Interactions for odd-ω gap singlet superconductors

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A class of singlet superconductors with a gap function $\Delta(\mathbf{k},\omega_n)$ that is *odd* in both momentum and Matsubara frequency was proposed recently [Phys. Rev. B **45**, 13 125 (1992)]. To show an instability in the *odd* gap channel, a model phonon propagator was used with the p-wave interaction strength larger than the s-wave strength. We argue that the positive scattering matrix element entering the Eliashberg equations leads to a constraint on the relative strength of p- and s-wave interactions, which inhibits odd pairing. However, a general spin-dependent electron-electron interaction can satisfy all constraints and produce the odd singlet gap. A possibility that may lead to an odd gap is a strongly antiferromagnetically correlated system, such as a high-T_c material.

Balatsky and Abrahams[1] recently proposed a class of singlet superconductors which exhibit unconventional symmetry of the pairing order parameter $\Delta(\mathbf{k},\omega_n)$, where ω_n is the Matsubara frequency. While Δ for conventional superconductors is an even function of frequency, the class is one in which Δ is odd in ω_n and, as a consequence of the Pauli principle, odd under parity as well: $\Delta(-\mathbf{k},\omega_n) = -\Delta(\mathbf{k},\omega_n)$. Thus, this class can have spin singlet p-wave pairing in contrast to the triplet p-wave pairing which occurs in ^{3}He with a gap which is even in ω_n.

In this paper we show that a stable odd (i.e., odd in ω_n) singlet pairing is unlikely to occur for a spin-independent effective potential, e.g., a phonon interaction. This is because renormalization effects reduce the dressed p-wave coupling below the threshold value for $T_c > 0$, regardless how strong the bare coupling is. However, this difficulty can be overcome if spin-dependent terms are added to the interaction, such as may occur in high-T_c superconductors because of antiferromagnetic fluctuations, or other strongly correlated systems. Whether this situation is realized in nature remains unclear.

We first consider a phonon model

$$H = \sum_{\mathbf{k},\sigma} \epsilon_{\mathbf{k}} n_{\mathbf{k}\sigma} + \sum_{\mathbf{k},\mathbf{k}',\sigma} g_{\mathbf{k},\mathbf{k}'}\, c^\dagger_{\mathbf{k}'\sigma} c_{\mathbf{k}\sigma} (a_{\mathbf{k}-\mathbf{k}'} + a^\dagger_{\mathbf{k}'-\mathbf{k}}) + \sum_{\mathbf{q}} \omega_{\mathbf{q}} N_{\mathbf{q}}, \tag{1}$$

where $a_{\mathbf{k}} + a^\dagger_{-\mathbf{k}}$ is the phonon coordinate. At $T = T_c$, the Eliashberg equations for spin singlet odd l pairing [e.g., $\Delta_{\mathbf{k}}(\omega_n) = \Delta_1(\omega_n)\,\hat{\mathbf{k}}\cdot\hat{\mathbf{d}}$ for p-wave pairing] are

$$\Delta_l(\omega_n) = \pi T \sum_{n'} \frac{K_l(\omega_n - \omega_{n'})}{|Z(\omega_n)||\omega_{n'}|} \Delta_l(\omega_{n'}), \tag{2}$$

$$Z(\omega_n) = 1 + \frac{1}{\omega_n} \pi T \sum_{n'} K_0(\omega_n - \omega_{n'}) \frac{\omega_{n'}}{|\omega_{n'}|}. \tag{3}$$

The interaction kernels are defined by

$$K_l(\omega_n - \omega_{n'}) = N_0 \int d\mu\, P_l(\mu) g_\mu^2 \frac{2\Omega_\mu}{(\omega_n - \omega_{n'})^2 + \Omega_\mu^2}. \tag{4}$$

We have set $|\mathbf{k}| = |\mathbf{k}'| = k_F$ and defined $\mu = \mathbf{k}\cdot\mathbf{k}'/k_F^2$. P_l is the Legendre polynomial.

For Δ_l even in ω_n, T_c is nonzero regardless of how small K_l is, so long as K_l is positive (attractive), i.e., the Cooper instability. As discussed in Ref. 1, for the odd case, only the part of $K_l(\omega_n - \omega_{n'})$ which is odd in ω_n enters; this suppresses the density of states near the Fermi surface by a factor ω_n^2 (one power from K and one from Δ_l) requiring a finite value of the interaction for T_c to be nonzero.

At first sight, it would appear that one could choose g_0^2 to be small so that $Z \simeq 1$; then by increasing $g_{l>0}^2$ above threshold, $T_c > 0$ can be obtained. However, we note that the derivability of K from the phonon Hamiltonian requires, because of the positivity of the phonon spectral function, that

$$K_{\mathbf{k}-\mathbf{k}'}(\omega_n - \omega_{n'}) = \sum_{l=0}^{\infty}(2l+1)P_l(\mu)K_l(\omega_n - \omega_{n'}) \geq 0. \tag{5}$$

For example, if only K_0 and K_1 are nonzero, then

$$|K_1| < |K_0|/3. \tag{6}$$

In this case, $T_c \equiv 0$ since the effective interaction $K_1^{\rm eff} = K_1/Z = K_1/(1+K_0)$ must be larger than 1 for $T_c > 0$.[1] However, from Eq. (6) it follows that $K_1^{\rm eff} < (K_0/3)/(1+K_0) < \frac{1}{3}$.[2]

To avoid this difficulty, we consider a general electron-electron coupling. In that case, the low-energy behavior will be determined by an effective interaction which is retarded and spin dependent. Explicitly, we introduce a general spin- and frequency-dependent coupling

$$\gamma(\alpha k;\beta k'|\gamma p;\delta p') = \gamma^c(k-p)\delta_{\alpha\beta}\delta_{\gamma\delta} + \gamma^s(k-p)\sigma^i_{\alpha\beta}\sigma^i_{\gamma\delta}, \tag{7}$$

where α, β, etc., are spin indexes; k, p, etc., are four-vectors, three of which are independent.

In this case, the Eliashberg equations in the spin singlet l-wave channel become

$$\Delta_l(\omega_n) = -\pi T \sum_{n'} [\gamma_l^c(\omega_n - \omega_{n'}) - 3\gamma_l^s(\omega_n - \omega_{n'})] \times \frac{\Delta_l(\omega_{n'})}{|Z(\omega_n)||\omega_{n'}|}, \tag{8}$$

$$Z(\omega_n) = 1 - \pi T \sum_{n'} [\gamma_0^c(\omega_n - \omega_{n'}) + 3\gamma_0^s(\omega_n - \omega_{n'})] \frac{\omega_{n'}}{\omega_n|\omega_{n'}|}. \tag{9}$$

The change of sign of the γ^s interaction in Eqs. (8) and (9) provides the possibility of density and spin couplings adding in the pairing channel yet opposing each other in the normal self-energy channel, so that, as we shall see below, Z remains ~ 1 or even < 1.

In addition, there is no apparent restriction on $\gamma^{c,s}$ analogous to Eq. (5) for a general four-point vertex. Therefore, $T_c > 0$ for odd Δ_l may be realized for systems having strong spin-dependent interactions.

For example, within the random-phase approximation (RPA) for the Hubbard model,

$$\gamma^{c(s)}(\mathbf{k}-\mathbf{k}', \omega_n - \omega_{n'}) = \pm \frac{U/2}{1 \pm U\chi_0(\mathbf{k}-\mathbf{k}', \omega_n - \omega_{n'})}, \tag{10}$$

where plus and minus signs correspond to charge (c) and spin (s) channels. In the high-T_c materials, it is observed experimentally that γ^s is enhanced for $\mathbf{k}-\mathbf{k}' \simeq Q = (\pm\pi, \pm\pi)$. This condition corresponds to backscattering. The p-wave part of $\gamma^{s(c)}$ in two dimensions is

$$\gamma_1^{s(c)}(\omega_n - \omega_{n'}) = \frac{1}{\pi}\int_0^{\pi} \cos\theta \, d\theta \, \gamma^{s(c)}(\mathbf{k}-\mathbf{k}', \omega_n - \omega_{n'}), \tag{11}$$

while the s-wave part is

$$\gamma_0^{s(c)}(\omega_n - \omega_{n'}) = \frac{1}{\pi}\int_0^{\pi} d\theta \, \gamma^{s(c)}(\mathbf{k}-\mathbf{k}', \omega_n - \omega_{n'}). \tag{12}$$

Since γ^s is negative for all θ but is peaked for $\theta \sim \pi$, where $\cos\theta = -1$, it follows that $\gamma_1^s > 0$ while $\gamma_0^s < 0$. On the other hand, $\gamma_0^c > 0$ and we expect that γ_1^c is small.

Here, because of the frequency summations in Eqs. (8) and (9), only the odd $\omega_n, \omega_{n'}$ parts of $\gamma^{c(s)}(\mathbf{k}-\mathbf{k}', \omega_n - \omega_{n'})$ enter the Eliashberg equations. Now consider the strong-correlation regime in the presence of the shadow upper and lower Hubbard bands. Then $\partial\Sigma(\omega_n)/\partial\omega_n > 0$ over most of the frequency range up to $\omega_n \sim U$,[3] in contrast to the conventional Fermi liquid in which $\partial\Sigma(\omega_n)/\partial\omega_n < 0$. Consequently for those frequencies high enough to be relevant for odd pairing, $Z(\omega_n) = 1 - \Sigma(\omega_n)/\omega_n \sim 1$. This is sufficient to have Z of order unity while still having an attraction in the p-wave spin singlet pairing channel.

Under the circumstances being considered here, standard BCS s-wave singlet pairing is impossible because the interaction is repulsive in that channel.

We mention, in connection with the above remarks, that a recent Monte Carlo simulation study for the Cooper-pair t matrix in the two-dimensional Hubbard model[4] shows that the dominant singlet pair eigenvalues occur in the $d_{x^2-y^2}$ (even gap) and p-wave (odd-gap) channels.

In summary, we have shown that pairing by phonons is unlikely to give the odd gap singlet superconductor discussed in Ref. 1. We have shown how a general electron-electron can mediate such pairing and have given a concrete example of how this can work in the context of high-T_c superconductivity, i.e., for the Hubbard model in two dimensions.

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[1] A.V. Balatsky and E. Abrahams, Phys. Rev. B **45**, 13 125 (1992).

[2] A complete proof of this result is available, P.B. Allen (unpublished).

[3] A.P. Kampf and J.R. Schrieffer, Phys. Rev. B **42**, 7967 (1990).

[4] N. Balut, D.J. Scalapino, and S.R. White (unpublished).