtain the analytical result for the zero-temperature linear response. Finally, in Sec. V we discuss our results and future problems.

II. CORRELATION FUNCTION OF THE DENSITY OF STATES

In this section we will derive the correlation function of the density of states in the nonperturbative regime where the level repulsion is strongly manifested. The expression for the correlation function $K(\varepsilon_1,\varepsilon_2) = \langle \nu(\varepsilon_1)\nu(\varepsilon_2)\rangle - \langle \nu(\varepsilon_1)\rangle\langle \nu(\varepsilon_2)\rangle$ in the perturbative regime was obtained in Ref. 18 as

$$K_{\mathrm{pert}}(\varepsilon_1,\varepsilon_2)=\frac{s^2}{\pi^2\mathcal{V}^2}\mathrm{Re}[-i(\varepsilon_1-\varepsilon_2)+\gamma]^2$$
$$=\frac{\Delta^2\nu_0^2}{\pi^2}\mathrm{Re}[-i(\varepsilon_1-\varepsilon_2)+\gamma]^{-2}\,, \quad (1)$$

where $\mathcal{V}$ is the sample volume, $\Delta=[\nu_0\mathcal{V}]^{-1}$ is the mean interlevel separation, ν_0 is the mean density of states, γ is the inelastic level broadening, and angular brackets denote averaging over the impurity configurations. Only the zero-mode contribution is represented here which is sufficient in the temperature regime of interest to us (see I).

The validity of the perturbative approach in this derivation is defined by the condition that $\gamma \gg \Delta$, which implies that the single-electron levels are smeared out into bands. In this circumstance, it is clear that the level repulsion will only be observed at the energy scales of $|\xi| = |\varepsilon_1 - \varepsilon_2| \geq \gamma$ and will be relatively small. On the other hand, it was demonstrated in Ref. 16 that in the absence of level broadening the repulsion is very strong on the energy scales $|\xi| \leq \Delta$. A simple modification of the technique of Ref. 16 allows us to obtain the result applicable to both regimes. Indeed, even for zero level broadening the term containing $-i\omega$ in the action of the supersymmetric nonlinear σ model[16] has to be modified to $-i\omega+\delta$ for convergence purposes; at the end of the calculation δ is set to zero. For finite broadening, the substitution $-i\omega+\gamma$ should be maintained throughout the calculation. For simplicity, we first illustrate the outcome of this procedure for the unitary ensemble. The perturbative result is just a half of the result given by Eq. (1). The complete expression is as follows:

$$K_{\mathrm{unit}}(\xi)=\frac{s^2}{2\pi^2\mathcal{V}^2}\mathrm{Re}\frac{1}{(-i\xi+\gamma)^2}$$
$$\times\left[1-\exp\left[\frac{2\pi}{\Delta}(i\xi-\gamma)\right]\right]. \quad (2a)$$

The correlation function of Eq. (2) is shown in Fig. 1. For large, γ, it approaches the perturbative result. For small γ, it approaches the results of Ref. 16:

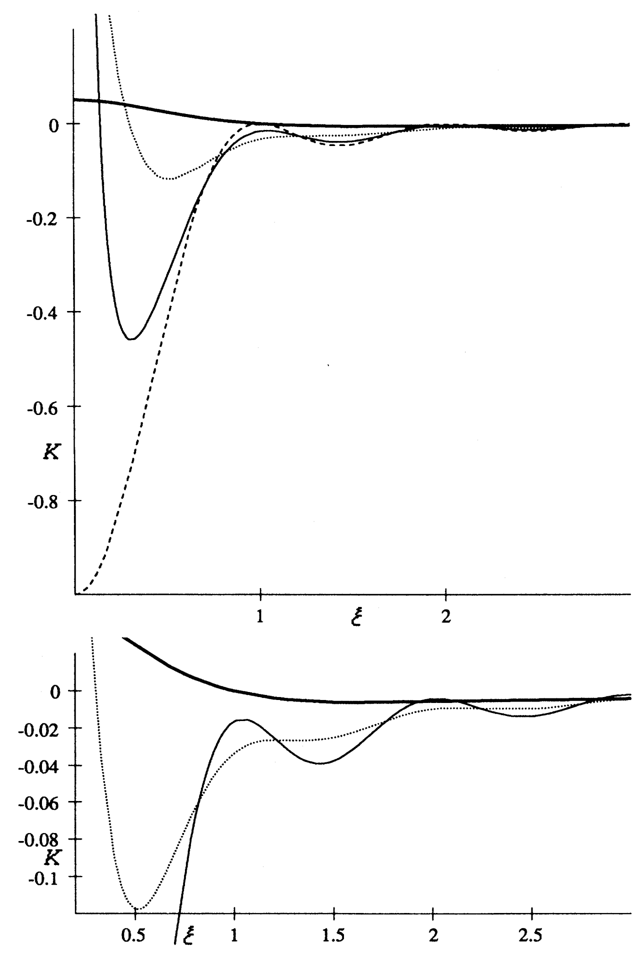

FIG. 1. Correlation function of the density of states in the unitary case [see Eq. (2a)] for $\gamma=1$ (heavy line), $\gamma=\frac{1}{4}$ (normal line), $\gamma=\frac{1}{16}$ (dotted line), and $\gamma=0$ (dashed line), respectively. The inset magnifies the onset of oscillations with the decrease of level broadening characteristic of the discrete spectrum.

$$K_{\rm unit}(\xi)\approx\begin{cases}\frac{1}{2}K_{\rm pert}(\xi), & \gamma\gg\Delta\ ,\\ -\nu_0^2\frac{\sin^2(\pi\xi/\Delta)}{(\pi\xi/\Delta)^2}, & \gamma=0\ .\end{cases} \tag{2b}$$

Notice that the limits $\gamma\to 0$ and $\xi\to 0$ do not commute so that for $\gamma\le\Delta$ only the scales $|\xi|\ge\gamma$ are relevant.

The problem of matching the perturbative and nonperturbative regimes has been extensively discussed in the problem of quantum chaos. There the Fourier transform of Eq. (1), $K(\tau)$, can be obtained by using the quasiclassical sum rule of Hannay–Ozorio de Almeida. The latter is based on Gutzwiller's representation of the level density in terms of the sum over periodic orbits[17] and the cancellation between the exponential smallness of the amplitudes of such orbits and their exponentially large number for a classically chaotic ergodic system. The energy averaging utilized in that approach eliminates the phase incoherence, just as in the disorder averaging for mesoscopic phenomena. Unfortunately, the diagonal approximation, whereby the interference between periodic paths is excluded,[17] extends the similarity to the inability of reproducing the discreteness of the level structure as well. In fact, the correlation function of the level density obtained in the diagonal approximation coincides with the perturbative result for disordered metals given above. The "analytical bootstrap" is an identity[17] which relates the level density correlation function (as well as the higher cumulants) to the mean level density. To satisfy this identity the correlation function is forced to have the correct asymptotic behavior leading to the form of $K(\tau)$ predicted by the random matrix theory.[16] The generalization of the expressions found from the random matrix theory to finite level broadening, followed by the Fourier transform, is a convenient way of deriving $K(\varepsilon_1,\varepsilon_2)$. Below we illustrate this procedure for the orthogonal case.

The proposed generalization is easily achieved via the multiplication of $K(\tau)$ by $\exp(-2\pi\gamma\tau)$. Indeed, the Fourier transform of Eq. (1) is just $K(\tau)=2\tau\exp(-2\pi\gamma\tau)$. Using now the complete expression[16] for $K(\tau)$ for the orthogonal circumstance and hereafter setting $\Delta=1$, we obtain the following formula:

$$K_{\rm orth}(\tau)=\exp(-2\pi\gamma\tau)\begin{cases}\tau+\tau[1-\ln(2\tau+1)]\ , & 0\le\tau\le 1\\ 1+\left[1-\tau\ln\left[\frac{2\tau+1}{2\tau-1}\right]\right]\ , & 1\le\tau\le\infty\ '\end{cases} \tag{3}$$

where the terms in front of the square brackets are the contributions that survive the breaking of time-reversal symmetry and give the answer for the unitary case—the diffuson contribution in the language of the perturbative expansion. Taking the Fourier transform we obtain

$$K_{\rm orth}(\xi)=K_{\rm unit}(\xi)+k(\xi)\ ,$$

$$k(\xi)=\frac{s^2}{2\pi^2\mathcal{V}^2}{\rm Re}E_1[\pi(-i\xi+\gamma)]\frac{d}{d\xi}\left[\frac{2i\sinh[\pi(-i\xi+\gamma)]}{-i\xi+\gamma}\right]\ , \tag{4a}$$

where $K_{\rm unit}(\xi)$ is given by Eq. (2a) and E_1 is the exponential integral. Again, in the language of perturbation theory $k(\xi)$ corresponds to the Cooperon contribution. It is easy to verify that

$$k(\xi)\approx\begin{cases}\frac{1}{2}K_{\rm pert}(\xi)\ , & \gamma\gg 1\ ,\\ \nu_0^2\frac{d}{dx}\left[\frac{\sin x}{x}\right]si(x)\ , & \gamma=0\ .\end{cases} \tag{4b}$$

The generalization of Eq. (4a) to the presence of magnetic field is accomplished by the substitution $\gamma\to\gamma+\tau_H^{-1}$, where $\tau_H^{-1}\propto H^2$ (see Sec. III), in the expression for $k(\xi)$ in Eq. (4a). Clearly, $k(\xi)\to 0$ for a sufficiently large field, $\tau_H^{-1}\gg 1$, leading to $K(\xi)\to K_{\rm unit}(\xi)$.

III. ORBITAL RESPONSE IN THE PERTURBATIVE REGIME

According to the results of Ref. 15, which we also used in I, the correction to the free energy which needs to be evaluated to account for the fixed number of particles in the system is given by

$$\delta F(\phi)=\tfrac{1}{2}\Delta\langle[\delta N(\phi)]^2\rangle\ , \tag{5}$$

where ϕ is the magnetic flux through the sample.[19] As in I, we shall assume the two-dimensional samples since the generalization to three dimensions is trivial. Thus, $\nu_0=ms/2\pi$ ($\hbar=1$), and $\mathcal{V}=La$ for the ring and $\mathcal{V}=L^2/4\pi$ for the disk. Here L is the circumference, a is the width of the ring, and s is the spin degeneracy. The dependence of the correlation function of the density of states on the flux is crucial for the derivation of the response since

$$\langle[\delta N(\phi)]^2\rangle=\int\int d\varepsilon_1 d\varepsilon_2 K(\varepsilon_1,\varepsilon_2;\phi)f(\varepsilon_1)f(\varepsilon_2)\ , \tag{6}$$

where $f(\varepsilon)$ is the Fermi distribution function. In I we found that it is given by

$$K(\varepsilon_1,\varepsilon_2;\phi)=\frac{s^2}{2\pi^2\mathcal{V}^2}{\rm Re}[-i(\varepsilon_1-\varepsilon_2)+\gamma+\tau_H^{-1}]^{-2}\ , \tag{7a}$$

for the disk, while for the ring it is given by[8,15]

$$K(\varepsilon_1,\varepsilon_2;\phi)=\frac{s^2}{2\pi^2 \mathcal{V}^2}\sum_{n=-\infty}^{\infty} \text{Re}[-i(\varepsilon_1-\varepsilon_2)+\gamma+\pi^2 E_c(n+2\phi/\phi_0)^2]^{-2} . \quad (7b)$$

Here $\phi_0=2\pi/e$ is the flux quantum ($c=1$), $\tau_H^{-1}=2\pi^2 E_c(\phi/\phi_0)$, $E_c=D/L^2$, and D is the diffusion coefficient. Since the correlation function decreases with the flux, the orbital response under consideration should have the paramagnetic sign.

In I we derived an expression for the magnetic moment of the disk using the perturbative approximation of Eq. (7a) in terms of an integral which is evaluated numerically (exactly for the zero-field response). We also derived the formula for the ring based on Eq. (7b). However, in the latter evaluation the Matsubara sum was approximated by an integral yielding a zero-field susceptibility $\pi^2/6$ smaller than the exact result (a factor of 2 was also erroneously omitted). A numerical Matsubara summation for several values of the parameter $\xi=(2\pi T/E_c)^{1/2}$ (as in I it assumed that $2\pi T \gg \gamma$ in a metal) results in the dependence of the ring moment on the flux shown in Fig. 2(a). Figure 2(b) depicts the magnetic moment of the disk in the same flux range. The moments are shown scaled by the respective inverse conductances, meaning that for the weak disorder the orbital moment of the ring should be larger than the moment of the disk of equal circumference. Notice that the zero-field susceptibility of the ring is twice that of the disk. Aside from a direct calculation,

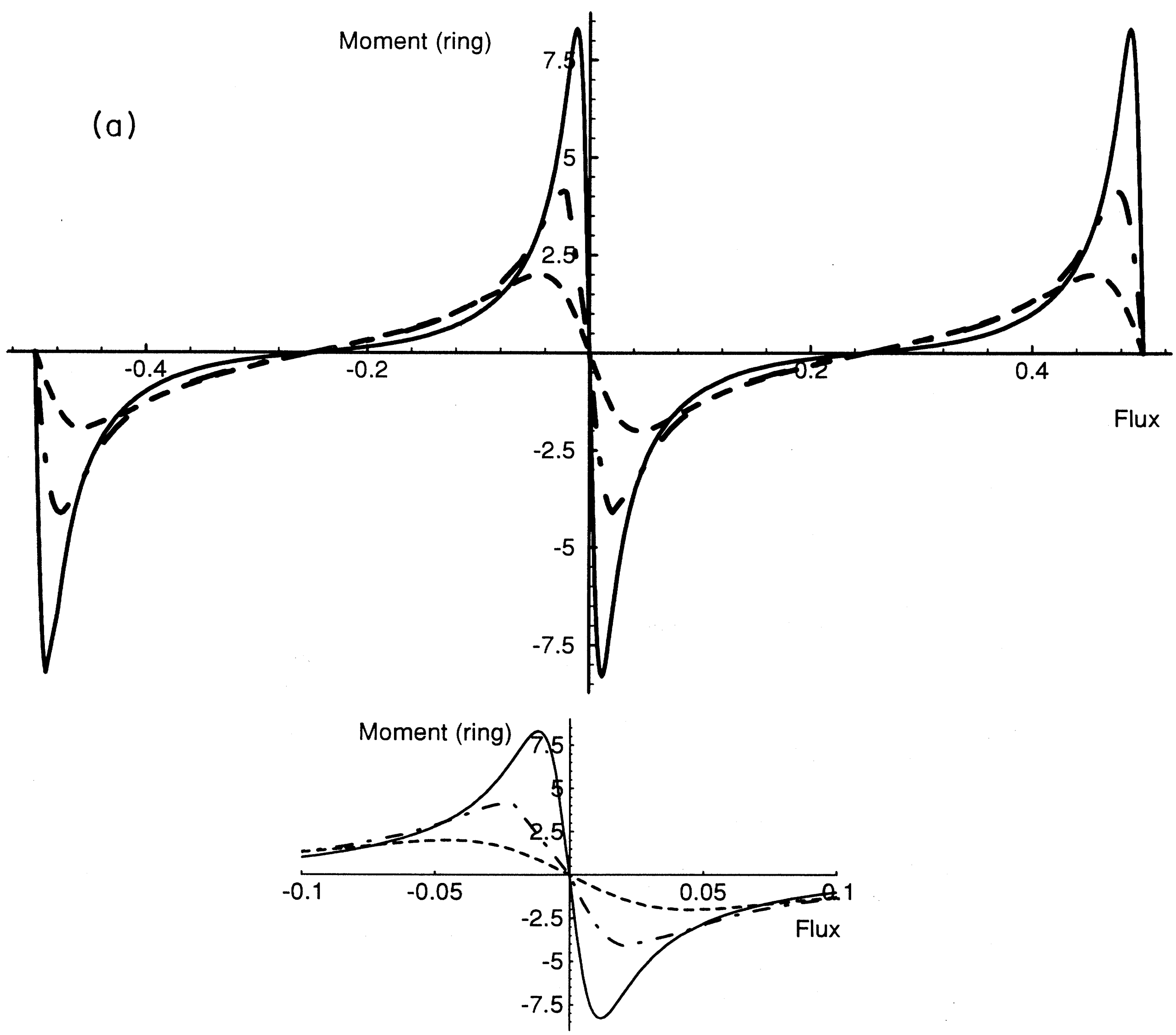

FIG. 2. (Negative) magnetic moment from the perturbative calculation of the (a) ring and (b) disk for $\xi=1$ (dashed line), $\xi=0.5$ (dash-dotted line), and $\xi=0.25$ (normal line), respectively.

this can be easily seen also from the following general expression which should apply to all quantum coherent contributions to the orbital magnetic moment:

$$M_{\rm ring}(\phi)=\sum_{n=-\infty}^{\infty}\exp\left[in\frac{4\pi\phi}{\phi_0}\right]\int_{-\infty}^{\infty}d\phi\, M_{\rm disk}(\phi)\exp\left[-in\frac{4\pi\phi}{\phi_0}\right], \tag{8}$$

with the substitution $E_c \rightarrow 2E_c$ in the right-hand side [compare Eq. (7b) for $n=0$ with Eq. (7a)]. Equation (8) is consistent with the factor of 2 difference in the zero-field susceptibility of the disk and of the ring. It also points to the position of the maximum being $\sqrt{2}$ closer to the origin for the ring than for the disk.

IV. ORBITAL RESPONSE IN THE NONPERTURBATIVE REGIME

We now turn to the evaluation of the orbital response in the nonperturbative regime. In view of the relation (8), it is sufficient to consider only the response of a disk. Using the results of Sec. II, we arrive at the following formula for $\delta F(\phi)$ in Eq. (5):

$$\delta F(\phi)=\pi\Delta T\sum_{\omega_m>0}\omega_m K_{\rm orth}(i(\omega_m+\tau_H^{-1})) , \tag{9}$$

where $K_{\rm orth}(\xi)$ is given by Eq. (4a). Neglecting γ as before, we find the following expression—immediately suitable for a numerical evaluation—for the magnetic moment of the disk:

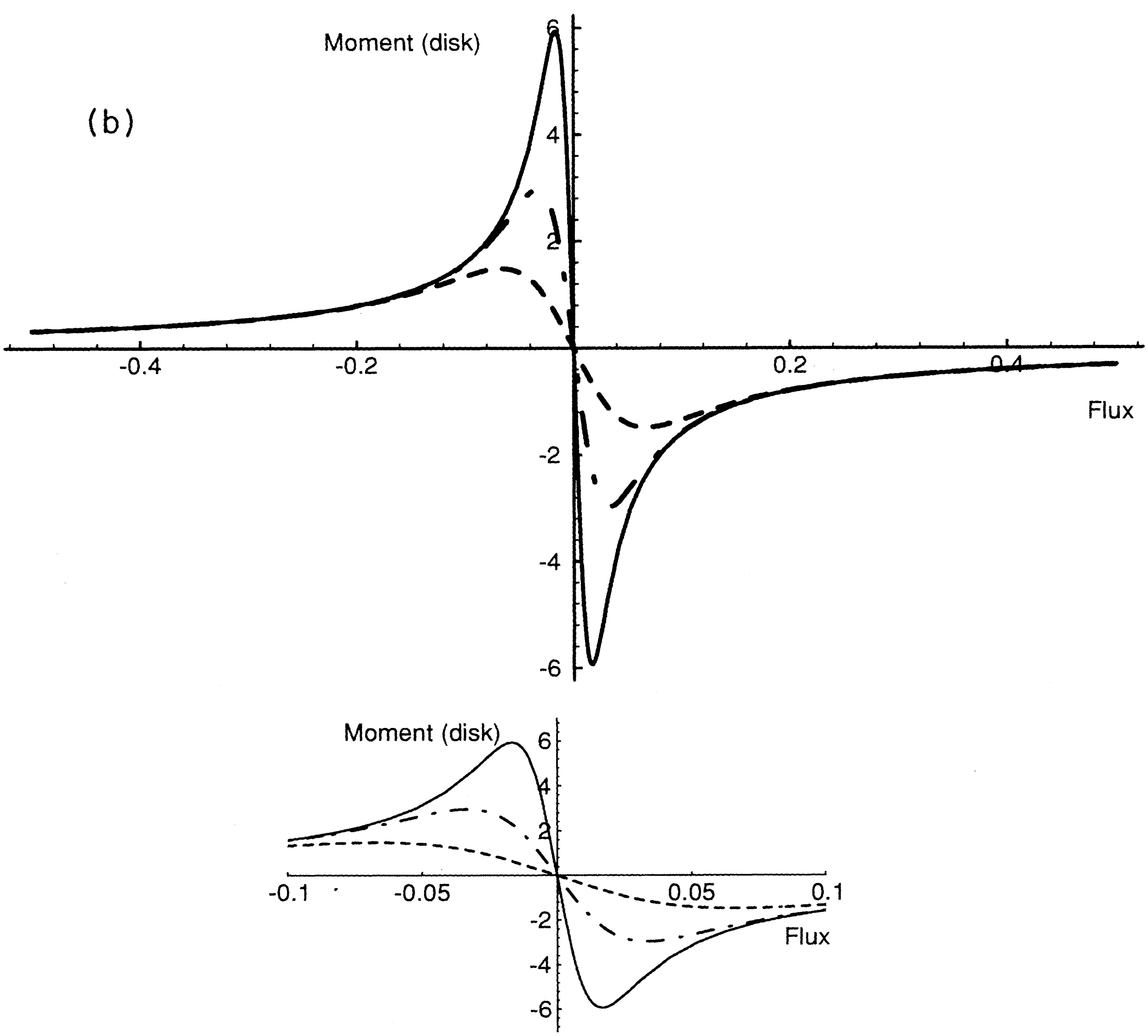

FIG. 2. (*Continued*).

$$-\frac{M}{\mu_B \frac{s^2}{\pi}\frac{2}{s}} = \frac{2\phi_0}{\phi}(2\pi^2\xi^2)^2\left[\frac{E_c}{\Delta}\right]^3 \times \sum_{m=1}^{\infty} m\frac{d}{dx}\left[\frac{d}{dx}E_1(x)\right],$$
$$x=\pi\frac{E_c}{\Delta}\left[m\xi^2+2\pi^2\left[\frac{2\phi_0}{\phi}\right]^2\right]. \tag{10}$$

In this equation we maintained the units of the moment used in I (aside from the factor $2/s$ omitted there) and explicitly exposed $\Delta=1$. The numerical evaluation of the sum in Eq. (10) leads to the magnetic moments depicted in Fig. 3 for $\xi=1$ and $E_c/\Delta=10$ and $E_c/\Delta=100$, respectively. The perturbative result obtained in I is also plotted for comparison. Clearly, at this temperature the orbital response is almost entirely insensitive to the interlevel spacing. Notice that the saturation to the perturbative result for the fields $\tau_H^{-1} \gg T$ (the downturn of the curve past the maximum, see I for details) improves with the number of terms in the sum; in this calculation we used 200 terms. In Fig. 4 the magnetic-field dependence of the moment is shown for $E_c/\Delta=10$ for $\xi=1$, 0.5, 0.25, and 0.125, respectively. Saturation to the asymptotic dependence clearly takes place as the temperature becomes comparable to the interlevel spacing Δ. A better approximation to the asymptotic curve is shown in Fig. 5 for $\xi=0.025$ using 3200 Matsubara terms.

The linear (zero-field) response can be evaluated exactly, yielding

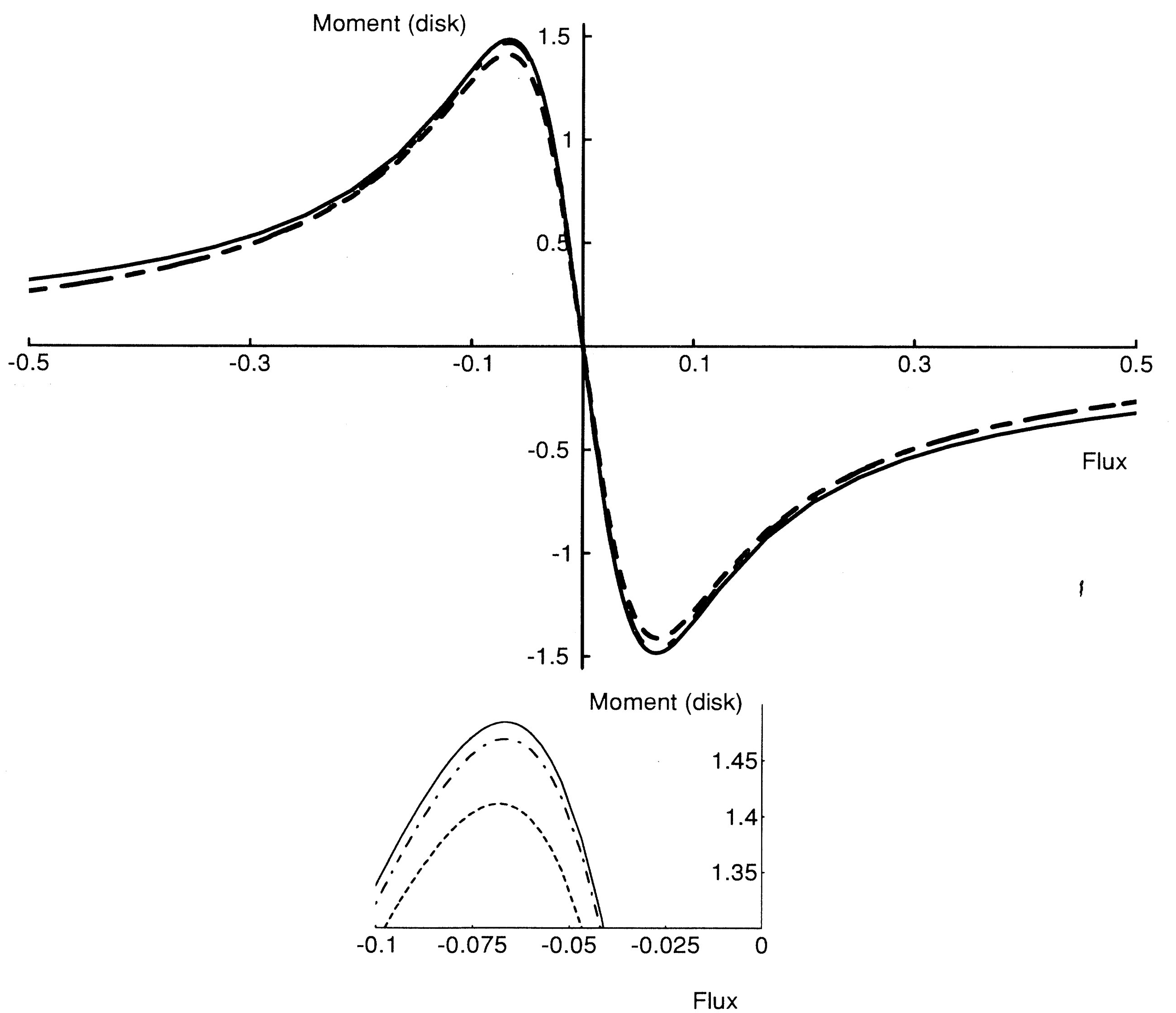

FIG. 3. (Negative) magnetic moment of the disk for $\xi=1$ from the perturbative calculation (normal line), and from the truncation of the Matsubara sum in the exact expression [see Eq. (10) in text] for $E_c/\Delta=10$ (dashed line) and $E_c/\Delta=100$ (dash-dotted line), respectively.

$$\delta F(\phi) \cong -\frac{s^2}{2\pi}\tau_H^{-1}(1-\ln 2)\,,$$
$$-M(\phi) \cong \mu_B \frac{s}{\pi}(\varepsilon_F \tau)\left[\frac{\phi}{\phi_0}\right] s(1-\ln 2)\,, \qquad (9')$$

for the free energy and the magnetic moment respectively. The former should be compared to the perturbative result,

$$\delta F_{\text{pert}}(\phi) \cong -\frac{s^2}{2\pi}\frac{\Delta}{12T}\tau_H^{-1}\,, \qquad (11)$$

which sets the condition $2\pi T/\Delta \approx 1.7$ as the transition point from the perturbative to the nonperturbative regime. The latter should be compared with the sample-specific (mesoscopic) fluctuation of the moment found in I,

$$\frac{M}{\delta M} \approx \frac{s(1-\ln 2)}{\sqrt{3}\ln(E_c/\Delta)}\,. \qquad (12)$$

The rms fluctuation δM was evaluated in the perturbative approximation and can be generalized using the results of this work. However, due to the weak logarithmic dependence it is clear that the prediction of Eq. (12) that it should be roughly an order of magnitude larger than the

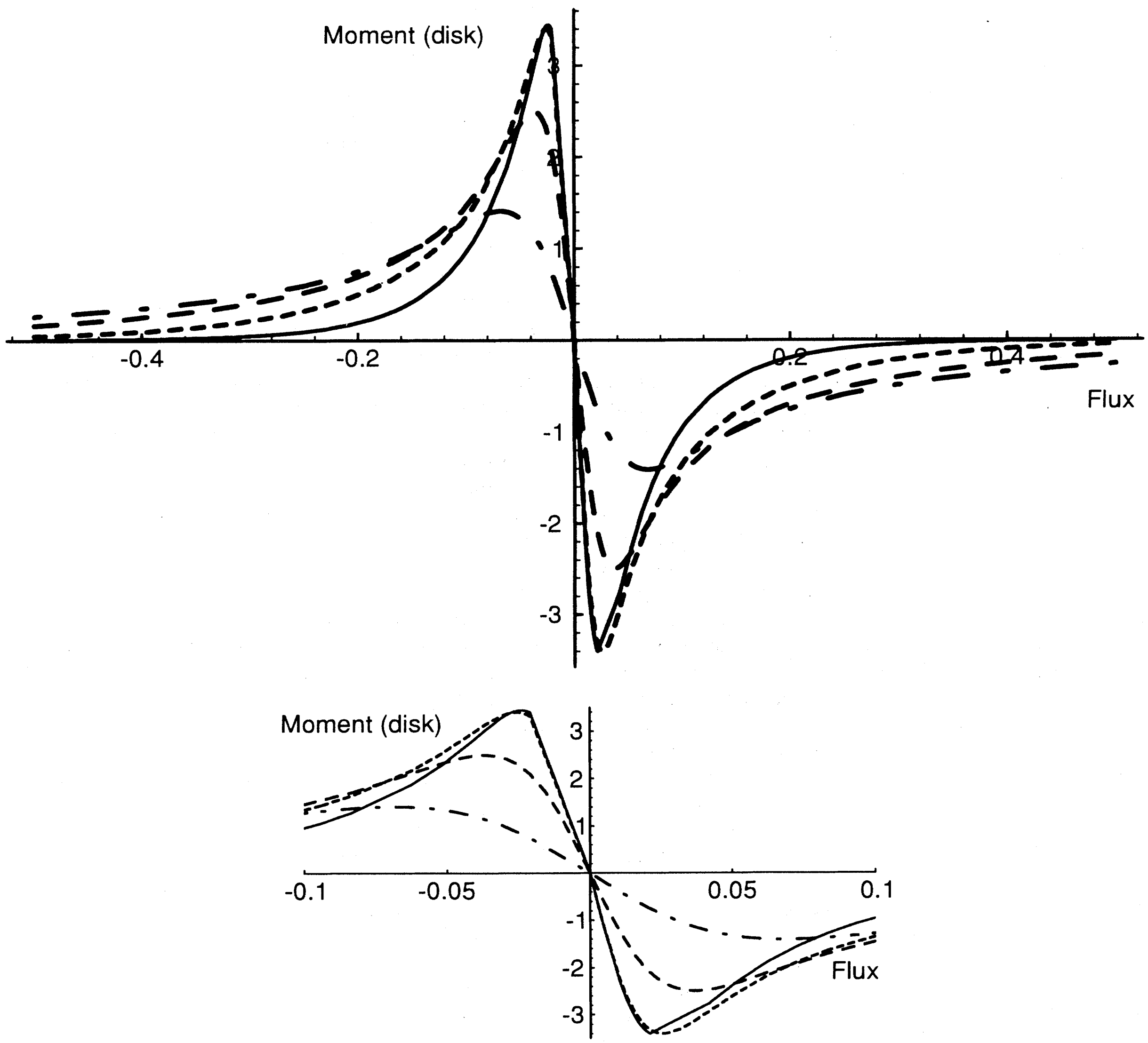

FIG. 4. (Negative) magnetic moment from the truncation of the Matsubara sum in the exact expression for $E_c/\Delta=10$, for $\xi=1$ (dash-dotted line), $\xi=0.5$ (long-dashed line), $\xi=0.25$ (short-dashed line), and $\xi=0.125$ (normal line), respectively. The larger deviation from the saturation curve for $\xi=0.125$ (the lowest temperature) on the downturn is due to the relatively small number of Matsubara terms (200) used in this calculation. In general, for lower temperature more terms are required to approach the saturation curve for $\tau_H^{-1} \gg \Delta$.

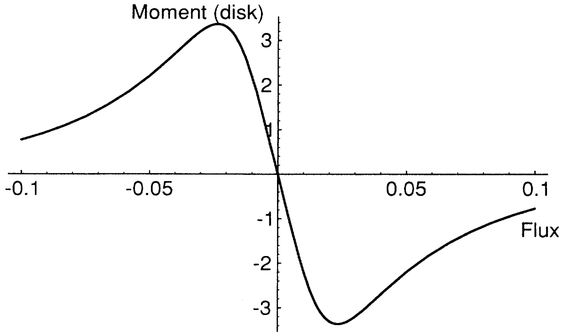

FIG. 5. Approximation of the saturation dependence obtained for $\xi=0.025$ near the origin with the use of 3200 Matsubara terms.

average value will remain unscathed. This is in agreement with the simulations of Ref. 20.

V. DISCUSSION

The central result of this work is the generalization of the correlation function of density of states to finite level broadening which provides the correct limits of the perturbation theory for large broadening and of the random matrix theory for small broadening. The correct account of level correlations at low temperatures, when the temperature and inelastic level broadening are less than the average interlevel spacing, has been made and the orbital response has been shown at near-zero temperature to approach an asymptotic dependence as a function of magnetic field.

The paramagnetic sign of the average response deserves a special notice. The prediction of the paramagnetic response[21] for all even harmonics of clean one-dimensional rings emerges as a consequence of the distinctive structure of the free electron spectrum in a one-dimensional periodic potential. Its presence in weakly disordered narrow multichannel rings[22] could be possibly a "leftover" of one dimensionality. On the other hand, the paramagnetic response has been predicted on the basis of numerical simulations of the Anderson model in wide disordered rings as well, including the universality across several regimes with various degrees of disorder.[20] This extends to the regime with Wigner-Dyson spectral rigidity. At this time we do not fully understand this result.

Moreover, the average paramagnetic response has also been predicted for the systems with Poisson statistics which allow level "bunching" on the basis of numerical simulations[11] on a rectangle. We would like to approach this issue in terms of the competition between the precession diamagnetic and the Van Vleck paramagnetic contributions to the total response[12] which was exploited both in I for disordered systems and in Ref. 11. Whereas the precession contribution is always diamagnetic and is due to the "shrinkage" of orbits in the magnetic field, the Van Vleck response is always paramagnetic due to its origin as the second-order term in the perturbation expansion. The diamagnetic contribution should be a smooth function. Conversely, the Van Vleck contribution should be greatly sensitive to the level arrangement and hence should exhibit large fluctuations. This can be appreciated easily from the expression for the zero-field Van Vleck susceptibility,

$$\chi_{\rm VV}=-\frac{\mu_B^2}{\mathcal{V}}\sum_{k\neq 0}\frac{|\langle k|\mathcal{L}_z|0\rangle|}{E_k-E_0}\ ,$$

where E_0 is the ground-state energy of the system and $\mathcal{L}_z$ is the projection of the total moment. This can be rewritten in terms of the single-level energies and it is clear that when the first unoccupied level is close to the Fermi level there should be a surge of the paramagnetic response. Notice, however, that in a disordered metal (classically chaotic system) the repulsive energy spectrum does not favor such a circumstance. The rectangle, on the other hand, is an integrable system whose spectrum is described by the Poisson statistics[17] wherein the levels are completely uncorrelated so that the narrow gaps at the Fermi level are as likely as the wide gaps. Consequently, the tendency towards paramagnetism should be significantly larger for integrable systems.

It should be emphasized that the averaging for the rectangle was performed over the number of electrons or, alternatively, the position of the Fermi level. In the disordered circumstance it is the disorder averaging. However, if the ergodic hypothesis is valid, the results obtained here should be applicable to averaging over the position of the Fermi level as well. Then, the interpretation of the results of the simulations on the rectangle is unclear for disordered systems. Vice versa is also true since the effect considered here is closely tied to the repulsive level statistics, while there are no level correlations in the rectangle up to very large[17] energy scales. This problem will be analyzed in a future work.

Another issue relegated to future analysis is the formulation of the "analytical bootstrap" in quantum chaos in terms of the supersymmetric nonlinear σ model and the first principle derivation of the relation between the even cumulants of the level density and the mean level density. It is also of interest to understand the connection between the breakdown of the diagonal approximation and the strong level repulsion at the smallest energy scales. Finally, the relative strength of the Langevin and Van Vleck contributions remains unresolved. In the case of systems with the Wigner-Dyson spectrum it might be possible to use an ansatz for the wave functions used in nuclear physics.[23] It is also intriguing to find out whether their relative strength is actually independent of the level statistics.

Note added. After the completion of this work, we received a manuscript by Altland, Iida, Müller-Groeling, and Weidenmüller who found the dependence of the persistent current of a ring at zero temperature as a function of the magnetic field. The position of the maximum in their plot appears to be in agreement with our result. However, the magnitude of the maximal persistent current is roughly 4 times smaller than our estimate.

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