Many-body effects on the symmetric-antisymmetric gap in double quantum wells in strong magnetic fields

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We study the effect of the electron-electron interaction on the symmetric-antisymmetric energy gap in double quantum well systems in strong magnetic field at total filling factor $\nu=1$. We sum self-energy diagrams up to second order, and also we evaluate the self-energy in the $GW\Gamma$ approximation. We find that in the $GW\Gamma$ approximation the symmetric-antisymmetric energy gap collapses when the charge-density excitation develops a soft mode. Then the charge-density wave instability, which occurs in the double quantum well system at $\nu=1$, is accompanied by a transition to a metallic system.

I. INTRODUCTION

The two-dimensional electron-gas (2DEG) in the presence of a strong perpendicular magnetic field **B** exhibits the integral[1] and fractional[2] quantum Hall effects (QHE's). The explanation of these effects requires an incompressible ground state, i.e., the existence of an energy gap between the ground state and the lowest excited state of the system. In the integer QHE this gap is due to the quantization of the kinetic energy or to the freezing of the electron spin by the Zeeman energy. In the fractional QHE this gap is created by the electron-electron interaction.

In recent years, there has been much interest in the study of the evolution of the QHE states of a quasi-2DEG when the third dimension is introduced. This degree of freedom can be introduced into the system in different ways: by means of a new periodicity (superlattice) in the third direction,[3] by growing a wide parabolic quantum well,[4] or by fabricating a double quantum well (DQW) (Ref. 5) or wide single wells.[6]

Boebinger *et al.* have studied experimentally the integer QHE in DQW and they have found that the plateaus associated with the symmetric-to-antisymmetric (SAS) energy gap in the DQW are destroyed when the barrier thickness between the wells, the magnetic field, or the electron density increases.[5] Similar effects have been observed in wide single wells.[6]

For total filling factors 1 and 3, which correspond to the SAS energy gap, calculations in the time-dependent Hartree-Fock approximation[7,8] (TDHFA) and in the single-mode approximation[9] found that the DQW system undergoes a phase transition as the distance between the wells increase. In the unrestricted Hartree-Fock approximation the new ground state corresponds to a correlated Wigner crystal.[10,11] Although the new correlated Wigner crystal is expected to have an energy gap in the excitation spectrum, this gap does not allow the existence of a plateau in the Hall resistance since one might expect the Wigner crystal to become pinned by the impurities. Then, this phase transition could be the responsible for the experimental lack of a Hall plateau.

In this work we study, for the case of a DQW at $\nu=1$, many-body corrections, beyond the Hartree-Fock approximation, to the SAS energy gap. We calculate this gap in three different approaches: second-order perturbation theory, in the GW approximation,[12] and in the $GW\Gamma$ approximation.[13] We obtain that in the $GW\Gamma$ approximation the SAS energy gap ($\Delta^{GW\Gamma}$) collapses as the charge-density excitation (CDE), calculated in the TDHFA, develops a soft mode. Then, the charge-density wave (CDW) instability is accompanied by a disappearance of the SAS energy gap and therefore by a insulator-metal transition. This result questions the validity of the Hartree-Fock approximation for describing the properties of the DQW system at $\nu=1$, beyond the CDW instability.

This paper is organized as follows. In Sec. II we give expressions for the wave functions and energies in the Hartree-Fock (HF) approximation and we describe the random-phase approximation and the TDHFA for the calculation of the charge-density response function. In Sec. III we write expressions for the self-energies including up to second-order diagrams, and we describe and show results of the GW and $GW\Gamma$ approximations for the self-energies. In Sec. IV we give a brief summary of our results.

II. ENERGIES, WAVE FUNCTIONS, RESPONSE FUNCTIONS, AND CDE OF A DQW

In this section we write, for a pure DQW at filling factor $\nu=1$, expressions for the wave functions and energies obtained in the HF approximation. We give also expressions for the charge-density response function and for the CDE spectrum calculated in the random-phase approximation[14] (RPA) and in the TDHFA.

A. Energies and wave functions

We treat the electrons in the effective-mass approximation: they move with an effective mass m^* and they are in a medium characterized by a dielectric constant ϵ. The barrier between the wells has a height V_b and a thickness d_b. The width of the wells is d_w. We take m^* and ϵ as constants across the DQW system. In this mod-

el the tunneling between wells is inherently included. We take the z direction as the growth direction and assume translational invariance in the x-y plane. The magnetic field is applied in the z direction and we choose the Landau gauge, $\mathbf{A}=(0,Bx,0)$. Throughout this paper we will work within the lowest Landau level, and assume all the electrons to be in the lowest spin state, which is appropriate in the strong field limit.[15] For the remainder of this work we ignore the spin variable and the Landau index of the electrons. With these approximations the electron wave functions take the form

$$\psi_{i,k_y}(\mathbf{r})=\left[\frac{1}{\pi l^2 L^2}\right]^{1/4}\exp\left[ik_y y-\frac{(x+k_y l^2)^2}{2l^2}\right]\phi_i(z)\ . \tag{1}$$

Here L is the linear sample dimension, l is the magnetic length $l=\sqrt{\hbar c/eB}$, k_y is a discrete variable with $L^2/2\pi l^2$ possible values, i stands for symmetric (s) or antisymmetric (a), and $\phi_i(z)$ are the wave function of the electrons in the z direction. For describing the motion of the electrons in the z direction we restrict our basis of functions to the symmetric and antisymmetric states. We have checked that, for the DQW systems of interest, this is a very good approximation.

In the one-electron approximation the energies corresponding to the wave functions (1) are $\frac{1}{2}\hbar\omega_c+\varepsilon_s$ and $\frac{1}{2}\hbar\omega_c+\varepsilon_a$. Here ε_s and ε_a are, respectively, the energies of the symmetric and antisymmetric states of the DQW system and $\omega_c=eB/m^*c$ is the cyclotron frequency. The energies do not depend on k_y, and therefore each state is $L^2/2\pi l^2$ times degenerated. In the one-electron approximation the SAS energy gap is $\Delta^0=\varepsilon_a-\varepsilon_s$.

In the Hartree-Fock approximation we have to include exchange processes in the Hamiltonian. Assuming translational invariance in the x-y plane, so that the energies do not depend on k_y, and at filling factor $\nu=1$, the self-energies take the form[15]

$$E_s=\tfrac{1}{2}\hbar\omega_c+\varepsilon_s-g_{ssss}(q=0)\ , \tag{2}$$

and

$$E_a=\tfrac{1}{2}\hbar\omega_c+\varepsilon_a-g_{ssaa}(q=0)\ , \tag{3}$$

where

$$g_{ijkl}(q)=\frac{e^2}{\varepsilon l}\int_0^\infty d(kl)e^{-k^2l^2/2}J_0(kql^2)\int dz\int dz' e^{-k|z-z'|}\phi_i(z)\phi_l(z)\phi_k(z')\phi_j(z')\ , \tag{4}$$

and J_0 is a Bessel function. In obtaining Eqs. (2) and (3) we have assumed that the shape of the wave functions do not change when the exchange term is included in the Hamiltonian, and that the Hartree term only represents a shift in the energies, which can be neglected. We have checked that in our problem, finding that these are very good approximations. From Eqs. (2) and (3), the SAS energy gap in the HF approximation takes the form

$$\Delta^{\mathrm{HF}}=\Delta^0-g_{ssaa}(q=0)+g_{ssss}(q=0)\ . \tag{5}$$

B. Response function and collective excitations

We treat the response function in the TDHFA, i.e., we keep only the terms that correspond to a simple exciton present at all times, neglecting terms with two or more excitons present.[15–17] The diagrammatic representation of the charge-density response function is given in Fig. 1. Note that, since the TDHFA follows from a functional differentiation of the HF equation of motion, the use of this approach for the response function should be accompanied by the use of a HF Green function.[18,19] In our case, the charge-density function, in the TDHFA, takes the form[8,15]

$$\chi^{\mathrm{TDHFA}}(z,z',q;\omega)=\frac{1}{\pi l^2}e^{-q^2l^2/2}\phi_s(z)\phi_a(z)\phi_a(z')\phi_s(z')\times\frac{\Delta^{\mathrm{HF}}-g_{saas}(q)+g_{aass}(q)}{\omega^2-\omega_C^2(q)}\ , \tag{6}$$

where

$$\omega_C(q)=\{[\Delta^{\mathrm{HF}}-g_{saas}(q)+g_{ssaa}(q)]\times[\Delta^{\mathrm{HF}}-g_{saas}(q)-g_{ssaa}(q)+2v_{ssaa}(q)]\}^{1/2}\ , \tag{7}$$

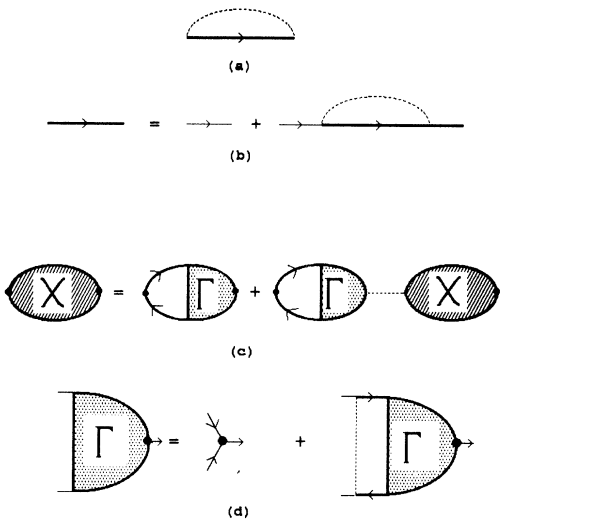

FIG. 1. Diagrammatic representation of (a) Hartree-Fock self-energy, (b) Green function in the Hartree-Fock approximation, (c) charge-density response function calculated in the TDHFA, and (d) vertex corrections that appear in the TDHFA. The thin dashed line represents the bare Coulomb interaction, the thin solid line represents the one-electron Green function, and the thick solid line the Hartree-Fock Green functions.

and

$$v_{ijkl}(q)=\frac{e^2}{\epsilon l}\frac{1}{ql}e^{-q^2l^2/2} \times\int dz\int dz' e^{-q|z-z'|}\phi_i(z)\phi_l(z)\phi_k(z')\phi_j(z') \ . \tag{8}$$

The poles of the response function, $\omega_C(q)$, give us the CDE spectrum of the system. In Fig. 2 we plot the dispersion relation of the CDE for a DQW at $\nu=1$. For $ql<1$, the dispersion relation decreases with the wave vector, developing a minimum at values $ql\sim 1.5$. By increasing the distance between the wells or the density of electrons in the DQW, this minimum becomes deeper, and for a critical distance or a critical density the dip becomes a soft mode, which indicates that the system undergoes a phase transition.[7–9,15]

If we calculate the response function in the RPA, i.e., we neglect excitonic effects in Eqs. (6) and (7), the charge-density response function takes the form

$$\chi^{\mathrm{RPA}}(z,z',q;\omega)=\frac{1}{\pi l^2}e^{-q^2l^2/2}\phi_s(z)\phi_a(z)\phi_a(z')\phi_s(z') \times\frac{\Delta^0}{\omega^2-\omega^2_{\mathrm{RPA}}(q)} \ , \tag{9}$$

where

$$\omega_{\mathrm{RPA}}(q)=\{\Delta^0[\Delta^0+2v_{ssaa}(q)]\}^{1/2} \ . \tag{10}$$

Diagrammatically the RPA for the response function corresponds to neglecting the second term in Fig. 1(d), i.e., to approaching the vertex function for the identity. Since the RPA comes from a functional differentiation of the Hartree approximation, the use of the RPA for the calculation of the response function requires us to neglect the exchange contributions to the self-energies.[18,19]

In Fig. 2 we plot the poles of the RPA charge-density response function, $\omega_{\mathrm{RPA}}(q)$. Note that in this lower approximation the CDE spectrum does not show any minimum as a function of the wave vector: The instability of the DQW at $\nu=1$ is due to the excitonic effects and, for describing the occurrence of this instability, it is essential to include excitonic effects in the response function.

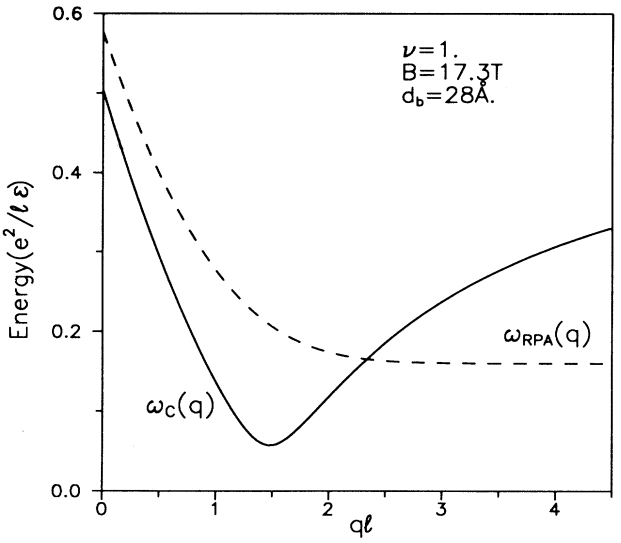

FIG. 2. Dispersion relation of the CDE for a DQW at $\nu=1$. The solid line corresponds to the TDHFA for the collective excitation, $\omega_C(q)$, and the dashed line corresponds to the RPA for the collective excitation, $\omega_{\mathrm{RPA}}(q)$. The parameters of the DQW are $d_w=139$ Å, $d_b=28$ Å, $V_b=250$ meV, $B=17.3$ T, $m^*=0.067$, and $\epsilon=12.5$.

III. SAS GAP BEYOND THE HF APPROXIMATION

The SAS energy gap obtained in the HF approximation is bigger than the SAS gap in the one-electron approximation.[8] In this section we study many-body corrections to the value of the SAS energy gap and study the behavior of this gap when the CDW instability occurs. We use three different approaches to calculate self-energies: (a) the sum of second-order diagrams, (b) the *GW* approximation, and (c) the *GWΓ* approximation.

A. Calculation of second-order diagrams

In Fig. 3 the two possible second-order self-energy diagrams are shown. For a DQW at $\nu=1$ the contributions of diagrams (a) and (b) of Fig. 3 to the self-energies of the s and a states are

$$\Sigma_s^{2(a)}(\omega)=\frac{g^2_{ssaa}(q=0)}{2E_a^{(2)}-E_s^{(2)}-\omega} \ , \tag{11}$$

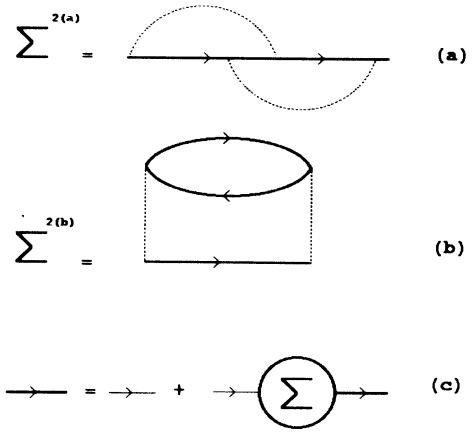

FIG. 3. In (a) and (b) are shown the second-order diagrams that contribute to the self-energy. In (c) we show the diagrammatic representation of Dyson's equation, which has to verify the Green function. The thin dashed line represents the bare Coulomb interaction, the thin solid line represents the one-electron Green function, and the thick solid line the dressed Green function.

$$\Sigma_a^{2(a)}(\omega)=\frac{g_{ssaa}^2(q=0)}{2E_s^{(2)}-E_a^{(2)}-\omega}\ , \tag{12}$$

$$\Sigma_s^{2(b)}(\omega)=\frac{\tilde{V}(q=0)}{E_s^{(2)}-2E_a^{(2)}+\omega}\ , \tag{13}$$

$$\Sigma_a^{2(b)}(\omega)=\frac{\tilde{V}(q=0)}{E_a^{(2)}-2E_s^{(2)}+\omega}\ . \tag{14}$$

In these expressions

$$\tilde{V}(q)=\left[\frac{e^2}{\epsilon l}\right]^2\int_0^\infty \frac{dq}{q}e^{-q^2l^2}\left[\int dz\int dz' e^{-q|z-z'|}\phi_s(z)\phi_a(z)\phi_s(z')\phi_a(z')\right]^2\ , \tag{15}$$

and $E_s^{(2)}$ and $E_a^{(2)}$ are obtained from Dyson's equation [see diagram (c) of Fig. 3]

$$E_i^{(2)}=E_i^{\rm HF}+\Sigma_i^{2(a)}(E_i^{(2)})+\Sigma_i^{2(b)}(E_i^{(2)})\ . \tag{16}$$

Here i stands for s and a. Note that $\Sigma_i^{(2)}$ depends on both the s and the a energies. From Eqs. (11)–(16), we can write an equation for the SAS energy gap in second order

$$\Delta^{\rm 2nd}=\Delta^{\rm HF}+\frac{\tilde{V}(q=0)-g_{ssaa}(q=0)}{\Delta^{\rm 2nd}}\ . \tag{17}$$

We have solved self-consistently this equation for a wide range of magnetic fields and the barrier's width, and we have always found that the second-order correction to the SAS energy gap is always less than 5% of the HF gap. The reason for this is that the interactions that appear in second order, $\sqrt{\tilde{V}}$ and g_{ssaa}, tend to cancel each other, and furthermore these second-order interactions are much smaller than the interaction g_{ssss} that controls the HF SAS energy gap. Then the second-order corrections to the self-energies almost do not correct the HF SAS energy gap and do not show any strange behavior when the CDW instability occurs.

B. *GW* and *GWΓ* approximations

Different approximations for the self-energy operator may be generated following an approach introduced by Baym.[20,21] Write down a free-energy functional of the dressed Green function and the Coulomb interaction, then generate an approximation for the self-energy by functional differentiation of the free energy with respect to the Green function, and compute the Green's function self-consistently with this self-energy. This way of obtaining the self-energy guarantees the use of a conserving approximation, i.e., an approximation consistent with microscopic conservation laws for particle number, energy, and momentum.[20,21]

If we use the RPA approximation for the free energy, the functional differentiation with respect to the Green function gives us the *GW* approximation. In this approximation for the self-energy the interactions in the exchange term are the Coulomb interaction dynamically screened with the RPA dielectric constant. The *GW* approximation was first introduced by Hedin[12,22] by expanding the electron self-energy in a perturbation series in the screened Coulomb interaction W. The *GW* approximation has been successfully applied to metals,[12] semiconductors,[23,24] and 2DEG.[25,26]

A higher approach to the self-energy can be generated by introducing excitonic interaction in the bubbles of the RPA free energy. The functional differentiation of this free energy with respect to the Green function gives us the *GWΓ* approximation for the self-energy. In this case the Coulomb interaction should be screened by the TDHFA dielectric constant, it being necessary to include the vertex function in the self-energy. The *GWΓ* approximation can be also obtained as a higher term in the series expansion of the self-energy in the screened Coulomb interaction.[12,27] The *GWΓ* approximation has been applied to metals,[13] given similar results for the *GW* approximation. Also recently it has been applied to quasi-2DEG.[28] the *GW* approximation can be obtained from the *GWΓ* approximation by neglecting excitonic interactions in the vertex function Γ and in the charge-density response function.

In Fig. 4 we plot the diagrammatic representation of

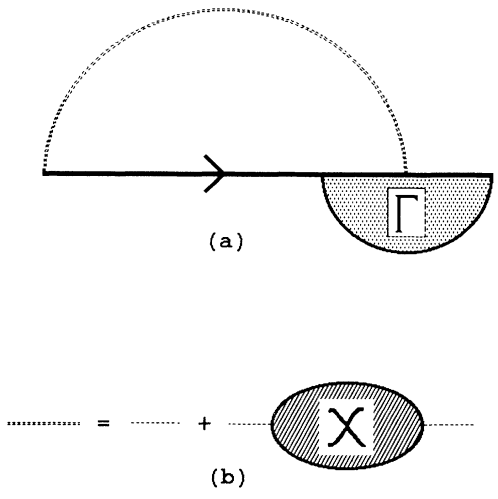

FIG. 4. (a) Diagrammatic representation of the *GWΓ* approximation for the self-energy. (b) Dynamically screened Coulomb interaction W. The thick solid line represents the dressed Green function. χ and Γ represent, respectively, the charge-density response function and the vertex function calculated in the TDHFA.

the $GW\Gamma$ approximation for the self-energy. The self-energies have to be obtained by solving self-consistently Dyson's equation [diagram (c) of Fig. 3]. For simplicity in the self-consistent process we neglect the changes in the charge-density response function and in the vertex function due to the renormalization of the Green function. In the case of a DQW at filling factor $\nu=1$ the self-energy of the state i takes the form

$$\Sigma_i^{GW\Gamma}(\omega)=\frac{i}{2\pi}\int_{-\infty}^{\infty} d\omega' e^{i\delta^+\omega'}\sum_{j,q} e^{-q^2l^2/4}\int dz\phi_i(z)\phi_j(z)\int dz'' G_j(\omega+\omega')\Gamma_{j,i}(q,z'',\omega')W(z,z'',q;\omega') , \tag{18}$$

where

$$W(z,z',q,\omega)=\frac{e^2}{\epsilon}\frac{2\pi}{q}\left[e^{-q|z-z'|}+\frac{e^2}{\epsilon}\frac{2\pi}{q}\int dz_1\int dz_2 e^{-q|z-z_1|}e^{-q|z'-z_2|}\chi^{\mathrm{TDHFA}}(z,z_2,q;\omega)\right] , \tag{19}$$

$$\Gamma_{i,i}(q,z,\omega)=e^{-q^2l^2/4}\phi_i(z)\phi_i(z) , \tag{20}$$

$$\Gamma_{\substack{a,s\\ s,a}}(q,z,\omega)=e^{-q^2l^2/4}\phi_a(z)\phi_s(z)\frac{\omega^2-(\Delta^{\mathrm{HF}})^2}{\omega^2-\omega_\Gamma^2(q)}\frac{\omega\pm[\Delta_{\mathrm{HF}}-g_{saas}(q)+g_{aass}(q)]}{\omega\pm\Delta^{\mathrm{HF}}} , \tag{21}$$

$$G_j(\omega)=(\omega-E_j^{GW\Gamma}-i\delta_j)^{-1} , \tag{22}$$

$$\omega_\Gamma^2=[\Delta^{\mathrm{HF}}-g_{saas}(q)]^2-g_{ssaa}^2(q) , \tag{23}$$

$\delta=0^+$, and $\delta_j=0^+$ for $j=s$ and 0^- for $j=a$. The energies $E_i^{GW\Gamma}$ are obtained from Dyson's equation

$$E_i^{GW\Gamma}=\varepsilon_i+\Sigma_i^{GW\Gamma}(E_i^{GW\Gamma}) . \tag{24}$$

It is important to note that when integrating Eq. (18) in frequencies the poles of the vertex function Γ do not contribute since they correspond to zeros of the dynamically screened Coulomb interaction W. This is only true in conserving approximations, when the same vertex function in the calculation of the charge-density response function and in the $GW\Gamma$ self-energy is used.

The GW approximation for the self-energy can be obtained from Eqs. (18)–(24) by changing χ^{TDHFA} by χ^{RPA}, Δ^{HF} by Δ^0, and $\omega_\Gamma(q)$ by Δ^{HF}. It is easy to obtain the following equation for the SAS energy gap in the GW approximation:

$$\Delta^{GW}=\Delta^{\mathrm{HF}}+\left[\frac{e^2}{\epsilon l}\right]^2 2\Delta^0\int\frac{dq}{q}\frac{e^{-q^2l^2}}{\omega_{\mathrm{RPA}}(q)}\frac{\left[\int dz\int dz' e^{-q|z-z'|}\phi_s(z)\phi_a(z)\phi_s(z')\phi_a(z')\right]^2}{\omega_{\mathrm{RPA}}(q)+\Delta^{GW}} . \tag{25}$$

In Fig. 5 is plotted, as a function of the magnetic field, the GW SAS energy gap, Δ^{GW}, obtained by solving self-consistently Eq. (25). Also the one-electron SAS energy gap, Δ^0, and the HF SAS energy gap, Δ^{HF}, are plotted. We see in this figure that contrary to the semiconductor case,[23,24] the energy gap obtained in the GW approximation is bigger than the HF energy gap. This is because, although the screened exchange[12] part of the self-energy tends to close the SAS energy gap, the Coulomb hole part[12] is stronger and tends to open it. In any case the correction is not very large. Also we can see that Δ^{GW} does not show any anomalous behavior in the vicinity of the CDW instability. The GW self-energies of the s and a states have a zero imaginary part, meaning that in this approximation the lifetimes of these states are infinity. Similar results are obtained when we study the SAS energy gap versus the thickness of the barrier of the DQW for a fixed magnetic field, Fig. 6.

Completely different results are obtained when we study the SAS energy gap in the $GW\Gamma$ approximation, $\Delta^{GW\Gamma}$. In Fig. 5 $\Delta^{GW\Gamma}$ as a function of the magnetic field is also plotted. We see that for small magnetic fields $\Delta^{GW\Gamma}$ is practically equal to Δ^{HF} and for magnetic fields near the CDW instability the real part of $\Delta^{GW\Gamma}$ decreases quickly and reaches a constant value just before the critical magnetic field where the CDW instability occurs. Also just before the instability the $GW\Gamma$ self-energies get an imaginary part. In Fig. 5 $\Delta^{GW\Gamma}$ minus its imaginary part is also plotted; we see that this quantity goes to zero at the critical magnetic field. Then in the $GW\Gamma$ approximation to the self-energy the SAS energy gap decreases and acquires a big imaginary part before the CDW instability: The SAS energy gap collapses at the critical magnetic field where the CDW instability occurs. We obtain the same results when we study the variations of $\Delta^{GW\Gamma}$ with respect to the thickness of the barrier d_b. The reason for this behavior is the dependence of the $GW\Gamma$ self-energies on the TDHFA charge-density response

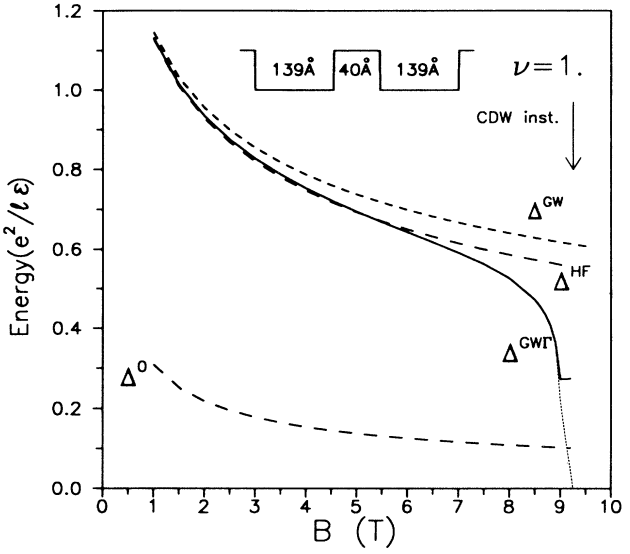

FIG. 5. Variation, as a function of the magnetic field, of the SAS energy gap calculated in different approximations (see text). The solid line represents the real part of the $\Delta^{GW\Gamma}$ and the dotted line corresponds to the real part of $\Delta^{GW\Gamma}$ minus its imaginary part. The parameters of the DQW are $d_w=139$ Å, $d_b=40$ Å, $V_b=250$ meV, $m^*=0.067$, and $\epsilon=12.5$. The filling factor is always 1. The arrow indicates the critical magnetic field where the CDW instability predicted by the TDHFA occurs.

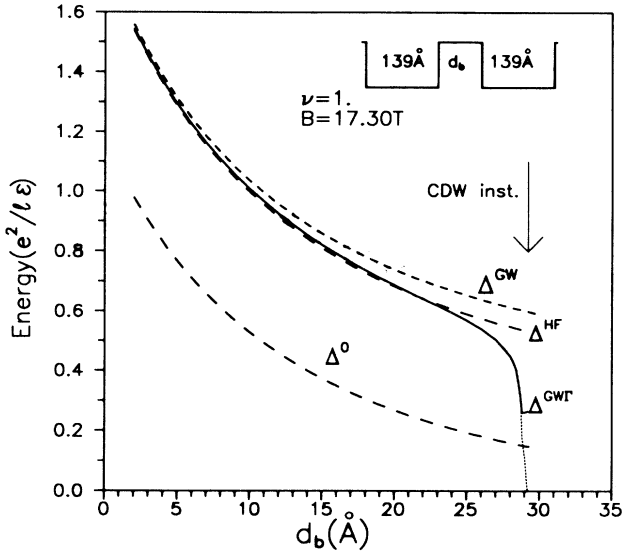

FIG. 6. Variation, as a function of the thickness of the barrier, of the SAS energy gap calculated in different approximations (see text). The solid line represents the real part of the $\Delta^{GW\Gamma}$ and the dotted line corresponds to the real part of $\Delta^{GW\Gamma}$ minus its imaginary part. The parameters of the DQW are $d_w=139$ Å, $V_b=250$ meV, $m^*=0.067$, $B=17.3$ T, and $\epsilon=12.5$. The filling factor is 1. The arrow indicates the critical thickness of the barrier where the CDW instability predicted by the TDHFA occurs.

function. In the $GW\Gamma$ approximation, when the response function χ^{TDHFA} diverges, the screened Coulomb interaction W also diverges. This dependence of the $GW\Gamma$ self-energies on χ^{TDHFA} produces the correlation between the CDW instability and the collapse of the SAS energy gap.

IV. SUMMARY

We have studied the effect of the electron-electron interaction on the symmetric-antisymmetric energy gap of a DQW in the presence of a strong magnetic field and a total filling factor $\nu=1$. We have found that the SAS energy gap in the HF approximation is much bigger than the HF SAS energy gap and that the correction of the second-order diagrams to the HF SAS energy gap is negligible. The GW approximation gives an energy gap slightly bigger than the HF SAS gap. In the $GW\Gamma$ approximation the SAS energy gap is smaller than the HF SAS energy gap and collapses when the DQW system becomes unstable against the formation of a CDW ground state. The CDW instability is accompanied by the closing of the SAS energy gap and the broadening of the symmetric and the antisymmetric states: The system behaves as a metal rather than as an insulator. The collapse of the SAS energy gap produces that second-order corrections to the self-energies diverge [see Eqs. (11)–(14)]; therefore this result questions the validity of the unrestricted Hartree-Fock approximation for describing the properties of the DQW at $\nu=1$ after the CDW instability. Then the character of the new ground state after these instabilities is not clear. It is possible that, in DQW systems with little tunneling between wells, after the vanishing of the SAS energy gap, a new phase with a gap produced entirely by many-body effects could appear.[29,30]

In this work we have not included the effect of impurities. We assume that in the experimental high-mobility DQW systems the electron-impurity interaction is very weak and can be treated as a second-order effect.[31]

Our analysis offers no explanation for the disagreement between the experimental activation energies[5] and the theoretical value of the SAS energy gap. In the presence of disorder, the activation energy can be related to the minimum in the collective excitations and a theory including the interaction of the charge-density excitations and the impurities should be required.[32,33]

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