Spin-flip excitations from Landau levels in two dimensions

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Dispersion relations for the spin-flip modes, in which an electron is both promoted from one Landau level to the next and its spin reversed, are calculated in the time-dependent Hartree-Fock approximation and in the generalized single-mode approximation for integer and noninteger filling factors. The energy of these modes can be shifted substantially by electron-electron interactions at $k=0$. This is in contrast to the magnetoplasma and spin-wave modes, which are not shifted at $k=0$. The calculated dispersion relations are compared with the results of recent inelastic light-scattering experiments near filling factor $\nu=1$, and predictions are made for other filling factors.

I. INTRODUCTION

At low temperatures, and for typical densities, the inversion layer electrons in GaAs heterojunctions are all in the lowest subband for motion perpendicular to the interface and, hence, their dynamics can be described by that of a two-dimensional electron gas (2DEG).[1] In the presence of a strong perpendicular magnetic field, B, the electronic motion is further quantized, and the density of states for the ideal 2DEG then consists of discrete, highly degenerate, spin-split Landau levels. Both the reduced dimensionality and the Landau quantization of the kinetic energy can lead to enhanced correlation effects due to electron-electron interactions. Also, in the best GaAs heterojunctions, the amount of disorder is small, making it possible to observe correlation effects experimentally. The fractional quantum Hall effect[2] and possible Wigner crystallization[3] are striking examples of ground states stabilized by correlation effects. However, the excitations can also exhibit strong correlation effects even when the ground state is not strongly perturbed by electron-electron interactions, such as when a single Landau level is fully occupied. For example, there is a large exchange enhancement of the g factor which can be observed by transport measurements,[1,4] and a number of features observed in cyclotron resonance experiments have been attributed to the effect of electron-electron interactions on the collective excitations of the system.[5] More recently, strong features in the spectra obtained by the inelastic light-scattering methods of Pinczuk *et al.*[6] have been attributed to collective excitations of the 2DEG. It appears that inelastic light-scattering techniques can probe excitations that are not active in spin or cyclotron resonance experiments, and can also probe over a wider range of wave vectors. In particular, it has been argued that the so-called "spin-flip" mode, in which an electron both is promoted from one Landau level to the next and has its spin reversed in the process, has been observed in these experiments.[7] This mode is strongly affected by electron-electron interactions and will be the main subject of this paper.

In the presence of a strong perpendicular magnetic field, the energy levels for an ideal 2DEG lie in discrete Landau levels, with spacing $\omega_c=eB/m^*c$, the cyclotron frequency. Within each Landau level there are two spin-split levels, separated by $\omega_z=|g\mu_B B|$, the Zeeman energy. The N electron ground state has electrons in the lowest $\nu=N/\mathcal{N}$ levels, where the highest occupied Landau level may be only partially filled. The degeneracy of the spin-split Landau level is $\mathcal{N}=(eB/2\pi c)A$, where A is the area of the sample, and $\hbar=1$. In the absence of interactions, excited states can be formed by removing an electron with spin s from Landau level n and placing it in an empty state in Landau level $n+m$, with spin $s+\delta S_z$. The energy of such an excited state is $m\omega_c+\delta S_z\omega_z$. When interactions are included, the excited states are labeled by wave vector $\mathbf{k}$, m, and δS_z.[8] In the strong-field approximation ($\omega_c \gg E_c$, where the typical Coulomb energy is $E_c=e^2/\epsilon l_0$, and the magnetic length is $l_0=\sqrt{c/eB}$) the excited states occur near integer multiples of ω_c, and the dispersion relations can be written as

$$E_\mu^{m,\delta S_z}(\mathbf{k})=m\omega_c+\delta S_z\omega_z+\Delta E_\mu^{m,\delta S_z}(k)\ , \tag{1.1}$$

where $\Delta E_\mu^{m,\delta S_z}(k)$ is of order E_c, and μ is an additional index which may be needed to distinguish between different branches of the dispersion relation. According to Kohn's theorem,[9] the frequency for cyclotron resonance is not affected by interactions, i.e., the only electric dipole allowed $\delta S_z=0$ (magnetoplasmon) mode at $\mathbf{k}=0$ occurs at energy ω_c. Kohn's theorem applies only to translationally invariant systems; the combined effects of disorder and electron-electron interactions can cause a shift from ω_c at $\mathbf{k}=0$ and so absorption can occur away from the cyclotron frequency. However, disorder is typically very small in the heterojunctions, and does not have a large effect on the dispersion relations at small wave vectors. An argument similar to Kohn's shows that in a spin rotationally invariant system the spin wave mode ($m=0$, $\delta S_z=1$), which is observed in electron spin resonance experiments,[10] occurs at ω_z for $\mathbf{k}=0$. (This is Larmor's theorem.) However, no such symmetry argument exists for an excited state in which an electron both changes Landau level and flips its spin. In this case, even in the absence of disorder, the mode can have energy different from $m\omega_c+\delta S_z\omega_z$ at $\mathbf{k}=0$. These spin-flip

modes have been studied at integer filling factor ν;[8] however, the effects of exchange were not treated correctly.[11] In the simplest case, when the Fermi level lies between the two spin-split levels in the lowest Landau level, the shift at $\mathbf{k}=0$, which is substantial, is equal to the difference in exchange self-energy between an electron with spin $s=\frac{1}{2}$ in the first excited Landau level, and an electron with $s=-\frac{1}{2}$ in the lowest Landau level. In this paper, we consider the spin-flip modes at other filling factors, including noninteger ν, and find in these cases that $\Delta E_{\mu}^{1,\pm 1}(\mathbf{k}=0)$ can be shifted from zero by Coulomb interactions.

Dispersion relations have been calculated previously for the magnetoplasmon modes ($\delta S_z=0$) near ω_c for integer filling using a diagrammatic approach,[8] which is equivalent to the time-dependent Hartree-Fock approximation.[12] These approximations are essentially exact in the strong-field limit for integer filling factor, but they ignore important correlations between the electrons in the partially filled Landau level when used for noninteger filling factors.[13] An approximation which can include the correlations within the partially filled Landau level is the generalized single-mode approximation (SMA), which has also been used to calculate dispersion relations for the magnetoplasmon modes.[13,14] The magnetoplasmon dispersion relations are relevant to light-scattering experiments, where structure is observed in the spectra at energies that correspond to critical points in the dispersion relations.[6] Spectra measured for filling factor $\nu=1$ also show a large peak at an energy corresponding to the shift of the spin-flip mode from ω_c.[6,7] When there is more than one mode for the magnetoplasmon or spin-flip excitations at a particular filling factor, as occurs for $\nu>1$, the modes may contribute unequally to structure in the light-scattering spectra. Oscillator weights, which are a measure of the relative contribution of each mode, have been calculated previously for magnetoplasmon modes for filling factors between 1 and 2,[13] and near $\mathbf{k}=0$, for filling factors between 2 and 3.[15] We have calculated oscillator weights for the magnetoplasmon modes for all $\mathbf{k}$ for filling factors between 2 and 3, as the weights are required to predict the result of a light-scattering experiment. We have also calculated dispersion relations in the Hartree-Fock approximation and oscillator weights for the spin-flip modes near $\omega_c \pm \omega_z$ for integer and noninteger filling factors. It is expected that these modes will cause structure in the light-scattering spectra. To illustrate the interplay between oscillator weights and the dispersion relations, we include a calculation of the dynamic structure factor, which is proportional to the differential cross section for light scattering.[16]

II. DISPERSION RELATIONS

A. Hartree-Fock approximation

The charge-density and spin-density response functions have been calculated previously using a diagrammatic approach,[8] which is equivalent to the Hartree-Fock approximation (HFA),[11] for integer filling factors. The poles of the response functions given the dispersion relations $E_{\mu}^{m,\delta S_z}(\mathbf{k})$. Here we present the main results of that calculation, with the extension to noninteger filling factors, and $\delta S_z \neq 0$, and refer the reader for details to Ref. 8.

The single-particle Hamiltonian for an electron confined to two dimensions, in a perpendicular magnetic field is

$$H_i=\frac{1}{2m^*}\left[p_i-\frac{e}{c}\mathbf{A}(\mathbf{r}_i)\right]^2+\omega_z S_z(i)\ , \tag{2.1}$$

where the magnetic field is $\mathbf{B}=\nabla\times\mathbf{A}$, and $\mathbf{S}(i)$ is the spin angular momentum operator for the ith electron. For the single-particle state $\phi_{n,l,s}(\mathbf{r}_i)$,

$$H_i\phi_{n,l,s}(\mathbf{r}_i)=[(n+\tfrac{1}{2})\omega_c+s\omega_z]\phi_{n,l,s}(\mathbf{r}_i)\ . \tag{2.2}$$

Here n is the Landau level index, and l distinguishes the different members of the degenerate set of states. The spin index, s, is $\pm\frac{1}{2}$.

The Hamiltonian for the interacting two-dimensional electron gas in a $\mathbf{B}$ field is

$$\mathcal{H}=\sum_i H_i+V\ ,$$

where the Coulomb interaction is

$$V=\frac{1}{2}\sum_{i\neq j}\int\frac{d\mathbf{q}}{(2\pi)^2}\tilde{u}(\mathbf{q})\rho_i(\mathbf{q})\rho_j(-\mathbf{q})\ . \tag{2.3}$$

The finite thickness of the electron layer can be taken into account by modifying the Coulomb interaction.[1] In this case, the Fourier transform of the effective interaction can be written as

$$\tilde{u}(\mathbf{q})=\frac{2\pi e^2}{q}F(q)\ , \tag{2.4}$$

where

$$F(q)=\frac{1}{8}\left[1+\frac{q}{b}\right]^{-3}\left[8+9\frac{q}{b}+3\left[\frac{q}{b}\right]^2\right]$$

modifies the interaction from its ideal two-dimensional form. The average distance of the inversion layer electrons from the interface is Z_0, and $b=3/Z_0$. The ideal two-dimensional case is recovered in the limit of infinite b. The density operator is

$$\rho(\mathbf{q})=\sum_i\rho_i(\mathbf{q})=\sum_i e^{i\mathbf{q}\cdot\mathbf{r}_i}\ . \tag{2.5}$$

For integer filling factors ν, the ground state is unchanged by interactions; in the strong magnetic field limit it is ν filled single spin Landau levels, with the remaining levels unoccupied. When ν is noninteger, electrons in the highest occupied level, which is partially filled, are strongly correlated.

In the HFA of Ref. 8, the dispersion relations of the collective excitations are found by locating the poles of the charge-density and spin-density response functions. The response functions are

$$\chi_A(\mathbf{k},\omega)=-i\int_0^{\infty}dt\,e^{i\omega t}[\Theta_A(\mathbf{k},t),\Theta_A(\mathbf{k},0)]\ , \tag{2.6}$$

where

$$\Theta_A(\mathbf{k},t)=e^{i\mathcal{H}t}\sum_i e^{i\mathbf{k}\cdot\mathbf{r}_i}\Theta_A e^{-i\mathcal{H}t} .$$

The charge-density response function has $A=\rho$, and $\Theta_\rho=1$; the spin-density response functions have $A=\sigma_z,\sigma_\pm$ and $\Theta_A=2S_z,2S_\pm$, where $S_\pm=S_x\pm iS_y$. In this approximation for the response functions, only diagrams with one exciton present at all times are considered. This results in poles for the response functions at real frequencies $\omega(\mathbf{k})$. This calculation does not produce decay rates which are given by $\mathrm{Im}(\omega(\mathbf{k}))$. When both spin states are equally occupied ($\nu=2,4,\ldots$) this approximation includes all contributions to order E_c, and so is essentially exact in the strong-field limit. For the spin-flip modes considered here, there are additional contributions which are of order E_c at integer filling factors when both spin states are unequally occupied ($\nu=1,3\ldots$) which are not included in this calculation. For example, a spin-flip excitation may decay into a magnetoplasmon and spin wave, conserving both energy and momentum. Contributions of this type are currently being studied and may produce poles at complex frequencies in the spin response functions, and lead to an estimate for decay rates.

In the HFA, the response functions have poles at frequencies $\omega(\mathbf{k})$ which are solutions to[8]

$$\sum_{\lambda\mu}\{\delta_{\alpha,\lambda}\delta_{\beta,\mu}[D(\omega)]^{-1}_{\alpha\beta}-\delta_{s_\alpha,s_\lambda}\delta_{s_\mu,s_\beta}\tilde{V}^{(1)}_{\alpha\mu\beta\lambda}(\mathbf{k})+\delta_{s_\alpha,s_\beta}\delta_{s_\mu,s_\lambda}\tilde{V}^{(2)}_{\alpha\mu\lambda\beta}(\mathbf{k})\}B_{\lambda\mu}(\mathbf{k})=0 , \tag{2.7}$$

where the Greek labels represent both Landau level and spin, i.e., $\alpha=(n_\alpha,s_\alpha)$. The basis states are $B_{\lambda\mu}(\mathbf{k})$. The two-particle propagator is

$$D_{\alpha\beta}(\omega)=\frac{\nu_\alpha[1-\nu_\beta]}{\omega+E_\alpha-E_\beta+i0^+}-\frac{\nu_\beta[1-\nu_\alpha]}{\omega+E_\alpha-E_\beta-i0^+} , \tag{2.8}$$

where the poles of the electron Green's function occur at energies

$$E_\alpha=(n_\alpha+\tfrac{1}{2})\omega_c+s_\alpha\omega_z+\Sigma_\alpha , \tag{2.9}$$

with the exchange self-energy of an electron labeled by α equal to

$$\Sigma_\alpha=-\sum_\beta\delta_{s_\alpha,s_\beta}\nu_\beta\tilde{V}^{(1)}_{\alpha\beta\alpha\beta}(0) . \tag{2.10}$$

The filling factor for level α is ν_α. The contribution of the direct or Hartree term (i.e., the bubble diagrams),

$$\tilde{V}^{(2)}_{\alpha\mu\lambda\beta}(\mathbf{k})=\frac{\tilde{u}(\mathbf{q})}{2\pi}G^{n_\alpha n_\beta}(k)G^{n_\lambda n_\mu}(k^*)e^{-|k|^2/2} , \tag{2.11}$$

is from interactions in which an electron-hole pair recombines, simultaneously exciting another electron-hole pair. These terms only contribute when the excited electron does not flip its spin, and do not contribute to the energy of the spin-flip and spin-wave modes. The contribution of the exchange or Fock term (i.e., the ladder diagrams),

$$\tilde{V}^{(1)}_{\alpha\mu\beta\lambda}(\mathbf{k})=\int\frac{d\mathbf{q}}{2\pi}e^{i\mathbf{q}\times\mathbf{k}\cdot\hat{\mathbf{z}}}\tilde{V}^{(2)}_{\lambda\mu\alpha\beta}(\mathbf{q}) , \tag{2.12}$$

is from the direct interaction of the excited electron and hole. The function $G^{nn'}(k)$ is

$$G^{nn'}(k)=\left[\frac{n!}{n'!}\right]^{1/2}\left[\frac{-ik}{\sqrt{2}}\right]^{n'-n}L_n^{n'-n}\left[\frac{|k|^2}{2}\right] , \tag{2.13}$$

where $k=k_x+ik_y$, and $L_n^{n'-n}$ is a generalized Laguerre polynomial. In the strong-field approximation, the sums in Eq. (2.7) are restricted to pairs of indices such that $(n_\lambda-n_\mu)=(n_\alpha-n_\beta)=m$, and $(s_\lambda-s_\mu)=(s_\alpha-s_\beta)=\delta S_z$.

B. Generalized single-mode approximation

The above HFA is exact to order E_c at integer filling factors for the magnetoplasmon and spin-wave modes. At noninteger filling factors there are many intra-Landau-level excitations with energy of order E_c. An exciton with energy near $m\omega_c+\delta S_z\omega_z$ can decay into another exciton plus a number of low-energy excitations, conserving energy and momentum. Although the diagrammatic approach has been extended to noninteger filling factors, it is unreliable since contributions which have more than one exciton present at a time are not included.

At special filling factors, the ground state is an incompressible, highly correlated electron fluid described by Laughlin's wave function.[2] The Hartree-Fock approximation does not take into account the special nature of the ground state. The correlations that are present in the ground state can be built into the excited-state wave functions by considering excited-state wave functions that are created by forming density-wave excitations in the ground state. This approach, known as the single-mode approximation, was first applied to finding the dispersion relations for intra-Landau-level excitations for filling factors less than 1 (magnetorotons).[17] It has since been generalized (GSMA), and has been applied to finding dispersion relations, and wave functions for inter-Landau-level excitations, and for $\nu>1$.[13,14] Here we generalize this approach further to find the dispersion relations and many-body wave functions for the excited states in which an electron's spin is flipped.

The approach begins by choosing a set of basis states all with the same m and δS_z. These are formed by operating on the ground state, ψ_0, with the operator

$$\rho^{\alpha+\Delta;\alpha}(\mathbf{k})=\sum_i\rho_i^{\alpha+\Delta;\alpha}(\mathbf{k}) , \tag{2.14}$$

where $\Delta=(m,\delta S_z)$, and $\alpha+\Delta=(n_\alpha+m,s_\alpha+\delta S_z)$. The generalized density operator is

$$\rho_i^{\alpha+\Delta;\alpha}(\mathbf{k})\phi_{n,l,s}(\mathbf{r}_i)=\delta_{n_\alpha,n}\delta_{s_\alpha,s}G^{n_\alpha+m,n_\alpha}(k)B_i(k)\times\phi_{n_\alpha+m,l,s_\alpha+\delta S_z}(\mathbf{r}_i) . \tag{2.15}$$

$B_i(k)$ is the part of the density operator, $\rho_i(\mathbf{k})$, which can be expressed in terms of intra-Landau-level operators, as in Ref. 14. When the symmetric gauge is used, the index l is the angular momentum, and

$$B_i(k)\phi_{n,l,s}(\mathbf{r}_i)=\sum_{l'}G^{l+l',l}(-k)\phi_{n,l+l',s}(\mathbf{r}_i) . \quad (2.16)$$

When $\delta S_z=0$, $\rho^{\alpha+\Delta;\alpha}(\mathbf{k})$ is the density operator projected between Landau levels n_α and $n_\alpha+m$, and the excited states are density waves. The definition of $\rho^{\alpha+\Delta;\alpha}(\mathbf{k})$ has been extended to include spin-wave excitations, with $\delta S_z=\pm 1$.

The basis states are

$$\Phi^{\alpha+\Delta;\alpha}(\mathbf{k})=[N^{\alpha+\Delta;\alpha}(\mathbf{k})]^{-1/2}\sum_i\rho_i^{\alpha+\Delta;\alpha}(\mathbf{k})\psi_0 , \quad (2.17)$$

normalized by

$$N^{\alpha+\Delta;\alpha}(\mathbf{k})=N\nu_\alpha[1-\nu_{\alpha+\Delta}]\frac{(n_\alpha+m)!}{n_\alpha!}\times|G^{n_\alpha,n_\alpha+m}(\mathbf{k})|^2e^{-k^2/2} . \quad (2.18)$$

The many-body excited-state wave functions

$$\Psi_\mu^\Delta(\mathbf{k})=\sum_\alpha C_\mu^{\alpha+\Delta;\alpha}(\mathbf{k})\Phi_\mu^{\alpha+\Delta;\alpha}(\mathbf{k}) \quad (2.19)$$

are formed by making linear combinations of the (μ) basis states all with the same Δ.

The dispersion relations, $\Delta E_\mu^\Delta(\mathbf{k})$, and excited-state wave functions, $\Psi_\mu^\Delta(\mathbf{k})$, are found by diagonalizing the $\mu\times\mu$ matrix with elements

$$E^{\alpha';\alpha}(\mathbf{k})=\langle\Phi^{\alpha'+\Delta;\alpha'}(\mathbf{k})|(V-V_0)|\Phi^{\alpha+\Delta;\alpha}(\mathbf{k})\rangle . \quad (2.20)$$

The unprojected density operators in the expansion of V in Eq. (2.3) can be written as

$$\rho_i(\mathbf{q})=\sum_{\beta,\beta'}\delta_{s_\beta,s_{\beta'}}\rho_i^{\beta;\beta'}(\mathbf{q}) , \quad (2.21)$$

since the Coulomb interaction does not flip the spin of an electron. The following identity from Ref. 14 can be used to simplify the matrix elements:

$$\sum_l G^{n,l}(k_1)G^{l,m}(k_2)=e^{-k_1^*k_2/2}G^{n,m}(k_1+k_2) . \quad (2.22)$$

The matrix elements, Eq. (2.20), can be written in terms of Fourier transforms of pair correlation functions, of the form

$$S_{\mathbf{k}}^{\alpha\beta;\lambda\mu}=\sum_{i\neq j}\frac{\langle\psi_0|\rho_i^{\alpha;\beta}(-\mathbf{k})\rho_j^{\lambda;\mu}(\mathbf{k})|\psi_0\rangle}{G^{\alpha\beta}(-k)G^{\lambda\mu}(k)} . \quad (2.23)$$

The required Fourier transforms are

$$S_{\mathbf{k}}^{\alpha\alpha;\alpha\alpha}=\mathcal{N}\nu_\alpha[\tilde{h}_\alpha(\mathbf{k})e^{-|\mathbf{k}|^2/2}+2\pi\nu_\alpha\delta^2(\mathbf{k})] , \quad (2.24a)$$

$$S_{\mathbf{k}}^{\alpha\alpha;\beta\beta}=2\pi\mathcal{N}\nu_\alpha\nu_\beta\delta^2(\mathbf{k}) , \quad (2.24b)$$

$$S_{\mathbf{k}}^{\alpha\beta;\beta\alpha}=-\mathcal{N}\nu_\alpha\nu_\beta e^{-|\mathbf{k}|^2/2} , \quad (2.24c)$$

with $\alpha\neq\beta$, and

$$\tilde{h}_\alpha(\mathbf{k})=e^{|\mathbf{k}|^2/2}[S_\alpha(\mathbf{k})-1] . \quad (2.25)$$

The static structure factor for electrons in Landau level α is

$$S_\alpha(\mathbf{k})=\int d\mathbf{r}e^{i\mathbf{k}\cdot\mathbf{r}}[g_\alpha(\mathbf{r})-1] , \quad (2.26)$$

where $g_\alpha(\mathbf{r})$ is the pair distribution function for electrons in Landau level α. For a full Landau level, $\tilde{h}_\alpha(\mathbf{k})=-1$. When $0<\nu_\alpha<1$, calculation of $S_{\mathbf{k}}^{\alpha\alpha;\alpha\alpha}$ requires knowledge of the two-particle distribution function for electrons within a partially filled Landau level. For filling factor $\nu_\alpha=1/m_\alpha$, with $m_\alpha=1,3,5,\ldots$, the two-particle distribution function is known from an analogy with a two-dimensional one-component plasma.[18] Equations (2.24b) and (2.24c) are unchanged when $\nu_\alpha,\nu_\beta<1$. The Hartree-Fock results are obtained by using Eqs. (2.24) with $\tilde{h}_\alpha(\mathbf{k})=-\nu_\alpha$ for $\nu_\alpha<1$.

When excited states are considered in which an electron is excited into a level that is already partially occupied, three-particle correlation functions of the form

$$S_{\mathbf{k}-\mathbf{q};-\mathbf{k};\mathbf{q}}^{\gamma\mu;\alpha\beta;\lambda\gamma}=\sum_{i\neq j\neq l}\frac{\langle\psi_0|\rho_i^{\gamma;\mu}(\mathbf{k}-\mathbf{q})\rho_j^{\alpha;\beta}(-\mathbf{k})\rho_l^{\lambda;\gamma}(\mathbf{q})|\psi_0\rangle}{G^{\gamma\mu}(\mathbf{k}-\mathbf{q})G^{\alpha\beta}(-\mathbf{k})G^{\lambda\gamma}(\mathbf{q})} \quad (2.27)$$

are required, where γ is different from the other superscripts. This can be written in terms of the single-particle distribution function for electrons in level γ, which is constant in a Laughlin state. The result is

$$S_{\mathbf{k}-\mathbf{q};-\mathbf{k};\mathbf{q}}^{\gamma\mu;\alpha\beta;\lambda\gamma}=-\nu_\gamma e^{q^*(k-q)/2}S_{\mathbf{k}}^{\alpha\beta;\lambda\mu} , \quad (2.28)$$

where the pair correlation function is one of the types in Eqs. (2.24).

Like the HFA, this approach only considers states which have a single exciton present at all times, however, the full correlations of the ground state are included through the static structure factor for the ground state $S(\mathbf{k})$. Because it does not allow for a single exciton to decay and emit two excitons, conserving momentum and energy, this approach also is not capable of producing an estimate for decay rates.

III. OSCILLATOR WEIGHTS

A generalized oscillator weight for the mode $\Psi_\mu^\Delta(\mathbf{k})$ can be defined as

$$\left|\langle\Psi_\mu^\Delta(\mathbf{k})|\sum_\alpha\rho^{\alpha+\Delta;\alpha}(\mathbf{k})|\psi_0\rangle\right|^2 .$$

Using Eqs. (2.17) and (2.19), this can be written as

$$\mathcal{F}_\mu^\Delta(\mathbf{k})=\left|\sum_\alpha[N_\mu^{\alpha+\Delta;\alpha}(\mathbf{k})]^{1/2}C_\mu^{\alpha+\Delta;\alpha}(\mathbf{k})\right|^2 . \quad (3.1)$$

In the case where only one value of n_α contributes to this sum [for example, when only the lowest Landau level is occupied, only $n_\alpha=0$ contributes in the sum for $\mathcal{F}_\mu^\Delta(\mathbf{k})$] the oscillator weight may be written as

$|G^{n_\alpha+m,n_\alpha}(\mathbf{k})|^2 F_\mu^\Delta(n_\alpha,\mathbf{k})$, where

$$F_\mu^\Delta(n,\mathbf{k})=\frac{(n_\alpha+m)!}{n_\alpha!}\left[\sum_{s_\alpha}[\nu_\alpha(1-\nu_{\alpha+\Delta})]^{1/2}C_\mu^{\alpha+\Delta;\alpha}(\mathbf{k})\right]^2 . \tag{3.2}$$

The sum over different modes μ of $F_\mu^{m,0}(n,\mathbf{k})$, and the sum over μ and $\delta S_z=\pm 1$ of $F_\mu^{m,\delta S_z}(n,\mathbf{k})$ are each equal to ν, the total filling factor.

In more general cases (general ν and m) it is possible to define an oscillator weight

$$F_\mu^\Delta(\mathbf{k})=\left[\sum_\alpha\left[\frac{(n_\alpha+m)!}{n_\alpha!}\nu_\alpha[1-\nu_{\alpha+\Delta}]\right]^{1/2}C_\mu^{\alpha+\Delta;\alpha}(\mathbf{k})\right]^2 \tag{3.3}$$

such that

$$\sum_\mu F_\mu^{m,0}(\mathbf{k})=\nu \tag{3.4}$$

and

$$\sum_{\delta S_z=\pm 1}\sum_\mu F_\mu^\Delta(\mathbf{k})=\nu . \tag{3.5}$$

In the limit of small $\mathbf{k}$,

$$\mathcal{F}_\mu^\Delta(\mathbf{k})\approx N\left[\frac{|\mathbf{k}|^2}{2}\right]^m\frac{e^{-|\mathbf{k}|^2/2}}{(m!)^2}F_\mu^\Delta(\mathbf{k}) . \tag{3.6}$$

The differential cross section for scattering to a state which couples to the ground state through the density operator is proportional to the dynamic structure factor,

$$S^\Delta(\mathbf{k},\omega)\propto\sum_\mu\mathcal{F}_\mu^\Delta(\mathbf{k})\delta(\omega-E_\mu^\Delta(\mathbf{k})) . \tag{3.7}$$

In light-scattering experiments, with a scattering wave vector of 10^{-4} cm^{-1}, peaks are seen in the spectra which correspond to structure in the dispersion relation at the wave vector of 10^{-6} cm^{-1}, implying a massive breakdown of wave-vector conservation, which is not yet understood. Completely ignoring wave-vector conservation, the dynamic structure factor for scattering at energy near $m\omega_c+\delta S_z\omega_z$ is

$$S^\Delta(\omega)=\sum_\mu\int\frac{dk\,2\pi k\,\mathcal{F}_\mu^\Delta(\mathbf{k})\delta(k-k_0(\omega))}{[(\delta E_\mu^\Delta/\delta k)^2+\gamma^2]^{1/2}} , \tag{3.8}$$

where $E_\mu^\Delta(k_0)=\omega$, and γ is the (small) lifetime of the collective excitation. The structure factor has peaks at energies corresponding to critical points of the dispersion relations, weighted by the oscillator weights which contain an $e^{-k^2/2}$ factor.

IV. RESULTS

A. $\nu<1$

The eigenstate for the spin-flip mode, $\Phi^{1,1/2;0,-1/2}(\mathbf{k})$, has energy shifted from $\omega_c+\omega_z$ by

$$\Delta E_{\mathrm{SMA}}^{1,1}(\mathbf{k})=\int\frac{d^2q}{(2\pi)^2}\tilde{u}(q)[G^{11}(q)e^{iq\times\mathbf{k}\cdot\hat{\mathbf{z}}}-1]\times e^{-q^2/2}\tilde{h}(q) \tag{4.1}$$

with $\tilde{h}(q)=e^{q^2/2}[S(q)-1]$, where $S(q)$ is the static structure factor for electrons in the partially filled level. The result in the Hartree-Fock approximation,

$$\Delta E_{\mathrm{HFA}}^{1,1}(\mathbf{k})=\nu[-\tilde{V}_{1001}^{(1)}(\mathbf{k})+\tilde{V}_{0000}^{(1)}(0)] , \tag{4.2}$$

is equal to ν times the result for the full Landau level, and can be obtained by using the uncorrelated value, $\tilde{h}(q)=-\nu$, in Eq. (4.1). These results are shown in Fig. 1(a) for $\nu=1/3$, using the structure factor for electrons in a Laughlin state. The pair correlation function, $g(\mathbf{r})$, was obtained from the fit to Monte Carlo data reported in

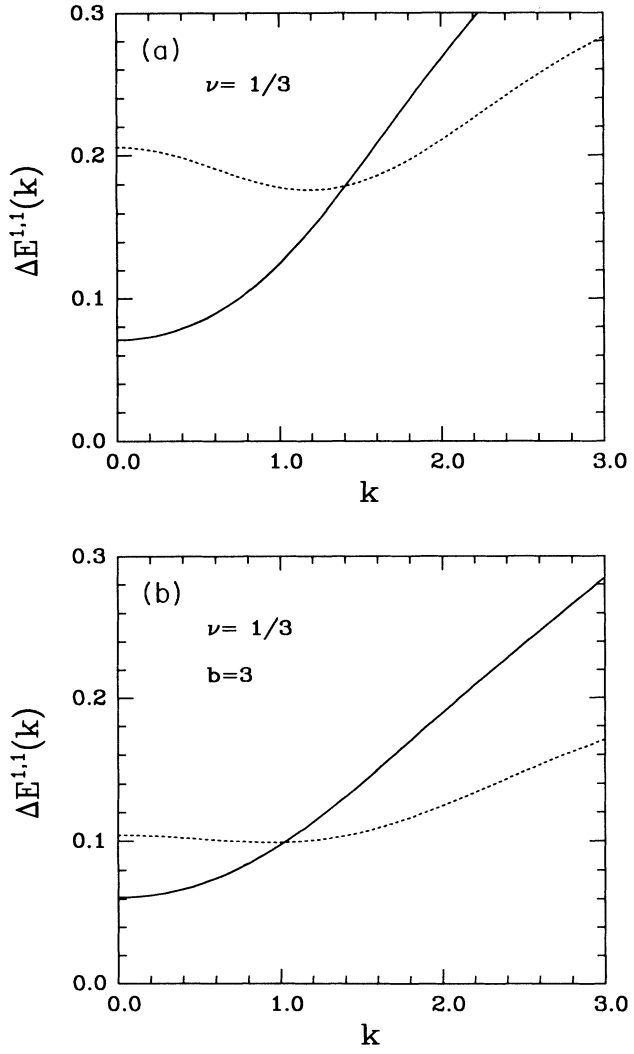

FIG. 1. (a) Energy shift from $\omega_c+\omega_z$ for the spin-flip mode, $\Delta E^{1,1}(k)$, for filling factor $\nu=1/3$, calculated for the ideal two-dimensional electron gas as a function of wave vector. The solid curve is the result in the SMA, and the dotted curve is the result in the HFA. The energy is in units of $e^2/\epsilon l_0$, and the wave vector is in units of l_0^{-1}. (b) As in (a), but for finite thickness of the electron layer, with $b=3$.

Ref. 17. In the SMA, the excited electron retains the correlations that it had with the electrons in the partially filled level in the ground state, so that the first term in Eq. (4.1), the direct interaction between the excited electron and hole, involves the structure factor for correlated electrons. The second term, $-\Sigma_{0,-1/2}$, is the loss of self-energy due to exchange within the partially filled level in the ground state, and also involves $S(q)$. In the SMA, since both of these terms are affected by the strong correlations in the partially filled level, each is larger in magnitude than the corresponding term in the HFA result. However, the terms have opposite signs, and for small k there is more cancellation between them in the SMA result, so that $\Delta E^{1,1}_{\rm SMA}(0) < \Delta E^{1,1}_{\rm HFA}(0)$. There is no theorem requiring these shifts to be zero at $k=0$, and indeed they are substantial, of order E_c. The dispersion curves in Fig. 1(a) were calculated for the ideal two-dimensional case, with $b=\infty$ in Eq. (2.4). The effect of finite thickness can be seen in Fig. 1(b), where the calculations were carried out for $b=3$. The rest of the calculations will be presented for $b=3$.

At $k\rightarrow\infty$, the energy of the excitation for $\nu=1$,

$$\Delta E^{1,1}(\mathbf{k})=\Sigma_{1,1/2}-\Sigma_{0,-1/2}-\frac{1}{k}\ , \tag{4.3}$$

is equal to the change in self-energy of the excited electron, and the energy of a well-separated electron-hole pair interacting through an r^{-1} potential.[8] For $\nu=\frac{1}{3},\frac{1}{5},\ldots$, it has been argued that at large k, there is an excitation with lower energy than that given by the SMA.[17] This excited state consists of an exciton formed from the fractionally charged quasielectron and quasihole of the Laughlin state.[8] The dispersion relation at large k of this excitation is

$$\Delta_{\rm ex}(k)=\Delta(\infty)-\frac{\nu^3}{k}\ , \tag{4.4}$$

where $\Delta(\infty)$ is the energy required to produce a well-separated quasielectron-quasihole pair. At some intermediate wave vector, the spectrum crosses over from the magnetoroton excitation of the SMA to the quasiexciton. There are excited states[19] for inter-Landau-level excitations, and for excitations in which the spin of the electron is flipped of the form

$$\Phi(\mathbf{k})=[N(\mathbf{k})]^{-1/2}\sum_i L_i\sum_j \rho_j^{\alpha;\alpha}(\mathbf{k})\psi_0\ , \tag{4.5}$$

where the density operator acting alone would produce a magnetoroton within the partially filled level α. The operator L_i is the Landau-level-raising operator, $a_i^\dagger$, the spin operator, $2S_+(i)$, or the product, $2a_i^\dagger S_+(i)$, to produce states with energies near ω_c, $\pm\omega_z$, or $\omega_c\pm\omega_z$, respectively. The energy spectrum from magnetorotons is repeated exactly at $\omega_c+\delta S_z\omega_z$. In general, for any excited state labeled by $\mathbf{k}$ formed within the partially filled level, there is a corresponding state formed by operating with $\sum_i L_i$ which has the same energy shift. At small k, the SMA is expected to produce good results, but at larger k, the repeated magnetoroton spectrum may have lower energy, and for $k\rightarrow\infty$, the lowest-energy state will have an energy shift given by Eq. (4.4). Since the large-k behavior is not experimentally accessible because of the factor of $e^{-|\mathbf{k}|^2/2}$ in the oscillator weight, $\mathcal{F}^{\Delta}_{\mu}(\mathbf{k})$, we will confine our attention to small and intermediate wave vectors and will assume that the SMA is valid in this regime. It is likely that the SMA is only valid for $k<1$, and that there are other modes at larger k that have lower energy than the result of the calculation in the SMA.

The SMA result for the magnetoplasmon mode, $\Phi^{1,-1/2;0,-1/2}(\mathbf{k})$, is[14]

$$\Delta E^{1,0}_{\rm SMA}(\mathbf{k})=\int\frac{d^2q}{(2\pi)^2}\tilde{u}(q)e^{-q^2/2}\{[G^{11}(q)e^{i\mathbf{q}\times\mathbf{k}\cdot\hat{\mathbf{z}}}-1]\tilde{h}(q)+G^{10}(-q)G^{01}(q)\tilde{h}(|\mathbf{k}+\mathbf{q}|)\}+\nu\tilde{V}^{(2)}_{1010}(\mathbf{k}) \tag{4.6}$$

and the HFA result, obtained using $\tilde{h}(q)=-\nu$, is

$$\Delta E^{1,0}_{\rm HFA}(\mathbf{k})=\nu[-\tilde{V}^{(1)}_{1001}(\mathbf{k})+\tilde{V}^{(1)}_{0000}(0)-\tilde{V}^{(1)}_{1010}(0)+\tilde{V}^{(2)}_{1010}(\mathbf{k})]\ . \tag{4.7}$$

In addition to the terms which also occur in the expressions for the spin-flip modes, these expressions include the self-energy of the excited electron, $\Sigma_{1,-1/2}(\mathbf{k})$, and the random-phase approximation energy. In the SMA, the excited electron retains its correlations with the electrons in the partially filled level, and the self-energy of the excited electron is $\mathbf{k}$ dependent because it arises from exchange between the excited electron of the electron-hole pair with momentum $\mathbf{k}$, and the other correlated electrons. In the HFA, which ignores the correlations among electrons in the partially filled level, this self-energy is a constant. In both the SMA and HFA, the shift from ω_c at $k=0$ is zero, as required by Kohn's theorem.

The spin-wave mode, $\Phi^{0,1/2;0,-1/2}(\mathbf{k})$, is not shifted from ω_z at $k=0$. In the SMA, the dispersion can be expressed in terms of $\tilde{h}(q)$:

$$\Delta E^{0,1}_{\rm SMA}(\mathbf{k})=\int\frac{d^2q}{(2\pi)^2}\tilde{u}(q)e^{-q^2/2}[e^{i\mathbf{q}\times\mathbf{k}\cdot\hat{\mathbf{z}}}-1]\tilde{h}(q)\ . \tag{4.8}$$

This reduces to the HFA result when $\tilde{h}(q)=-\nu$. In the HFA, the dispersion,

$$\Delta E^{0,1}_{\rm HFA}(\mathbf{k})=\nu[-\tilde{V}^{(1)}_{0000}(\mathbf{k})+\tilde{V}^{(1)}_{0000}(0)]\ , \tag{4.9}$$

is equal to ν times its value for a full Landau level. In the SMA, both the direct energy and the self-energy are affected by the strong correlations in the partially filled level, and both terms are larger in magnitude than the corresponding terms in the HFA. The spin-wave mode is shown in Fig. 2 for $\nu=1/3$ in the HFA and the SMA. Note that at larger wave vectors there will be a lower-

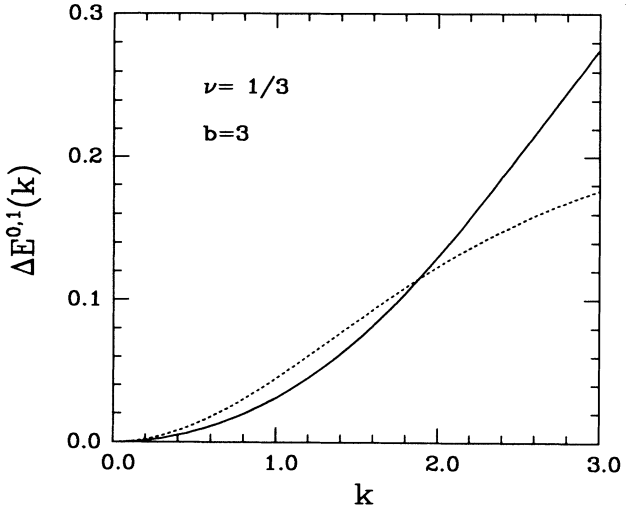

FIG. 2. The shift of the spin-wave mode from ω_z, calculated in the HFA (dotted curve), and in the SMA (solid curve), for $\nu=1/3$.

energy excitation than that shown in Fig. 2, corresponding to an exciton formed by a quasihole and a spin-reversed quasiparticle.[20]

The oscillator weight for each of these modes is equal to the filling factor ν.

B. $1<\nu\leq 2$

Here we discuss the dispersion relations for the modes at filling factor $\nu=1+\epsilon$, where ϵ is the filling factor for the minority spin electrons.

The $\delta S_z=1$ spin-flip mode, $\Phi^{1,1/2;0,-1/2}(\mathbf{k})$, has energy shifted from $\omega_c+\omega_z$ by

$$\Delta E^{1,1}(\mathbf{k})=-\tilde{V}^{(1)}_{1001}(\mathbf{k})+\tilde{V}^{(1)}_{0000}(0)-\epsilon\tilde{V}^{(1)}_{1010}(0)\ , \tag{4.10}$$

which is the same in both the SMA and HFA. The direct interaction between the excited electron and the hole in the full Landau level, and the self-energy due to exchange within the full level depend on $S(q)$ for a full Landau level, which is the same in the SMA and HFA. The last term, $\Sigma_{1,1/2}$, is the self-energy of the excited electron (which is correlated with the majority spin electrons), due to exchange with the minority spin electrons, and depends on single-particle distribution functions, which are the same in the HFA, and for the uniform density Laughlin state.

The shift from $\omega_c-\omega_z$ for the $\delta S_z=-1$ spin-flip mode, $\Phi^{1,-1/2;0,1/2}(\mathbf{k})$, is

$$\Delta E^{1,-1}_{\rm SMA}(\mathbf{k})=\int\frac{d^2q}{(2\pi)^2}\tilde{u}(q)e^{-q^2/2}[G^{11}(q)e^{i\mathbf{q}\times\mathbf{k}\cdot\hat{\mathbf{z}}}-1]\times\tilde{h}(q)-\tilde{V}^{(1)}_{1010}(0) \tag{4.11}$$

in the SMA. The first term in this expression is the direct energy between the electron excited from and the hole remaining in the partially filled level, and the second term is the loss of self-energy due to exchange within the partially filled level; both depend on the electron static structure factor for the partially filled level. The Hartree-Fock result,

$$\Delta E^{1,-1}_{\rm HFA}(\mathbf{k})=\epsilon[-\tilde{V}^{(1)}_{1001}(\mathbf{k})+\tilde{V}^{(1)}_{0000}(\mathbf{k})]-\tilde{V}^{(1)}_{1010}(0)\ , \tag{4.12}$$

is obtained using $\tilde{h}(q)=-\epsilon$. The third term in both of these expressions is $\Sigma_{1,-1/2}$, the self-energy of the excited electron; this depends on the correlations between majority and minority spin electrons, which are the same in both approximations. Again, there is no theorem that requires that these shifts are zero at $k=0$. In the HFA, $\Delta E^{1,1}_{\rm HFA}(0)=-\Delta E^{1,-1}_{\rm HFA}(0)$, while in the SMA, in which only two of the terms contributing to the energy shift are

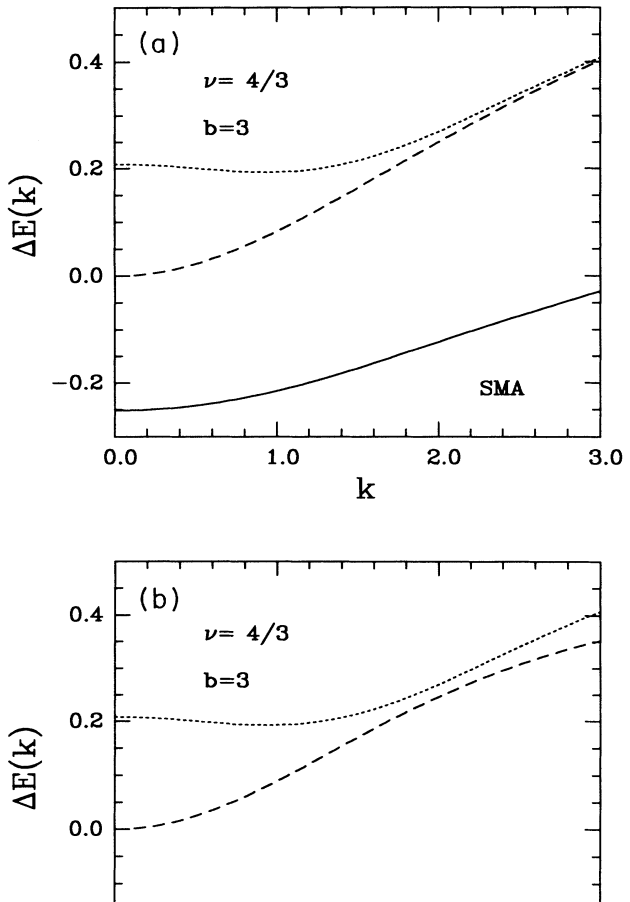

FIG. 3. (a) The dashed curve is the shift of the spin-wave mode from ω_z, $\Delta E^{1,0}(k)$, at $\nu=\frac{4}{3}$. The shifts of the spin-flip modes, $\Delta E^{1,\delta S_z}(k)$, from $\omega_c+\delta S_z\omega_z$ are the dotted curve (for $\delta S_z=1$) and the solid curve (for $\delta S_z=-1$). The calculations were done in the SMA for finite thickness of the electron layer. (b) As in (a), but in the HFA.

affected by the strong correlations in the partially filled level, the magnitudes of the shifts are not equal at $k=0$. The oscillator weight is 1 for the mode with $\delta S_z=1$, and ϵ for the mode with $\delta S_z=-1$. The results for the $\delta S_z=\pm 1$ modes are shown in Figs. 3 at $\nu=4/3$, using a Laughlin 1/3 state for the electrons in the partially filled level.

There are two basis states with $m=1$ and $\delta S_z=0$ at filling factor $\nu=1+\epsilon$: $\Phi^{1,\sigma;0,\sigma}(\mathbf{k})$ with $\sigma=\pm\frac{1}{2}$. The modes near ω_c are found by taking linear combinations of these two states which diagonalize the Hamiltonian.[13] The diagonal matrix elements are

$$E^{0,\sigma;0,\sigma}(\mathbf{k})=\nu_{0,\sigma}\tilde{V}^{(2)}_{1010}(\mathbf{k})+\int\frac{d^2q}{(2\pi)^2}\tilde{u}(q)e^{-q^2/2}\{[G^{11}(q)e^{i\mathbf{q}\times\mathbf{k}\cdot\hat{\mathbf{z}}}-1]\tilde{h}_{0,\sigma}(q)+G^{10}(-q)G^{01}(q)\tilde{h}_{0,\sigma}(|\mathbf{k}+\mathbf{q}|)\}\ . \tag{4.13}$$

The filling factors are $\nu_{0,-1/2}=1$, and $\nu_{0,1/2}=\epsilon$, and $\tilde{h}_{0,\sigma}$ is related to the electron static structure factor for the level $(0,\sigma)$. Each of the matrix elements involves only electrons of the same spin, and is equal to $\Delta E^{1,0}(\mathbf{k})$ for a single level with filling factor $\nu=\nu_{0,\sigma}$. The off-diagonal matrix elements are

$$E^{0,\sigma;0,-\sigma}(\mathbf{k})=(\nu_{0,\sigma}\nu_{0,-\sigma})^{1/2}\tilde{V}^{(2)}_{1010}(\mathbf{k})\ . \tag{4.14}$$

The dispersion relations for the two modes are

$$\Delta E^{1,0}_{\pm}(\mathbf{k})=\frac{E^{0,1/2;0,1/2}+E^{0,-1/2;0,-1/2}}{2}\pm\left[\left[\frac{E^{0,1/2;0,1/2}-E^{0,-1/2;0,-1/2}}{2}\right]^2+(E^{0,1/2;0,-1/2})^2\right]^{1/2}\ . \tag{4.15}$$

These dispersion relations are shown in Fig. 4 for $\nu=4/3$, calculated in the SMA using a Laughlin state for the electrons in the partially filled level, and in the Hartree-Fock approximation, together with the corresponding oscillator weights. The higher-energy mode, which for small k is an in-phase combination of the two basis states and has most of the oscillator weight, corresponds to a pole of the density response function $\chi_p(\mathbf{k},\omega)$. The lower-energy mode, an out-of-phase combination of the two basis states which has nearly zero oscillator weight for small k, corresponds to a pole of the spin response function $\chi_{\sigma_z}(\mathbf{k},\omega)$. At large k, the Coulomb interaction becomes small, and the modes decouple, and approach the value of the self-energy change of the excited electron. In this limit, the upper branches of the dispersion relations in the HFA and SMA, Eq. (4.13) with $\tilde{h}(q)=-1$, become equal, and the oscillator weights become 1 and ϵ. In the SMA, there is structure in the oscillator weights at intermediate k where the modes cross.

The spin-wave mode has an electron excited from the filled level into the partially filled level, and involves the three-particle correlation functions discussed in Sec. II B. The result of the SMA is

$$\Delta E^{0,1}_{\mathrm{SMA}}(\mathbf{k})=\frac{1-2\epsilon}{1-\epsilon}[-\tilde{V}^{(1)}_{0000}(\mathbf{k})+\tilde{V}^{(1)}_{0000}(0)]+\frac{\epsilon}{1-\epsilon}\int\frac{d^2q}{(2\pi)^2}\tilde{u}(q)e^{-q^2/2}[e^{i\mathbf{q}\times\mathbf{k}\cdot\hat{\mathbf{z}}}-1]\tilde{h}(q)\ . \tag{4.16}$$

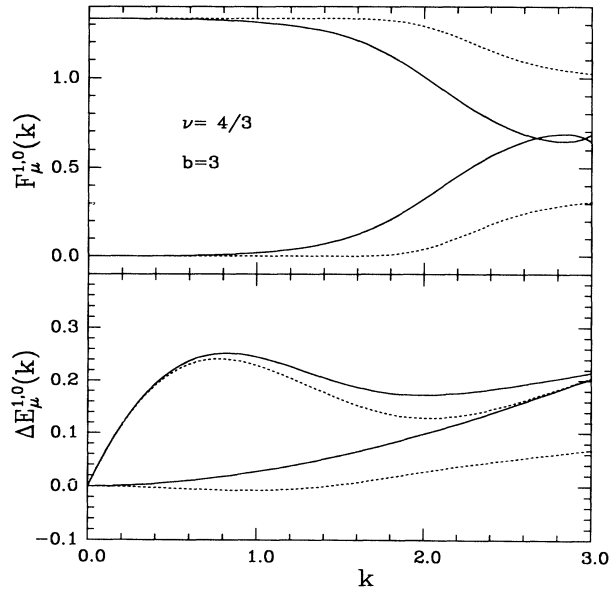

FIG. 4. Energy shifts and oscillator weights for the two $\delta S_z=0$ modes near ω_c, calculated in the HFA (dotted curves) and the SMA (solid curves) for $\nu=\frac{4}{3}$.

The electron excited into the partially filled level retains the correlations with the full Landau level, and is not corrected with the electrons in the partially filled level. The last two terms in this expression, which involve $\tilde{h}(q)$, arise from terms involving interaction and exchange with a third electron in the partially filled level. This expression reduces to the HFA result

$$\Delta E^{0,1}_{\mathrm{HFA}}(\mathbf{k})=(1-\epsilon)[-\tilde{V}^{(1)}_{0000}(\mathbf{k})+\tilde{V}^{(1)}_{0000}(0)] \tag{4.17}$$

if the uncorrelated value $(-\epsilon)$ is taken for $\tilde{h}(q)$. These dispersion relations for the spin-wave modes are shown in Fig. 3 for filling factor $\nu=\frac{4}{3}$.

C. $2<\nu\leq 3$

The spin-flip mode with $\delta S_z=-1$, $\Phi^{1,-1/2;0,1/2}(\mathbf{k})$, has an electron excited into the partially filled level but retaining correlations with the electrons in the full Landau level. The energy shift from $\omega_c-\omega_z$ for filling factor $\nu=2+\epsilon$ is

$$\Delta E^{1,-1}_{\rm SMA}(\mathbf{k})=-\frac{1-2\epsilon}{1-\epsilon}\tilde{V}^{(1)}_{1001}(\mathbf{k})+\tilde{V}^{(1)}_{0000}(0)-\tilde{V}^{(1)}_{1010}(0)-\frac{\epsilon}{1-\epsilon}\tilde{V}^{(1)}_{1111}(0)$$
$$+\frac{\epsilon}{1-\epsilon}\int\frac{d^2q}{(2\pi)^2}\tilde{u}(q)e^{-q^2/2}[e^{iq\times\mathbf{k}\cdot\hat{\mathbf{z}}}-G^{11}(q)]\tilde{h}(q)G^{11}(q) \quad (4.18)$$

in the SMA, where $\tilde{h}(q)$ can be written in terms of the static structure factor for electrons in the partially filled level. The terms which involve $\tilde{h}(q)$ are the energies of interaction and exchange involving a third electron in the partially filled level. The Hartree-Fock result,

$$\Delta E^{1,-1}_{\rm HFA}(\mathbf{k})=-(1-\epsilon)\tilde{V}^{(1)}_{1001}(\mathbf{k})+\tilde{V}^{(1)}_{0000}(0)$$
$$-\tilde{V}^{(1)}_{1010}(0)-\epsilon\tilde{V}^{(1)}_{1111}(0)\ , \quad (4.19)$$

is obtained if the uncorrelated value, $\tilde{h}(q)=-\epsilon$, is used. The oscillator weight for this mode is $(1-\epsilon)$. These dispersion relations are shown in Fig. 5, calculated for $\nu=7/3$.

The basis states for the spin-flip modes with $\delta S_z=1$ are $\Phi^{1,1/2;0,-1/2}(\mathbf{k})$ and $\Phi^{2,1/2;1,-1/2}(\mathbf{k})$. The diagonal matrix elements are

$$E^{0,-1/2;0,-1/2}(\mathbf{k})=-\tilde{V}^{(1)}_{1001}(\mathbf{k})+\tilde{V}^{(1)}_{0000}(0)$$
$$+\epsilon\tilde{V}^{(1)}_{1010}(0)-\tilde{V}^{(1)}_{1010}(0) \quad (4.20)$$

and

$$E^{1,-1/2;1,-1/2}(\mathbf{k})=\int\frac{d^2q}{(2\pi)^2}\tilde{u}(q)e^{-q^2/2}[e^{iq\times\mathbf{k}\cdot\hat{\mathbf{z}}}G^{22}(q)-G^{11}(q)]G^{11}(q)\tilde{h}(q)+\tilde{V}^{(1)}_{1010}(0)-\tilde{V}^{(1)}_{2020}(0)\ . \quad (4.21)$$

The off-diagonal matrix elements are

$$E^{1,-1/2;0,-1/2}(\mathbf{k})=\sqrt{\epsilon}\tilde{V}^{(1)}_{2110}(\mathbf{k})\ . \quad (4.22)$$

The eigenvalues of this matrix are the energy shifts, $\Delta E^{1,1}_{\pm}(\mathbf{k})$, shown in Fig. 5 for the HFA and SMA with the corresponding oscillator weights. These modes are substantially shifted from $\omega_c+\omega_z$ at $\mathbf{k}=0$. At small $\mathbf{k}$, the lower-energy mode, with energy $\Delta E^{1,1}_{-}(\mathbf{k})$, contributing most of the oscillator weight for the $\delta S_z=1$ modes, is an in-phase combination of the two basis states. The higher-energy mode, $\Delta E^{1,1}_{+}(\mathbf{k})$, which has nearly zero oscillator weight at small $\mathbf{k}$, and so would not contribute to the structure factor, is an out-of-phase combination of the two $\delta S_z=1$ basis states. At large $\mathbf{k}$, the normal modes decouple, and the oscillator weights are 2ϵ and 1; in this limit the higher-energy mode, given by Eq. (4.20), is the same in the HFA and SMA.

The eigenstates for the three modes near ω_c with $\delta S_z=0$ are linear combinations of $\Phi^{1,-1/2;0,-1/2}(\mathbf{k})$, $\Phi^{1,1/2;0,1/2}(\mathbf{k})$, and $\Phi^{2,-1/2;1,-1/2}(\mathbf{k})$. The dispersion relations are found by diagonalizing the 3×3 matrix, Eq. (2.20). The results in the SMA at $\nu=7/3$ are shown in Fig. 6(a), where the static structure factor for a Laughlin 1/3 states is used for the electrons in the partially filled level. The HFA results in Fig. 6(b) are obtained by using $\tilde{h}(q)=-\frac{1}{3}$. The corresponding oscillator weights for the three modes,

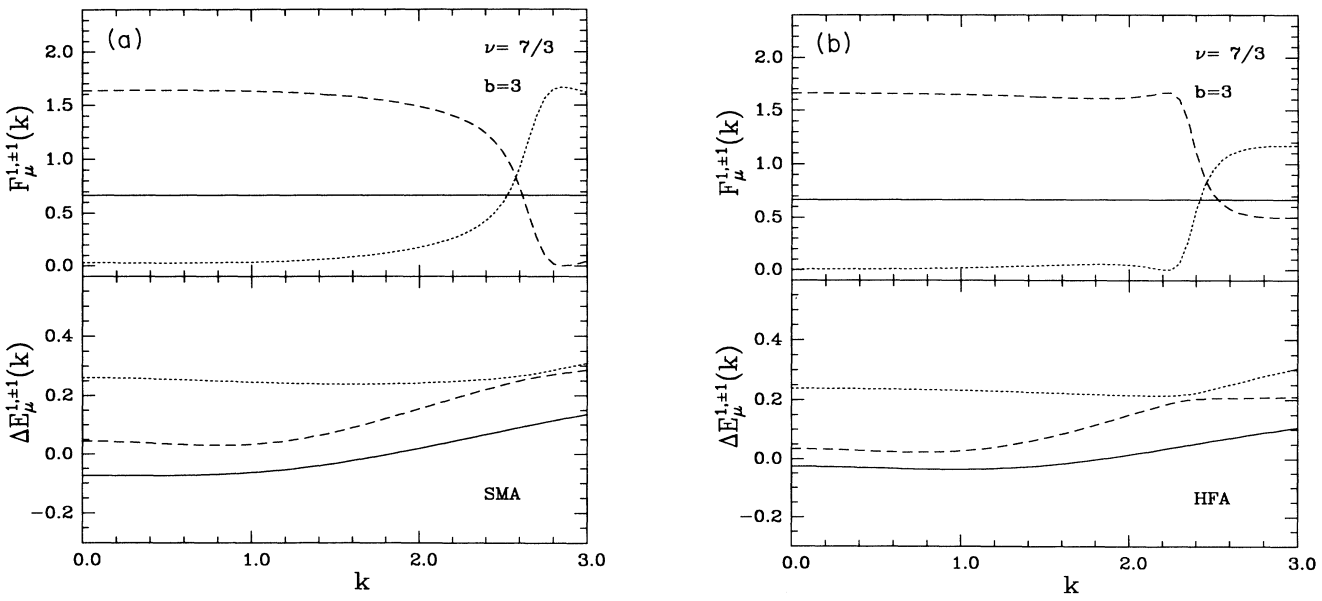

FIG. 5. (a) Energy shifts and oscillator weights for the spin-flip modes near $\omega_c+\delta S_z\omega_z$, calculated in the SMA for $\nu=\frac{7}{3}$. The two $\delta S_z=1$ modes are the dotted and dashed curves; the $\delta S_z=-1$ mode is the solid curve. (b) As in (a), but in the HFA.

$$F_{\mu}^{1,0}(\mathbf{k})=[\sqrt{1-\epsilon}C_{\mu}^{1,-1/2;0,-1/2}(\mathbf{k})+\sqrt{2\epsilon}C_{\mu}^{2,-1/2;1,-1/2}(\mathbf{k})+C_{\mu}^{1,1/2;0,1/2}(\mathbf{k})]^2 , \tag{4.23}$$

show that at small $\mathbf{k}$, nearly all the oscillator weight is contributed by one mode, an in-phase oscillation of the three normal modes. At $\mathbf{k}=0$, the energy of this mode is ω_c. At large $\mathbf{k}$, the excitations correspond to well-separated electron-hole pairs, and the Coulomb interaction is small, so that the matrix is diagonalized by the normal modes. In this limit, the oscillator weights become $(1-\epsilon)$, 2ϵ, and 1. In the region between the two limits, there is structure in the oscillator weights where the modes cross.

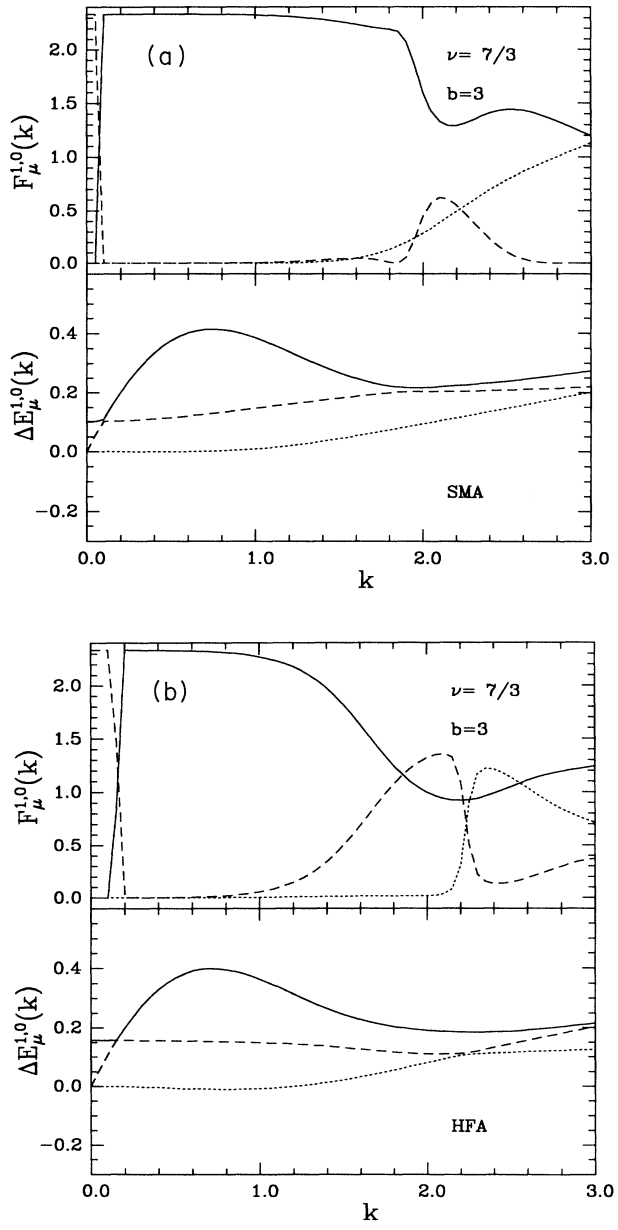

FIG. 6. (a) Energy shifts and oscillator weights for the three $\delta S_z=0$ modes near ω_c, calculated in the SMA for $\nu=\frac{7}{3}$. (b) As in (a), but in the HFA.

V. COMPARISON WITH EXPERIMENT

Pinczuk *et al.*[6,7] have reported the results of inelastic light-scattering experiments on the 2DEG in GaAs/AlGaAs heterojunctions. The spectra shown in Fig. 7 were measured on a sample with filling factor $\nu=0.98$. The excitation spectra, calculated within the HFA, are also shown in Fig. 7, for $\nu=1$ and $b=3$. The dispersion relations at $\nu=1$ can be compared with the experimental spectra in Fig. 7, and the features may be identified as follows.

The strongest peak, which is at 17.5 meV, is from the $\delta S_z=1$ spin-flip mode, which has a large density of states and a large oscillator weight.[7] Presumably this mode is active because of the presence of spin-orbit coupling in this system. The agreement between the position of this peak and the shift calculated in the HFA is excellent. This suggests that corrections to the HFA are small; in particular, the decay processes mentioned earlier should

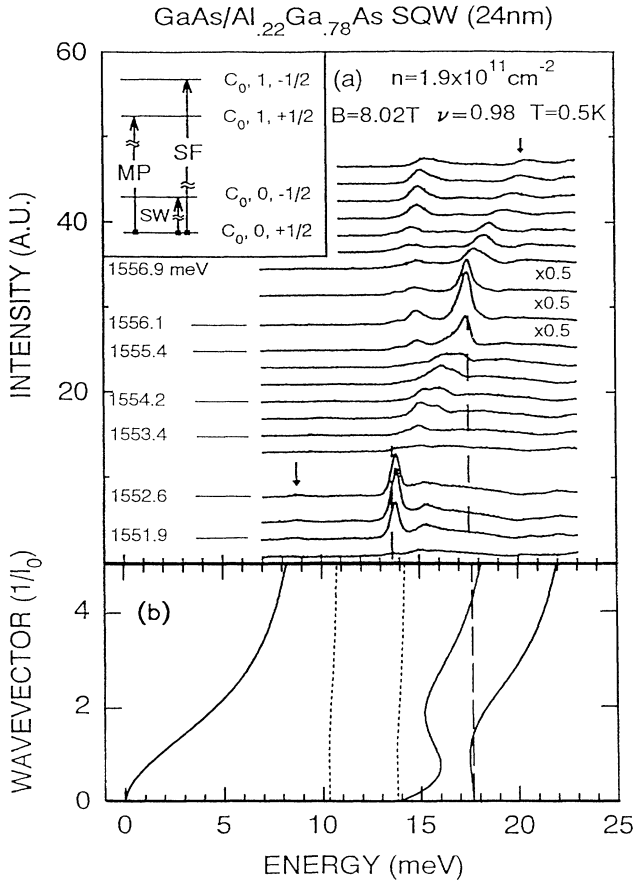

FIG. 7. (a) Results of the inelastic light-scattering experiments reported in Refs. 6 and 7. (b) Dispersion relations calculated in the HFA for $\nu=1$ (solid curves). The additional modes which appear when there are a finite number of minority spin electrons present were calculated in the HFA for $\nu=1.1$ (dotted curves).

not cause a large shift or broadening of this mode. The weaker peaks between 15 and 17 meV are associated with the extrema of the magnetoplasmon dispersion ($m=1$, $\delta S_z=0$).[6] This implies a massive breakdown in wave-vector conservation which is not yet understood.

The large peak at $\omega_c=13.8$ meV has been attributed to the cyclotron mode which can be active in the presence of disorder.[7] However, even in the presence of disorder, the density of states associated with the long-wavelength magnetoplasmon mode is very small.[21] Here we consider an alternative explanation for this strong peak. Disorder broadens the spin-split Landau levels and, hence, even at the low temperature, $T=0.5$ K, at which the experiments are done, there may be a small but finite occupation of the $(0,\frac{1}{2})$ level. In this case, two new modes appear, corresponding to excitations involving minority spin electrons. This is easiest seen by considering the case $\nu=1+\epsilon$, where ϵ is small, although the lowest spin level does not have to be fully occupied. These two modes are shown in Fig. 7 for $\epsilon=0.1$. A $\delta S_z=0$ mode appears which has nearly zero oscillator weight for $k<2$, but also has very little dispersion, and so has a large density of states. The oscillator weight for the cyclotron mode is nearly $1+\epsilon$ for $k<2$. The oscillator weights are to be multiplied by the weighting function $(k^2/2)e^{-k^2/2}$ to determine the contribution that the density of states for that mode will make to the spectrum. In the presence of disorder, these two modes will mix. The large peak at $\omega_c=13.8$ meV can be expected due to the large density of states and large oscillator weight of the $\delta S_z=0$ modes.

The weak feature near 9 meV may be from the $\delta S_z=-1$ excitation of the small number of minority spin electrons. The $\delta S_z=-1$ spin-flip mode has oscillator weight ϵ compared to the $\delta S_z=1$ mode, but is very flat, and so has a large density of states.

There is additional weak structure near 21 meV which may be from the excitation of the minority spin electrons with $m=2$, $\delta S_z=-1$. Within the HFA, this excitation occurs at a somewhat higher energy of 25 meV (and is almost dispersionless at $\nu=1.1$). However, there are several distinct decay processes for this mode which are not included in the HFA and which could substantially shift its frequency. We note that no structure is expected to be seen from the asymptotes of the dispersion curves because of the $e^{-k^2/2}$ falloff in oscillator weight.

As the filling factor decreases below 1, we can expect the large peak from the spin-flip mode to move to lower energy. The decrease suggested by the SMA calculation is larger than that expected from the HFA calculation. As the filling factor increases above 1, we expect the peak from the $\delta S_z=-1$ spin-flip mode to increase in strength, and to shift closer to ω_c. The peak from the $\Delta S_z=1$ spin flip mode is also expected to shift closer to ω_c as the filling factor increases to 2. At $\nu=2$ the shifts of the $\delta S_z=\pm 1$ modes become identical, and equal to the lower branch of the $\delta S_z=0$ mode.

It is hoped that the results presented here will stimulate further inelastic light-scattering experiments. Experimental studies which track the various peaks as a function of filling factor could resolve some of the uncertainties that remain in the identification of these features with collective excitations of the 2DEG. In addition, contributions left out of the HFA and SMA, corresponding to the possible decay mentioned earlier deserve further study.

ACKNOWLEDGMENTS

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