Absence of spontaneous persistent current for interacting fermions in a one-dimensional mesoscopic ring

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We address the issue of whether a system of interacting electrons confined to a one-dimensional ring can sustain a spontaneous persistent current in the absence of an externally applied flux. The current-current coupling between electrons, describing radiative back-action effects, is exactly treated in the formalism of quantum electrodynamics, where the electrons interact via the exchange of virtual photons. In addition, the instantaneous screened Coulomb potential is taken into account using the Luttinger liquid model including finite-size parity effects. The partition function is calculated exactly, with the result that the system does not possess a spontaneous persistent current. We show that, in the presence of an external flux, the amplitude of the (conventional) persistent current is reduced by quantum fluctuations of the internal transverse electromagnetic field. These corrections can be expressed in terms of the self-induction of the ring and are shown to be of first and higher order in the small dimensionless parameter $\alpha v_F^*/c$, where α is the fine-structure constant and v_F^* the Fermi velocity renormalized through Coulomb interactions.

I. INTRODUCTION

Persistent currents in mesoscopic normal metal rings have recently received much attention, in the light of the experimental observations of this phenomenon.[1,2] The pioneering work of Ref. 3 pointed out that a one-dimensional disordered metal ring threaded by an external flux has a Bloch-like energy spectrum which is periodic with the flux quantum $\phi_0 = hc/e$. For a metal ring which is small enough so that the electrons maintain sufficient phase coherence after traveling around the ring, a persistent current was predicted. While the description of electrons confined to a mesoscopic ring bears similarities with earlier works dealing with flux quantization in macroscopic superconducting rings by Byers and Yang,[4] Bloch,[5] and Gunter and Imry,[6] the persistent currents in normal metal rings differ in nature from their superconducting counterparts: a superconducting ring can trap flux, i.e., the current continues to flow in the absence of an externally applied magnetic flux. Nevertheless, in general the ground state of a superconducting system, in the absence of an externally applied flux, has no spontaneous persistent current, and therefore satisfies time reversal symmetry. It has been pointed out, however,[7,8] that a superconducting ring with a Josephson junction containing magnetic impurities (a so-called π junction) may have a ground state with a spontaneous persistent current. It is therefore natural to conjecture that such spontaneous persistent currents exist also in normal metals. The purpose of the present paper is to address the question of whether a spontaneous persistent current can occur in mesoscopic normal metal rings. This phenomenon, if at all possible, should arise from the zero-point motion of the internal electromagnetic field coupling the electrons of the metallic ring.

The possibility for spontaneous time reversal symmetry breaking in mesoscopic rings has recently been pointed out by Wohlleben *et al.*[9] In this proposal, the electrons are assumed to interact with each other only via the flux that they create on each other by circulating around the ring. Using a phenomenological model to describe this situation, Wohlleben *et al.* argue that it is possible for an ideal metallic ring to exhibit a magnetic phase transition to a low temperature state with a persistent current in the absence of an externally applied field.

There is reason to doubt whether this description is appropriate: after all, electrons interact with each other via the exchange of virtual photons, the vector particles of the electromagnetic interaction. While the phenomenological model of Ref. 9 contains the ingredients leading to spontaneous persistent currents, a question of this importance should be addressed from first principles. In particular, one has to describe the back-action of an electric current on itself: the current around the ring is affected by the flux created by this same current a moment earlier. The principal question then is whether this retarded back-action effect, expressed in terms of a current-current interaction, can lead to a spontaneous persistent current. We note in passing that this back-action effect is in analogy with the self-energy problem of the electron, which arises in classical and quantum electrodynamics.[10] It is the goal of the present investigation to study the thermodynamic properties of a system of N_0 spinless fermions constrained to move on a one-dimensional mesoscopic ring, coupled to an internal electromagnetic field. This radiation field possesses its own dynamics described by the Maxwell equations and effectively mediates the interaction between electrons, which, in the Coulomb gauge, leads to instantaneous density-density (Coulomb) interactions as well as to retarded current-current interactions. Reference 9 gives an effec-

tive description of the latter interaction, and neglects the former. As we shall see, our result does not support the prediction of Ref. 9 in one-dimensional systems, since we find that the ground state of an assembly of interacting fermions confined to a one-dimensional ring does not sustain a persistent current by itself. At any time in the following calculation, the short range repulsive interaction which models the density-density interaction can be switched to zero for comparison with Ref. 9.

The question whether there exists a spontaneous persistent current is related to the stability of the free energy F of the interacting fermions as a function of flux. As in magnetic systems, the symmetry of the Hamiltonian implies that the state with zero external flux gives a vanishing persistent current $J(\phi = 0) = 0$. Whether this situation corresponds to the actual ground state of the system, i.e., whether F is minimal, is a different issue. This issue should be addressed by probing the system with an external force so as to explore the phase space away from this zero-current state and find a possible ground state which is current carrying and has lower energy. For this purpose we switch on an external flux $\phi_{\rm ext}$, calculate the "conventional" persistent current and consider then the limit of vanishing flux:

$$J = -c \lim_{\phi_{\rm ext}\to 0} \frac{\partial F}{\partial \phi_{\rm ext}} \ . \tag{1}$$

If J does not vanish in this limit then there is a spontaneous persistent current which breaks time reversal symmetry. This approach is clearly analogous to that of a magnetic phase transition in ferromagnets, where the magnetization is probed by a vanishing external magnetic field. Unlike there, however, we cannot perform the thermodynamic limit of the whole expression given in Eq. (1) before we let the external field go to zero, since the persistent current is a mesoscopic effect and as such vanishes for large system size (even exponentially fast for finite temperatures, see below). Nevertheless, we still can perform the thermodynamic limit at appropriate places, i.e., replace sums by integrals when evaluating finite-size corrections. This replacement interchanges vanishing field limit with thermodynamic limit and could result in possible ground state minima with persistent currents, if such a situation exists at all. However, it will turn out that this interchange of the thermodynamic limit and zero external field limit does not cause or even indicate a phase transition and thus has no consequence on the persistent current. We should also emphasize that our calculation is not restricted to small values of external flux. Moreover, everything said above applies equally well to every other zero of the current with respect to the external flux.

In quantum electrodynamics, the Coulomb repulsion between electrons follows naturally from the elimination of the scalar potential that influences the electrons.[11] The remaining problem we are dealing with is therefore that of correlated electrons coupled to a vector potential, describing retardation effects. Restricting our considerations to a one-dimensional ring embedded into a three-dimensional space, we shall make use of the Luttinger liquid description for the matter degrees of freedom.[18–20] This description was recently applied and extended by one of us to study persistent currents for interacting electrons in the presence of an externally applied flux.[12] This Luttinger liquid approach takes the instantaneous short-ranged interaction between the electrons into account and, moreover, incorporates the topological effects associated with the even or odd number of electrons occupying the ring. For simplicity the spin of the electrons is neglected.

Another aspect of our treatment is to take proper account of the cylindrical symmetry of the system: an electron circulating around the ring will couple to the azimuthal electromagnetic modes on the ring. These electromagnetic modes, however, exist also outside and, in particular, inside the circumference of the ring. This situation adds to the complexity of our problem, since usually one deals with the Cartesian representation of the electromagnetic field being enclosed in a box with periodic boundary conditions. In contrast, the cylindrical symmetry of the problem imposes, e.g., that the 4-vector potential vanishes on the boundaries of a (large) cylinder of radius R which encloses the mesoscopic ring concentrically (see Fig. 1). Note that this proper account of the geometry of the problem goes beyond the standard assumption of a Cartesian system closed upon itself in one direction—the typical geometry used so as to "mimic" a ring. This difference is illustrated for a two-dimensional world in Fig. 2: In the cylindrical geometry, the 4-vector potential does also exist inside the circumference of the ring [Fig. 2(a)], whereas this is not the case for the Cartesian geometry with periodic boundary conditions [Figs. 2(b) and 2(c)]. We note in passing that the choice of the Cartesian geometry with periodic boundary conditions is justified when one considers a system where

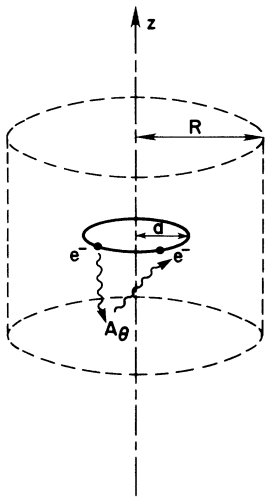

FIG. 1. Cylindrical geometry: we assume vanishing boundary conditions on the surface of the cylinder of radius R (dashed lines) which encloses the mesoscopic ring (thick line). An electron circling the ring interacts with other electrons firstly via Coulomb interaction and secondly by generating a flux through the ring, which consequently affects the motion of the other electrons.

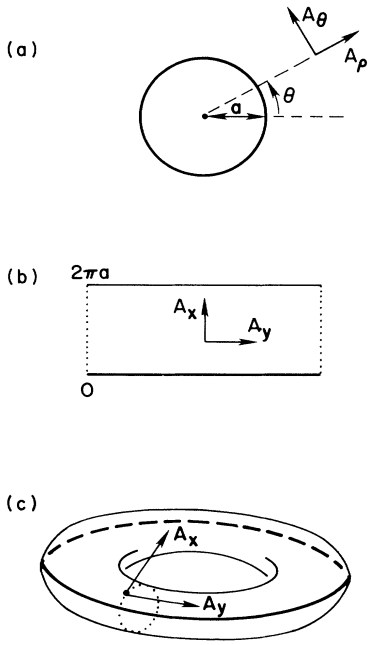

FIG. 2. Comparison between cylindrical and Cartesian geometry in two dimensions. (a) Cylindrical geometry: the radial and azimuthal component of the vector potential A_ρ and A_θ exist and vary inside and outside the perimeter of the ring. (b) Cartesian geometry: the wire (dotted line) is located along the x direction, on the left boundary of the rectangle. (c) Cartesian geometry with periodic boundary conditions: the rectangle of (b) is mapped onto the surface of a torus. The small radius of the torus is the radius of the wire. In this situation, the fields A_x, A_y can never be located inside or outside the loop.

both matter and field are confined to the same space dimensions (provided, of course, that the curvature of the finite system is negligible, which is typically the case in a mesoscopic ring). However, it will turn out that for both geometries, there is no spontaneous persistent current. As a matter of fact, the Cartesian geometry captures the essential physics of the problem, and no qualitatively new information is obtained with a full account of the cylindrical symmetry of the problem. For pedagogical purposes, we shall first present the results for the Cartesian case, which are simpler in structure, rendering the interpretation of the effects more transparent. We derive then the analogous results for the cylindrical geometry in a separate section. We formulate and evaluate our problem using functional integral techniques mainly in real space (not Fourier space); this, *inter alia*, has the great advantage that the discussion for the Cartesian geometry can be adapted to the cylindrical case in an obvious way.

Finally, the effect of an external electromagnetic environment on conventional persistent currents has been recently investigated by Park and Fu,[13] using the approach of Caldeira and Leggett[14] to incorporate the quantum fluctuations of the environment. There, it was found that the coupling to this external field actually suppresses the persistent current to zero. Despite the fact that we are interested here in a different issue—spontaneous persistent currents—our method of approach also allows us to draw conclusions about the conventional persistent current of interacting electrons coupled to an electromagnetic environment. We find that the introduction of coupling to an electromagnetic field slightly reduces the amplitude of the persistent current by a factor in the exponent which is proportional to $\alpha v_F^*/c$ (α is the fine-structure constant and v_F^* the renormalized Fermi velocity), but does not suppress it, in contrast to the result found in Ref. 13. Furthermore, we show that the radiative corrections can be expressed in terms of the classical self-inductance (per unit length) of the ring.

The paper is organized as follows: in Sec. II, we define the partition function of the fermions in the presence of the intrinsic electromagnetic field which is responsible for their interaction. Section III introduces the description of the interacting fermions in terms of the Luttinger liquid in the Cartesian geometry. Since the problem reduces then to Gaussian functional integrals, the calculation of the partition function is carried out explicitly. The discussion of our results is presented in Sec. IV, where we also calculate the magnitude of the corrections introduced by the fluctuations of the electromagnetic field. Section V is devoted to an analogous calculation in the cylindrical geometry. Conclusions are given in Sec. VI.

II. PRELIMINARIES

A. Functional integral formulation of the partition function

Our starting point is the functional integral representation of the partition function for electrons coupled to an intrinsic electromagnetic field,[11]

$$Z = \int DA_0 \int D\mathbf{A} \int Dq\ \Phi[\mathbf{A}, A_0] \exp(-S[A_0, \mathbf{A}, q]) \ , \tag{2}$$

where A_0 ($\mathbf{A}$) is the scalar (vector) potential, and the matter degrees of freedom are described by the variable q; the proper antisymmetrization of the fermions is left implicit in the integration measure. The gauge fixing functional[15,16] $\Phi[A_0, \mathbf{A}]$ specifies the choice of gauge for the electromagnetic field. Note that the internal electromagnetic field has its own dynamics and that the partition function includes the quantum fluctuations of the matter as well as those of the field. The Euclidean action S representing the coupled matter-field system can be separated into three contributions:

$$S[A_0, \mathbf{A}, q] = S_1[q] + S_2[A_0, \mathbf{A}] + S_3[A_0, \mathbf{A}, q] \ , \tag{3}$$

where S_1 is the action for the matter degrees of freedom, i.e., N_0 free electrons of mass m and charge e, confined to a one-dimensional ring of length L parallel to the x-axis, along a particular boundary of the rectangular box which constitutes the "universe" (periodic boundary conditions are assumed in all three space directions), see Fig. 2. Note that we are working in a canonical ensemble with fixed particle number N_0. The second term, S_2, is the

action for the free electromagnetic field, and finally S_3 describes the coupling between matter and field:

$$S_1 = \sum_{i=1}^{N_0} \frac{m}{2} \int_0^\beta d\tau \left(\frac{d\mathbf{q}_i}{d\tau}\right)^2 , \tag{4a}$$

$$S_2 = \frac{1}{8\pi} \int d^4r (\epsilon \mathbf{E}^2 + \mu^{-1}\mathbf{B}^2), \tag{4b}$$

$$S_3 = \int d^4r (i\rho A_0 + \frac{i}{c}\mathbf{j}\cdot\mathbf{A}') , \tag{4c}$$

with $\mathbf{E} = \frac{i\hbar}{c}\partial_\tau \mathbf{A} - \nabla A_0$ the electric field, and $\mathbf{B} = \nabla \wedge \mathbf{A}$ the magnetic field. As we are imposing an external flux ϕ_{ext} through the ring, $\mathbf{A}' = \mathbf{A} + \mathbf{A}_{\text{ext}}$ contains a constant component,[17] $\mathbf{A}_{\text{ext}} = \mathbf{e}_x \phi_{\text{ext}}/L$, with $\mathbf{e}_x$ the "tangential" unit vector in the Cartesian geometry. In Eq. (4b), we have included the dielectric constant ϵ and the permeability constant μ associated with free space. We use the compact notation $d^4r \equiv d\mathbf{r}\, d\tau$ (unless stated otherwise), with $d\mathbf{r}$ being the spatial volume element. The integrations run over the four-dimensional interval $\beta \times L_x \times L_y \times L_z$, with $L_x = L$ fixed and $\beta, L_{y,z}$ arbitrary (eventually approaching infinity in the zero-temperature limit and thermodynamic limit of the transverse degrees of freedom). Finally, ρ is the particle density and $\mathbf{j}$ the current density associated with the electrons.

In the following, we shall adopt the Coulomb (transverse) gauge $\nabla \cdot \mathbf{A} = 0$, and take this constraint into account by expressing the gauge fixing term as a functional integral over an auxiliary field λ:

$$\Phi[\mathbf{A}] = \int D\lambda \exp\left(-i \int d^4r \lambda \nabla \cdot \mathbf{A}\right) . \tag{5}$$

Using the Coulomb gauge and the periodicity in space and imaginary time of the electromagnetic field, we express S_2 in terms of the 4-vector potential:

$$S_2 = \frac{1}{8\pi} \int d^4r (-A_0 \Delta A_0 - \mathbf{A}\cdot\Box\mathbf{A}) , \tag{6}$$

with $\Box = \mu^{-1}\Delta + \epsilon\hbar^2 c^{-2}\partial_\tau^2$ being the D'Alembertian operator in imaginary time.

The above formulas constitute our starting point. The calculation of the persistent current is carried out via the evaluation of the free energy obtained from the partition function Eq. (2).

B. Elimination of the scalar potential

Our first step is to integrate out the scalar potential A_0. For this purpose, we consider the functional integral:

$$I_1[\rho] = \int DA_0 \exp\Big(\int dx\, [-(8\pi)^{-1} A_0(-\Delta)A_0 - i\rho A_0]\Big) . \tag{7}$$

This is a Gaussian functional integral which is given by

$$I_1[\rho] = c_1 \exp\left(-2\pi \int d^4r \int d^4r' \rho(r)(-\Delta)^{-1}(r,r')\rho(r')\right) \equiv c_1 \exp\{-2\pi\rho\cdot(-\Delta)^{-1}\rho\} , \tag{8}$$

where Δ^{-1} is the inverse Laplacian operator, and c_1 is an irrelevant integration constant given by $c_1 = [\det(-\Delta)]^{-1/2}$. The scalar product notation of Eq. (8), where the integration over r is implicit, shall be used in the remainder of this paper. The integration of the scalar potential leads to a density-density coupling which, of course, represents the Coulomb repulsion between electrons. As we are working in the Coulomb gauge, this Coulomb interaction is instantaneous. The retardation effect due to the finite velocity of the electromagnetic wave propagation is a higher order effect in terms of the fine-structure constant, and it is described by the vector potential $\mathbf{A}$ as discussed below. In what follows, we replace this instantaneous Coulomb interaction by a short-ranged repulsive potential which effectively takes into account the screening effects between electrons. We incorporate this screened potential into the part of the action of Eq. (3) which contains the matter degrees of freedom only. The action S_1 given in Eq. (4a) is therefore modified to

$$\tilde{S}_1 = S_1 + \frac{1}{2}\int d^4r \int d^4r' \rho(r) v(\mathbf{r}-\mathbf{r}')\delta(\tau-\tau')\rho(r') , \tag{9}$$

where $v(\mathbf{r}-\mathbf{r}')$ is the short-ranged potential.

III. PARTITION FUNCTION OF THE SYSTEM

In this section, we introduce the Luttinger liquid picture to describe a system of interacting electrons confined to a ring and with coupling to the vector potential of the internal radiation field. We follow the treatment of Ref. 12 where such a Luttinger liquid description is applied to a mesoscopic one-dimensional ring threaded by an external flux.

A. Effective action for the Luttinger liquid

A detailed review of the Luttinger liquid machinery[18–20] goes beyond the scope of this paper. A possible starting point of this method is a Hubbard model, where the repulsive on site interaction replaces the short-ranged potential v introduced in the preceding section. To summarize briefly, this approach amounts to a decomposition of the Fermi field operators into right and left going excitations, which both move with the Fermi velocity through the system. This decomposition is a consequence of the linearization of the free energy spectrum around the two Fermi points $\pm k_F$ and is based on the assumption that large energy fluctuations away from the Fermi surface can be neglected. In a subsequent bosonization procedure, the Lagrangian of the interacting electrons is then replaced by a harmonic fluid Lagrangian expressed in terms of a real scalar bosonic field φ. This bosonic field reproduces the long wavelength excitations (density fluctuations) of the system which are relevant at low temperatures. The interaction effects resulting from the screened Coulomb potential simply renormalize the parameters of this harmonic

fluid. As argued by Haldane[18] this Luttinger liquid approach does not depend on the microscopic details of the model as long as we are interested in its long wavelength properties only. In addition, it is possible to incorporate finite-size effects,[12] carried by the zero modes of the bosonic field (Haldane's topological excitations). These zero modes are essential for the calculation of the conventional persistent currents and their dependence on the parity of the particle number. In light of these results, it is now very natural to assume that if spontaneous persistent currents exist at all then they should also be present in the framework of the Luttinger liquid approach, where, in addition, the coupling to an internal electromagnetic vector potential is taken into account. Proceeding with this assumption we express the electronic part of the action, given by $\tilde{S}_1$ in Eq. (9) and the second term of S_3 in Eq. (4c), in terms of its corresponding Hamiltonian. Following Ref. 12 we then transform this Hamiltonian into its Luttinger liquid representation. This eventually leads to a new action, denoted by $\tilde{S}$, where now the bosonized current density, $\sqrt{\pi}\partial_\tau\varphi(x;\tau)$, couples to the "tangential" x component of the total vector potential $A'_x(x, y = 0, z = 0;\tau)$ evaluated along the ring. As a result, the partition function of Eq. (2) can now be expressed as

$$Z = c_1 \int D\mathbf{A} \int D\lambda \int D\varphi \exp\{-\tilde{S}[\mathbf{A}, \lambda, \varphi]\},$$
$$\tilde{S} = \int d^4r \left(-\frac{1}{8\pi}\mathbf{A}\cdot\Box^*\mathbf{A} + i\lambda\nabla\cdot\mathbf{A}\right) + S_{LL}[\varphi, \mathbf{A}] , \tag{10}$$

where the action for the matter degrees of freedom, expressed in terms of the bosonic field φ, is given by

$$S_{LL} = \int d^2r \left[K^*[(\nabla_x\varphi)^2 + (\partial_\tau\varphi)^2] + i\frac{\sqrt{\pi}\partial_\tau\varphi}{L}\left(\kappa_J + 2\frac{\phi_{\rm ext}}{\phi_0} + 2\frac{LA_x(x,0,0;\tau)}{\phi_0}\right)\right] . \tag{11}$$

Here the integration is limited to the imaginary time τ and the position coordinate x along the ring. The flux quantum is given by $\phi_0 = hc/e$. In Eq. (10), the measure $D\varphi$ contains implicitly the sum over the spatial and temporal winding numbers m, n, and the integers κ_M, κ_J (see below), which characterize the topological sector according to the parity of the total number of electrons N_0. κ_M and κ_J take the values 0, 1 and satisfy the following: $\kappa_M = \kappa_J$ if N_0 is odd, $\kappa_J = 1$, $\kappa_M = 0$ (and vice versa) if N_0 is even.

Note that for convenience the imaginary time has been rescaled, $\tau \to v^*\tau$, so that the first two quadratic terms of the harmonic fluid action S_{LL} are both multiplied by the same dimensionless coefficient K^*. K^* and v^* are Haldane's parameters, which take the value $\frac{1}{2}$ and v_F (the Fermi velocity), respectively, in the limit of a free Fermi liquid. For the Luttinger liquid, the switching on of a repulsive interaction results in a renormalization of these parameters.[18] The renormalized Fermi velocity v_F^* is related to the Haldane parameters via the relation $v_F^* = v^*/2K^*$. The rescaling of imaginary time implies that the D'Alembertian operator must now be redefined as

$$\Box^* = \frac{1}{\mu v^*}\Delta + \frac{\epsilon\hbar^2 v^*}{c^2}\partial_\tau^2 . \tag{12}$$

The bosonic field φ depends on the rescaled imaginary time and the azimuthal position along the wire (x) only, in contrast to the vector potential $\mathbf{A}$ which also depends on the transverse coordinates y and z. The winding numbers in imaginary time and in space, n and m, enter the boundary conditions of the bosonic field in the following way:

$$\varphi(\tau + k_0\beta v^*, x + k_1 L) = \varphi(\tau, x) + k_0\sqrt{\pi}n + k_1\sqrt{\pi}(2m + \kappa_M) , \tag{13}$$

where k_0, k_1 are arbitrary integers counting how many times one circles the (1+1)-dimensional space-time torus. Note that φ couples only to the azimuthal component of the vector potential A_x along the ring, reflecting the locality of matter-field interaction.

In summary, the partition function of Eq. (10) describes a system of interacting fermions (treated as Luttinger liquid), which in addition interacts with the transverse photon field accounting for the retardation effects. We note that, in addition to addressing the issue of spontaneous persistent currents, we are also in a position to study the effect of an external electrodynamic environment on the properties of a one-dimensional electron system. For this we simply reinterpret the radiation field as being provided by an external fluctuating source.

B. Elimination of the vector potential

In preparation for the next step of the calculation, which consists in integrating out the vector potential $\mathbf{A}$, it is convenient to keep the coupling in the form $\int d^4r\mathbf{j}\cdot\mathbf{A}$ as in Eq. (4c). We thus redefine the one-dimensional matter current density associated with the boson field φ as

$$\mathbf{j}(r) = \mathbf{e}_x\frac{2\sqrt{\pi}}{\phi_0}\delta(y)\delta(z)\partial_\tau\varphi(x;\tau) . \tag{14}$$

To integrate out the vector potential, we must perform the following Gaussian functional integral:

$$I_2[\lambda, \mathbf{j}] = \int D\mathbf{A}\exp\left(\int d^4r\ [\,(8\pi)^{-1}\mathbf{A}\cdot\Box^*\mathbf{A} - i\mathbf{A}\cdot(\nabla\lambda + \mathbf{j})]\right). \tag{15}$$

The operator $\Box^*$ acting on $\mathbf{A}$ is a diagonal operator ma-

trix in Cartesian coordinates. The Gaussian integral over the three components of the electromagnetic field A_x, A_y and A_z yields the result

$$I_2[\lambda,\mathbf{j}] = c_2 \ \exp\{2\pi(\nabla\lambda+\mathbf{j})\cdot\Box^{*-1}(\nabla\lambda+\mathbf{j})\}\ , \tag{16}$$

with $c_2 = (\det[-\Box^*])^{-3/2}$. In Cartesian coordinates, $(\Box^*)^{-1}$ depends only on the difference of coordinates, $r-r'$. Using this property, the exponent in the preceding equation is transformed into

$$S'[\lambda,\mathbf{j}] = 2\pi\lambda\cdot\Delta\Box^{*-1}\lambda+4\pi(\Box^{*-1}\nabla\cdot\mathbf{j})\cdot\lambda-2\pi\mathbf{j}\cdot\Box^{*-1}\mathbf{j} \tag{17}$$

Inserting this result into the partition function Eq. (10), and integrating by parts (with vanishing contribution from the boundary terms), we can now perform the Gaussian integral over the auxiliary field λ:

$$\begin{aligned}\int D\lambda\exp\{-S'[\lambda,\mathbf{j}]\} &= c_3\exp\{2\pi j_x\cdot\Box^{*-1}j_x - 2\pi\nabla_x j_x\cdot(\Box^*\Delta)^{-1}\nabla_x j_x\}\\ &= c_3\exp\{-2\pi j_x\cdot D_{xx}j_x\}\ ,\end{aligned} \tag{18}$$

with the irrelevant constant $c_3 = (\det[-\Delta\Box^{*-1}])^{-1/2}$. The transverse photon propagator in Cartesian coordinates is given by the standard expression

$$D_{\mu\nu} = -\Box^{*-1}[\delta_{\mu\nu} - \nabla_\mu\nabla_\nu\Delta^{-1}]\ , \tag{19}$$

where the second term in this equation accounts for the fact that the matter current can only couple to transverse photon modes (perpendicular to the direction of the electron motion). The partition function thus obtained now depends only on the matter degrees of freedom. The effective action associated with the partition function therefore reads

$$\begin{aligned}\tilde{S}_{LL}[\varphi] &= 2\pi j_x\cdot D_{xx}j_x + in\pi(\kappa_J + 2\phi_{\rm ext}/\phi_0)\\ &\quad + \int d^2r\ K^*(\nabla_\sigma\varphi)^2\ ,\end{aligned} \tag{20}$$

where the summation index σ is restricted to the space and time variables x and τ in the remainder of this section. We notice that the exchange of transverse virtual photons leads to an effective current-current coupling in the action which is now nonlocal in space and time (due to retardation). This form is, of course, expected as it also appears in Feynman's treatment of quantum electrodynamics.[11] This current-current interaction can be viewed in the following way: if the ring carries a current, this current produces a magnetic field (via Ampere's law) which acts back on the system (in the form of a flux through the ring) and generates a current itself. The crucial question at this point is whether this back action can sustain a spontaneous persistent current in the absence of an external Aharonov-Bohm flux. We shall now demonstrate that such a spontaneous persistent current does not occur in our model.

C. Integration of the matter field

In view of the boundary condition Eq. (13), the bosonic field φ is then written as a sum of a contribution φ_p which is periodic in imaginary time and space, and a contribution which is linear in these arguments to reproduce Eq. (13). The last term in the action (20) leads to a contribution of the form

$$\begin{aligned}\int d^2r\ (\nabla_\sigma\varphi)^2 \ &= \int d^2r\ (\nabla_\sigma\varphi_p)^2 \ + n^2\frac{\pi L}{\beta v^*}\\ &\quad +(2m+\kappa_M)\frac{\pi\hbar^2\beta v^*}{L}.\end{aligned} \tag{21}$$

The last term in this expression can be neglected if we restrict ourselves to temperatures small compared to $T_1 = K^*\hbar v^*/Lk_B$, which is determined by the energy level spacing at the Fermi surface. T_1 is of the order of $1K$ for the parameters of a mesoscopic gold loop[12] used in Ref. 2. This allows us to neglect the spatial winding numbers, retaining only $\kappa_M = m = 0$.

After integration by parts in imaginary time and by use of the above decomposition into periodic and aperiodic components of the field, the partition function now becomes

$$\begin{aligned}Z = c_4\sum_{n=-\infty}^{+\infty}\int D\varphi_p\ \exp\Big\{\ &\varphi_p\cdot\left[\frac{2\alpha}{c}\delta_\perp\partial^\tau D_{xx}\partial_\tau\delta_\perp + K^*\delta_\perp\nabla_\sigma^2\right]\varphi_p + nd\cdot\varphi_p\\ &-n^2\frac{K^*\pi L}{\hbar\beta v^*}(1+\eta_1) - in\left(\kappa_J+\frac{\phi_{\rm ext}}{\phi_0}\right)\Big\}\ ,\end{aligned} \tag{22}$$

where we have introduced the fine-structure constant $\alpha = e^2/\hbar c$, and the transverse "vector" $\delta_\perp$ has components $\delta_\perp(r) = \delta(y)\delta(z)$. In Eq. (22) we have introduced the quantities

$$\begin{aligned}d(r) &= \frac{4\sqrt{\pi}\alpha}{\hbar\beta v^* c}\int d^4r'\delta_\perp(r')\partial_{\tau'}D_{xx}(r',r)\delta_\perp(r)\\ &\equiv d_\perp(x,\tau)\delta_\perp(r)\end{aligned} \tag{23a}$$

$$\eta_1 = \frac{2\alpha}{\hbar\beta v^* K^* cL}\delta_\perp \cdot D_{xx}\delta_\perp \ , \tag{23b}$$

and the irrelevant prefactor is given by $c_4 = c_1 c_2 c_3$.

Performing the Gaussian integration over the field φ_p, we obtain the partition function in its final form:

$$Z = c_5 \sum_{n=-\infty}^{+\infty} \exp\Big(-\frac{\pi L}{2\hbar\beta v_F^*} n^2(1+\eta_1+\eta_2) + in\pi(\kappa_J + 2\phi_{\rm ext}/\phi_0)\Big)$$
$$= c_5\ \Theta_3[(1+\eta_1+\eta_2)T/T_0; \phi_{\rm ext}/\phi_0 + \kappa_J/2] \ , \tag{24}$$

where Θ_3 is the Jacobi theta function, the characteristic temperature is defined by $T_0 = 2\hbar v_F^*/k_B L$ and the numerical factor $c_5 = c_4(\det[2\alpha/c\delta_\perp \partial_\tau D_{xx}\partial_\tau \delta_\perp + K^*\delta_\perp \nabla_\sigma^2])^{-1/2}$. The integration of the periodic field thus leads to a second correction, η_2, which is given by

$$\eta_2 = \frac{\hbar\beta v^*}{4\pi L K^*} d\cdot\left(\frac{2\alpha}{c}\delta_\perp \partial_\tau D_{xx}\partial_\tau \delta_\perp + K^*\delta_\perp \nabla_\sigma^2\right)^{-1} d \ . \tag{25}$$

We notice that the partition function describing a system of fermions interacting with an internal electromagnetic radiation field has precisely the same form as the partition function of conventional persistent currents in the Luttinger liquid picture, as was calculated in Ref. 12. The effect of the virtual transverse-photon exchange can therefore be absorbed in a redefinition of the characteristic quantities describing the system such as the Fermi velocity $v_F{}^*$ or the temperature T_0. Note that η_1 is of order α and η_2 of order α^2 and higher. We shall compute these corrections in the next section explicitly.

IV. DISCUSSION OF THE RESULTS

A. The issue of spontaneous persistent current

From the partition function of Eq. (24) we may now study the stability properties of the free energy. Using the properties of the logarithmic derivative of the Jacobi Θ_3 function, we obtain the persistent current in the presence of an applied flux:

$$J(\phi_{\rm ext}) = \frac{e k_B T}{\hbar}\sum_{l=1}^{\infty}(-1)^{lN_0}\frac{\sin(2\pi l\phi_{\rm ext}/\phi_0)}{\sinh(lT/\tilde{T}_0)} \ , \tag{26}$$

where the correction factors $\eta_{1,2}$ have been absorbed in the definition of $\tilde{T}_0 = (1+\eta_1+\eta_2)^{-1}T_0$. The oscillating sign factor accounts for the parity effect, i.e., for the fact that the current is diamagnetic if the number of electrons, N_0, is odd, and paramagnetic otherwise. Our conclusion is therefore

$$\lim_{\phi_{\rm ext}\to 0} J(\phi_{\rm ext}) = 0 \ , \tag{27}$$

i.e., there is no spontaneous persistent current, as J is an analytic function of $\phi_{\rm ext}$ for all temperatures $T > 0$, and thus vanishes with vanishing external flux [we had remarked earlier that $J(\phi_{\rm ext} = 0) = 0$ by symmetry]. Equation (27) is no longer true at $T = 0$, where the right-hand side of Eq. (26) becomes the Fourier series representation of the sawtooth function which vanishes at (half) integral values of $\phi_{\rm ext}/\phi_0$. As a consequence, $J(\phi)$ becomes discontinuous and, for N_0 odd, develops a finite jump at $\phi = 0$ due to the double degeneracy of the ground state. This degeneracy, however, is not caused or affected by the presence of the radiative corrections and is well known to occur already in the free system.[4] Thus, this jump of $J(\phi)$ at $T = 0$ is of no importance in the present context. [Note moreover that in real systems the double degeneracy is lifted by the presence of impurities, with $J(\phi)$ becoming continuous also at $T = 0$.] Note also that Eq. (27) is valid for arbitrary parity of the particle number. As long as $1 + \eta_1 + \eta_2 > 0$, which is indeed the case as shown below, the only effect associated with the internal electromagnetic fields is to change the *amplitude* of the (conventional) persistent current.

B. Magnitude of the effect

Aside from the issue of spontaneous persistent currents which was addressed in Sec. IV A, an estimate of the magnitude of the correction terms η_1 and η_2 is important for the following reasons. First, we must make sure that $1 + \eta_1 + \eta_2 > 0$, which is required for the series of Eq. (24) to be convergent. Note that $\eta_{1,2}$ are obviously real quantities since all operators involved are real and symmetric. Second, the evaluation of the correction factors provides useful information on what degree of importance one should attribute to internal (or external) electromagnetic fluctuations. Are these corrections finite, or divergent quantities? Having η_1 or $\eta_2 \to +\infty$ would imply that the quantum fluctuations of the electromagnetic field suppress the amplitude of the conventional persistent current to zero. We shall see now that this is not the case and that $\eta_{1,2}$ is finite.

In Eq. (23b), η_1 is explicitly given by

$$\eta_1 = \frac{2\alpha}{\hbar\beta v^* K^* Lc}\int d^2r \int d^2r'\ D_{xx}(x,0,0,\tau;x',0,0,\tau') \ . \tag{28}$$

With the help of Eq. (19), the Cartesian geometry with periodic boundary conditions allows us to write D_{xx} in the Fourier representation (suppressing the vanishing arguments)

$$D_{xx}(x,\tau;x',\tau') = \int \frac{d^4k}{(2\pi)^4}\ e^{ik_\sigma(x_\sigma - x'_\sigma)} \times \frac{1 - k_x^2/\mathbf{k}^2}{\epsilon v^* k_\tau^2/c^2 + (\mu v^*)^{-1}\mathbf{k}^2} \ , \tag{29}$$

where we restrict σ to the variables τ, x, as in the preceding section. Note that, strictly speaking, the above k-space integral over $k_x = 2\pi n_x/L$ and $k_\tau = 2\pi n_\tau/\beta v^*$ has to be interpreted as a discrete sum over $n_{x,\tau}$. However, the error made by replacing sums by integrals is of negligible order for small temperatures and a micrometer-sized ring. The transverse dimensions, $L_{y,z}$, are approaching infinity. It is this replacement which amounts to interchanging the thermodynamic limit with the limit of vanishing external flux. Obviously, this interchange

does not cause or indicate any phase transition here and thus has no consequences for the persistent current. Next, the imaginary time and space integrals in Eq. (28) generate two Kronecker δ's constraining $k_\tau = k_x = 0$. The first order correction therefore becomes, setting the dielectric constant $\epsilon = 1$:

$$\eta_1 = \frac{2\alpha\mu v^*}{(2\pi)^2 cK^*} \int dk_y dk_z \, [m^2 + k_y^2 + k_z^2]^{-1} \, . \tag{30}$$

To avoid unphysical infrared divergences, we have regularized the integral of Eq. (30) by introducing a small photon mass m, which we choose to be of the order of $2\pi/L$, the radius of the ring. This choice is physically motivated as will be seen shortly. Furthermore, for condensed matter systems, a short wavelength (ultraviolet) cutoff can be assumed on physical grounds, appealing to the fact that we have no pretension of describing the microscopic behavior of our system below, say, the scale of an interatom spacing. In the context of our problem, the short wavelength cutoff arises from the fact that the mesoscopic ring has a finite thickness. Denoting by b the typical radius of the wire which constitutes the ring (the small radius of a toroidal ring with large radius a), we therefore choose the upper bound of the integrals of Eq. (30) to be of the order of b^{-1}.

The resulting integral appearing in Eq. (30) can be performed trivially:

$$\eta_1 = \frac{1}{\pi}\frac{v_F^*}{c}\alpha\mu \ln\left[\frac{b^{-2} + (2\pi/L)^2}{(2\pi/L)^2}\right] \approx \frac{2}{\pi}\frac{v_F^*}{c}\alpha\mu \ln[L/2\pi b] \, , \tag{31}$$

and we therefore obtain a finite result for the first correction. Note that the logarithmic dependence, $\mu \ln[L/2\pi b]$, of η_1 in Eq. (31) has the same origin as the self-inductance of a torus in classical electrodynamics. Indeed, the formula for the self-inductance M of a torus with large radius a and small radius b is given by

$$\begin{aligned} M &= \frac{4\pi}{c^2} a\{\mu(\ln[8a/b] - 2) + \mu'\} \\ &\approx \frac{4\pi}{c^2} a\{\mu \ln[a/b] + \mu'\} \, , \end{aligned} \tag{32}$$

where μ' is the permeability of the ring, and we identify the radius a of the ring with $L/2\pi$. For a metal ring in vacuum with $\mu = 1$ and a large aspect ratio $a \gg b$, the dominant contribution comes from the term $\mu \ln[a/b]$, which appears explicitly in Eq. (31).

Next, we turn to the calculation of the second correction parameter η_2. We first write Eq. (25) in the Fourier representation:

$$\begin{aligned} \eta_2 &= \frac{\hbar\beta v^*}{4\pi LK^*} \int d^2r \int d^2r' d_\perp(r) G^{-1}(r - r') d_\perp(r') \\ &= \frac{\hbar\beta v^*}{4\pi LK^*} \int \frac{d^2k}{(2\pi)^2} d_\perp(-k) G^{-1}(k) d_\perp(k) \, , \end{aligned} \tag{33}$$

where $G^{-1}(r - r') = [(2\alpha/c)\delta_\perp \partial_\tau^2 D_{xx} \delta_\perp + K^* \delta_\perp \nabla_\sigma^2]^{-1}(r - r', 0, 0, \tau - \tau')$, and $d_\perp(k)$, $G^{-1}(k)$ are the two-dimensional Fourier transforms of $d_\perp(r)$, $G^{-1}(r)$. $G^{-1}(k)$ is the inverse of $G(k)$ [with $G^{-1}(k) = 1/G(k)$], where the latter is given by the expression

$$\begin{aligned} G(k_\tau, k_x) = &-K^*(k_\tau^2 + k_x^2) \\ &- \frac{2\alpha\mu v^*}{c} \int \frac{dk_y dk_z}{(2\pi)^2} \frac{k_\tau^2(1 - k_x^2/\mathbf{k}^2)}{\mathbf{k}^2 + \mu\epsilon v^{*2} k_\tau^2/c^2 + m^2} \, . \end{aligned} \tag{34}$$

Note that we encounter the same type of divergences as in the calculation of η_1. We therefore invoke the same regularization procedure for long and short wavelengths, leading to

$$\begin{aligned} G(k_\tau, k_x) = &-K^*(k_\tau^2 + k_x^2) - k_\tau^2 \frac{\alpha}{2\pi}\mu\frac{v^*}{c} \ln\left[\frac{b^{-2} + k_x^2 + \mu\epsilon v^{*2} k_\tau^2/c^2 + m^2}{k_x^2 + \epsilon v^{*2} k_\tau^2/c^2 + m^2}\right] \\ &+ k_x^2 k_\tau^2 \frac{\alpha}{2\pi}\mu\frac{v^*}{c} (\mu\epsilon v^{*2} k_\tau^2/c^2 + m^2)^{-1} \ln\left[\frac{k_x^2 + \mu\epsilon v^{*2} k_\tau^2/c^2 + m^2}{b^{-2} + k_x^2 + \mu\epsilon v^{*2} k_\tau^2/c^2 + m^2} \frac{k_x^2 + b^{-2}}{k_x^2}\right] \, . \end{aligned} \tag{35}$$

The expression on the right-hand side of this equation is now well behaved in the limit $k_x, \, k_\tau \to 0$. In a similar way, we calculate the Fourier transform of $d_\perp(x, \tau)$, setting $\epsilon = 1$, with the result

$$d_\perp(k_\tau, k_x) = -i\frac{4\pi^{3/2}\mu\alpha}{\hbar\beta c} k_\tau \delta(k_\tau)\delta(k_x) \ln\left[\frac{b^{-2} + m^2}{m^2}\right] \, , \tag{36}$$

with the understanding that we set $k_\tau = 0$ only at the end of the calculation. Inserting Eqs. (35) and (36) in the Fourier representation of η_2 [Eq. (33)], we get the final result, setting $m = 2\pi/L$,

$$\begin{aligned} \eta_2 &= -\left(\frac{\alpha\mu v_F^*}{\pi c}\right)^2 \ln^2\left[\frac{b^{-2} + m^2}{m^2}\right] \\ &\quad \times \left\{1 + \frac{\alpha\mu v_F^*}{\pi c} \ln\left[\frac{b^{-2} + m^2}{m^2}\right]\right\}^{-1} \\ &\approx -\left(\frac{2\alpha\mu v_F^*}{\pi c}\right)^2 \ln^2[L/2\pi b] \\ &\quad \times \left\{1 + 2\frac{\alpha\mu v_F^*}{\pi c} \ln[L/2\pi b]\right\}^{-1} \, . \end{aligned} \tag{37}$$

Note that this correction has a sign opposite to that of η_1 and can also be expressed in terms of the above self-inductance per unit length. It is now clear that this correction is small compared to η_1, as η_2 is of the order of

$(\alpha v_F^*/c)^2$ (and higher). Note that η_2 can be viewed as being obtained by resumming the infinite series of all higher order corrections in a perturbation expansion in $\alpha v_F^*/c$. Obviously, $\alpha v_F^*/c$ is the small parameter associated with electromagnetic corrections. It is instructive to regroup the contributions of η_1 and η_2 in the following form:

$$1+\eta_1+\eta_2 = \frac{1+2(\alpha\mu v_F^*/\pi c)\ln[1+(mb)^{-2}]}{1+(\alpha\mu v_F^*/\pi c)\ln[1+(mb)^{-2}]} \approx \frac{1+4(\alpha\mu v_F^*/\pi c)\ln[L/2\pi b]}{1+2(\alpha\mu v_F^*/\pi c)\ln[L/2\pi b]} \approx \frac{1+2(\alpha c v_F^*/\pi)\bar{M}}{1+(\alpha c v_F^*/\pi)\bar{M}} , \tag{38}$$

where we have expressed the result as a function of the self-inductance per unit length, $\bar{M} \equiv M/L$, in the last line of Eq. (38). Note that this expression is always positive and finite, $1 \le 1+\eta_1+\eta_2 \le 2$, regardless of the value of the coupling constant or of the choice of the lower and upper cutoffs (of course, $a > b$): this guarantees that the sum of Eq. (24) is well defined. We therefore find that the amplitude of the persistent current in the presence of an external flux is *reduced* from that of a ring without electromagnetic effects taken into account. In particular, if we consider the strong coupling limit ($\alpha \to \infty$), the amplitude of the zero temperature (conventional) persistent current is exactly half of its uncoupled value.

As discussed in Ref. 12, the renormalized Fermi velocity is not changed much by the interactions, the change being of order unity. Thus, to get an estimate for $\eta_{1,2}$ we consider a Au loop with (unrenormalized) Fermi velocity $v_F = 1.4 \times 10^8$ cm/s, giving $\alpha v_F^*/c \simeq 3.4 \times 10^{-5}$, while $\ln[L/2\pi b] \simeq 4.6$ for an aspect ratio $a/b = 100$, and the radiative corrections $\eta_{1,2}$ can be safely neglected. As pointed out in the Introduction, the fact that the electromagnetic corrections are finite leads to a different conclusion from that of Ref. 13, in which a vanishing persistent current is obtained due to the fluctuations of the electromagnetic environment.

V. CYLINDRICAL GEOMETRY

In this section, we retrace the steps of Sec. III, taking full account of the cylindrical symmetry of our problem, as illustrated in Fig. 1. The circumference of the ring is given by $L = 2\pi a$, with a the radius of the ring. The four-dimensional volume element is now replaced by $d^4r = \rho d\theta d\rho dz d\tau$, with θ the azimuthal angle, and ρ the radial coordinate. The current around the ring couples to the azimuthal component of the vector potential A_θ only. Note that in the cylindrical geometry there is also a radiation field inside the ring circumference which, *a priori*, could be a possible source for the creation of spontaneous persistent currents. The following analysis, however, will demonstrate that this is not the case.

We start the discussion of the cylindrical geometry at the level of the Luttinger liquid partition function of Eq. (10). This is justified as we assumed in the previous sections that the Coulomb repulsion between electrons is short ranged, due to screening effects. Translated into the case of cylindrical geometry, this assumption allows us to neglect the interaction between electrons located, say, at opposite sides of the ring, and we only consider the interaction between two points on a short quasi-one-dimensional arclength of the ring. The only noticeable change in Eq. (10) in the cylindrical geometry is the fact that the variable x is replaced by $a\theta$, which identifies a position along the perimeter of the ring.

As before, we need to integrate out the vector potential:

$$I_2[\lambda,\mathbf{j}] = \int D\mathbf{A} \exp\left(\int d^4r \, [\,(8\pi)^{-1}\mathbf{A}\cdot\Box^*\mathbf{A} - i\mathbf{A}\cdot(\nabla\lambda+\mathbf{j})]\right) , \tag{39}$$

where now the current density is defined by

$$\mathbf{j}(r) = \mathbf{e}_\theta \frac{2\sqrt{\pi}}{\phi_0}\delta(\rho-a)\delta(z)\partial_\tau\varphi(\theta;\tau) . \tag{40}$$

In contrast to the Cartesian geometry, we take special care of the fact that in cylindrical coordinates, the operator $\Box^*$ "mixes" the azimuthal and the radial components of the vector potential A_r and A_θ. Performing this functional integral in the cylindrical geometry implies finding the inverse of the D'Alembertian operator matrix defined by

$$G(r,r') = \rho \times \begin{pmatrix} -\Box^* - (v^*\rho^2)^{-1} & -2(v^*\rho^2)^{-1}\partial_\theta & 0 \\ 2(v^*\rho^2)^{-1}\partial_\theta & -\Box^* - (v^*\rho^2)^{-1} & 0 \\ 0 & 0 & -\Box^* \end{pmatrix} \times \delta^4(r-r')\ \rho' , \tag{41}$$

where $\Box^*$ is expressed in cylindrical coordinates, using Eq. (12) and $\Delta = \rho^{-1}\partial_\rho\rho^{-1}\partial_\rho + \rho^{-2}\partial_\theta^2 + \partial_z^2$. The factors ρ and ρ' have been incorporated in the definition of G in anticipation of the Gaussian integration over $\mathbf{A}$: their origin is related to the volume element expressed in cylindrical coordinates. Note that the matrix which appears in Eq. (41) is a real, symmetric matrix and that it can therefore be inverted. In the following, we shall do this matrix operator inversion step formally, and express our results in terms of the inverse operator matrix G^{-1}. Defining by $G^{-1}_{\mu\nu}$ (with $\mu,\ \nu = \rho,\ \theta,\ z$) the elements of the inverse, the integral of Eq. (39) becomes

$$I_2[\lambda,\mathbf{j}] = \tilde{c}_2 \exp\{-2\pi\rho(\nabla\lambda+\mathbf{j})_\mu \cdot G^{-1}_{\mu\nu}(\nabla\lambda+\mathbf{j})_\nu\rho\} , \tag{42}$$

with the constant prefactor $\tilde{c}_2 = (\det G)^{-1/2}$, and we use from now on the Einstein summation convention over the spatial indices μ and ν.

The next step in the calculation is the integration of

the auxiliary field λ, which is again a Gaussian integral:

$$I_3[\varphi] = \int D\lambda \ \exp\{-2\pi a^2 \lambda \cdot \nabla_\mu G^{-1}_{\mu\nu} \nabla_\nu \lambda + 4\pi a^2 \lambda \cdot \nabla_\mu G^{-1}_{\mu\theta} j_\theta\}$$
$$= \tilde{c}_3 \exp\{2\pi\hbar a^2 (\nabla_\mu G^{-1}_{\mu\theta} j_\theta) \cdot \nabla^{-1}_\xi G_{\xi\zeta} \nabla^{-1}_\zeta (\nabla_{\mu'} G^{-1}_{\mu'\theta} \ j_\theta)\} \ , \tag{43}$$

with $\tilde{c}_3 = (\det[a^2 \nabla_\mu G^{-1}_{\mu\nu} \nabla_\nu])^{-1/2}$. The effective action associated with the partition function in the cylindrical geometry therefore reads

$$\tilde{S}_{LL}[\varphi] = 2\pi a^2 j_\theta \cdot D_{\theta\theta} j_\theta + in\pi(\kappa_J + 2\phi_{\rm ext}/\phi_0)$$
$$+ \int_0^{2\pi} a \, d\theta \int_0^{\beta v^*} d\tau \ K^* (\nabla_\sigma \varphi)^2 \ , \tag{44}$$

where the index σ is restricted to the variables τ, θ in the remainder of this section. In Eq. (44), we have introduced the photon propagator in cylindrical geometry:

$$D_{\theta\theta} = G^{-1}_{\theta\mu} \{\delta_{\mu\theta} - \nabla_\mu \nabla^{-1}_\nu G_{\nu\xi} \nabla^{-1}_\xi \nabla_\zeta G^{-1}_{\zeta\theta}\} \ . \tag{45}$$

Note that the structure of Eq. (44) is quite similar to that of Eq. (20). The second term in Eq. (45) restricts the propagator to transverse modes, as in Eq. (19).

The integration of the matter field is performed in a way which is similar to Sec. III C: the bosonic field φ is decomposed into a contribution which is periodic in the variables θ and τ (with respective periods 2π and βv^*), together with a linear contribution which gives rise to the azimuthal and temporal winding numbers m and n in the partition function. For sufficiently low temperatures, only n survives, and we obtain, omitting now irrelevant constants,

$$Z \propto \sum_{n=-\infty}^{+\infty} \int D\varphi_p \ \exp\left\{ \varphi_p \cdot \left[\frac{2\alpha a^2}{c} \delta_\perp \partial^2_\tau D_{\theta\theta} \delta_\perp + K^* \delta_\perp \nabla^2_\sigma \right] \varphi_p + nd \cdot \varphi_p - n^2 \frac{K^* \pi L}{\hbar \beta v^*} (1 + \tilde{\eta}_1) - in \left(\kappa_J + \frac{\phi_{\rm ext}}{\phi_0} \right) \right\} \ , \tag{46}$$

where the two-dimensional transverse δ function has components $\delta_\perp(r) = \delta(\rho - a)\delta(z)$, and we have introduced the quantities

$$d(r) = \frac{4\sqrt{\pi} a^2}{\hbar \beta v^* c} \alpha \int d^4 r' \delta_\perp(r') \partial_{\tau'} D_{\theta\theta}(r', r) \delta_\perp(r)$$
$$\equiv d_\perp(\theta, \tau) \delta_\perp(r) \tag{47a}$$

$$\tilde{\eta}_1 = \frac{2\alpha a^2}{\pi \hbar \beta v^* K^* c L} \delta_\perp \cdot D_{\theta\theta} \delta_\perp \ , \tag{47b}$$

in analogy with the Cartesian case. Finally, we perform the last Gaussian integral over the periodic component of the bosonic field φ_p, and get the final result:

$$Z \propto \ \Theta_3[(1 + \tilde{\eta}_1 + \tilde{\eta}_2) T/T_0; \phi_{\rm ext}/\phi_0 + \kappa_J/2] \ , \tag{48}$$

with the second order correction

$$\tilde{\eta}_2 = \frac{a^4 \hbar \beta v^*}{4\pi L K^*} d_\perp \cdot \left(\frac{2\alpha a^2}{c} \delta_\perp \partial^2_\tau D_{\theta\theta} \delta_\perp + K^* \delta_\perp \nabla^2_\sigma \right)^{-1} d_\perp \ . \tag{49}$$

As predicted, the results for the cylindrical geometry have a form which is equivalent to the Cartesian case and therefore lead to the same conclusions as stated before. Since no new insight can be gained by an explicit evaluation of the radiative corrections, we shall not develop this calculation further.

VI. SUMMARY AND CONCLUSION

In this paper, we have examined the possibility that a one-dimensional ring could generate a spontaneous persistent current indicating spontaneous time reversal symmetry breaking. Our reasoning was that if such an effect exists, it should arise from the electrodynamic interaction between the electrons, in particular, from radiative corrections accounting for time-retarded back-action effects. Using the framework of quantum electrodynamics, as outlined in Ref. 11, we calculated the partition function of the coupled matter-field system, treating the instantaneous interaction between fermions (the screened Coulomb potential) in the Luttinger liquid model. With this approach, we found that the inclusion of retardation effects amounts only to a change in the magnitude of the conventional persistent current. As a consequence this persistent current vanishes with vanishing external flux threading the ring, and we therefore conclude that there is no spontaneous persistent current for our one-dimensional model of interacting spinless fermions. It is well known that in one-dimensional quantum systems there is a complete separation between charge and spin degrees of freedom, and we therefore do not expect that the inclusion of spin affects our conclusions found for the spinless case in a qualitative way. Nevertheless, this issue should be the subject of further investigation.

We relied on the Cartesian geometry to get an estimate for the magnitude of the correction introduced by the retardation effects: the results for the cylindrical geometry—which constitutes the "proper" way of describing the system—can be cast into the same form as in the Cartesian case. In the computation of this correction, we stressed the importance of the size of the ring, for removing both short and long distance divergences inherent to the bare transverse photon propagator. As

a consequence, the radiative corrections are finite and can be expressed in terms of the classical self-inductance (per unit length) of the ring. Clearly, the fact that electromagnetic corrections do not suppress the persistent current to zero is in agreement with the experimental situation,[1,2] where a finite persistent current is measured in the presence of an external flux.

It should be mentioned that several ingredients have been left out of the problem. Firstly, we have assumed the ring to be strictly one dimensional, and taking into account several transverse channels would bring us closer to the situation encountered in experiments. The inclusion of transverse channels presents a real technical challenge in the Luttinger liquid picture, and we do not know how to address this problem at the moment. The effect of transverse hopping between one-dimensional Hubbard chains is still in debate, and has been addressed recently.[21] Nevertheless, it is reasonable to expect that a ring with a small number of transverse channels will effectively behave like a one-dimensional system, and can be described as a Luttinger liquid. Secondly, we have ignored the effects associated with disorder in the ring. This issue should be addressed separately for (conventional) persistent currents of normal metal rings in the presence of an external flux.

The fact that we can obtain the partition function of the interacting fermions coupled to virtual electromagnetic modes in a closed form is in itself quite remarkable. In most problems dealing with degrees of freedom coupled to a bath of oscillators, the bath coordinates are integrated out, resulting in an action for the matter degrees of freedom with a matter-matter coupling term representing the influence of the environment. The integration of the matter degrees of freedom using this effective action represents in general a formidable task, because of the complexity of this new coupling: one often resorts to a perturbative analysis to draw some conclusions on how the dissipative environment affects the system. On the contrary, the integration of the matter degrees of freedom can be performed analytically here. This is a consequence of the fact that we have used the Luttinger liquid machinery to describe the fermions: the current operator, being quadratic in the original fermion field operators, becomes a quantity which is *linear* in the bosonic field φ. While this technique allows one in principle to treat a more complex type of problem—interacting fermions—the bosonic excitations which are generated by this theory are themselves uncoupled, allowing for an exact integration of the matter fields in the partition function. This surprising result leads us to believe that the Luttinger liquid approach could be applied to other problems of dissipative quantum systems, such as, for example, the coupling of interacting fermions to phonons. This will be the subject of a subsequent publication.

Finally, we wish to mention the observation that the current-current coupling induced by the electromagnetic gauge potential adds a quadratic contribution to the harmonic Luttinger liquid action which, however, mixes spatial and temporal derivatives. This radiative contribution therefore leads to a further renormalization of the Haldane parameters K^* and v^*, besides the renormalization caused by the (screened) Coulomb interaction. This renormalization does not affect our conclusion concerning the absence of spontaneous persistent currents, as it influences only the bulk properties of the system (the persistent current is a finite-size effect). The explicit calculation of this radiative renormalization [basically determined by the propagator given in Eq. (35)] will be given elsewhere.[22] Consequently, even in the absence of Coulomb interactions, the radiative corrections drive the system away from an ordinary Fermi liquid behavior towards a true Luttinger liquid state characterized, e.g., by the absence of a finite Migdal jump in the momentum distribution at the Fermi surface. This is true, no matter how small the correction is which arises from the field induced current-current interaction. We note that the same effect has been found in a perturbation treatment within the random phase approximation,[23] predicting a vanishing jump at the Fermi surface even in higher dimensions.

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