U-j relationship in type-II superconductors

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(Received 22 June 1992; revised manuscript received 23 September 1992)

The Anderson-Kim model has been modified in terms of the *U-j* relationship for high-T_c superconductors. We have obtained *U-j* curves by an approach based on the temperature dependence of j_c. Our results are consistent with the physical picture of flux creep. Our data is obtained from magnetic-relaxation experiments on $YBa_2Cu_3O_x$ single crystals with the direction of the field parallel to the *c* axis. The decay of magnetization was studied for both increasing and decreasing fields, used to obtain the current density.

INTRODUCTION

The decay of magnetization with time has been reported in type-II superconductors.[1-9] Anderson and Kim explained this phenomenon by using a thermally activated flux-creep model.[10] In their model they assumed that the fluxoid motion is thermally activated and that the rate at which the flux bundles jump over the pinning barriers can be described by an Arrhenius-type expression,

$$\nu=\nu_0 \exp\{-U(j)/kT\} , \tag{1}$$

where ν_0 is the attempt frequency, $U(j)$ is the effective activation energy, and T is the temperature of the system. Because of pinning, a nonuniform distribution of vortices exists at the critical state. At the critical state, $j\sim j_c$ (where j_c is the critical current density) and $U(j_c)\sim 0$. As the system relaxes to achieve a uniform distribution of vortices, the current density decays and the activation energy grows. The activation energy gives a measure of the barrier to the flux motion; that is, $U(j)$ is small when the system is relaxing quickly, and vice versa. The rate equation can be written as

$$dj/dt = A \exp(-U/kT) . \tag{2}$$

Experimentally, one can extract $U(j)$ by studying the time dependence of magnetization. Recently, Maley *et al.*[11] developed an expression for the effective activation energy $U(j)$, without the approximation $U/kT \gg 1$, which can be written as

$$U(j)/k = -T \ln|dM/dt| + T \ln(B\nu\omega/\pi d) , \tag{3}$$

where d is the thickness of the sample, M is the magnetization, B is the magnetic induction, and ω is the average distance that a flux bundle can hop. In this approach one first calculates $T \ln|dM/dt|$ for various temperatures at a given field. Then, by adjusting the constant $C = \ln(B\nu\omega/\pi d)$, one obtains the *U-j* relationship. For obtaining a best fit, one uses a scaling factor of $g(t)$, where $t = T/T_x$ is the reduced temperature and T_x is a characteristic temperature of the system.[12]

As reported earlier,[13,14] one can also expand $U(j)$ in the neighborhood of some current density j_0,

$$\begin{aligned} U(j) &= U(j_0)+[\partial U/\partial j]_0(j-j_0) \\ &\quad +\tfrac{1}{2}[\partial^2 U/\partial j^2]_0(j-j_0)^2+\cdots \\ &\approx U(j_0)+\alpha(j-j_0)+\beta/2(j-j_0)^2 . \end{aligned} \tag{4}$$

Substituting Eq. (4) in Eq. (2) and integrating between time t_0 to t, one obtains

$$j(t)=j(0)+a \ln(t/t_0)+b \ln^2(t/t_0) , \tag{5}$$

where $a = kT/\alpha$ and $b = -(k^2T^2/2\alpha^3)\beta$. Both a and b can be determined from relaxation experiments and used to calculate α and β. With the knowledge of the slope α and curvature β, one can construct *U-j* curves using Eq. (4). Figure 1 schematically illustrates a typical *U-j* curve, which has been experimentally established by previous studies of various high-T_c systems.

If the *U-j* curve is constructed from magnetic-relaxation data taken at different temperatures and with the assumption that j_c is temperature independent (i.e., j_c = constant), we would expect a curve shown in Fig. 1. This curve implies that the magnetic relaxation, at each temperature, starts from the same critical state. This leads to a situation where U increases with temperature. However, it is well established that the critical current density is a function of temperature T and applied magnetic field H [i.e., $j_c = f(T,H)$]. In reality, the barrier height should always decrease with increasing temperature (assuming that there is no peak effect in the critical current density).

McHenry *et al.*[12] used a scaling function $g(t)$ to scale the implicit temperature dependence of U to $T=0$ K.[12] This makes $U/g(t)$ temperature independent. They found that $g(t)$ is consistent with the Ginzburg-Landau temperature dependencies of various pinning-related superconducting parameters (i.e., coherence length and

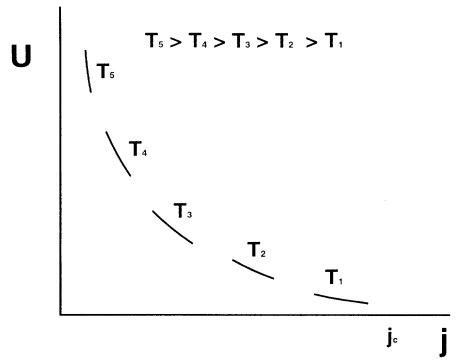

FIG. 1. Schematic diagram of U-j curve as established by previous studies.

fluxon lattice spacing) at low temperatures. However, instead of determining the reduced temperature scale by T_c, they assumed it to be scaled with a temperature T_x, which is a characteristic of the system. For the $YBa_2Cu_3O_x$ system T_x is the field-dependent irreversibility line temperature T_{ir}. For $Bi_2Sr_2CaCu_2O_x$, on the other hand, one has to use two T_x depending on the field and temperature. The two different T_x's characterize the three-dimensional (3D) and the 2D regimes for the system.[15]

In this paper, we have developed a different approach that can establish temperature-dependent U-j curves by considering the temperature and field dependence of j_c as shown in Fig. 2, based on which the relationship between the current density and flux-creep activation energy is physically meaningful. As evident from Fig. 2, at a given j, U always decreases as the temperature increases.

EXPERIMENTAL DETAILS

Magnetic-relaxation experiments were performed by using a quantum design superconducting quantum interference device (SQUID) magnetometer. The samples were zero-field cooled to the desired temperature below T_c. The experiments were performed for both increasing and decreasing fields. For increasing fields, a magnetic field of 3 T was applied after stabilization of the temperature, and the magnetization was recorded as a function of time. For decreasing fields a field of 5 T was first applied and then decreased to 3 T. The initial magnetization value was recorded 180 s after stabilization of the magnetic field. The field was always applied parallel to the c axis of the $YBa_2Cu_3O_x$ single crystal. In order to reduce the effects of field inhomogeneity, a scanning length of 3 cm was used in the experiment. The current density was estimated with a standard Bean's model.[16]

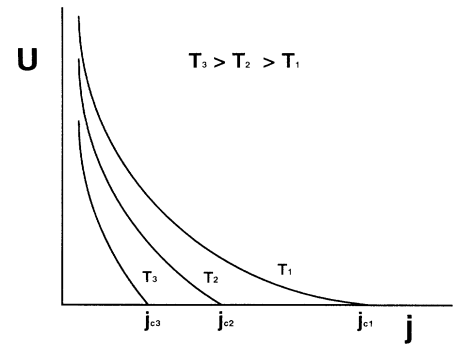

FIG. 2. Schematic diagram of U-j curves by considering the temperature and field dependence of j_c, which are physically correct representations of U-j relationship.

RESULTS AND DISCUSSION

To ensure that the samples were fully penetrated by the applied field so that the Bean model application is valid, we used a previously established method.[17–19] By plotting $dj/d\ln(t/to)$ vs T, we observe that the creep rate experiences a monotonic decrease with increasing temperature at 3 T, as shown in Fig. 3, indicating that the sample is indeed fully penetrated at all temperatures studied.

The magnetization measured at any time t can be written as

$$M(t)=M_{\rm rev}+M_{\rm irr}\ , \tag{6}$$

where $M_{\rm rev}$ is the reversible magnetization and $M_{\rm irr}$ is the irreversible magnetization. $M_{\rm rev}$ is the magnetization related to the Meissner effect.[20] $M_{\rm irr}$ arises because of the current density resulting from flux pinning. One can separate the contribution of $M_{\rm rev}$ and $M_{\rm irr}$ as follows. If M_+ and M_- are magnetization at increasing and decreasing fields, respectively, then

$$M_{\rm rev}=(M_++M_-)/2\ , \tag{7a}$$

$$M_{\rm irr}=(M_--M_+)2=\Delta M/2\propto j\ . \tag{7b}$$

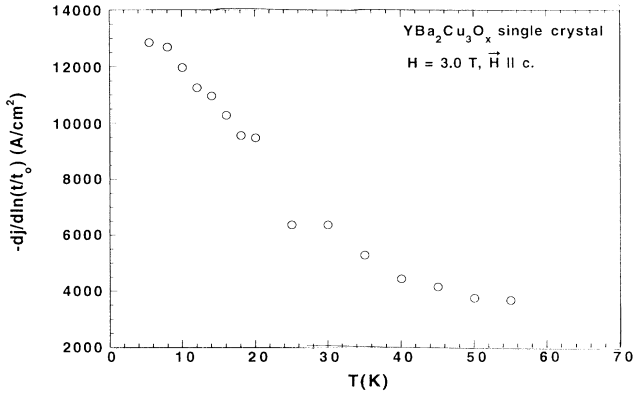

FIG. 3. $-dj/d\ln(t/t_0)$ vs T plot for a $YBa_2Cu_3O_x$ single crystal with $H=3.0$ T applied parallel to the c axis. The monotonous decrease of $-dj/d\ln(t/t_0)$ ensures that the field is fully penetrated in the sample.

Thus, by studying the relaxation of $M_+(t)$ and $M_-(t)$, one can calculate the relaxation of $M_{\rm irr}(t)$ and $j(t)$. In this paper we report the activation energy $U(j)$ as a function of current density j in a $YBa_2Cu_3O_x$ single crystal with $H\|c$ extracted by studying magnetization for both increasing and decreasing fields.

Theoretically, it has been predicted from the collective pinning theory that for $j \ll j_c$, $U(j)$ should grow with decreasing current according to a power law,[21]

$$U(j) \approx U_0 (j_c/j)^{\alpha} , \tag{8}$$

where U_0 is the characteristic energy scale and the exponent α depends on the dimensionality of the problem and on the particular regime of the flux creep.

The Anderson-Kim model predicts that

$$U(j) = U_0 (1 - j/j_c) . \tag{9}$$

Also, experimentally it has been observed that

$$U(j) = U_0 \ln(j_c/j) . \tag{10}$$

Equation (10) is a good approximation for single vortex creep in the 3D case where $\alpha \sim \frac{1}{7} \ll 1$.[22] Equation (10) may also arise from Eq. (8) if the exponent α varies with j. Thus, in general, one can write

$$U(j) = U_0 f(j/j_c) . \tag{11}$$

The current density $j = f(t,T,H)$ decays with time, whereas the critical current density $j_c(T,H)$ is a material property that depends on temperature and applied magnetic field only. Figure 2 schematically shows U-j curves for various j_c. Here j_c is changed either by changing magnetic fields or temperature.

To obtain a U-j curve with a significantly large range of j at constant temperature and field, one has to study long-time relaxation since the relaxation is logarithmic (approximately). In other words, one has to wait for years to get a considerable portion of the U-j curve. Alternatively, we can take advantage of Eq. (11) and construct a considerable portion of a U-j curve by studying isothermal magnetic relaxation at constant field for various temperatures or equivalently by studying magnetic relaxation at constant temperature for various fields. In all cases we change the critical current density j_c. We first construct a U-j curve for the highest j_c (lowest temperature or field). Using Eq. (11), we then calculate the change in activation energy resulting from the change in current density (achieved by changing either the temperature or the magnetic field). Next we add that value to the U value for the lower j_c for the same j value. Mathematically, one can write

$$U(j/j_{c0}) = U(j/j_{c1}) + C(j_{c0}, j_{c1}) , \tag{12}$$

where C is a constant depending on j_{c0} and j_{c1} and $j_{c0} > j_{c1}$. To check the validity of Eq. (12), let us write $z_0 = j/j_{c0}$ and $z_1 = j/j_{c1}$. We now expand $U(z_0)$ with a Taylor's series in the neighborhood of z_1,

$$U(z_0) \approx U(z_1) + (z_0 - z_1)[\partial U/\partial z_0]_{z1} + \cdots . \tag{13}$$

For Eq. (12) to be valid, $(z_0 - z_1)[\partial U/\partial z_0]_{z1}$ must be a constant, that is,

$$(z_0 - z_1)[\partial U/\partial z_0]_{z1} = K ,$$

or

$$\partial U/\partial z_0 = (K/z_0)(1 - z_1/z_0) .$$

Now j_{c0} and j_{c1} are constants, and hence $(1 - z_1/z_0) = (1 - j_{c0}/j_{c1})$ is also a constant. Therefore

$$\partial U/\partial z_0 = K'/z_0 ,$$

where $K' = K/(1 - z/z_0)$.

Integrating, we get

$$U = K' \ln(z_0) = K' \ln(j/j_{c0}) . \tag{14}$$

Thus, if U varies logarithmically with j/j_0, one can use Eq. (12) to construct a hypothetical U-j curve. In principle one can always use Eq. (12) to construct the U-j curve even if Eq. (14) is not satisfied. In that case, C is not a constant anymore but becomes a function of j. We have used Eq. (4) to determine the constant C for our case. Equation (4) can always be approximated by Eq. (14) if $\beta \sim -\alpha/j$ and $|(j/j_0 - 1)| \ll 1$, which was found to be satisfied in our measurements.

In Fig. 4 we plot U as a function of j at various temperatures. Experimentally it is not possible to measure magnetic relaxation exactly from the critical state ($U \sim 0$). Therefore the U plotted in Fig. 4 is actually the change in activation energy when the current density changes from j_0 (initial current density at $t = t_0$) to j_f (current density at $t = t_f$). This initial current density depends on critical current density and hence is a function of temperature and field. The solid lines in Fig. 4 are the fits to $\ln(j)$. Thus we can write $U \sim K \ln(j/j_0)$.

Figure 5 shows the U-j curve at $T = 5.5$ K constructed by using Eq. (12) for a $YBa_2Cu_3O_x$ single crystal with H parallel to the c axis. The smoothness of the curve and the overlapping of the points for some current density level further support the validity of Eq. (12).

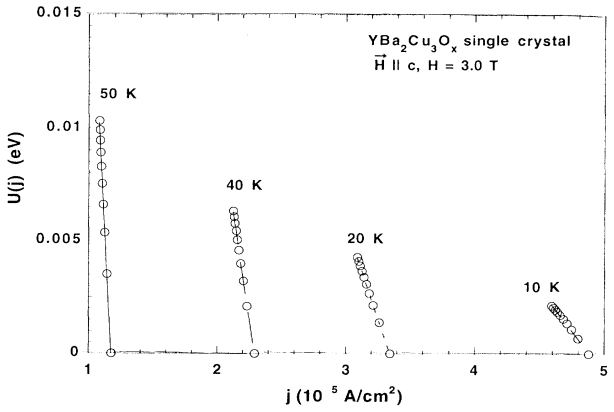

FIG. 4. U-j curves of various temperatures for a $YBa_2Cu_3O_x$ single crystal with $H = 3.0$ T applied parallel to the c axis. The lines are the fit to $\ln j$.

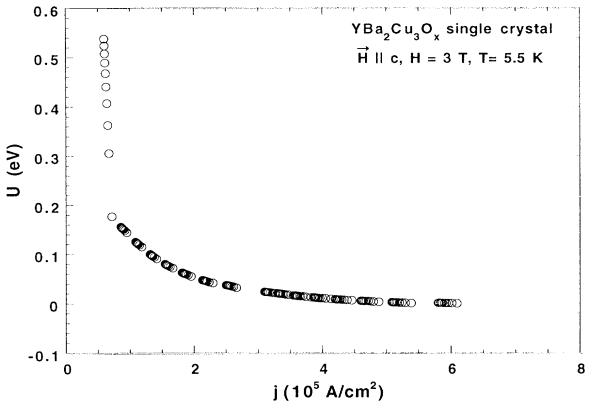

FIG. 5. U-j curve for a $YBa_2Cu_3O_x$ single crystal with $H=3.0$ T applied parallel to the c axis at $T=5.5$ K.

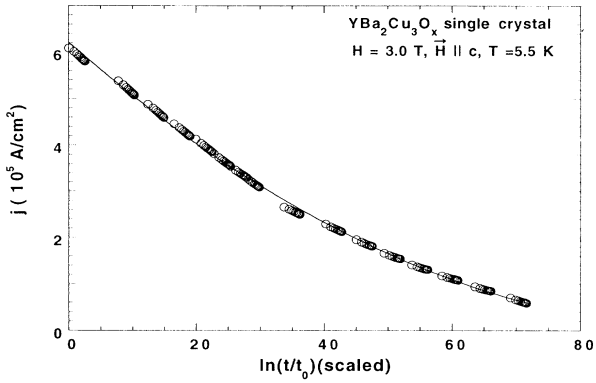

FIG. 6. j vs $\ln(t/t_0)$ at $T=5.5$ K for a $YBa_2Cu_3O_x$ single crystal with $H=3.0$ T applied parallel to the c axis. This curve is equivalent to the U-j curve shown in Fig. 5.

As pointed out earlier, U gives the measure of the barrier to the flux motion and hence does not reflect the true pinning force but rather the forces opposing the flux motion (a situation analogous to the one between static and dynamic friction in mechanics). In other words, the terminology used in the studies of flux pinning has different physical implications. In transport measurements where flux lines are somehow static, the flux-pinning energy U_p is more relevant, since the depinning process occurs as j increases from zero to j_c. On the other hand, in magnetic-relaxation measurements, the effective activation energy U is a more appropriate parameter, since the flux lines are already in motion. U will reach U_p as j approaches zero, where flux motion stops. Therefore, U and U_p are two different but related pinning parameters.

If we assume the preexponential factor A to be constant in Eq. (2), it will lead to a situation where $U\sim\infty$ as $j\sim 0$, which is also a natural consequence from collective-creep theory[21,23] or vortex-glass theory.[24] However, at low j or low driving force, A is not constant but is a function of j. Therefore, a finite U_p may be obtained assuming a j-dependent preexponential factor A. This regime has been considered by Kes *et al.*[25] Experimentally, when the relaxation rate becomes negligibly small, U becomes significantly large. This situation can be clearly seen in Fig. 5 where as the current density approaches a small value, U starts to increase rapidly, resulting in a discontinuity in the slope near 0.18 eV. Although it is difficult to obtain U_p by suppressing j to zero, a finite effective activation energy U at a given driving force would still be physically meaningful in studying the flux-pinning mechanisms. For instance, the U-j curves for materials with higher pinning strength will have larger U at the same current level j (or driving force). Physically, this means that at the same driving force, the j of the strongly pinned material would decay more slowly (equivalently, the flux lines would encounter larger barrier to motion).

In Fig. 6 we plot j as a function of $\ln(t/t_0)$ at $T=5.5$ K. This curve is constructed by measuring the relaxation at different temperatures (5.5–70 K). Theoretically such a curve can also be ideally obtained by doing relaxation measurements for an extremely long time; however, such measurements are not practical in reality. Figure 6 is actually equivalent to the U-j curve plotted in Fig. 5, as evident from Eqs. (4) and (5). The scaled time (t/t_0) was calculated by extrapolating the j vs $\ln(t/t_0)$ curve of the lower temperature to the higher temperature. Physically this would mean that one changes the temperature-dependent characteristic time of the relaxation and scales it to that of the lowest temperature. Hence the j vs $\ln(t/t_0)$ curve shown in Fig. 6 is the curve for $T=5.5$ K.

CONCLUSION

We have used a new approach to construct U-j curves for a single crystal of $YBa_2Cu_3O_x$ by considering the temperature dependence of j_c. We have discussed the physical meaning of the true pinning energy U_p and effective activation energy U obtained from magnetic-relaxation measurements. We conclude that j_c must be treated as a temperature- and field-dependent parameter in establishing physically meaningful relationships between U and j in the vortex state. We have shown that the j-$\ln(t/t_0)$ curve constructed by changing the temperature is equivalent to the U-j curves.

ACKNOWLEDGMENTS

This work was supported by the U.S. Department of Energy, Basic Energy Sciences-Materials Sciences, under Contract No. W-31-109-ENG-38 (D.S., Z.W., and M.S.). S.S. and P.M. acknowledge support from the Midwest Superconductivity Consortium (DOE Contract DE-FG02-90ER45427). We are grateful to Amit Roy and Sergei Sergeenkov for valuable discussions during this work.

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